What is inside MC generators...
...and why it is wrong

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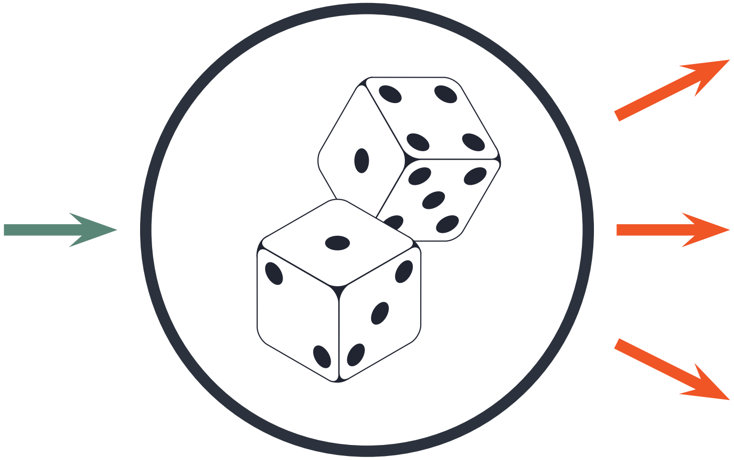
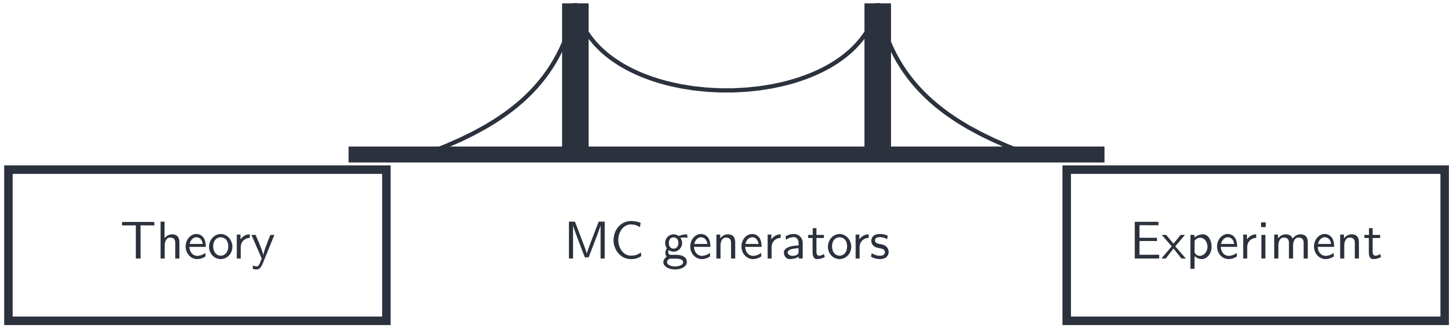
Monte Carlo neutrino event generators

Monte Carlo event generators

- Monte Carlo generators simulate interactions
- Physicists have been using them since ENIAC
- Some common generators used in neutrino community:
  - transport of particles through matter: **Geant4**, **FLUKA**
  - high-energy collisions of elementary particles: **PYTHIA**
  - neutrino interactions: **GENIE**, **GIBUU**, **NEUT**, **NUANCE**, **NuWro**



Why do we need them?

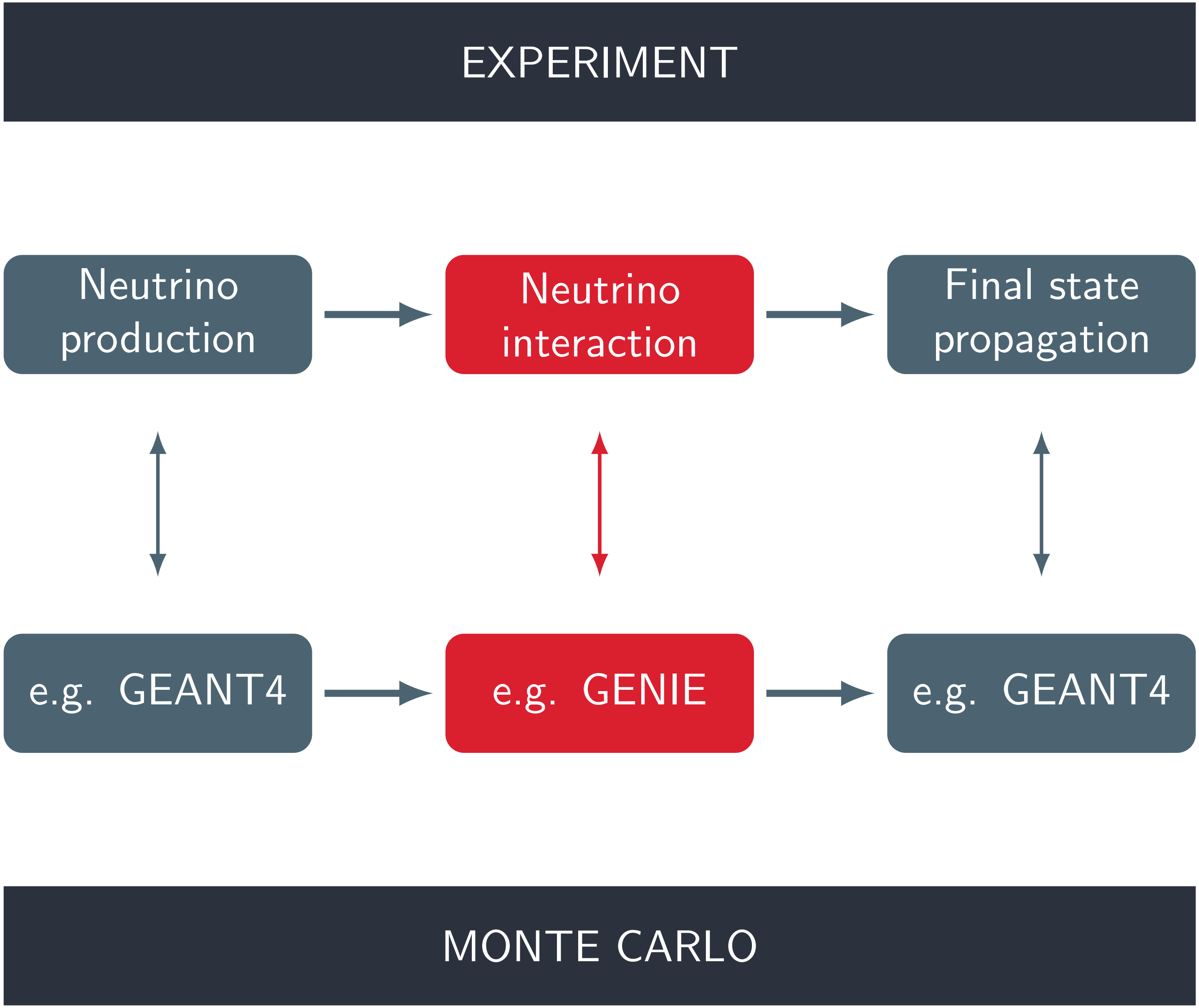


Theory

MC generators

Experiment

- Monte Carlo event generators connect experiment (what we see) and theory (what we think we should see)
- Any neutrino analysis relies on MC generators
- From neutrino beam simulations, through neutrino interactions, to detector simulations
- Used to evaluate systematic uncertainties, backgrounds, acceptances...



EXPERIMENT

Neutrino production → Neutrino interaction → Final state propagation

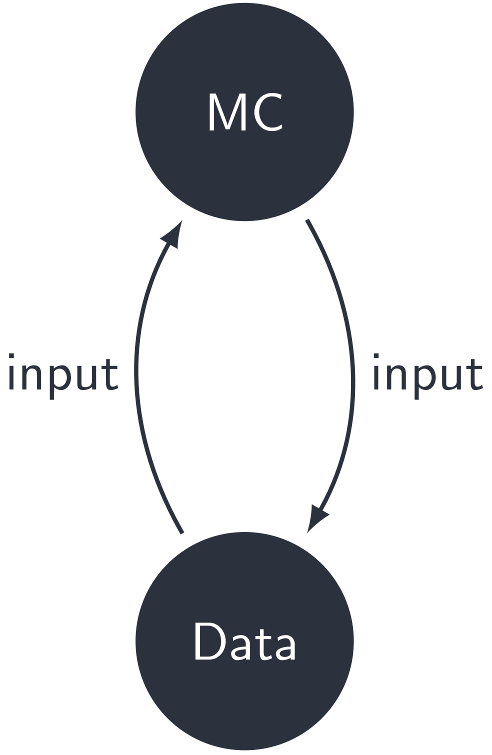
e.g. GEANT4 → e.g. GENIE → e.g. GEANT4

MONTE CARLO

What is the main problem?

> *"You use Monte Carlo until you understand the problem"*
> *Mark Kac*

- In perfect world MC generators would contain "pure" theoretical models
- In real world theory does not cover everything
- Neutrino and non-neutrino data are used to tune generators

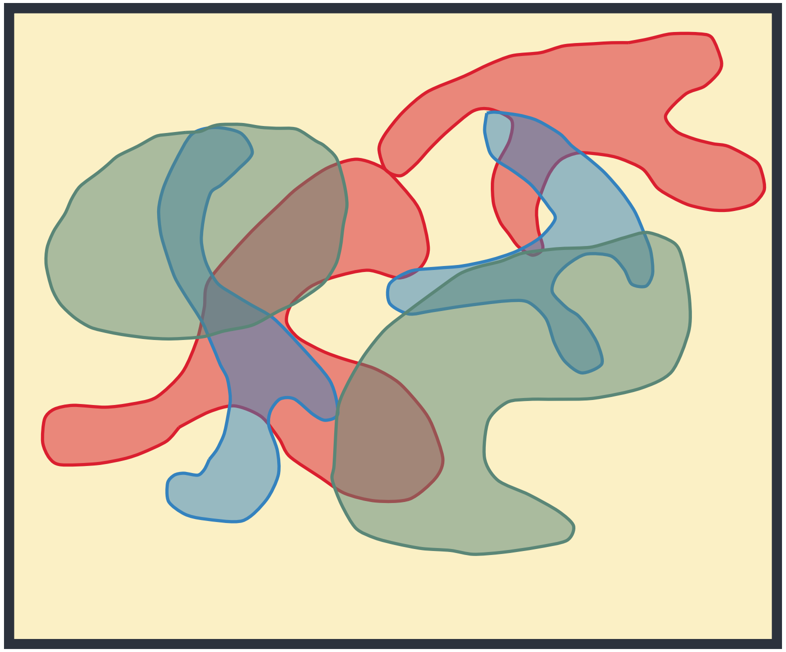


How to build generator

INGREDIENTS:

RECIPE:

Phase space



theory

ν data

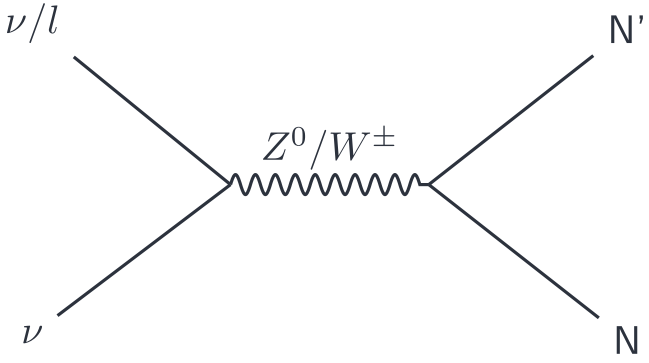
other data

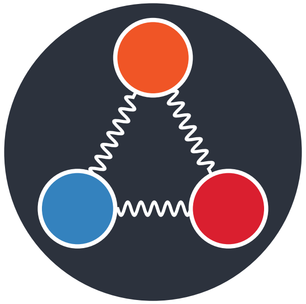
educated guesses

Neutrino interactions: free nucleon

(Quasi-)elastic scattering

- Llewellyn-Smith model is usually used for charged current quasi-elastic scattering
- Not much difference here between generators (but default parameters)





- Nucleon structure is parametrized by form factors

- Vector → Conserved Vector Current (CVC)
- Pseudo-scalar → Partially Conserved Axial Current (PCAC)
- Axial → dipole form with one free parameter (axial mass, M_A)

Rein-Sehgal model

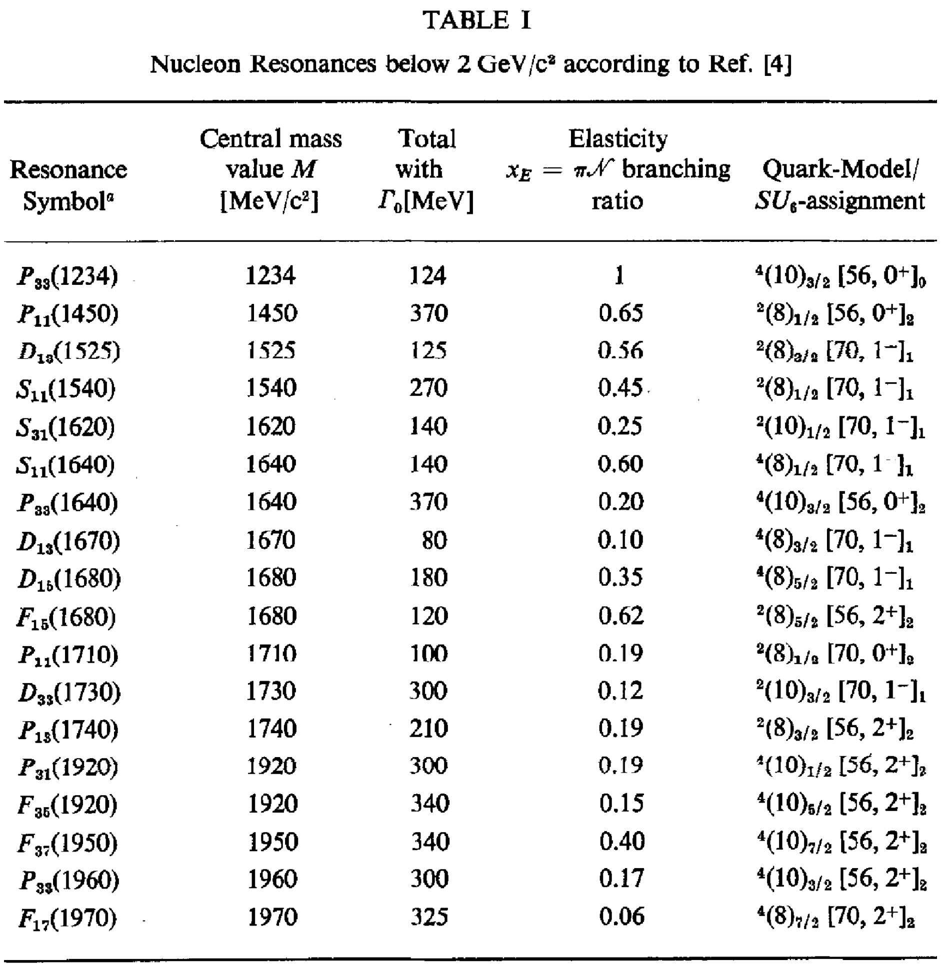
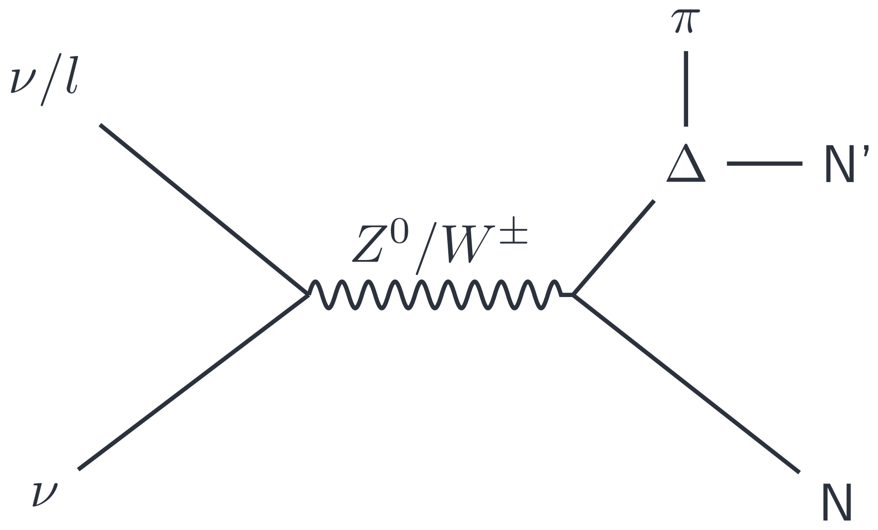


TABLE I

Nucleon Resonances below 2 GeV/c² according to Ref. [4]

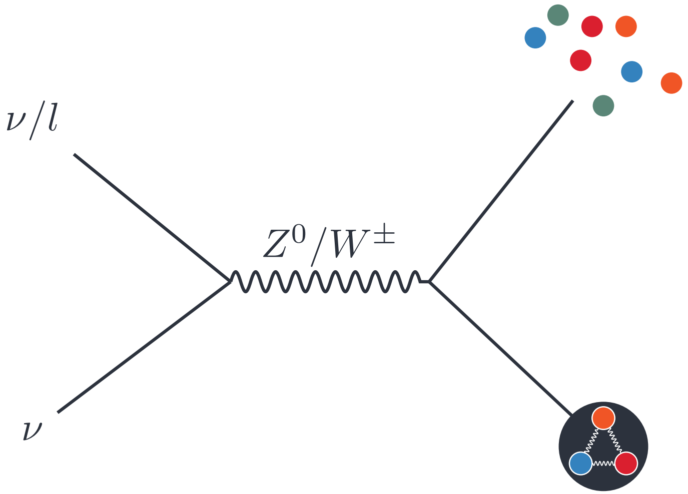
Resonance Symbol[a]	Central mass value M [MeV/c²]	Total with Γ_0[MeV]	Elasticity $x_E = \pi\mathcal{N}$ branching ratio	Quark-Model/ SU_6-assignment
$P_{33}(1234)$	1234	124	1	$^4(10)_{3/2}\ [56, 0^+]_0$
$P_{11}(1450)$	1450	370	0.65	$^2(8)_{1/2}\ [56, 0^+]_2$
$D_{13}(1525)$	1525	125	0.56	$^2(8)_{3/2}\ [70, 1^-]_1$
$S_{11}(1540)$	1540	270	0.45	$^2(8)_{1/2}\ [70, 1^-]_1$
$S_{31}(1620)$	1620	140	0.25	$^2(10)_{1/2}\ [70, 1^-]_1$
$S_{11}(1640)$	1640	140	0.60	$^4(8)_{1/2}\ [70, 1^-]_1$
$P_{33}(1640)$	1640	370	0.20	$^4(10)_{3/2}\ [56, 0^+]_2$
$D_{13}(1670)$	1670	80	0.10	$^4(8)_{3/2}\ [70, 1^-]_1$
$D_{15}(1680)$	1680	180	0.35	$^4(8)_{5/2}\ [70, 1^-]_1$
$F_{15}(1680)$	1680	120	0.62	$^2(8)_{5/2}\ [56, 2^+]_2$
$P_{11}(1710)$	1710	100	0.19	$^2(8)_{1/2}\ [70, 0^+]_2$
$D_{33}(1730)$	1730	300	0.12	$^2(10)_{3/2}\ [70, 1^-]_1$
$P_{13}(1740)$	1740	210	0.19	$^2(8)_{3/2}\ [56, 2^+]_2$
$P_{31}(1920)$	1920	300	0.19	$^4(10)_{1/2}\ [56, 2^+]_2$
$F_{35}(1920)$	1920	340	0.15	$^4(10)_{5/2}\ [56, 2^+]_2$
$F_{37}(1950)$	1950	340	0.40	$^4(10)_{7/2}\ [56, 2^+]_2$
$P_{33}(1960)$	1960	300	0.17	$^4(10)_{3/2}\ [56, 2^+]_2$
$F_{17}(1970)$	1970	325	0.06	$^4(8)_{7/2}\ [70, 2^+]_2$



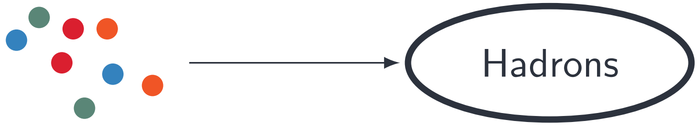
- Rein-Sehgal model describes single pion production through baryon resonances below $W = 2$ GeV
- It is used by GENIE and NEUT
- However, GENIE includes only 16 resonances and interference between them is neglected

Deep inelastic scattering [DIS]

- Quark-parton model is used for deep inelastic scattering
- Bodek-Young modification to the parton distributions at low Q^2 is included by most generators



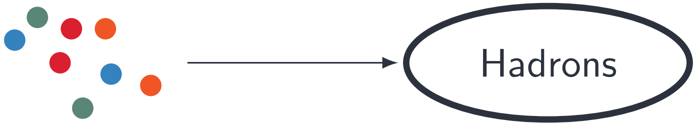
Hadronization



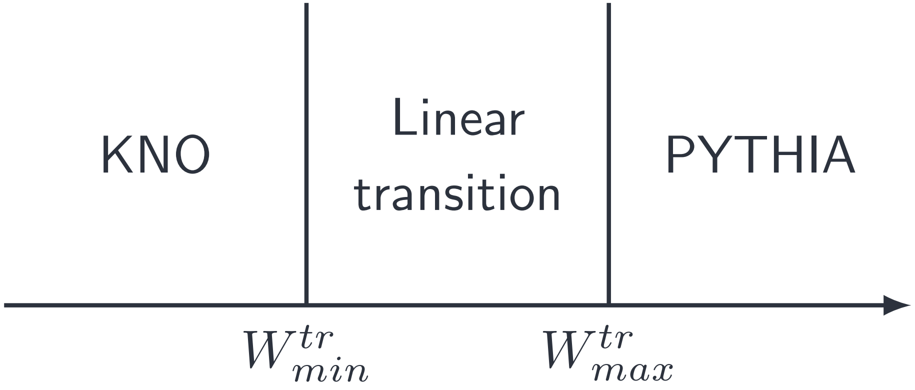
- Hadronization is the process of formation hadrons from quarks
- Pythia is widely used at high invariant masses

Andreopoulos-Gallagher-Kehayias-Yang model

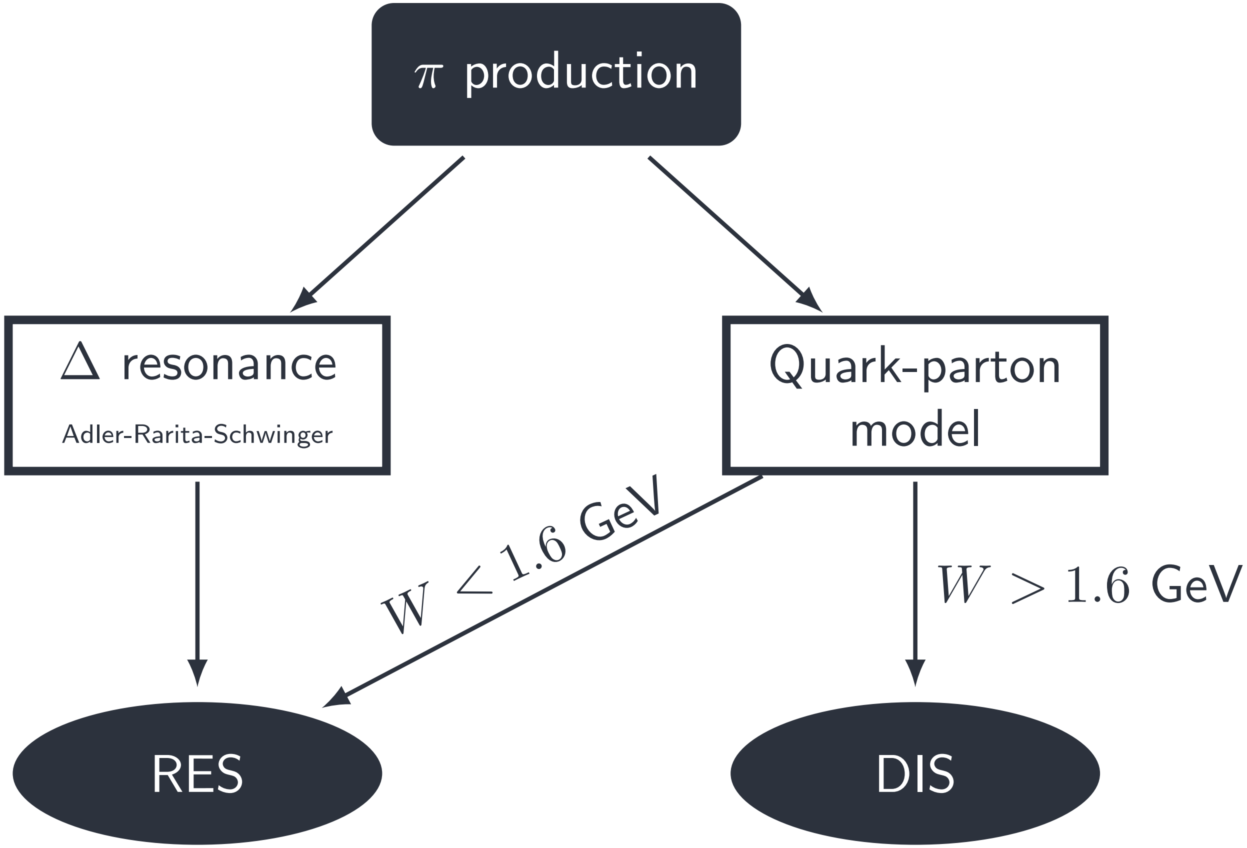
- AGKY hadronization model is used in GENIE



- It includes phenomenological description of the low invariant mass based on Koba-Nielsen-Olesen (KNO) scaling
- Pythia is used for the high invariant mass
- The smooth transition between two models is made in a window $W \in [2.3, 3.0]$ GeV



Pion production in NuWro



RES/DIS distinguish is arbitrary for each MC generator!

Transition region

- We factorized the reality to RES and DIS
- We must be careful to avoid double counting
- The smooth transition between RES and DIS is performed by each generator (but in slightly different way)
- E.g. in GENIE:

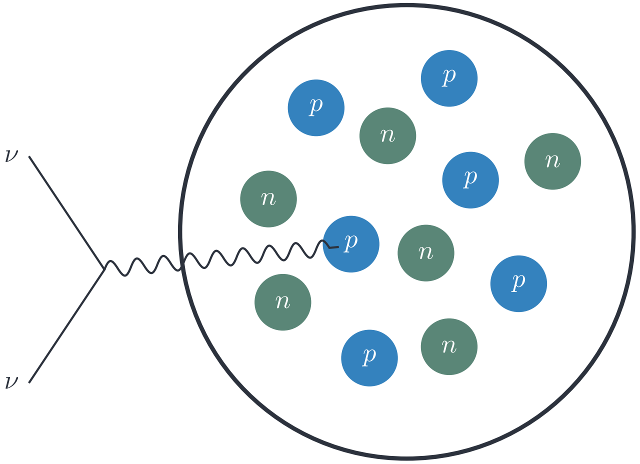
$$\begin{aligned}
\frac{d^2\sigma^{RES}}{dQ^2dW} &= \sum_k \left(\frac{d^2\sigma^{R-S}}{dQ^2dW}\right)_k \cdot \Theta(W_{cut} - W) \\
\frac{d^2\sigma^{DIS}}{dQ^2dW} &= \frac{d^2\sigma^{DIS,BY}}{dQ^2dW} \cdot \Theta(W - W_{cut}) + \frac{d^2\sigma^{DIS,BY}}{dQ^2dW} \cdot \Theta(W_{cut} - W) \cdot \sum_m f_m
\end{aligned}$$

where k - sum over resonances in Rein-Sehgal model, m - sum over multiplicity, $f_m = R_m \cdot P_m$ with P_m - probability of given multiplicity (taken form hadronization model), R_m - tunable parameter

Neutrino interactions: nucleus

Impulse approximation

- In impulse approximation neutrino interacts with a single nucleon



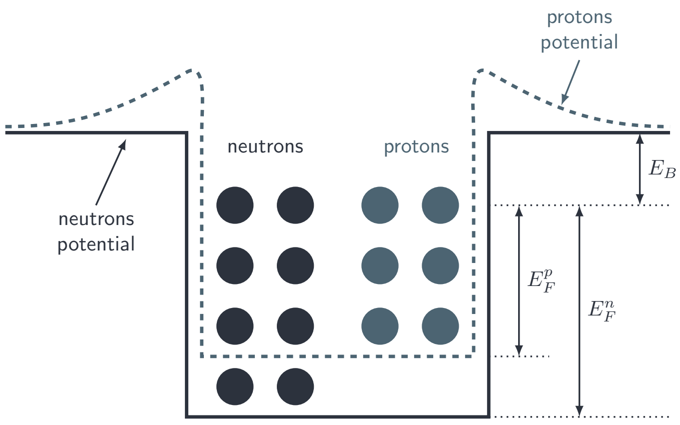
- If $|\vec{q}|$ is low the impact area usually includes many nucleons
- For high $|\vec{q}|$ IA is justified
- Squares of transition matrices are summed up and interference terms are neglected

$$\sigma^A = \sum_{i=1}^{Z} \sigma_p + \sum_{i=1}^{A-Z} \sigma_n$$

- High $|\vec{q}|$ means more than 400 MeV. However, IA is always assumed

Fermi gas

Nucleons move freely within the nuclear volume in constant binding potential.

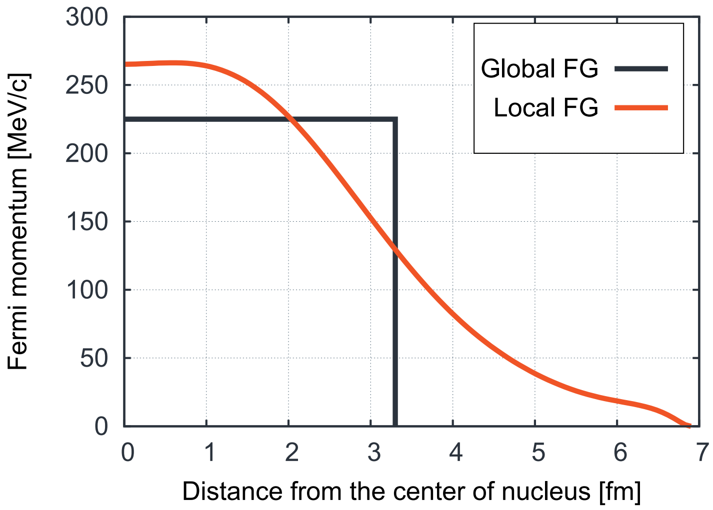


Global Fermi Gas

$$p_F = \frac{\hbar}{r_0}\left(\frac{9\pi N}{4A}\right)^{1/3}$$

Local Fermi Gas

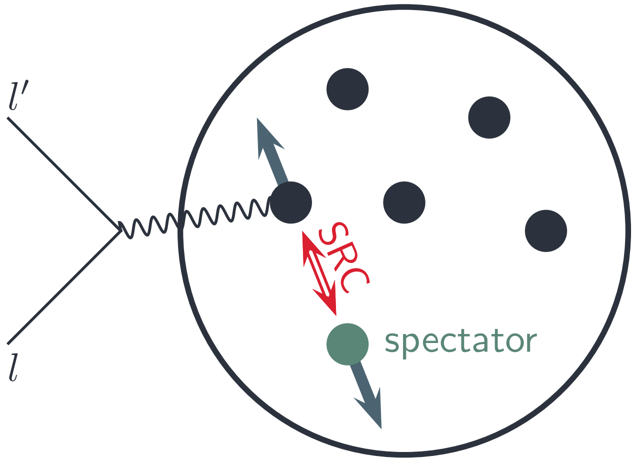
$$p_F(r) = \hbar\left(3\pi^2\rho(r)\frac{N}{A}\right)^{1/3}$$

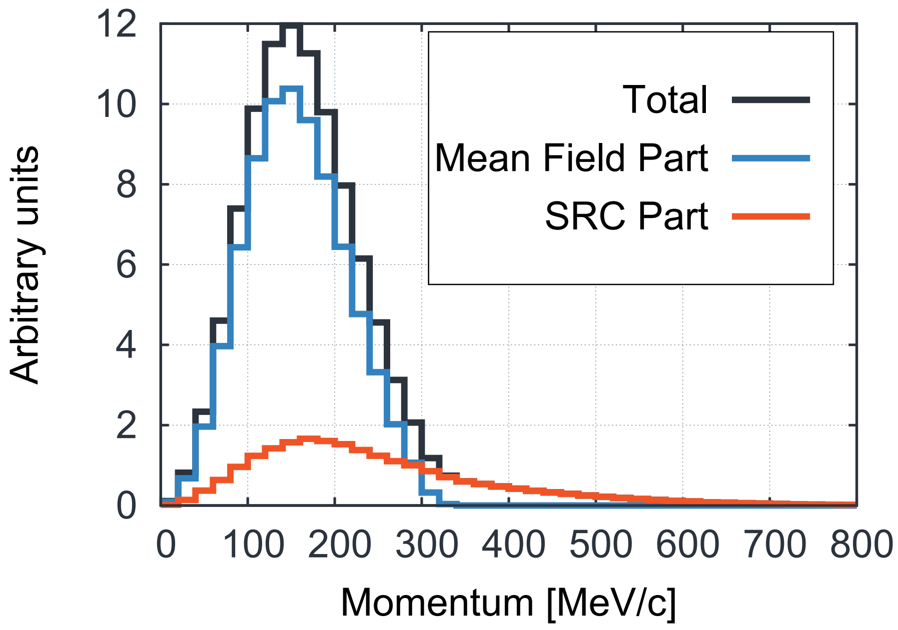


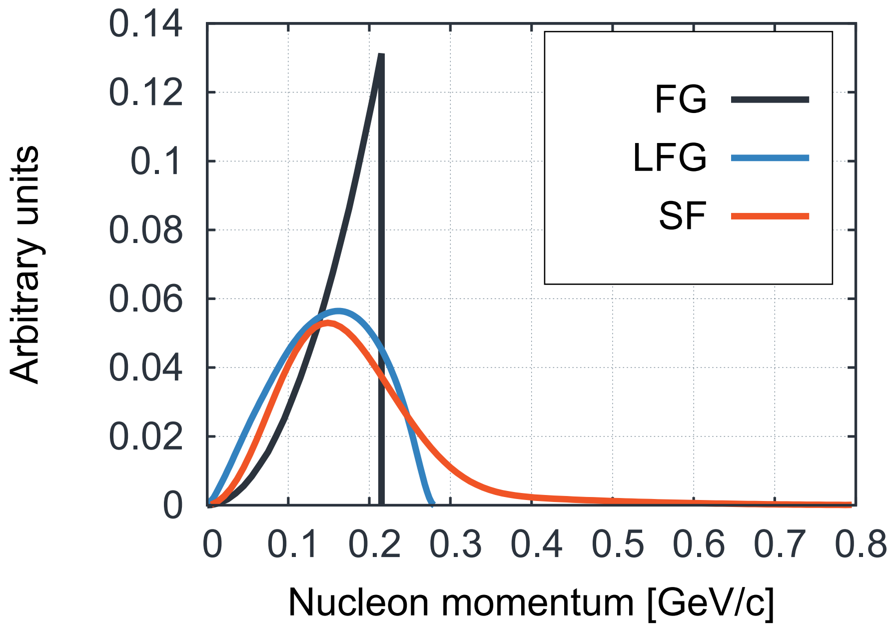
Spectral function

The probability of removing of a nucleon with momentum $\vec{p}$ and leaving residual nucleus with excitation energy E.

$$P(\vec{p}, E) = P_{MF}(\vec{p}, E) + P_{corr}(\vec{p}, E)$$





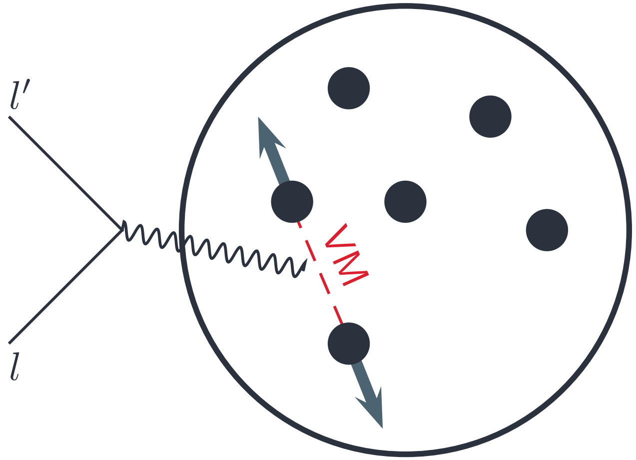


Two-body current interactions

Two Body Current

2 particles - 2 holes (2p-2h)

Meson Exchange Current (MEC)



Models in generators

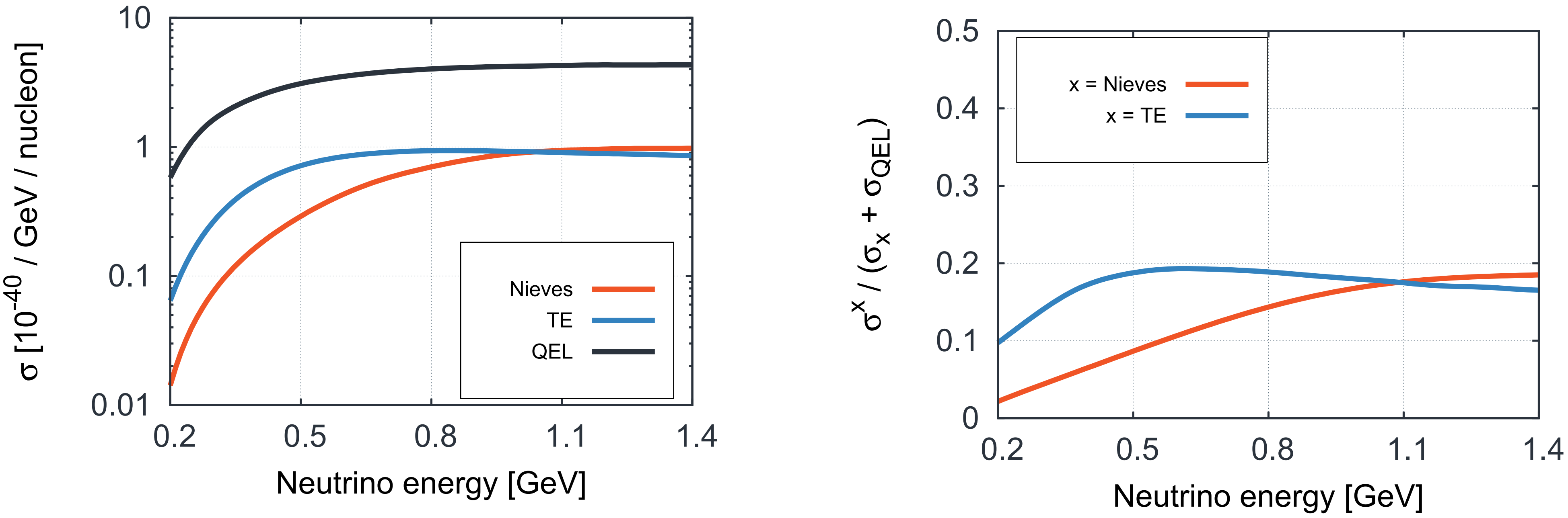
- Nieves model (GENIE, NEUT, NuWro)
- Transverse Enhancement (TE) model by Bodek (NuWro)
- Dytman model (GENIE)

Two-body current interactions

- Nieves model is microscopic calculation
- TE model introduce $2p - 2h$ contribution by modification of the vector magnetic form factors

Total MEC cross section

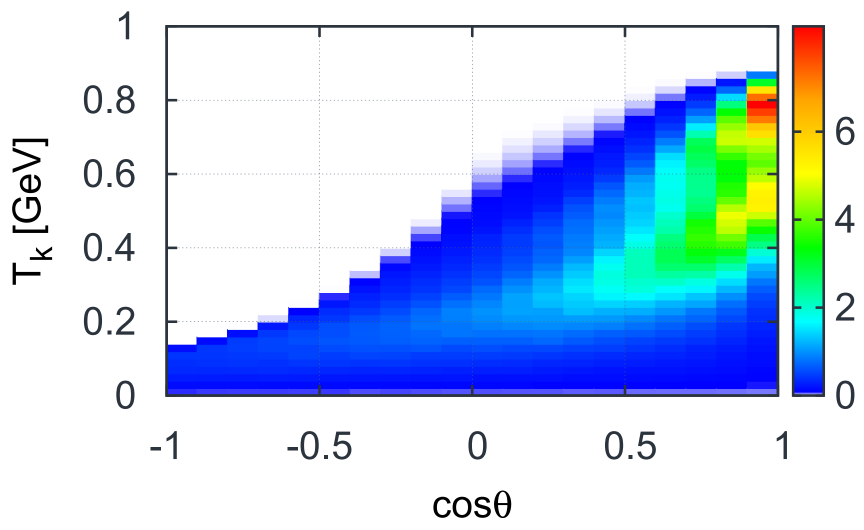
MEC / (QEL + MEC)



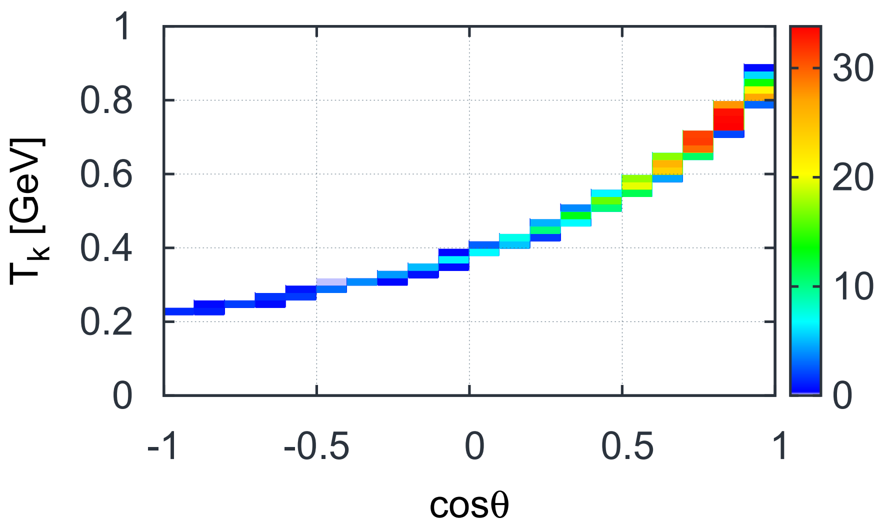
Two-body current interactions

- Both models provide only the inclusive double differential cross section for the final state lepton
- Final nucleons momenta are set isotropically in CMS

Nieves



Transverse Enhancement

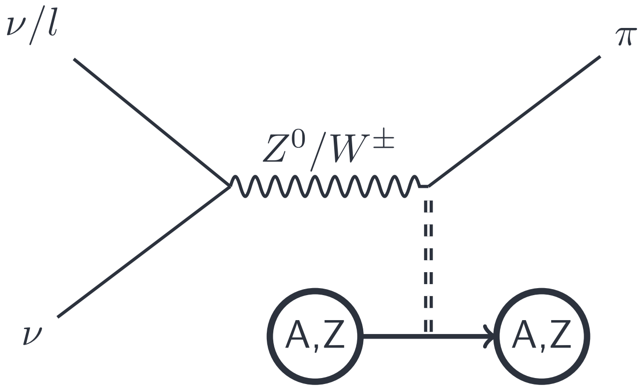


Coherent pion production

- Rein-Sehgal model is commonly used for coherent pion production

- Note: it is different model than for RES

- Berger-Sehgal model replaces RS (NuWro, GENIE)

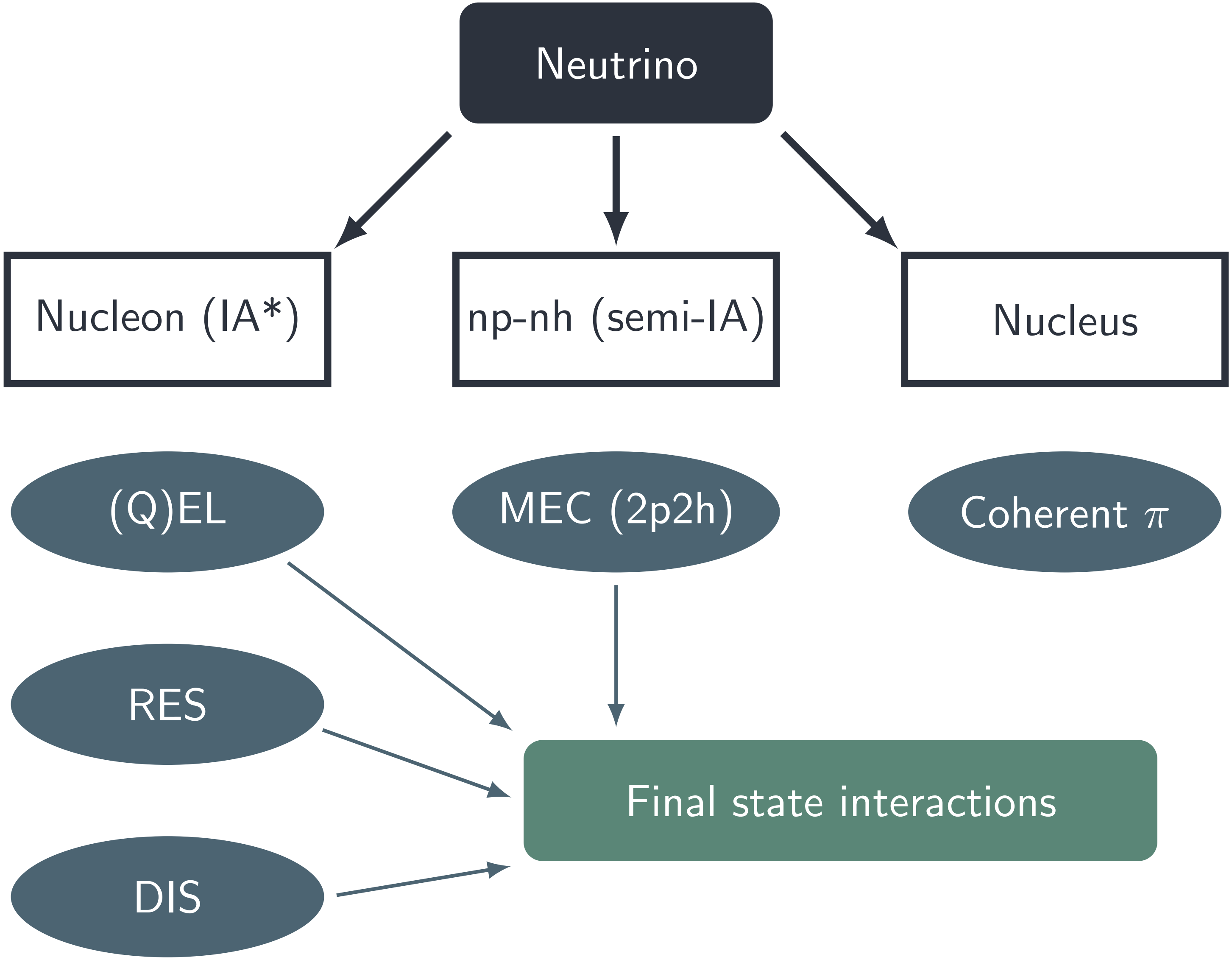


Comments

- In COH the residual nucleus is left in the same state (not excited)

- The interaction occurs on a whole nucleus - no final state interactions

Neutrino interactions - summary

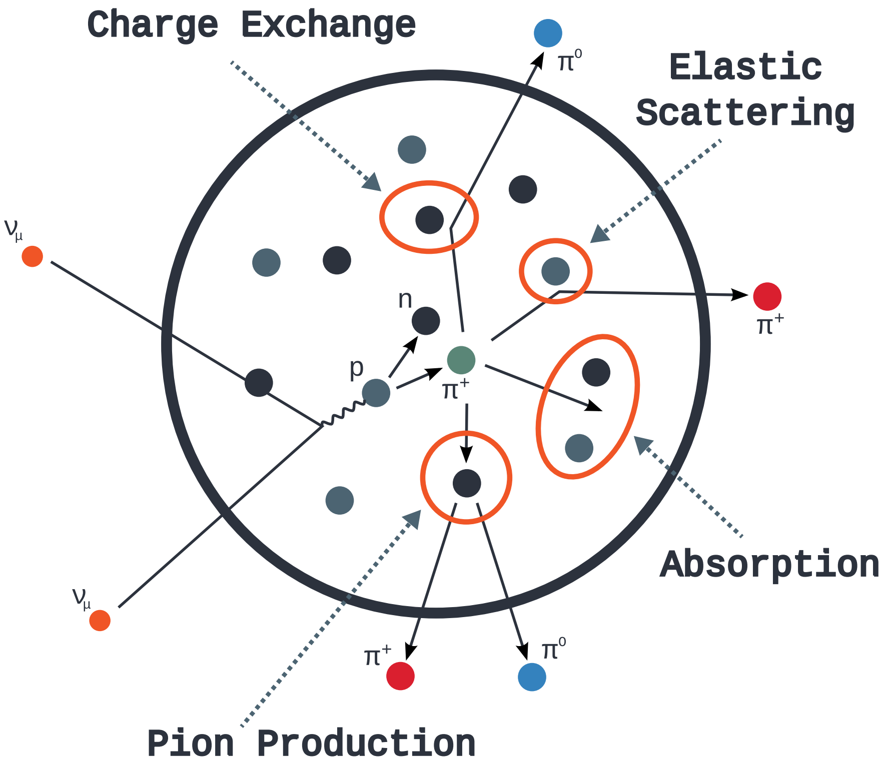


*IA = Impulse Approximation

Final state interactions

Final state interactions

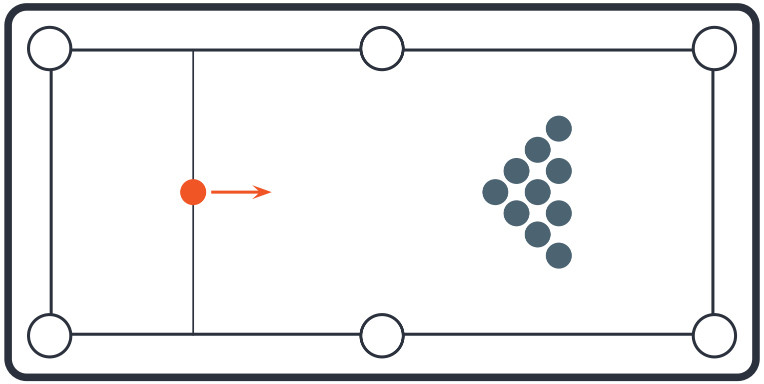
FSI describe the propagation of particles created in a primary neutrino interaction through nucleus



All MC generators (but GIBUU) use intranuclear cascade model

Intranuclear cascade

- In INC model particles are assumed to be classical and move along the straight line.



- The probability of passing a distance λ (small enough to assume constant nuclear density) without any interaction is given by:

$$P(\lambda) = e^{-\lambda/\tilde{\lambda}}$$

$\tilde{\lambda} = (\sigma\rho)^{-1}$ - mean free path

σ - cross section

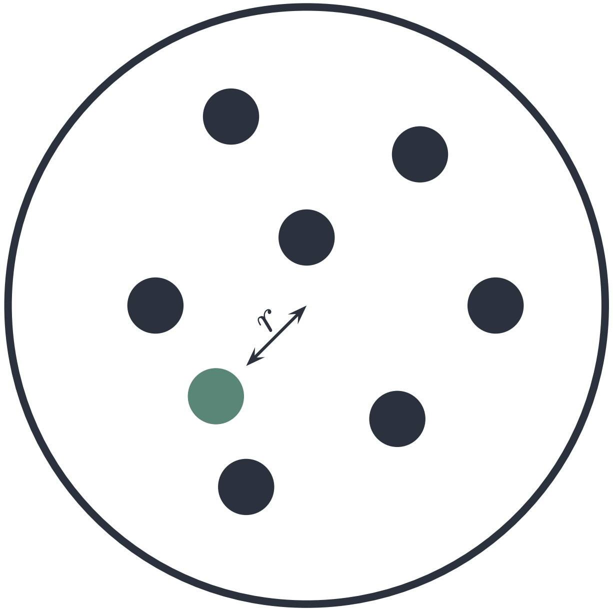
ρ - nuclear density

Can be easily handled with MC methods.

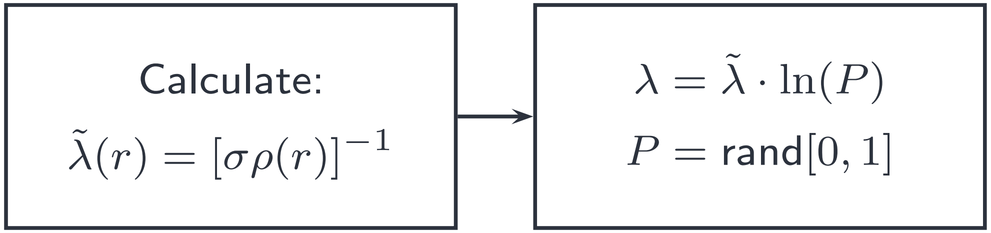
The algorithm for intranuclear cascade

Calculate:

$$\tilde{\lambda}(r) = [\sigma \rho(r)]^{-1}$$



The algorithm for intranuclear cascade

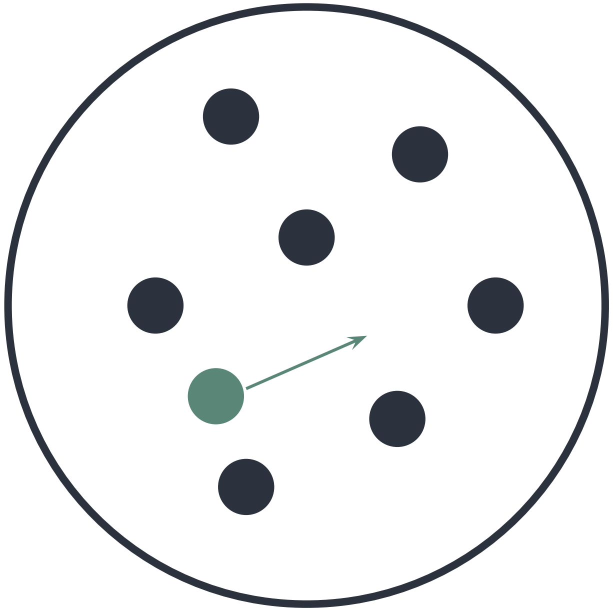


Calculate:

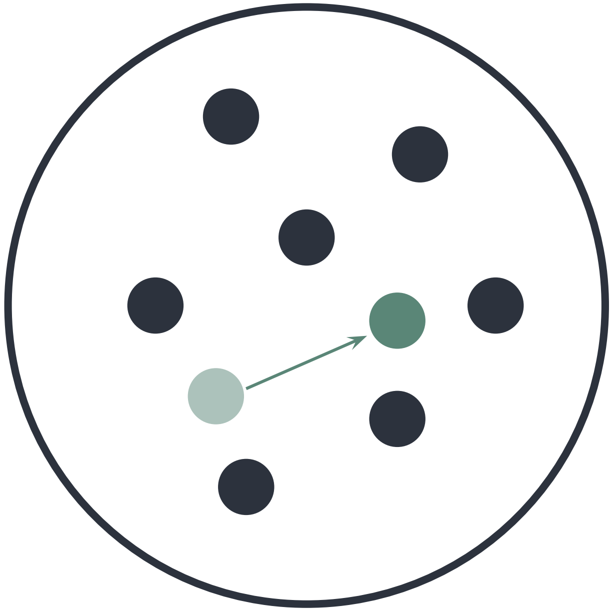
$\tilde{\lambda}(r) = [\sigma\rho(r)]^{-1}$

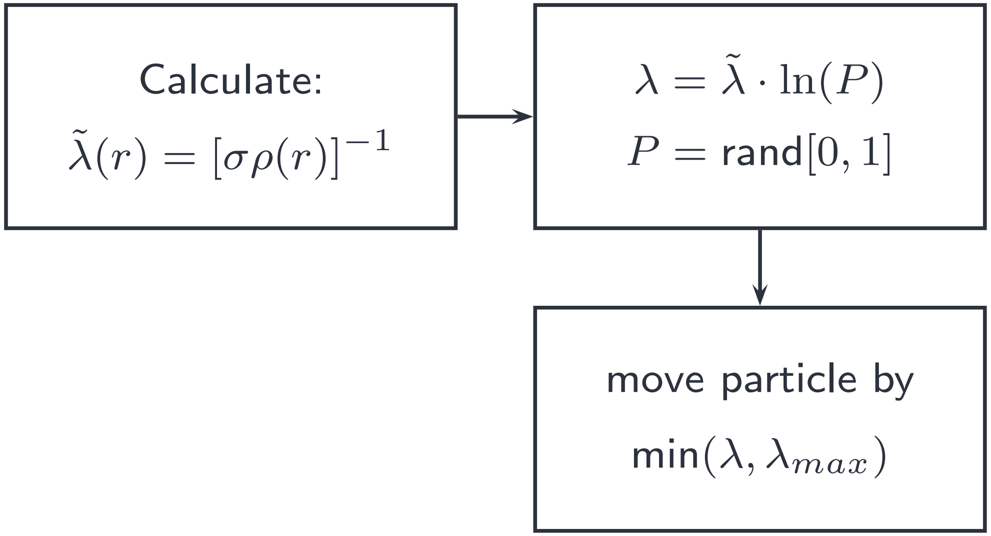
$\lambda = \tilde{\lambda} \cdot \ln(P)$

$P = \text{rand}[0, 1]$



The algorithm for intranuclear cascade





Calculate:

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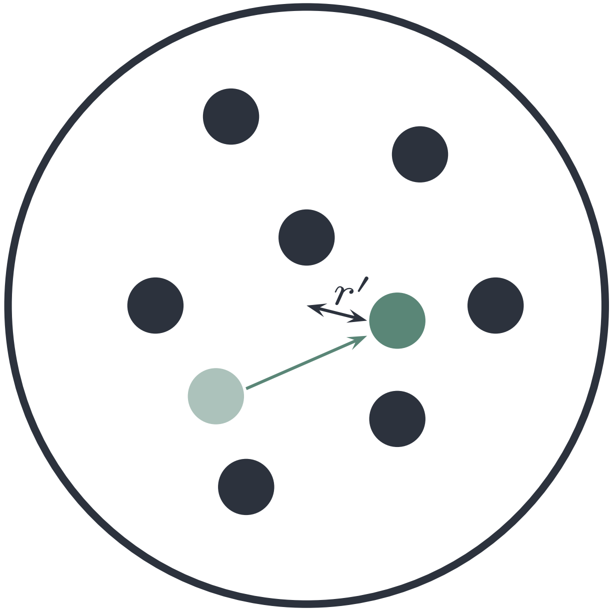
$\lambda = \tilde{\lambda} \cdot \ln(P)$

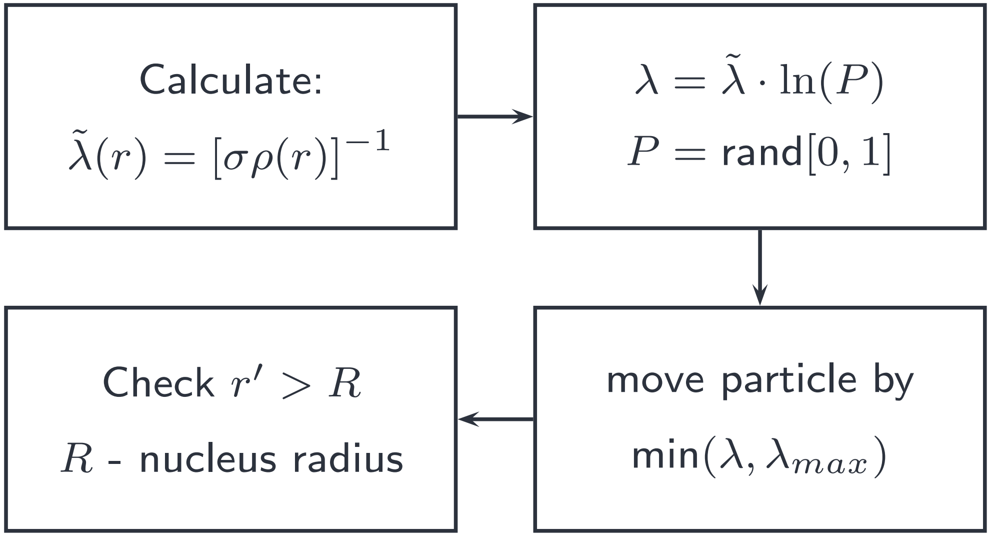
$P = \text{rand}[0, 1]$

move particle by

$\min(\lambda, \lambda_{max})$

The algorithm for intranuclear cascade





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$\lambda = \tilde{\lambda} \cdot \ln(P)$

$P =$ rand$[0, 1]$

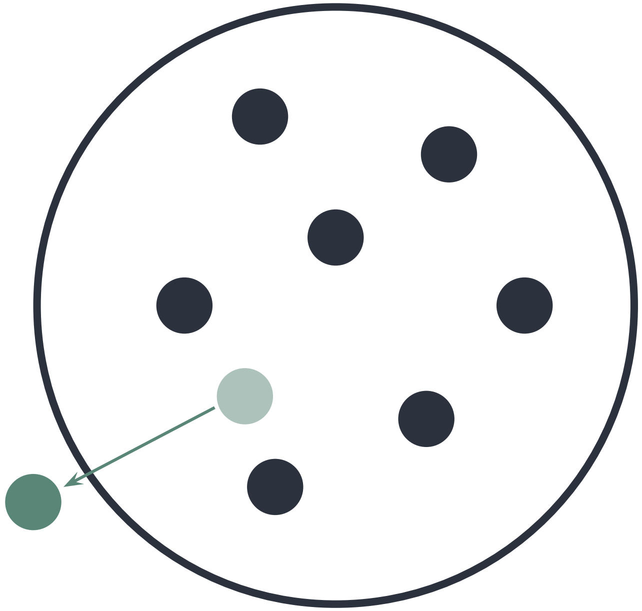
move particle by

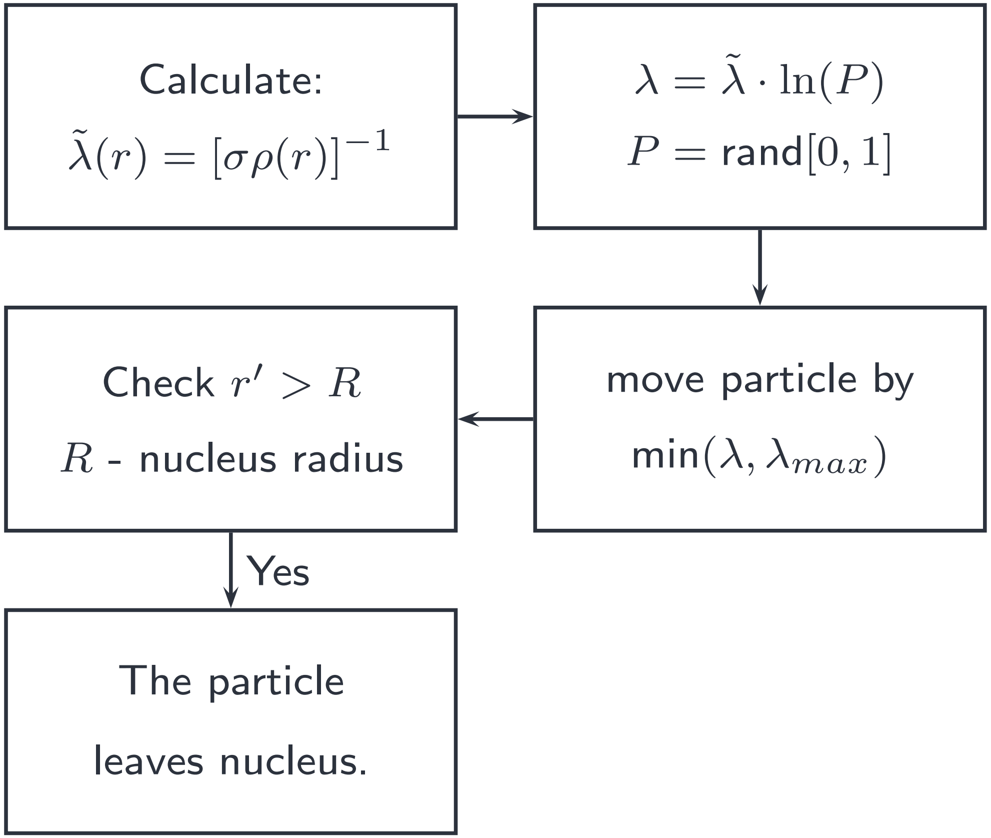
min(λ, λ_{max})

Check $r' > R$

R - nucleus radius

The algorithm for intranuclear cascade





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$P =$ rand$[0, 1]$

move particle by

min(λ, λ_{max})

Check $r' > R$

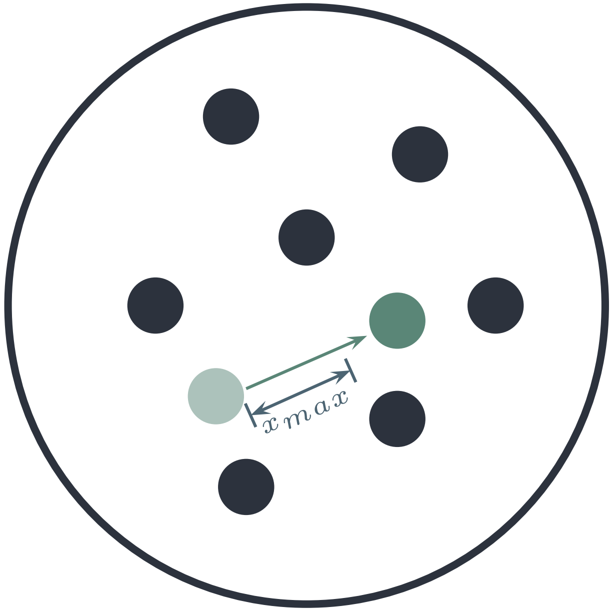
R - nucleus radius

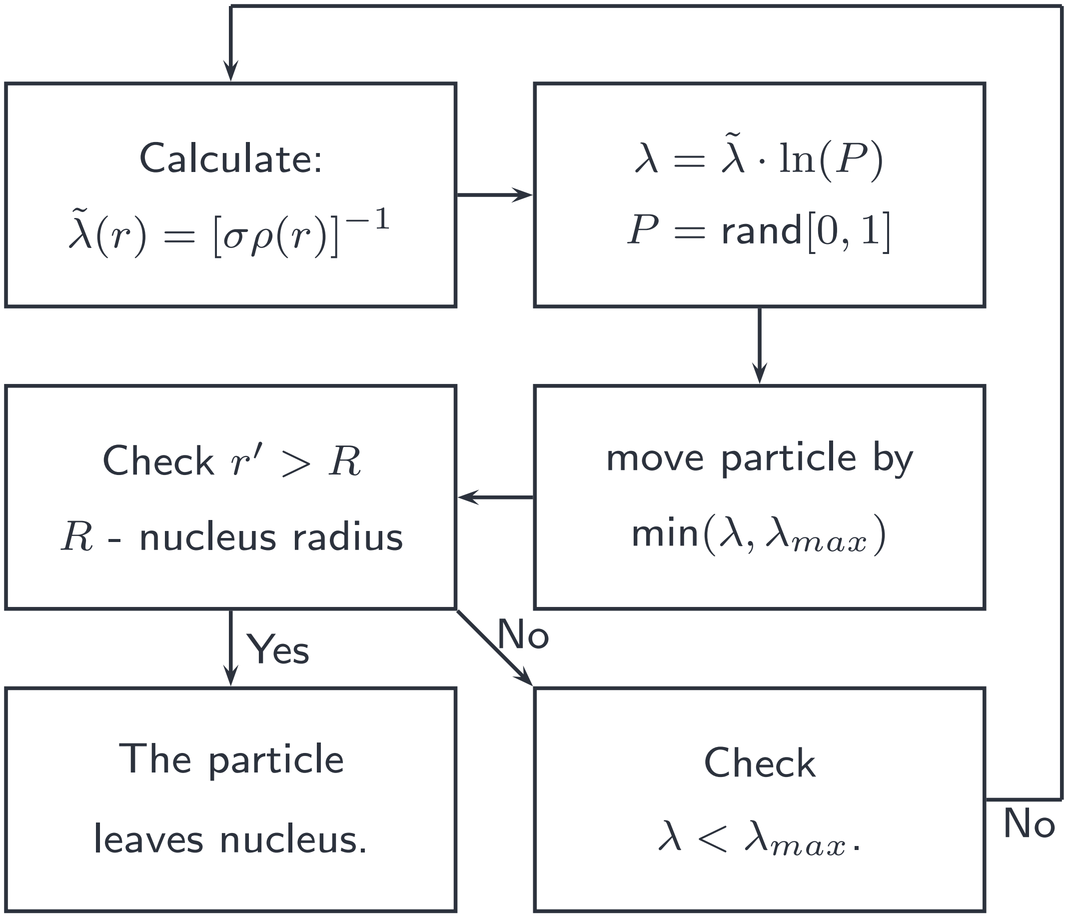
Yes

The particle

leaves nucleus.

The algorithm for intranuclear cascade





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$\lambda = \tilde{\lambda} \cdot \ln(P)$

$P = \text{rand}[0, 1]$

move particle by

$\min(\lambda, \lambda_{max})$

Check $r' > R$

R - nucleus radius

Yes

No

The particle

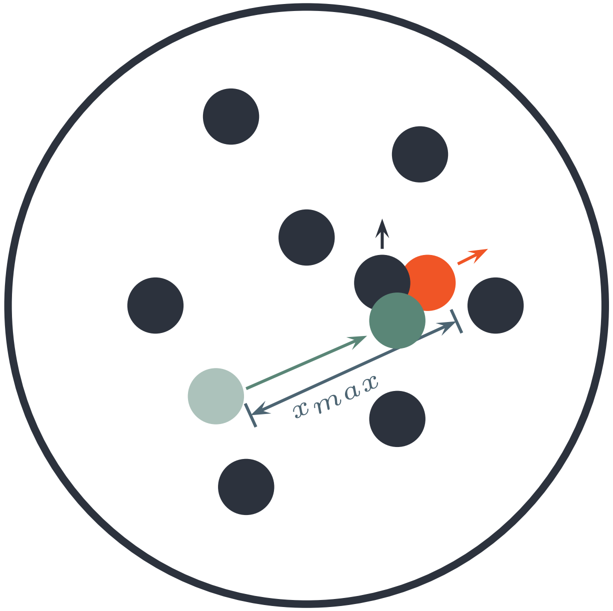
leaves nucleus.

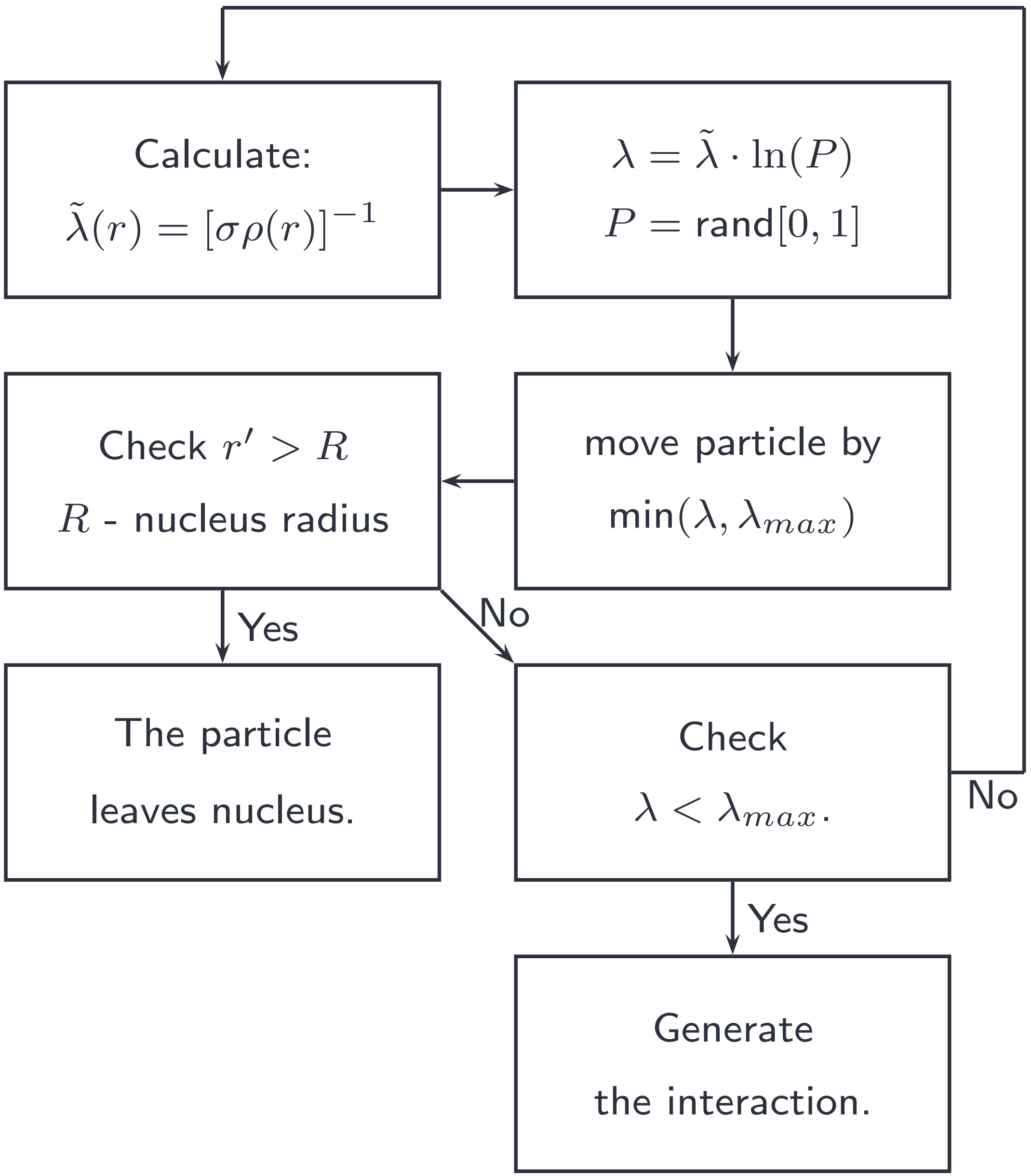
Check

$\lambda < \lambda_{max}$.

No

The algorithm for intranuclear cascade





Calculate:

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$P = \text{rand}[0, 1]$

move particle by

$\min(\lambda, \lambda_{max})$

Check $r' > R$

R - nucleus radius

Yes

The particle leaves nucleus.

No

Check

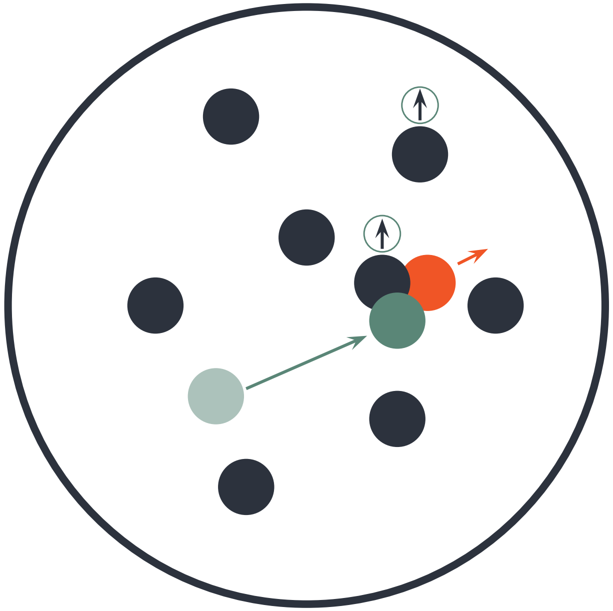
$\lambda < \lambda_{max}$.

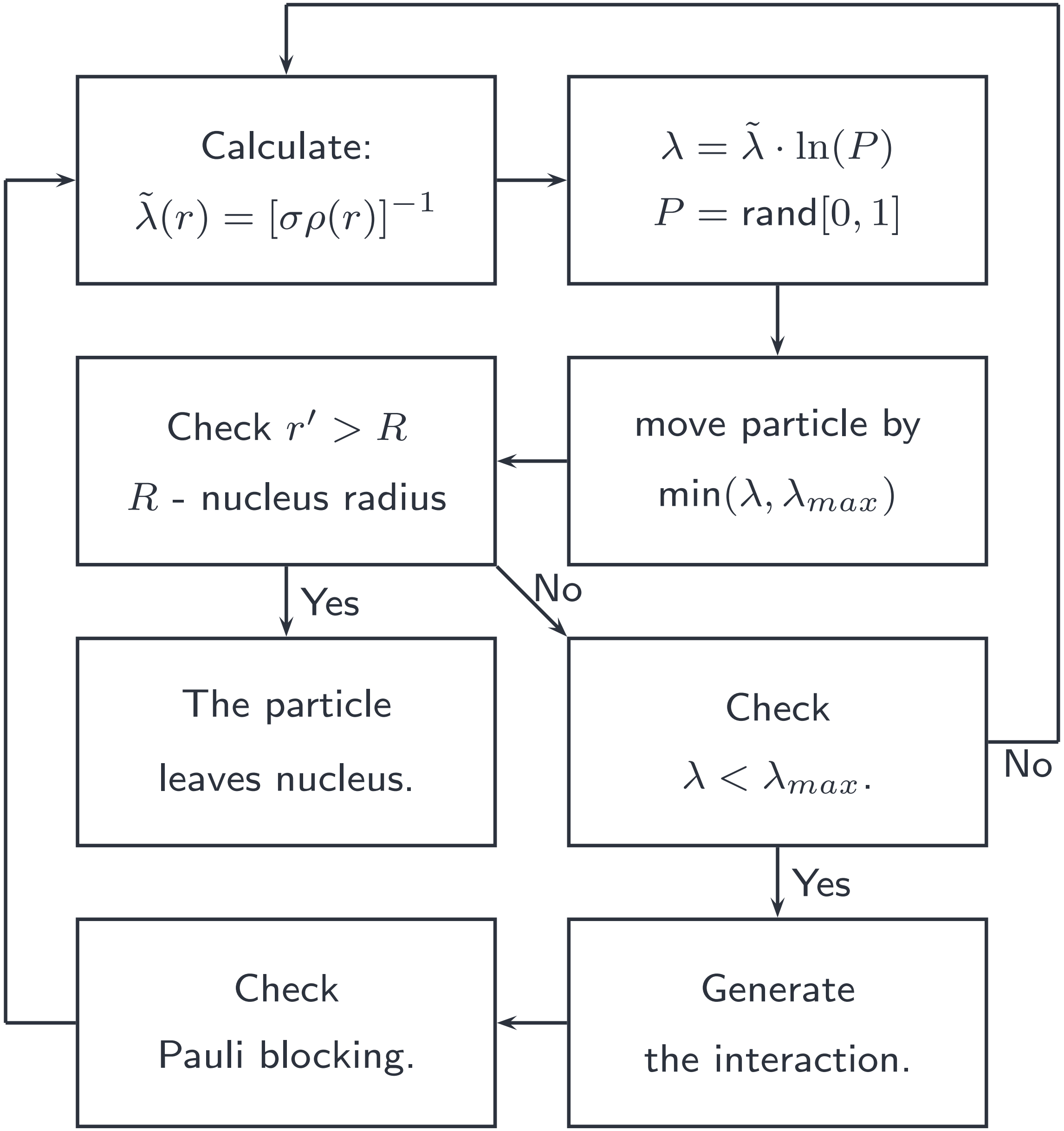
No

Yes

Generate the interaction.

The algorithm for intranuclear cascade





Calculate:

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$\lambda = \tilde{\lambda} \cdot \ln(P)$

$P =$ rand$[0, 1]$

move particle by

min(λ, λ_{max})

Check $r' > R$

R - nucleus radius

Yes

The particle

leaves nucleus.

No

Check

$\lambda < \lambda_{max}.$

No

Yes

Generate

the interaction.

Check

Pauli blocking.

INC input

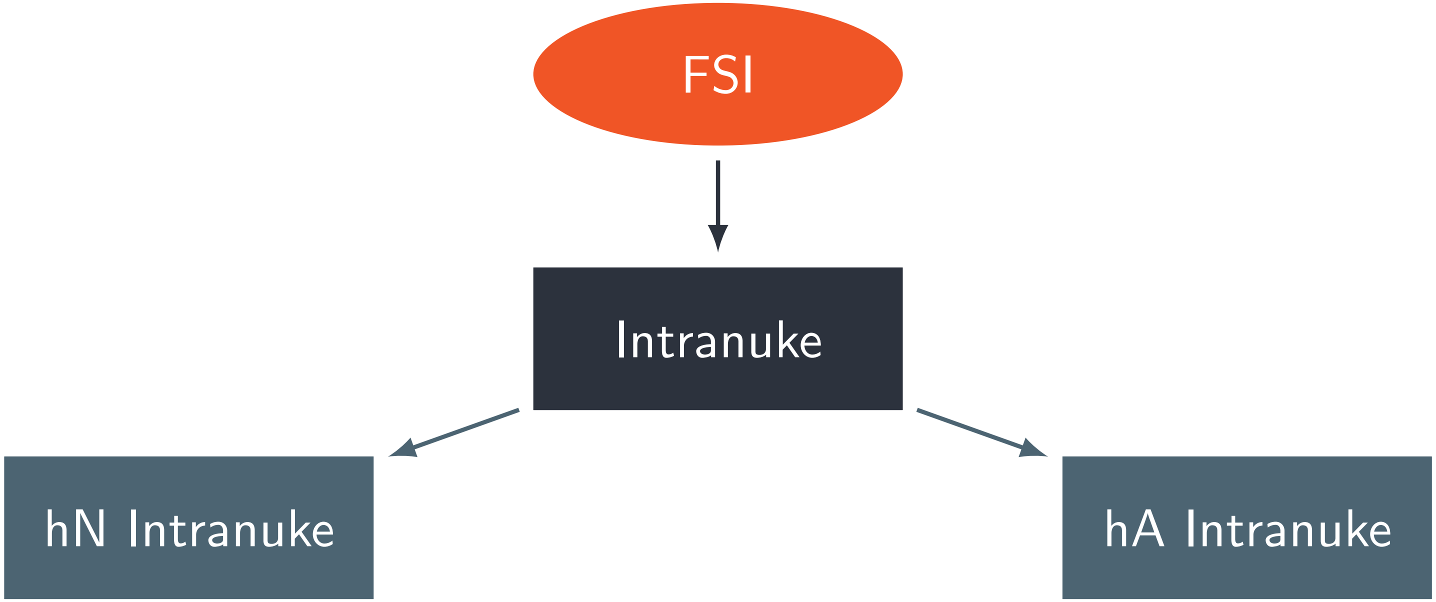
- The main input to the INC model is the particle-nucleon cross section
- Total cross section affects the mean free path
- Ratios of cross sections

$$\frac{\sigma_{qel}}{\sigma_{total}}, \quad \frac{\sigma_{cex}}{\sigma_{total}}, \quad \frac{\sigma_{abs}}{\sigma_{total}}, \quad \ldots$$

are used to determine what kind of scattering happened

- NuWro and Neut use Oset model for low-energy pions and data-driven cross sections for all other cases
- GENIE has two models of FSI

FSI in GENIE


FSI

Intranuke

hN Intranuke

hA Intranuke

- intranuclear cascade
- data-driven cross sections
- Oset model for pions

- INC-like with one "effective" interaction
- tuned do hadron-nucleus data
- easy to reweight

Formation time

Landau Pomeranchuk effect

- The concept of formation time was introduced by Landau and Pomeranchuk in the context of electrons passing through a layer of material.
- For high energy electrons they observed less radiated energy then expected.
- The energy radiated in such process is given by:

$$\frac{\mathrm{d}I}{\mathrm{d}^3k} \sim \left| \int_{-\infty}^{\infty} \vec{j}(\vec{x},t) e^{i(\omega t - \vec{k}\cdot\vec{x}(t))} \mathrm{d}^3x\mathrm{d}t \right|^2$$

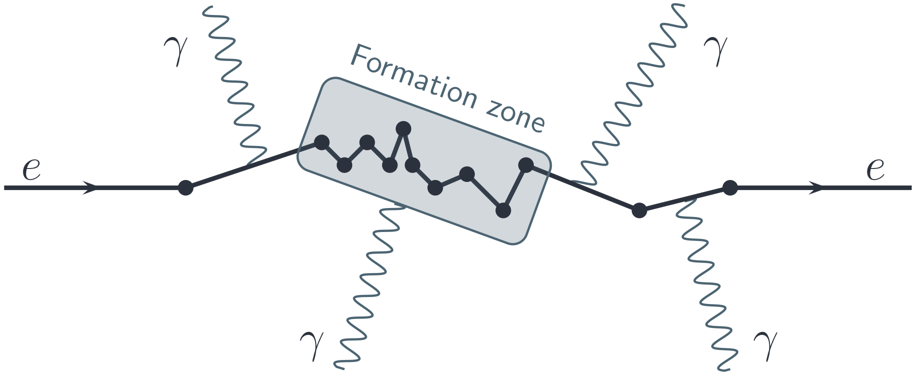
$\vec{x}(t)$ describes the trajectory of the electron.

ω, $\vec{k}$ are energy and momentum of the emitted photon.

Landau Pomeranchuk effect

- Assuming the trajectory to be a series of straight lines (the current density $j \sim \delta^3(\vec{x} - \vec{v}t)$) the radiation integral is:

$$\sim \int_{path} e^{i\left(\vec{k}\vec{v}-\omega\right)t}\mathrm{dt}$$



- Formation time is defined as:

$$t_f \equiv \frac{1}{\omega - \vec{k}\vec{v}} = \frac{E}{kp} = \frac{E}{m_e}\frac{1}{\omega_{r.f.}} = \gamma T_{r.f.}$$

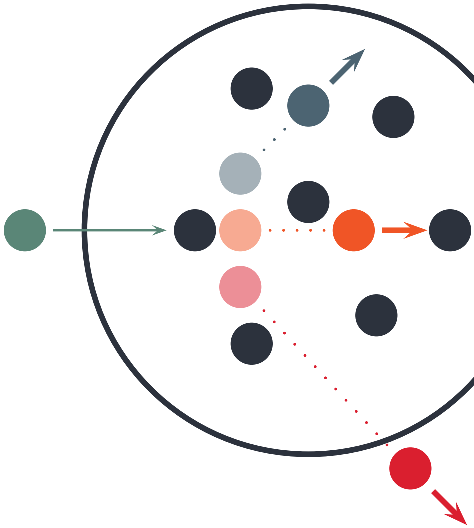
k, p - photon, electron four-momenta
$\omega_{r.f.}$ - photon frequency in the rest frame of the electron

- Formation time can be interpreted as the "birth time" of photon.

- If time between collisions $t >> t_f$, there is no interference and total radiated energy is just the average emitted in one collision multiplied by the number of collisions.

- If $t << t_f$, a photon is produced coherently over entire length of formation zone, which reduces the bremsstrahlung.

Formation time in INC



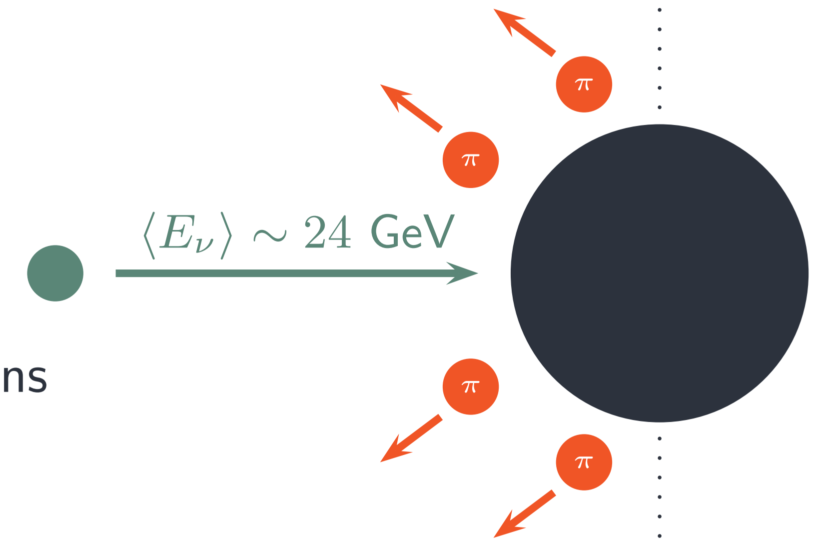
- One may expect a similar effect in hadron-nucleus scattering.
- In terms of INC it means that particles produced in primary vertex travel some distance, before they can interact.
- There are several parametrization used in MC generators
- Ranft parametrization:

$$t_f = \tau_0 \frac{E \cdot M}{\mu_T^2}$$

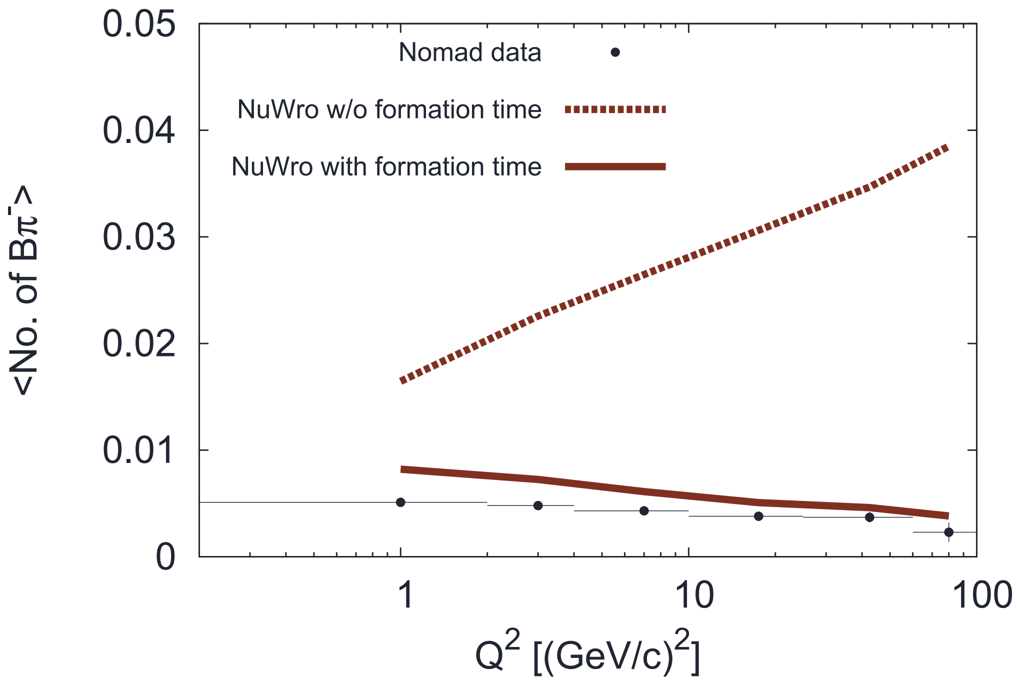
where E, M - nucleon energy and mass, $\mu_T^2 = M^2 + p_T^2$ - transverse mass

- SKAT parametrization (similar but with $p_T = 0$)
- NEUT and GENIE use SKAT parametrization
- NuWro uses Ranft parametrization for DIS and includes a Δ lifetime for RES

Comparison with NOMAD data



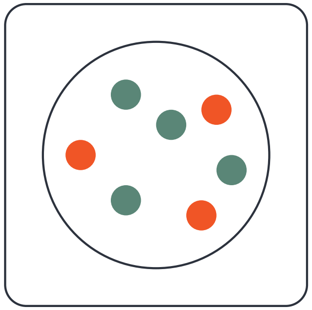
- Nomad data from Nucl. Phys. B609 (2001) 255.
- The average number of backward going negative pions with the momentum from 350 to 800 MeV/c.
- In this neutrino energy range $B\pi^-$ are an effect of FSI.
- The observable is very sensitive to formation time effect.



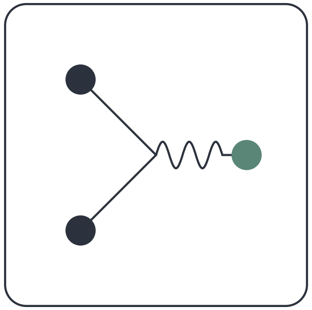
Summary

Neutrino-nucleus interactions

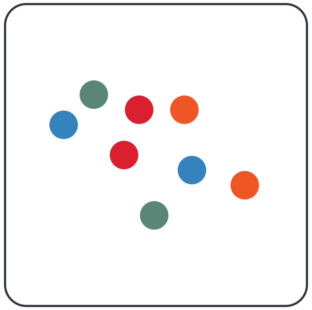
For all channels (but coherent) neutrino interactions are factorized in the following way



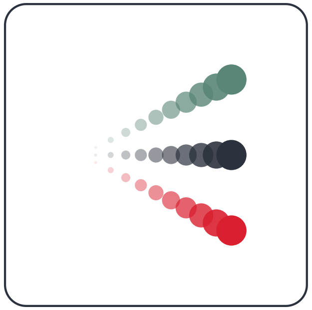
IA



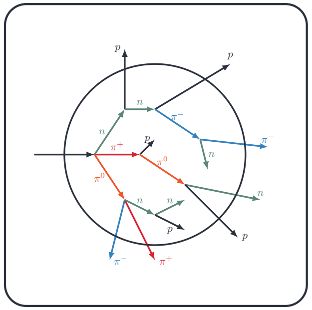
νN



hadronization



formation time

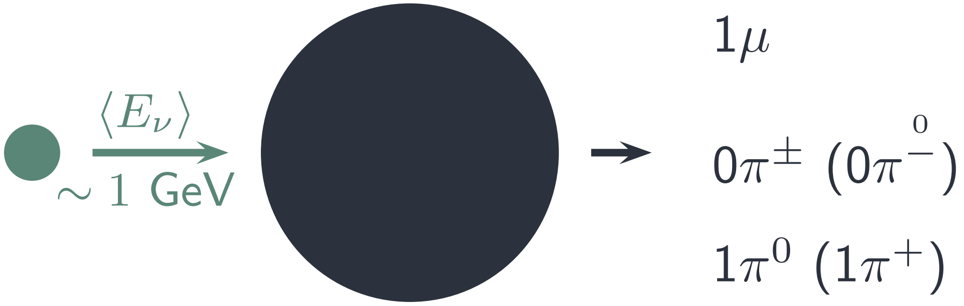


FSI

- Is the physics really factorized this way?
- This factorization is common for all generators
- However, some pieces are done in different way

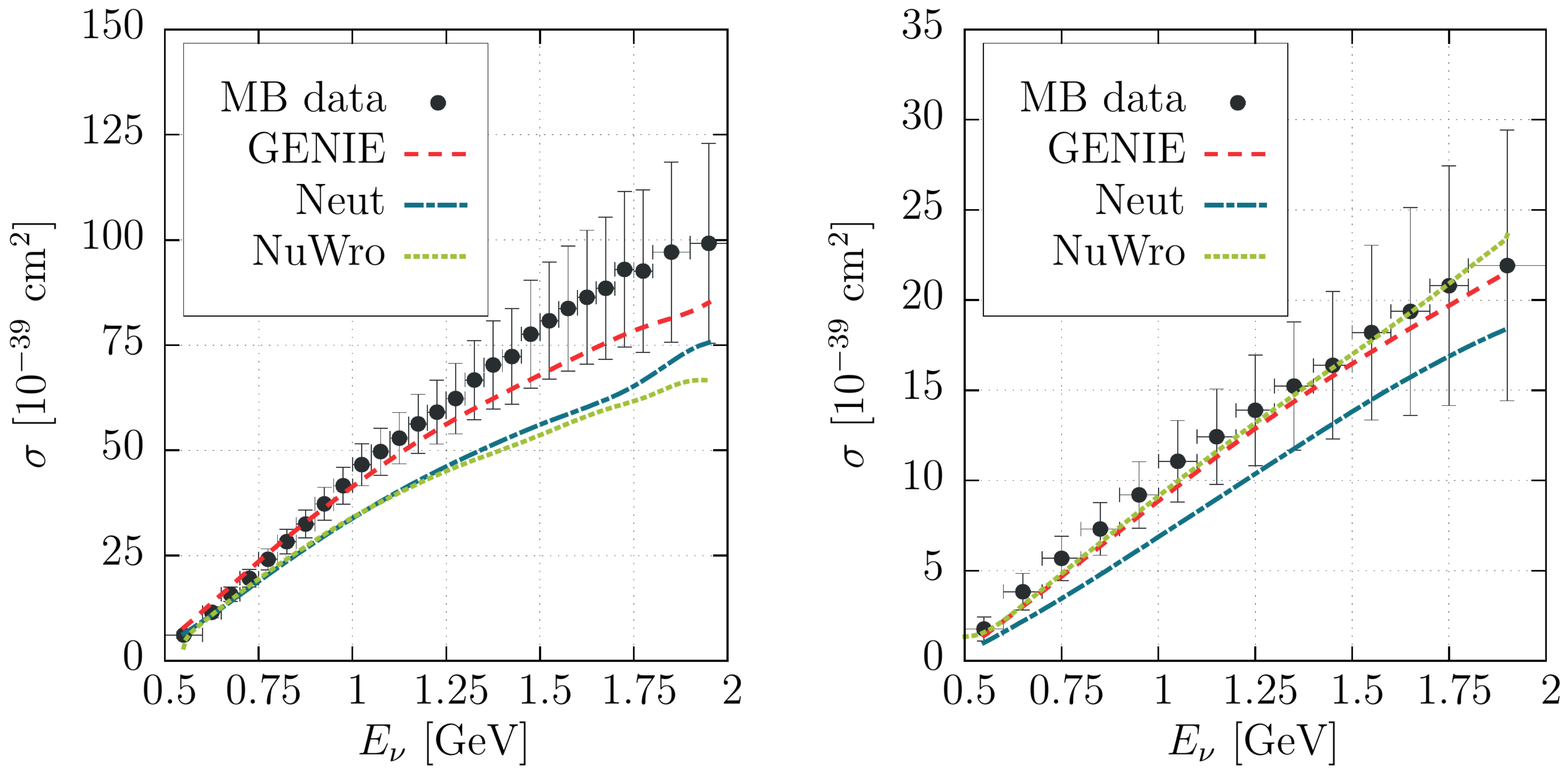
MiniBooNE data for CC π production

- The cross section for π^0 (π^+) production through charge current measured by MiniBooNE



- The signal is defined as: charged leptons, no charged pions and one neutral pion (one positive pion and no other pions) in the final state.
- The result depends on primary vertex and FSI, as pion can be:
  - produced in primary vertex
  - produced in FSI
  - affected by charge exchange
  - absorbed

MiniBooNE data for CC π production

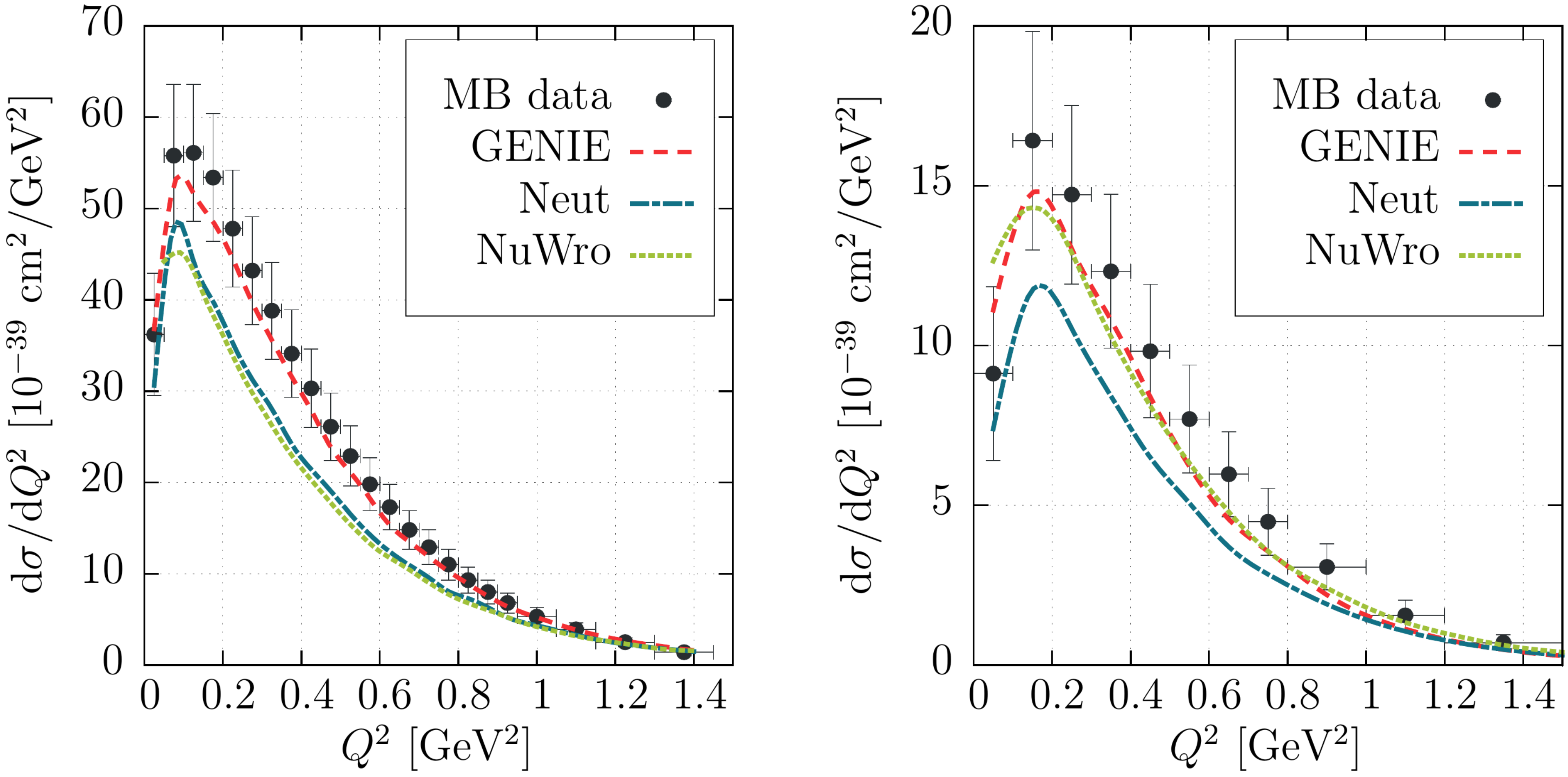


(a) $1\pi^+$ production

(b) $1\pi^0$ production

Figure 3.1: The total CC cross section for single pion production.

MiniBooNE data for CC π production

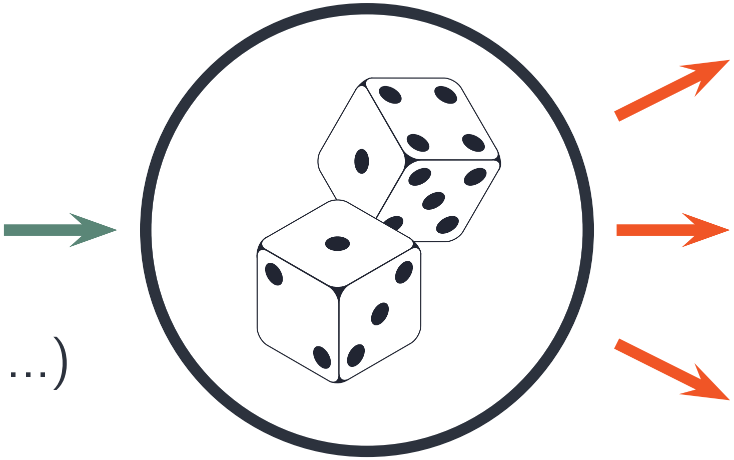


(a) $1\pi^+$ production

(b) $1\pi^0$ production

Figure 3.2: The differential CC cross section over Q^2 for single pion production.

Summary



- MC generators are irreplaceable tools in high-energy physics
- People use them before experiment exists (feasibility studies, requirements ...)
- And during data analysis (systematics uncertainties, backgrounds ...)
- There are several neutrino event generators and they all differ slightly
- And, unfortunately, there is no one right generator