Neutrino induced pion production reaction 1

Toru Sato

Osaka University

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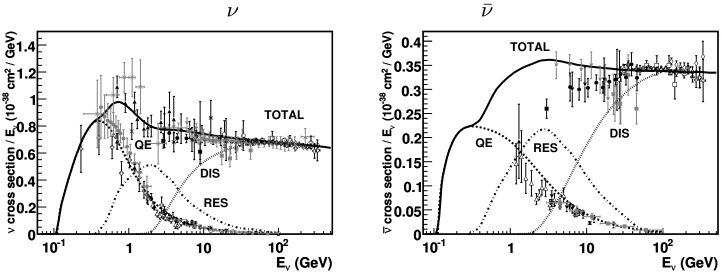
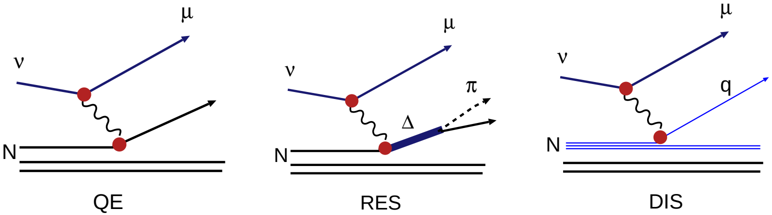


Total cross section. J. A. Formagglo and G. P. Zeller Rev. Mod. Phys. 84 (2012) 1307.



Plan of lectures

- How pion is produced from nucleon ? [1,2,3]
    - Weak current and iso-spin structure
    - Chiral effective Lagrangian and non-resonant mechanism
    - $\Delta_{33}(1232)$ production
    - Higher mass resonances

- How pion interacts in nuclei ? [3]
    - Propagation of pion and Δ in nuclei
    - Coherent pion production

The structure of the nucleon, A. W. Thomas, W. Weise, ISBN: 978-3-527-40297-7
Dynamics of the standard model, J. Donoghue, E. Golowich, B. R. Holstein, ISBN-10: 0521476526
S. X. Nakamura et al., Rep. Prog. Phys. 80 (2017) 056301 doi.org/10.1088/1361-6633/aa5e6c

Weak current and iso-spin structure

- Electroweak current in Standard model
- Iso-spin symmetry and hadron matrix element

Electroweak current in Standard model

Hadron current in standard model(u, d, s quarks)

$$
\begin{aligned}
J^\mu_{em} &= \frac{2}{3}\bar{u}\gamma^\mu u - \frac{1}{3}\bar{d}\gamma^\mu d - \frac{1}{3}\bar{s}\gamma^\mu s \\
J^\mu_{cc} &= V_{ud}\bar{u}\gamma^\mu(1-\gamma_5)d + V_{us}\bar{u}\gamma^\mu(1-\gamma_5)s \\
&= V_{ud}J^\mu_{CC(\Delta S=0)} + V_{us}J^\mu_{CC(\Delta S=1)} \\
J^\mu_{nc} &= \frac{1}{2}[\bar{u}\gamma^\mu(1-\gamma_5)u - \bar{d}\gamma^\mu(1-\gamma_5)d - \bar{s}\gamma^\mu(1-\gamma_5)s] - 2\sin^2\theta_W J^\mu_{em}
\end{aligned}
$$

Hadron matrix element

Nucleon axial vector current

$$
< p|\bar{u}\gamma^\mu\gamma_5 d|n >= \bar{u}(p')[g_A(Q^2)\gamma^\mu\gamma_5 + g_P(Q^2)q^\mu\gamma_5]u(p))
$$

Pion production amplitude

$$
< \pi N|\bar{u}\gamma^\mu\gamma_5 d|N >= \sum_{k=1}^{8} A_k(W, Q^2, \theta_\pi)\bar{u}(p')\mathcal{O}_k u(p)
$$

- $\mathcal{O}_k, \gamma^\mu\gamma_5, q^\mu\gamma_5$ ensure Lorentz structure of current
- $A_k(Q^2, W, \theta_\pi), g_A(Q^2)$: Scalar functions and include non-perturbative QCD dynamics

Iso-spin symmetry and hadron matrix element

Introduce Iso-vector, Iso-scalar current

$$V_i^\mu = \bar{q}\gamma^\mu \frac{\tau_i}{2} q, \quad A_i^\mu = \bar{q}\gamma^\mu\gamma_5 \frac{\tau_i}{2} q, \quad V_{IS}^\mu = \frac{1}{6}[\bar{q}\gamma^\mu q - 2\bar{s}\gamma^\mu s]$$

$$q = \begin{pmatrix} u \\ d \end{pmatrix}, \tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$V_3^\mu = \bar{q}\gamma^\mu \tfrac{\tau_3}{2} q = \tfrac{1}{2}(\bar{u}\gamma^\mu u - \bar{d}\gamma^\mu d)$$

Rewrite EM,CC,NC current

$$\begin{aligned} J_{em}^\mu &= V_3^\mu + V_{IS}^\mu \\ J_{CC}^\mu &= V_{1+i2}^\mu - A_{1+i2}^\mu \quad (\Delta S = 0 \text{ current without CKM}) \\ J_{NC}^\mu &= V_3^\mu - A_3^\mu - 2\sin^2\theta_W J_{em}^\mu - \frac{1}{2}\bar{s}\gamma^\mu(1-\gamma_5)s \end{aligned}$$

Assuming isospin symmetry (neglect electromagnetic correction, $m_p = m_n$..)

- ν and $\bar{\nu}$ amplitudes are related with each other.
- V_{CC}^μ is related to iso-vector EM current through iso-spin rotation.

- ν and $\bar{\nu}$ amplitude.

  Using $|\pi^+ p >= |I = 3/2, I_3 = 3/2 >, |\pi^- n >= |I = 3/2, I_3 = -3/2 >$
  and $J^\mu_{CC} = J^\mu_{1+i2}, (J^\mu_{CC})^\dagger = J^\mu_{1-i2}$

$$< \pi^+ p | J^\mu_{CC} | p >= - < \pi^- n | (J^\mu_{CC})^\dagger | n >$$

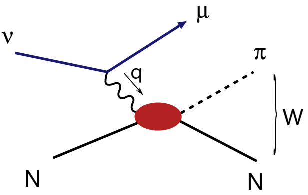
  Note: $\sigma(\nu p \to l^- \pi^+ p) \neq \sigma(\bar{\nu} n \to l^- \pi^- n)$ due to $A - V$ interference term W_3.

$$\frac{d\sigma}{dE_l d\Omega_l} = \frac{(G_F V_{ud})^2 E_l^2}{2\pi^2}[\cos^2 \frac{\theta_l}{2} W_2 + \sin^2 \frac{\theta}{2}(2W_1 \pm \frac{E_\nu + E_l}{M_N} W_3)] \text{ for } \nu/\bar{\nu}$$

- V^μ_{CC} and iso-vector EM

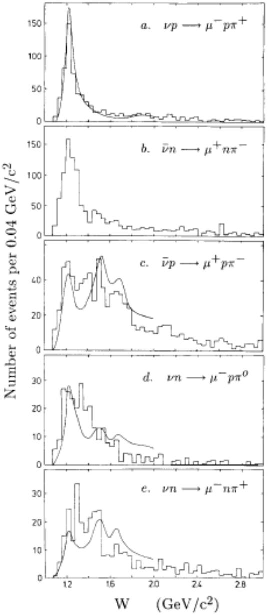
$$< \pi^+ p | V^\mu_{CC} | p >= -[\sqrt{2} < \pi^0 p | J^\mu_{em} | p > + < \pi^+ n | J^\mu_{em} | p >]$$

Single pion production(CC)



$$W^2 = (p_N(f) + p_\pi)^2 = (p_\nu + p_N(i) - p_\mu)^2$$

$$Q^2 = -q^2 = -(p_\nu - p_\mu)^2$$



D. Allasia et al. Nucl. Phys. B343(1990)285

Chiral effective Lagrangian and non-resonant mechanism

- Pion and chiral symmetry
- Chiral Effective Lagrangian
- Weak pion production

Pion and chiral symmetry

In the limit of massless quark, $\mathcal{L}_{QCD}(m_q = 0)$ is invariant under chiral $(SU(2)_L \otimes SU(2)_R$, global) transformation

$$q_{R/L} \to R/L q_{R/L} \quad \text{with} \quad q_{R/L} = \begin{pmatrix} \frac{1 \pm \gamma_5}{2} u \\ \frac{1 \pm \gamma_5}{2} d \end{pmatrix} \quad \text{and} \quad R/L = e^{i \vec{\theta}_{R/L} \cdot \vec{\tau}}$$

Chirality: $\gamma_5 q_{R/L} = \pm q_{R/L}$,
Explicit symmetry breaking: $\partial \cdot A_{1+i2} = i(m_u + m_d)\bar{u}\gamma_5 d$ (PCAC)

The symmetry realized in nature $SU(2)_L \otimes SU(2)_R \to SU(2)_V$

- Vector charge $Q_V = Q_R + Q_L$: 'trivial vacum'

$$Q_V |0> = 0$$

- Axial vector charge $Q_A = Q_R - Q_L$: Nambu-Goldstone mode

$$< 0|[Q_A, \phi]|0 > \neq 0$$

- pion as Nambu-Goldstone boson, pion interactions at low energy are constrained from chiral symmetry.

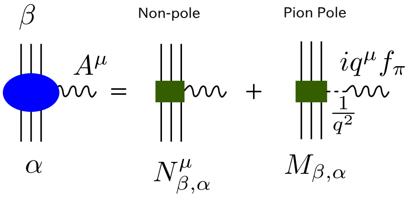
Useful consequences for us:

- axial vector current annihilates pion

$$< 0|A^i_\mu|\pi^i(q) >= if_\pi q_\mu, \quad f_\pi \sim 93MeV$$

- matrix element of axial vector current is related to pion production amplitude

$$< \beta|A^\mu_i|\alpha >= N^\mu_{\beta,\alpha} + \frac{if_\pi q^\mu}{q^2} M_{\beta,\alpha}$$



conservation of axial vector current $q \cdot A = 0$ gives

$$M_{\beta,\alpha} = \frac{iq_\mu}{f_\pi} N^\mu_{\beta,\alpha}$$

example Goldberger-Treiman relation $g_{\pi NN} = Mg_A/f_\pi$

Chiral Effective Lagrangian

Effective Lagrangian using relevant defrees of freedom of QCD at low energy(pion) and low energy expansion(power of momentum(derivative) and small symmetry breaking term (m_π).

Introduce U, under chiral rotation $U \to RUL^\dagger$ (non-linear σ model)

$$U = e^{i\vec{\pi}\cdot\vec{\tau}/f_\pi}$$

Lowest order Lagrangian(pion)

$$\mathcal{L}_{eff} = \frac{f_\pi^2}{4} Tr[\partial_\mu U^\dagger \partial^\mu U] + \frac{f_\pi^2 m_\pi^2}{4} Tr(U + U^\dagger)$$

Example:

$$\begin{aligned} U &= 1 + i\vec{\pi}\cdot\vec{\tau}/f_\pi + (i\vec{\pi}\cdot\vec{\tau}/f_\pi)^2/2 + ... \\ \partial^\mu U &= i\partial^\mu\vec{\pi}\cdot\vec{\tau}/f_\pi + ... \\ \to \mathcal{L}_{eff} &= \frac{1}{2}[\partial^\mu\vec{\pi}\cdot\partial_\mu\vec{\pi} - m_\pi^2\vec{\pi}\cdot\vec{\pi}] + [\pi\pi \text{ interactions}] \end{aligned}$$

Including nucleon

$$\mathcal{L}_{eff} \quad = \quad \bar{N}[i\gamma^\mu D_\mu - m_N + ig_A\gamma^\mu\gamma_5\Delta_\mu]N$$

where

$$D_\mu = \partial_\mu + \frac{1}{2}[\xi^\dagger, \partial_\mu \xi] = \partial_\mu + \frac{i}{4f_\pi^2}(\vec{\pi} \times \partial_\mu \vec{\pi}) \cdot \vec{\tau} + ..$$

$$\Delta_\mu = \frac{1}{2}\{\xi^\dagger, \partial_\mu \xi\} = \frac{i}{2f_\pi}\partial_\mu \vec{\pi} \cdot \vec{\tau} + ...$$

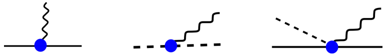
here $U = \xi^2, \xi = e^{i\vec{\pi}\cdot\vec{\tau}/2}$. parameters g_A, f_π.

- $\mathcal{L}_{eff} \rightarrow \pi N$ interactions
- Vector and axial vector current using Noether theorem when $\phi(x) \rightarrow \phi(x) + \epsilon(x)\delta\phi(x)$

$$J^\mu(x) = \frac{\partial}{\partial(\partial_\mu \epsilon(x))}\mathcal{L}(\phi, \partial\phi)$$

Relevant Interaction and current

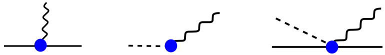
Vector current



$$\vec{V}^\mu = \bar{N}\gamma^\mu \frac{\vec{\tau}}{2} N + \vec{\pi}\times\partial^\mu\vec{\pi} + \frac{g_A}{2f_\pi}\bar{N}\gamma^\mu\gamma_5\vec{\tau}N\times\vec{\pi}$$

Contact interaction can be generated from πNN using Gauge invariance

Axial vector current



$$\vec{A}^\mu = g_A\bar{N}\gamma^\mu\gamma_5 \frac{\vec{\tau}}{2} N - f_\pi\partial^\mu\vec{\pi} + \frac{1}{2f_\pi}\bar{N}\gamma^\mu\vec{\tau}N\times\vec{\pi}$$

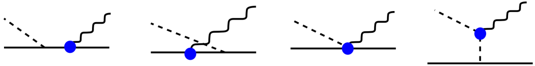
πN interaction

$$-\frac{g_A}{2f_\pi}\bar{N}\gamma^\mu\gamma_5\vec{\tau}N\cdot\partial_\mu\vec{\pi} - \frac{1}{4f_\pi^2}\bar{N}\gamma^\mu\vec{\tau}N\times\vec{\pi}\cdot\partial_\mu\vec{\pi}$$

Pseudo vector πNN, s-wave πN(Weinberg-Tomozawa interaction)

Weak pion production

Putting parts together, construct Born diagram of vector current



$$< \pi^i(k)N(p')|V^j_\mu(q)|N(p) > \quad = \bar{u}(p')[\Gamma^i_\pi \frac{1}{\not p + \not q - m_N}\Gamma^{\mu,j}_V + \Gamma^{\mu,j}_V \frac{1}{\not p - \not k - m_N} \not k \Gamma^i_\pi$$
$$+\epsilon_{ijk}\tau^k(\frac{g_A}{2f_\pi})(\gamma_\mu\gamma_5 + (2k-q)^\mu \frac{1}{(k-q)^2 - m_\pi^2}(\not k - \not q)\gamma_5)]u(p)$$

$q^\mu = (p' + k - p)^\mu$

- πNN interaction: $\Gamma^i_\pi = i\frac{g_A}{2f_\pi} \not k \gamma_5 \tau_i$
- Vector current of nucleon: $\Gamma^{\mu,j}_V = \gamma_\mu \frac{\tau^j}{2}$
  - anomalous magnetic moment $\quad \gamma_\mu \to \gamma_\mu + i\frac{\mu_p - \mu_n}{2M}\sigma^{\mu\nu}q_\nu$
  - Q^2 dependence $\quad F_V(Q^2) = 1/(1 + Q^2/M_V^2)^2$
- Near threshold 'Kroll-Ruderman' term is dominant contribution for s-wave pion production.

$$\frac{g_A}{2f_\pi}\gamma^\mu\gamma_5 \to \frac{g_A}{2f_\pi}\vec{\sigma}$$

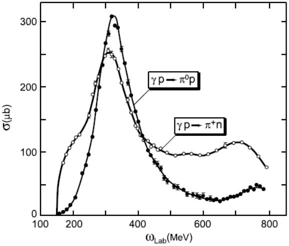
Charged pion photoproduction(electromagnetic vector current)

Main contribution for the threshold s-wave charged pion production(E_{0+}) is given by contact Kroll-Ruderman term.
(Isovector electromagnetic current V_3^μ, $|\pi^0>=|\pi^3> \to [\vec{\tau}\times\vec{\pi}_3]_3 = 0.$)

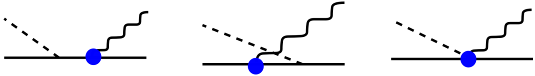
For $\gamma + p \to \pi^+ n$

$$E_{0+} = \frac{eg_A}{4\sqrt{2}f_\pi}(1 - \frac{3}{2}\frac{m_\pi}{m_N}) \sim 26.3 \times 10^{-3}/m_\pi$$

$$E_{0+}^{exp} = (27.9 \pm 0.5) \times 10^{-3}/m_\pi$$

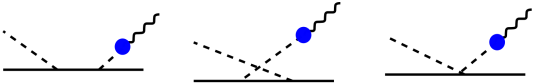


Construct Born diagram of axial vector current

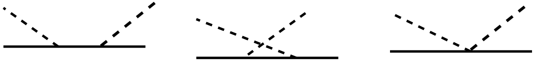


$$< \pi^i N | A^j_{\mu NP} | N > = \bar{u}(p')[\Gamma^i_\pi \frac{1}{\not{p} + \not{q} - m_N} \Gamma^{\mu,j}_A + \Gamma^{\mu,j}_A \frac{1}{\not{p} - \not{k} - m_N} \not{k} \Gamma^i_\pi - (\frac{1}{2f_\pi}) \epsilon_{ijk} \tau^k \gamma_\mu] u(p)$$

- Axial vector current of nucleon: $\Gamma^{\mu,j}_A = g_A \gamma_\mu \gamma_5 \frac{\tau^j}{2}$ and introduce Q^2 dependence $F_A(Q^2) = 1/(1 + Q^2/M_A^2)^2$
- Pion pole terms. $A^j_\mu = A^j_{\mu,NP} - \frac{q^\mu q \cdot A^j_{NP}}{q^2 - m_\pi^2}$



- Axial vector current and pion-nucleon scattering amplitude



$$< \pi^i(k) N | T | \pi^j(q) N > = \frac{i q^\mu}{f_\pi} < \pi^i(k) N | A^j_{\mu NP}(q) | N >$$

Q^2 dependence of E_{0+} and pion electroproduction

$$< \pi^i N|V^j_\mu|N > \sim < N|[Q^i_5, V^j_\mu]|N > = i\epsilon_{ijk} < N|A^k_\mu|N >$$

(Heavy baryon chiral perturbation theory)

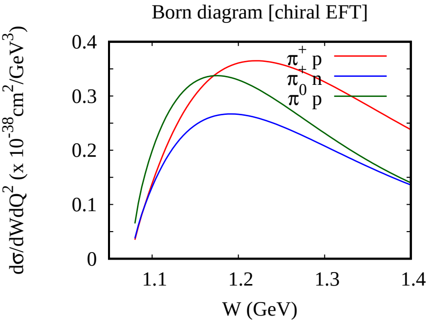
$$E^-_{0+}(m_\pi = 0, q^2) = \frac{eg_A}{8\pi f_\pi}[1 + \frac{q^2}{6} < r^2 >_A + \frac{q^2}{4m_N^2}(\kappa_V + \frac{1}{2}) + \frac{k^2}{128 f_\pi^2}(1 - \frac{12}{\pi^2})]$$

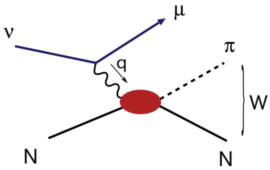
- Axial vector mass M_A can be obtained from pion electroproduction.
- $M_A = (1.026 \pm 0.021) GeV$ from neutrino scattering
  $M_A = (1.069 \pm 0.016) GeV$ from electron scattering
  higher order contibution from chiral perturbation theory gives $\Delta M_A = 0.055 GeV$ brings agreement between M_A extracted from neutrino and electron scattering data.

V. Bernard, L. Elouadrhiri, U. Meissner, J. Phys. G 28 R1 (2002)

Cross sections using lowest order amplitudes of chiral EFT

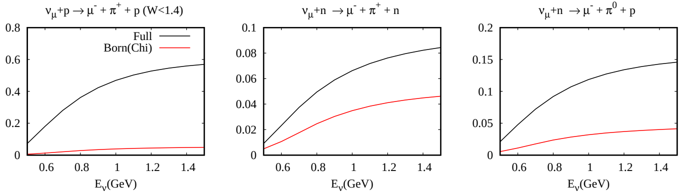
$$\frac{d\sigma}{dW\,dQ^2}\Big|_{E_\nu=2GeV,Q^2=0.1GeV^2}$$





$$\nu_\mu + p \to \mu^- + \pi^+ + p$$
$$\nu_\mu + n \to \mu^- + \pi^+ + n$$
$$\nu_\mu + n \to \mu^- + \pi^0 + p$$

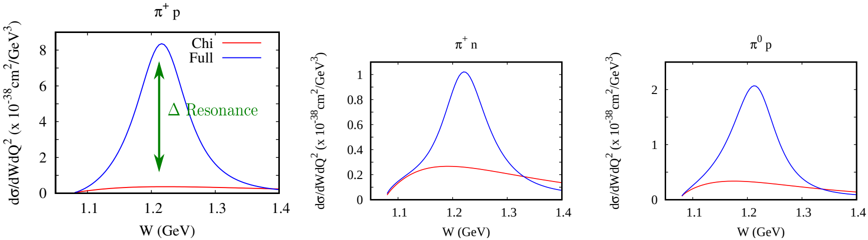
Total cross sections(compare with 'data'(full model))



- contirbution of chiral EFT is small for $\pi^+ p$, about a half of $\pi^+ n$ cross section

$$\frac{d\sigma}{dW dQ^2}$$

$\nu_\mu p \to \mu^- \pi^+ p$ $\nu_\mu n \to \mu^- \pi^+ n$ $\nu_\mu n \to \mu^- \pi^0 p$



$E_\nu = 2GeV, Q^2 = 0.1GeV^2$

- tree diagrams of chiral EFT account near threshold cross section.
- missing strength is due to resonance production.

Summary

- Iso-spin symmetry gives relations among amplitudes of charge states, $\nu/\bar{\nu}$.
- Low energy interactions and currents of pion and nucleon are derived from chiral effective Lagrangian.
- Non-resonant Chiral EFT mechanisms account near threshold pion production and will play important non-resonant mechanism in resonance region for some channels.