Neutrino induced pion production reaction 2

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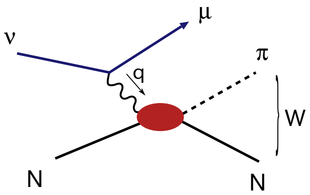
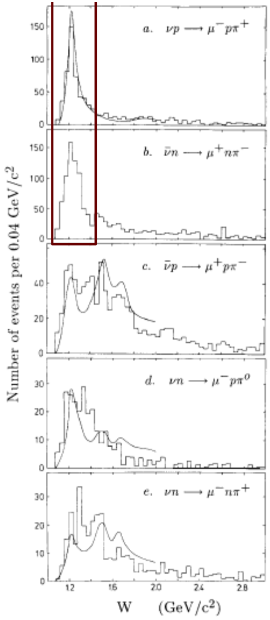
Single pion production(CC)



$$W^2 = (p_N(f) + p_\pi)^2 = (p_\nu + p_N(i) - p_\mu)^2$$

$$Q^2 = -q^2 = -(p_\nu - p_\mu)^2$$



D. Allasia et al. Nucl. Phys. B343(1990)285

plan of lect2

- $\Delta_{33}(1232)$ resonance
- Unitarity and phase of amplitude
- Reaction model of weak pion production

$\Delta_{33}(1232)$ resonance

Total cross seciton of $\pi^- p, \pi^+ p$

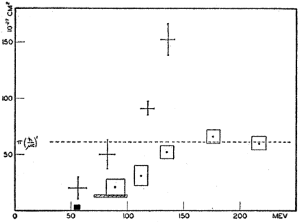
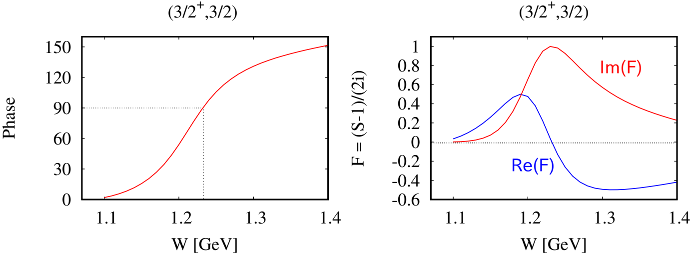


FIG. 1. Total cross sections of negative pions in hydrogen (sides of the rectangle represent the error) and positive pions in hydrogen (arms of the cross represent the error). The cross-hatched rectangle is the Columbia result. The black square is the Brookhaven result and does not include the charge exchange contribution.

H. L. Anderson et al. Phys. Rev. 85 934,936 (1952)

- Birth of N^*, Δ physics
- Spin-Parity and Isospin of Δ: $J^P = 3/2^+$,$I = 3/2$
  p-wave πN resonance

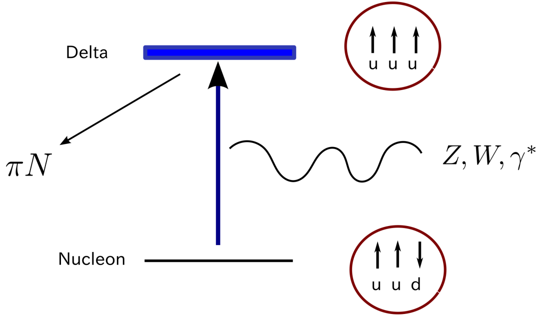
Elastic πN scattering phase shift and amplitude $\mathcal{F}$ of $(3/2^+, 3/2)$



$$\mathcal{F} = \frac{e^{2i\delta} - 1}{2i} = e^{i\delta}\sin\delta = -\frac{\Gamma/2}{W - M + i\Gamma/2} + BG$$

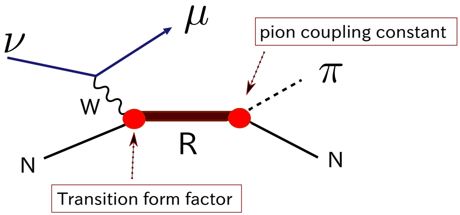
- $\delta = \pi/2$ at $W = 1.232$GeV $\qquad M = 1.21$GeV, $\Gamma = 0.1$GeV

Delta excitation and pion production



- In quark model, Δ is excited state of nucleon. Excitation energy is about 300MeV.
- Δ can be excited by W, Z, γ^*, flipping spin and isospin of quark
- Excited Δ decays almost 100% into πN immediately
- real Δ is unstable state with life time $\sim 1/\Gamma$

Resonance and Breit-Wigner formula



Amplitude of resonance production $(\gamma^* + N \to R \to \pi + N)$ of partial wave $(J^\pi I)$

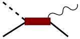
$$\boxed{\frac{g_{\pi NR} \quad g_{JNR}}{W - M_R + i\Gamma_R/2}}$$

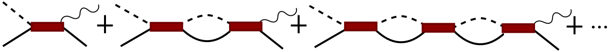
- Breit-Wigner formula
- Resonance is characterized quantum number (J^π, I) and Mass(M_R), Width(Γ_R)
- Coupling constants $g_{\pi NR}, g_{JNR}$ show how strongly the resonance couples with $\pi - N$ and $Z, W, \gamma^* - N$ channels.

explanation of BW formula

Use simplified notation:

$$g_{\pi N\Delta} =< \pi N|V|\Delta >, g^*_{\pi N\Delta} =< \Delta|V|\pi N >, g_{JN\Delta} =< \Delta|J^\mu_\alpha|N >$$

- lowest order  $t_1 = g_{\pi N\Delta}\dfrac{1}{W - M^0} g_{JN\Delta}$
- second order 
  $t_2 = g_{\pi N\Delta}\dfrac{1}{W - M^0}\sum_{\pi N} g^*_{\pi N\Delta}\dfrac{1}{W - E_\pi - E_N + i\epsilon} g_{\pi N\Delta}\dfrac{1}{W - M^0} g_{JN\Delta}$
- add all order terms (using $1 + x + x^2 + .. = 1/(1 - x)$)



$$t = t_1 + t_2 + ... = g_{\pi N\Delta}\frac{1}{W - M^0 - \Sigma(W)} g_{JN\Delta}$$

Here $\Sigma(W) = \sum_{\pi N} g^*_{\pi N\Delta}\dfrac{1}{W - E_\pi - E_N + i\epsilon} g_{\pi N\Delta}$

Focus resonance 'propagator' $1/(W - M_0 - \Sigma(W))$.

- Resonance mass and width

  $M = M_0 + Re(\Sigma(M))$ and $\Gamma/2 = Im(\Sigma(M))$

  (more carefully, we should find pole of amplitude at second Riemann sheet.)

  $$\rightarrow \qquad \frac{g_{\pi N\Delta} \quad g_{JN\Delta}}{W - M + i\Gamma/2}$$

- Γ of Δ propagator is decay probability of Δ into πN given by using Fermi's Golden rule,

  $$\Gamma = 2Im(\Sigma(M)) = 2\pi \sum_{\pi N} \delta(M - E_N - E_\pi)|g_{\pi N\Delta}|^2$$

  $(1/(W - E_N - E_\pi + i\epsilon) = P/(W - E_N - E_\pi) - i\pi\delta(W - E_N - E_\pi))$

  $\pi N\Delta$ coupling constant can be obtained from the width of the Δ.

$N\Delta$ transition form factors

- How many form factors of $< \Delta|V^\mu|N >, < \Delta|A\mu|N >$?
- How we can determine form factors

Since $I_N = 1/2, I_\Delta = 3/2$, only iso-vector current contributes.

$< \Delta|V^\mu|N >$

Parametrization of iso-vector vector current $N\Delta$ transition form factors. (complicated!) Three independent form factors.

$$< \Delta(p_\Delta)|\vec{V}^\mu(q)|N(p) >= \bar{u}^\nu(p_\Delta)\Gamma_{\mu\nu}\vec{T}u(p)$$

$$\begin{aligned}
\Gamma^V_{\mu\nu} &= \frac{m_\Delta + m_N}{2m_N}\frac{1}{(m_\Delta + m_N)^2 - q^2} \\
&\times [(G_M - G_E)3\epsilon_{\mu\nu\alpha\beta}P^\alpha q^\beta + G_E i\gamma_5 \frac{12}{(m_\Delta - m_N)^2 - q^2}\epsilon_{\mu\lambda\alpha\beta}P^\alpha q^\beta \epsilon^\lambda{}_{\nu\alpha\delta}p^\gamma_\Delta q^\delta \\
&+ G_C i\gamma_5 \frac{6}{(m_\Delta - m_N)^2 - q^2} q_\mu(q^2 P_\nu - q\cdot P q_\nu)]
\end{aligned}$$

There are another expression using C_1, C_2, C_3.

$< \Delta|A^\mu|N >$

Parametrization of Axial vector current (Charged current), four independent form factors.

$$
\begin{aligned}
< \Delta^+(p_\Delta)|A_1^\mu + iA_2^\mu|n(p) > &= \bar{u}(p_\Delta)_\nu[\frac{C_3^A}{m_N}(g^{\mu\nu} \not{q} - q^\nu\gamma^\mu) \\
&+ \frac{C_4^A}{m_N^2}(g^{\mu\nu}(q\cdot p_\Delta) - q^\nu {p_\Delta}^\mu) + C_5^A g^{\nu\mu} + \frac{C_6^A}{m_N^2} q^\mu q^\nu]u(p)
\end{aligned}
$$

or

$$
\begin{aligned}
< \Delta(p_\Delta)|A_i^\mu|N(p) > &= \bar{u}(p_\Delta)_\nu[d_1 g^{\mu\nu} + \frac{d_2}{m_N^2}(p_\delta + p)_\alpha(q^\alpha g^{\mu\nu} - q^\nu g^{\alpha\mu}) \\
&- \frac{d_3}{m_N^2} p^\nu q^\mu + i\frac{d_4}{m_N^2}\epsilon^{\mu\nu\alpha\beta}(p_\Delta + p)_\alpha q_\beta \gamma_5]T_i u(p)
\end{aligned}
$$

- vector current

		Quark model
G_M	Magnetic Dipole(M1)	$\sqrt{\frac{72}{25}}\mu_N^{IV}$
G_E	Electric Quadrupole(E2)	0
G_C	Coulomb Quadrupole(C2)	0

In naive quark model, $G_E = G_C = 0$(No deformation), since both N, Δ are spherical wave function.

- Axial vector current

		Quark model
$G_A \sim d_1$	'Gamow Teller'	$\frac{1}{2}\sqrt{\frac{72}{25}}g_A$
d_3	Pion pole	
d_4, d_2	Quadrupole	0

- G_M, G_E, G_C can be determined from the analysis of $(e, e'\pi)$ reaction.(we will discuss later)

- There are few experimental information to constrain $G_A(Q^2)$. We sometimes rely on PCAC or prediction from the effective model of QCD.

Unitality and phase of amplitude

We have $t_{non-res}$(Chiral model) and t_{res}(Breit Wigner form). Proceed to analyze $(e, e'\pi)$ and $(\nu, l^-\pi)$ reactions.

$$T = t_{non-res} + t_{res}e^{i\chi} \quad (??)$$

The above naive procedure does not satisfy unitarity (conservation of probability).

Watson Theorem

$J^\mu + N \to \pi N$ amplitudes for $\alpha = (J^\pi I)$

$$\boxed{\mathcal{F}^\alpha_{J\pi} = e^{i\delta^\alpha_{\pi N}} |\mathcal{F}^\alpha_{J\pi}|}$$

- $S^\dagger S = 1$, First order in electromagnetic interaction.
  Valid below two-pion production threshold(two-channel problem). Only useful for us up to Δ energy.
- DWIA $\quad t =< \pi N^{(-)}|J^\mu|N >\propto e^{i\delta_{\pi N}}$
- Recipe to implement Watson theorem: Tune phase χ so that whole amplitue T has phase $\delta_{\pi N}$.

$$T = t_{non-res} + t_{res}e^{i\chi}$$

Isobar Model

Breit-Wigner formula for partial wave ($\alpha = (J^\pi, I)$)

$$t_\alpha = \frac{g_{\pi NR} \;\; g_{JNR}}{W - M_R + i\Gamma_R/2}$$

- Mass(M_R), Width(Γ_R) of resonance R from PDG.
- Coupling constants $g_{\pi NR}, g_{VNR}$ can be estimated from branching ratio(B_α), g_{ANR}: use quark model estimation or use $g_{\pi NR}$ assuming PCAC.

  $$g_{\pi NR} = \sqrt{\frac{\Gamma}{2} B_\pi}, \quad g_{VNR} = \sqrt{\frac{\Gamma}{2} B_\gamma}$$
- No control of relative phases between non-res $\leftrightarrow$ res, (J^π, I) channels.

  $$\sigma \sim \sum_\alpha c_\alpha |t_\alpha|^2, \quad d\sigma \sim \sum_{\alpha,\beta} c_{\alpha,\beta} t_\alpha t^*_\beta$$
- example model of Rein and Sehgal, implemented in neutrino generators.

Reaction model of electroweak meson production

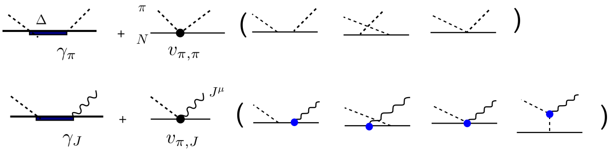
Meson exchange model of electroweak pion production reaction

- Below $W < 2GeV$, description of hadron dynamics in terms of meson exchange picture would be most economical and useful.
- Model of meson production is developed to investigate nucleon excited states.
- Since mechanism of strong interaction and electromagnetic current is well tested, the approach has a capability to predict for weak pion production.
- Amplitudes of the model satisfies unitarity. (extended model goes beyond single pion production)

Reaction model in Δ region

Hamiltonian of π, N, Δ system and current $J^\mu = J^\mu_{em}, V^\mu, A^\mu$

$H = H_0 + V$



Solve Lippman-Schwinger equation : Fock space $|\pi N>, |\gamma N>$

$$T(W) = V + V\frac{1}{W - H_0 + i\epsilon}T(W)$$

scattering wave function of quantum mechanics $\psi^+ \to e^{ikz} + \frac{f}{r}e^{ikr}$

$$(H_0 + V)\psi^+ = E\psi^+ \to \psi^+ = \phi_0 + \frac{1}{E - H_0 + i\epsilon}V\psi^+$$

multiply V from left and $V\psi^+ = T\phi_0$ gives LS equation.

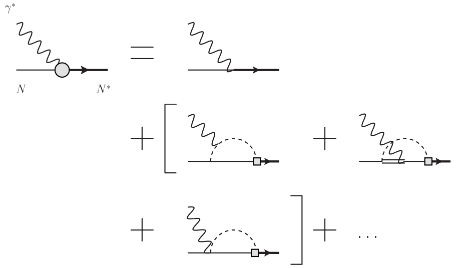
Electroweak pion production amplitude (first order in em/weak interaction J^μ)

Full amplitude can be written as sum of 'non-resonant' amplitude and 'resonant' amplitude.

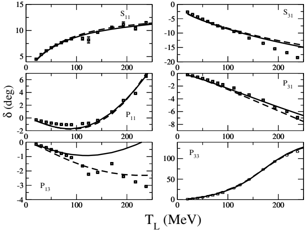
$$T_{\pi,J}(W) = t_{\pi,J}(W) + \frac{\tilde{\gamma}_\pi(W)\tilde{\gamma}_J(W)}{W - m_\Delta^0 - \Sigma(W)}$$

- Both non-resonant and resonant amplitudes have phases. The total amplitude satisfies Watson theorem.
- non-resonant t-matrix modifies resonance couplings and self-energy.

$$\text{dressed } \Delta N J \text{ vertex } \tilde{\gamma}_J = \gamma_J + t_{\pi,\pi}(W)\frac{1}{W - H_0 + i\epsilon}v_{\pi,J}$$



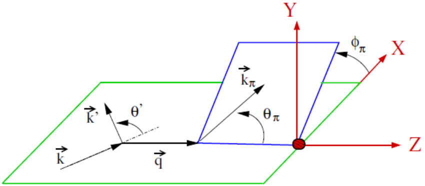
Step 1: πN phase shift



Step 2: Pion photoproduction and electroproduction

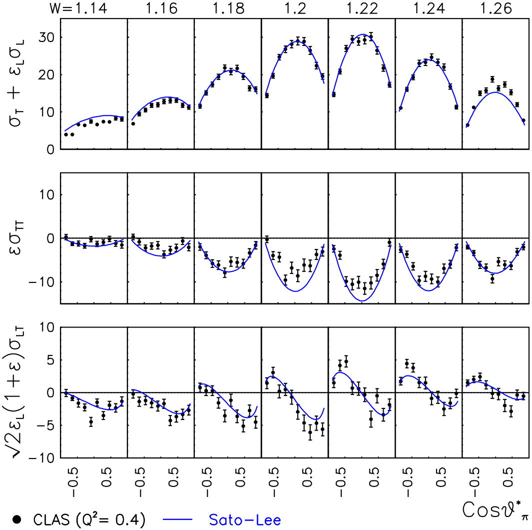
$$\frac{d\sigma}{dE_e d\Omega_e d\Omega_\pi} = \Gamma \frac{d\sigma}{d\Omega_\pi}$$

$$\frac{d\sigma}{d\Omega_\pi} = \frac{d\sigma_T}{d\Omega_\pi} + \epsilon \frac{d\sigma_L}{d\Omega_\pi} + \cos\phi_\pi \sqrt{2\epsilon(1+\epsilon)} \frac{d\sigma_{LT}}{d\Omega_\pi} + \cos 2\phi_\pi \epsilon \frac{d\sigma_{TT}}{d\Omega_\pi}$$

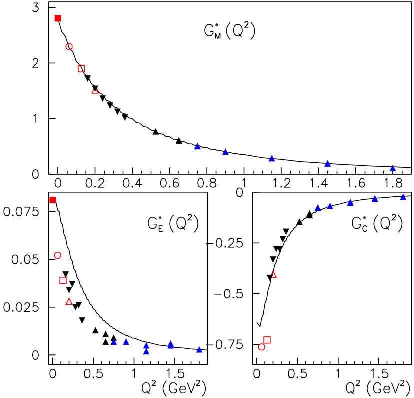


E. Hernandez, J.Nieves, M. Valverde, PRD76, 033005 (2007)

Pion electroproduction

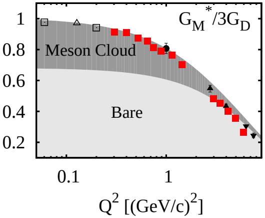


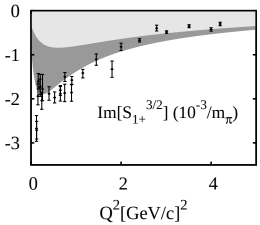
Extracted transition form factors(including vertex correction)

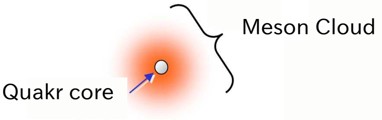


Sato,Lee PRC 63 (2001) 055201
B. Julia-diaz, T.-S. H. Lee, C. Smith, T. Sato PRC75(2007)015205
Extensive data from JLab, Mainz, Graal, MIT-Bates, LEGS of pion photo and electroproduction.

Interpretation of $N\Delta$ transition form factors







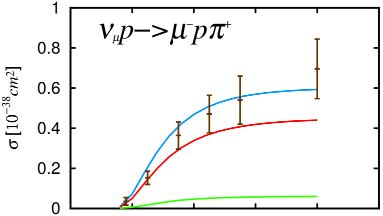
$$|N^*\rangle = |qqq\rangle + |\mathrm{m.c.}\rangle$$

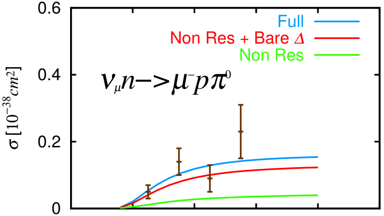
Compilation of Low energy ($E < 30GeV$) Neutrino Cross Section
(http://hepdata.cedar.ac.uk/review/neutrino/)

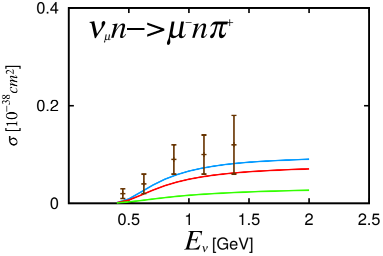
				1π	2π
GGM	Lerche 1978	ν	Propane	1-10	
	Bolognese 1979	$\bar{\nu}$	Propane-Freon	1-7.5	
BEBC	Allen 1986	ν	p	10-80	
	Allasia 1990	$\nu, \bar{\nu}$	d	5-150	
BNL	Kitagaki 1986	ν	d	0.5 - 3	0.5 - 3
ANL	Barish 1979	ν	p,d	0.4 - 6	
	Radecky 1982	ν	d	0.5 - 1.5	
	Day 1983	ν	d		0.75-5.55
FNAL	Bell 1978	ν	p	15-40	
SKAT	Ammosov 1988	ν	Freon(CF_3Br)	4-18	
	Grabosch 1989	$\nu, \bar{\nu}$	Freon(CF_3Br)	3.5-6	

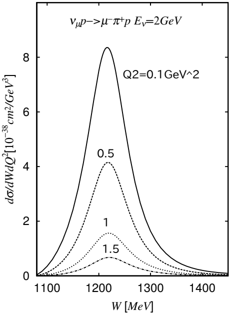
- Reanalysis of ANL/BNL data
  (C. Wilkinson et al. PRD90 (2014), Rodrigues et al. arXiv:1601.01888)
- Final state interaction in neutrino-deuteron reaction(Jia-jun Wu et al. PRC91 (2015))

Comparison with ANL data of single pion production



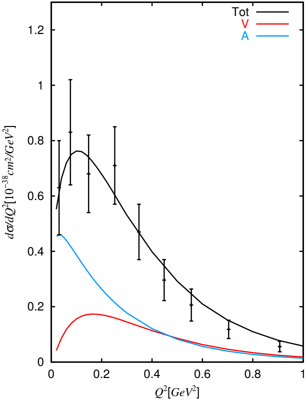


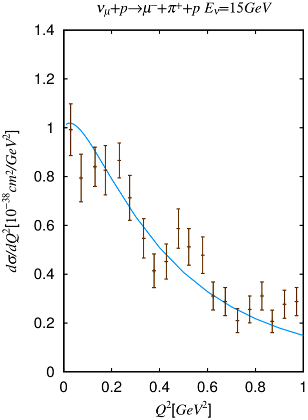




- quark model for G_A , dipole form factor

Q^2 dependence





Flux averaged $(0.5 < E_\nu < 6\text{GeV})$(Data ANL)

Summary of pion production at Delta resonance region

- $N\Delta$ transtion form factors are determined well for electromagnetic(Iso-vector Vector) current from pion photo and electroproduction. G_M is dominant term and G_E, G_C are small $< 10\%$.
- Axial $N\Delta$ form factor is not well determined as nucleon form factors from reaction model. (looking forward to see new experiment of higher precision / Lattice QCD)
- Model of electroweak pion production reaction is then used as a building block of the microscopic theoretical studies of neutrino-nucleus reaction.

Appendix A

Unitarity and Fermi-Watson theorem

From unitarity, phase of pion photoproduction amplitude is given by the phase shift of pion-nucleon scattering

S-matrix of $\gamma N, \pi N, \pi\pi N, ...$ reactions. (for given channel $j^\pi i$)

$$S = \begin{pmatrix} S_{\gamma,\gamma} & S_{\gamma,\pi} & S_{\gamma,2\pi} & ... \\ S_{\pi,\gamma} & S_{\pi,\pi} & S_{\pi,2\pi} & ... \\ S_{2\pi,\gamma} & S_{2\pi,\pi} & S_{2\pi,2\pi} & ... \\ ... & ... & ... & ... \end{pmatrix}$$

Unitarity $S^\dagger S = 1$ gives relation among matrix elements.

E. Fermi, Suppl. Nuovo Cimento 2 (1955), 17.
K. M. Watson, Phys. Rev. 95 (1954), 228.

Energy below $\pi\pi N$ threshold:

$$S = \begin{pmatrix} S_{\gamma,\gamma} & S_{\gamma,\pi} \\ S_{\pi,\gamma} & S_{\pi,\pi} \end{pmatrix}$$

The unitarity relation is given as

$$\begin{pmatrix} S^*_{\gamma,\gamma} & S^*_{\pi,\gamma} \\ S^*_{\gamma,\pi} & S^*_{\pi,\pi} \end{pmatrix} \times \begin{pmatrix} S_{\gamma,\gamma} & S_{\gamma,\pi} \\ S_{\pi,\gamma} & S_{\pi,\pi} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

we obtain

$$S^*_{\gamma,\gamma} S_{\gamma,\pi} + S^*_{\pi,\gamma} S_{\pi,\pi} = 0$$

- First order of e: $S_{\gamma,\gamma} \sim 1$
- T-invariance: $S_{\gamma,\pi} = S_{\pi,\gamma}$

$$S^*_{\gamma,\gamma} S_{\gamma,\pi} + S^*_{\pi,\gamma} S_{\pi,\pi} \sim S_{\pi,\gamma} + S^*_{\pi,\gamma} S_{\pi,\pi} = 0$$

- Below $\pi\pi N$ threshold: $S_{\pi,\pi} = e^{2i\delta_{\pi N}}$
- Pion photoproduction amplitude $t_{\pi,\gamma}$: $S_{\pi,\gamma} = 0 - it_{\pi,\gamma}$

$$\rightarrow \quad \frac{t_{\pi,\gamma}}{t^*_{\pi,\gamma}} = e^{2i\delta_{\pi N}}$$

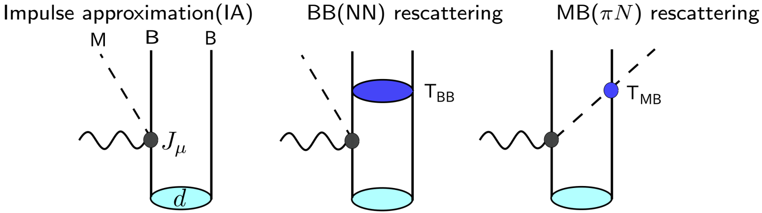
Fermi-Watson theorem:Phase of the pion photoproduction amplitude is given by the phase shift of pion-nucleon elastic scattering.

$$t_{\pi,\gamma} = e^{i\delta_{\pi N}} |t_{\pi,\gamma}|$$

Appendix B

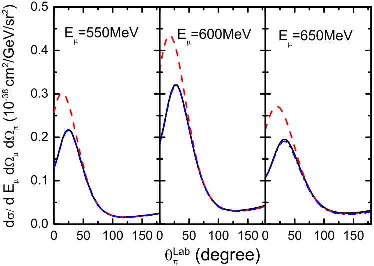
Extracting $\nu - N$ from $\nu - d$

- All theoretical models of neutrino induced pion production(CC) on nucleon are tuned to describe the ANL and BNL deuterium bubble chamber data.
- Cross sections are extracted from data by selecting QE(IA) events. No final state interactions are taken into account.(as far as we can read from the paper.)
- IF large FSI $\rightarrow \sigma(\nu N) \rightarrow \sigma(\nu A) \rightarrow$ neutrino flux at far detector.



FSI can be indeed large

- Jia-Jun Wu, TS, T.-S.H. Lee(PRC91 2015) have shown large effects of NN FSI. (red-dash IA, blue-solid with FSI)



$E_\nu = 1$GeV, $\theta_\mu = 25$deg. ('QE' delta production)

- Important to extend this analysis for full phase space!

Neutrino induced pion production on deuteron:Results

How can we extract $\sigma(\nu p)$ and $\sigma(\nu n)$ from $\sigma(\nu d)$ by taking into account FSI.

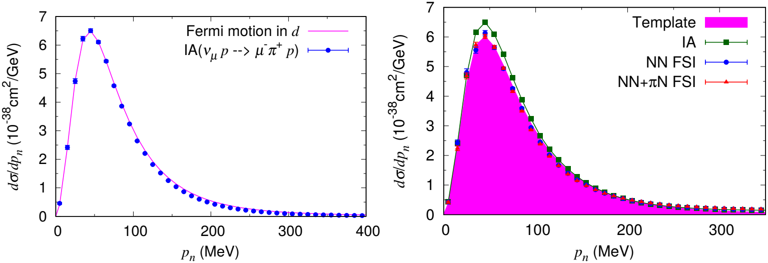
- With ANL, BNL exp. condition, re-analyze data with FSI.
  $\rightarrow$ not possible (information is lost). not practical(heavy numerical task)
- Parametrize FSI effect.
  Relation assuming IA would be modified by FSI($N \neq 1$).

$$\frac{d\sigma(\nu d)}{dp_n} = N|\psi_d(p_n)|^2\sigma(\nu p)$$

- $\nu_\mu + d \to \mu^- + p + n$ reaction
  π^+ is produced from p and n
  $\sigma(\nu n) \sim \sigma(\nu p)/9$
  Initial d(pn bound state),Final pn scattering state

$\sigma(\nu p \to \mu^- \pi^+ p)$ from $\sigma(\nu d \to \mu^- \pi^+ pn)$

Spectator (neutron) momentum distribution in IA(Left) and with FSI(right) ($E_\nu = 1GeV$)

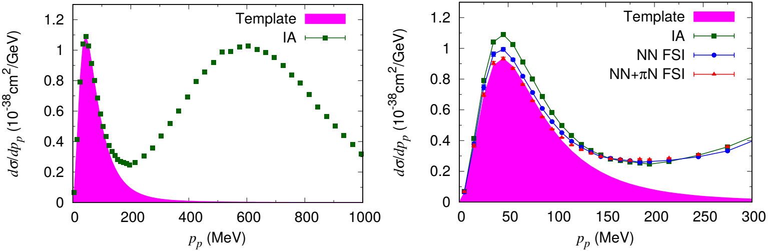


$$\boxed{\frac{d\sigma(\nu d)}{dp_n} = N|\psi_d(p_n)|^2 \sigma(\nu p)}$$

- For IA, $N(E_\nu = 1GeV, p_n) \sim 1$
- FSI cannot be neglected for this leading channel.
- Fit of shape or integrate $d\sigma/dp_s$ in $0 < p_{spectator} < 50MeV$ gives $N = 0.9 \sim 0.95$ for $E_\nu = 0.5 \sim$ 2GeV.

$\sigma(\nu n \to \mu^- \pi^+ n)$ from $\sigma(\nu d \to \mu^- \pi^+ pn)$

Spectator (proton) momentum distribution in IA(Left) and with FSI(right)
[weaker channel ($n\pi^+$)]



- For IA, $N(E_\nu = 1GeV, p_p) \sim 1$
- $N = 0.8 \sim 0.9$ for $E_\nu = 0.5 \sim 2GeV$.
- We can provide useful tables of N, however choice of $p_{spectator}$ and possible cuts depends on condition of experiments.