



The Practical Beauty of Neutrino-Nucleus Interactions

Gabriel N. Perdue

NuSTEC School

2017 / November / 7

Why do we climb mountains?



Why do we climb mountains?



Because they're *there*.

Why do we study neutrino scattering?



To support the oscillations program, yes, but also as *probes of fundamental physics*.

Goals

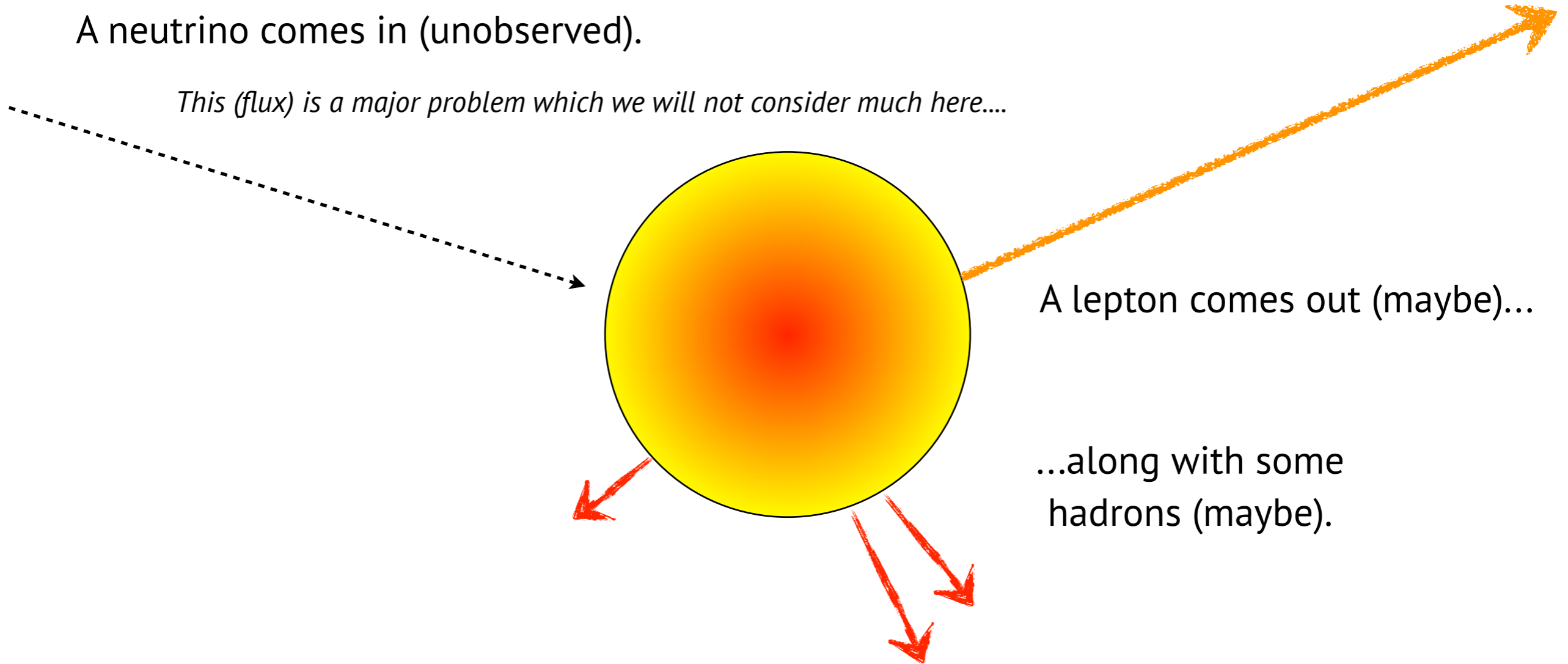
- Sketch the challenges imposed by our oscillation physics program, and highlight the need for high quality cross section measurements and calculations.
- Discuss the basic formalism and jargon.
- Highlight a handful of active neutrino cross section experimental programs (at a very superficial level).
- Explain the role of event generators in accelerator-based neutrino physics, and mention a few of the more popular codes.

Disclaimer: I work in experiment (MINERvA), on generators, and in computing. These facts shape my perspective... I tried to make connections to many of the lectures coming later in this school, but some of those were easier for me than others...

The Basic Problem

A neutrino comes in (unobserved).

This (flux) is a major problem which we will not consider much here....



A lepton comes out (maybe)...

...along with some
hadrons (maybe).

What was the neutrino's energy?

We really want flavor too...

Why do we need the energy?

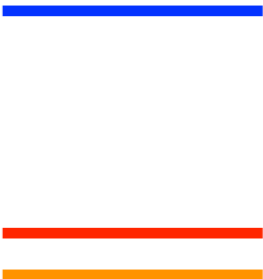
$$\begin{matrix} \nu_\alpha = \text{Flavor} \\ \text{Eigenstates} \end{matrix} \begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix} \begin{matrix} \nu_i = \text{Mass} \\ \text{Eigenstates} \end{matrix}$$

PMNS matrix...

- 3 x 3 Unitary Matrix
 - 3 “Euler Angles”, 1 Complex Phase*

$\theta_{12}, \theta_{23}, \theta_{13}, \delta_{CP}$

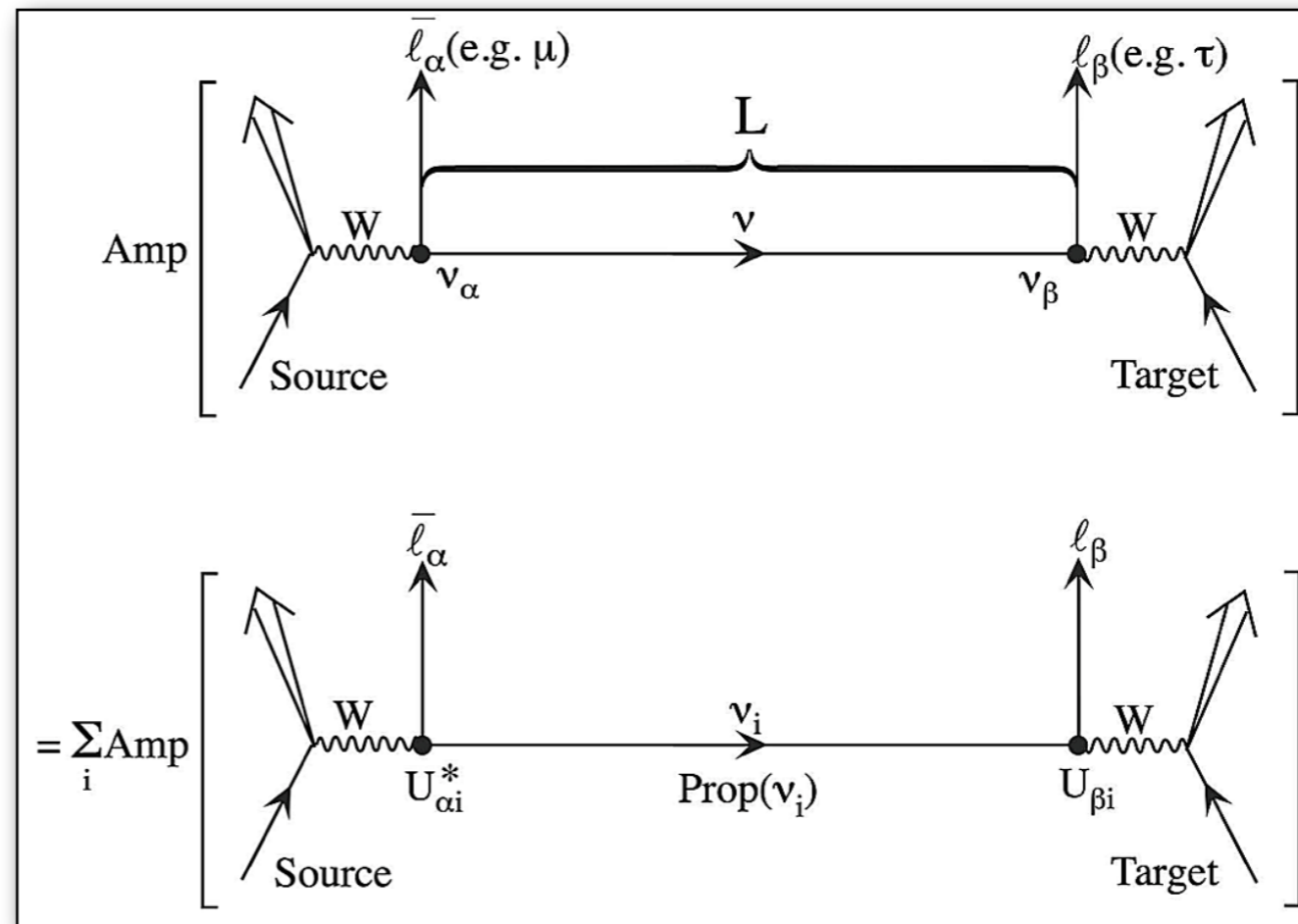
- 3 Masses
 - 2 Independent Splittings

 m_c
 m_b
 m_a

*Plus two Majorana phases - Insanely important!

$$\text{Prop}(\nu_j) \sim e^{(-im_j \tau_j)}$$

$$m_1 \neq m_2 \neq m_3$$



B. Kayser, arXiv
0804.1121

- Flavor eigenstates interact. Flavor states are superpositions of mass states.
 - Different masses \Rightarrow Different propagators.

$$P(\nu_\alpha \rightarrow \nu_\beta) = |\langle \nu_\beta | \nu(L) \rangle|^2 = \left| \sum_j U_{\alpha j}^* e^{-im_j^2 \frac{L}{2E}} U_{\beta j} \right|^2$$

- \Rightarrow Flavor composition evolves with time.

How do we measure PMNS?

$$\begin{aligned} P(\nu_\mu \rightarrow \nu_e) &= \left| U_{\mu 1}^* e^{-im_1^2 L/2E} U_{e1} + U_{\mu 2}^* e^{-im_2^2 L/2E} U_{e2} + U_{\mu 3}^* e^{-im_3^2 L/2E} U_{e3} \right|^2 \\ &= \left| 2U_{\mu 3}^* U_{e3} \sin \Delta_{31} e^{-i\Delta_{32}} + 2U_{\mu 2}^* U_{e2} \sin \Delta_{21} \right|^2 \\ &\simeq \left| \sqrt{P_{atm}} e^{-i(\Delta_{32} + \delta)} + \sqrt{P_{sol}} \right|^2 \end{aligned}$$

- We beat these probabilities against each other.
- $\delta \rightarrow -\delta$ for antineutrinos.
- Compare neutrinos to antineutrinos to measure CP violation and the mass hierarchy.

$$\Delta_{ij} = 1.27 \Delta m_{ij}^2 L / E$$

Probabilities

In MATTER:

$$P_{atm} \sim \sin^2 \theta_{23} \sin^2 2\theta_{13} \sin^2 (\Delta_{31} - aL) \left(\frac{\Delta_{31}}{\Delta_{31} - aL} \right)^2$$

$$P_{sol} \sim \cos^2 \theta_{23} \sin^2 2\theta_{12} \sin^2 (aL) \left(\frac{\Delta_{21}}{aL} \right)$$

$$a = \pm G_F N_e / \sqrt{2} \sim (4000 \text{ km})^{-1}$$

- The probabilities are a function of the matrix parameters, the mass splittings, and the *neutrino energy*!

$$\Delta_{ij} = 1.27 \Delta m_{ij}^2 L / E$$

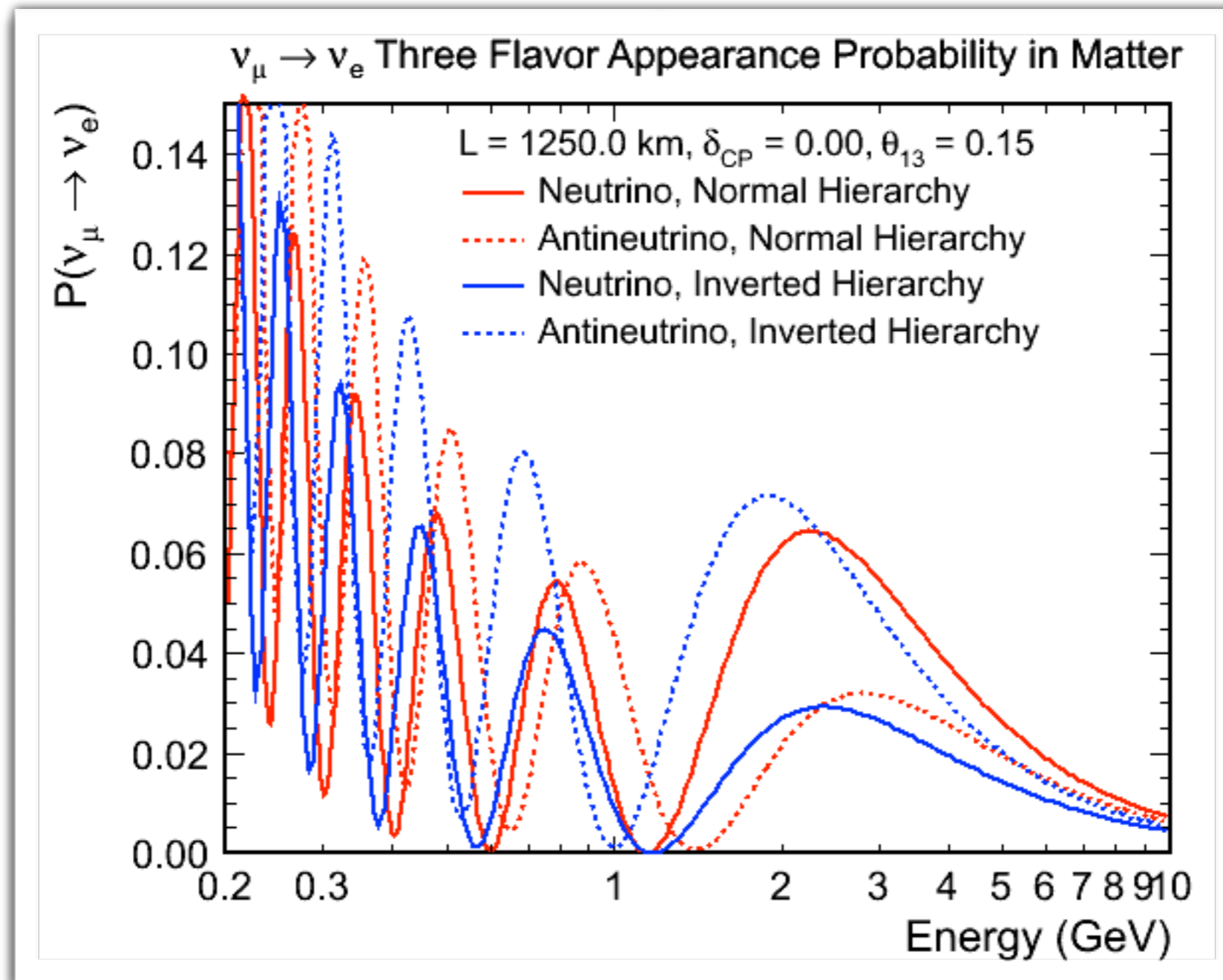
$$P \sim 2\sqrt{P'_{atm}}\sqrt{P'_{sol}} \cos \Delta_{32} \cos \delta_{CP} \mp 2\sqrt{P'_{atm}}\sqrt{P'_{sol}} \sin \Delta_{32} \sin \delta_{CP}$$

$$\delta_{CP} : 0 \rightarrow 2\pi$$

m_2
 m_1
 m_3

Inverted

Antineutrino: $-\delta$



m_3
 m_2
 m_1

Normal

Neutrino: δ

How do we measure these probabilities?

Experiments

We do not measure the probability directly - we measure a *rate*:

$$R(E_{\text{vis}}) = N \int dE \Phi_{\alpha}(E) \sigma_{\beta}(E, E_{\text{vis}}) \epsilon_{\beta}(E) P(\nu_{\alpha} \rightarrow \nu_{\beta}, E)$$

- N = overall normalization (e.g., mass)
- Φ_{α} = flux of ν_{α}
- σ_{β} = cross section for ν_{β}
- ϵ_{β} = detection efficiency for ν_{β}
- NOTE: $\sigma_{\beta}\epsilon_{\beta}$ always appear together. Define:

$$\tilde{\sigma}_{\beta} = \sigma_{\beta} \epsilon_{\beta}$$

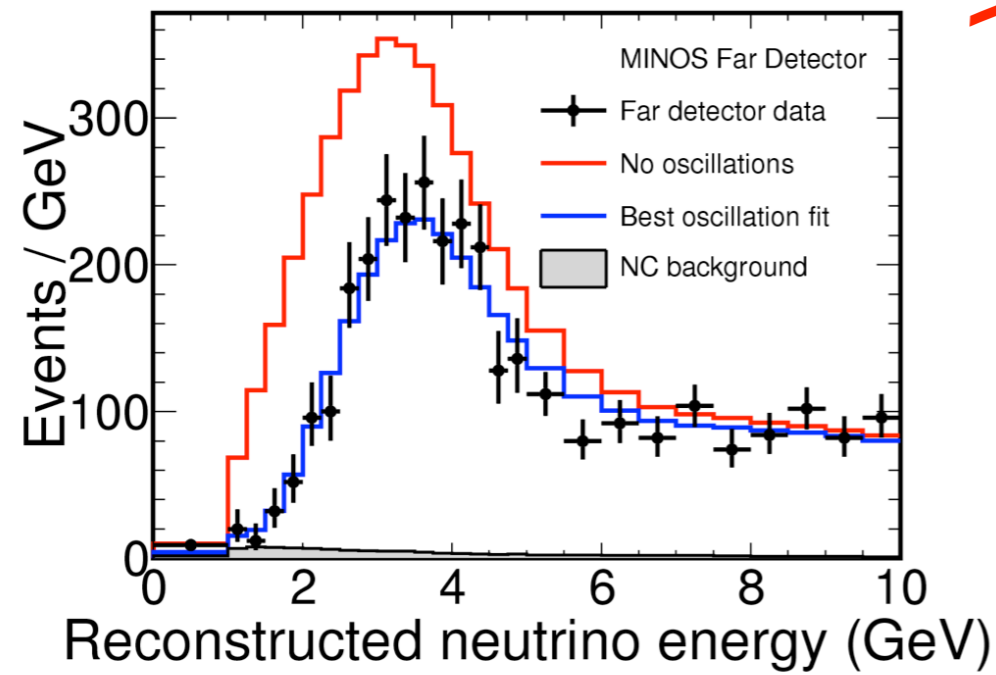
Problem & Solution

- How do we know the components of the integral?
- How can we even determine the ratios for flavor and helicity combinations?
- Even if we know the cross sections from theory, we don't know efficiency ratios.
- But, we can measure ratios if we use two detectors!

$$\frac{R_{\alpha \rightarrow \alpha}(\text{Far}) L^2}{R_{\alpha \rightarrow \alpha}(\text{Near})} = \frac{N_{\text{far}} \Phi_{\alpha} \tilde{\sigma}_{\alpha} P(\nu_{\alpha} \rightarrow \nu_{\alpha})}{N_{\text{near}} \Phi_{\alpha} \tilde{\sigma}_{\alpha}}$$
$$\frac{R_{\alpha \rightarrow \alpha}(\text{Far}) L^2}{R_{\alpha \rightarrow \alpha}(\text{Near})} = \frac{N_{\text{far}}}{N_{\text{near}}} P(\nu_{\alpha} \rightarrow \nu_{\alpha})$$

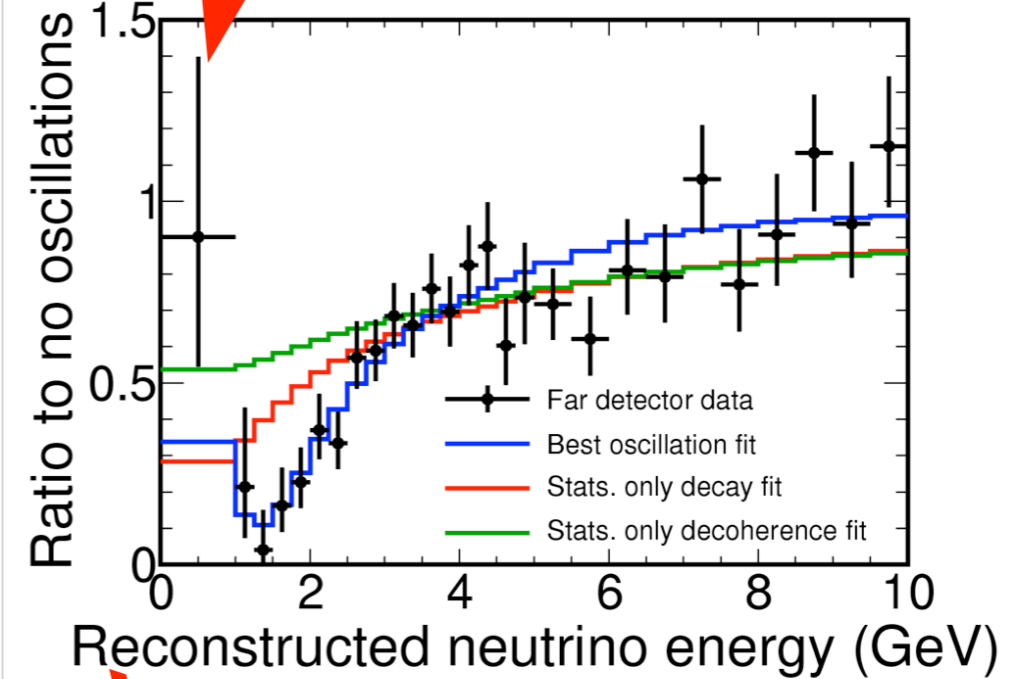
$$P(\nu_\mu \rightarrow \nu_\mu) = 1 - \sin^2 2\theta_{32} \sin^2 [1.27 \Delta m_{32}^2 (L/E)]$$

Measure "Near"/Far

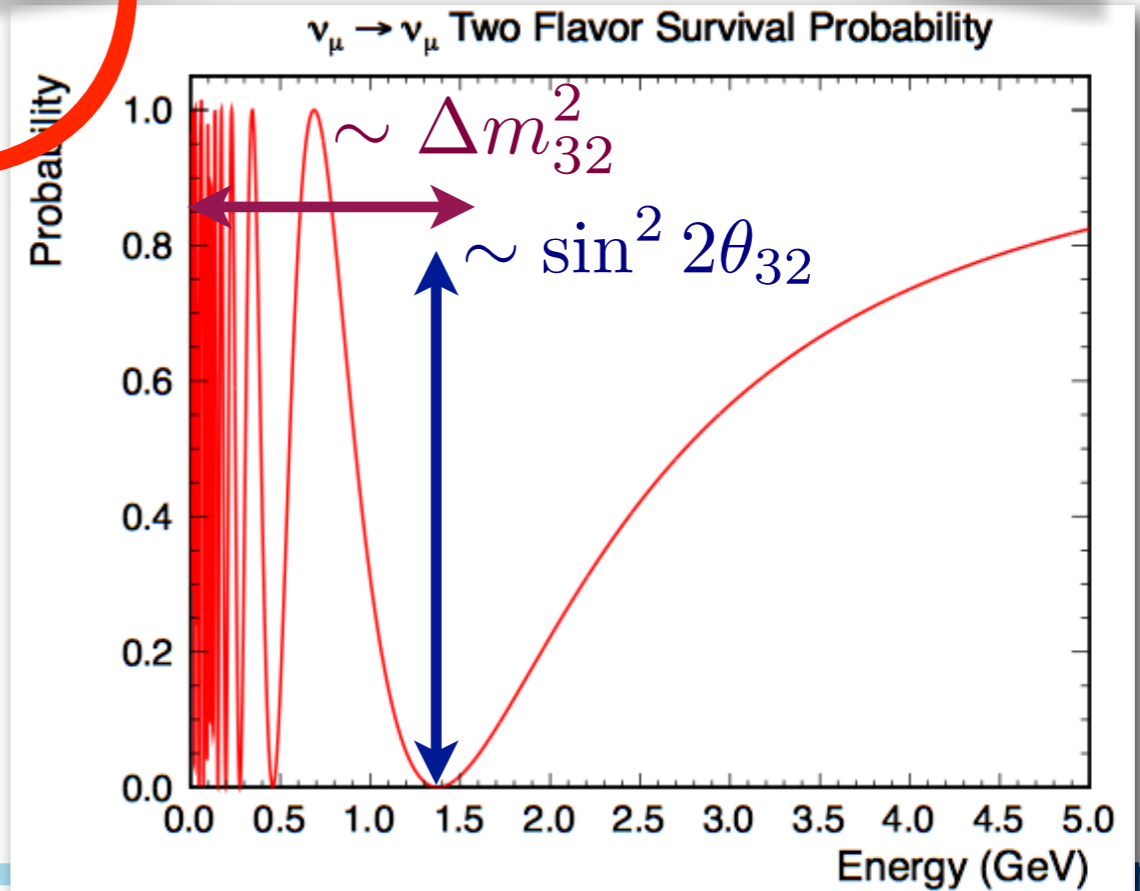
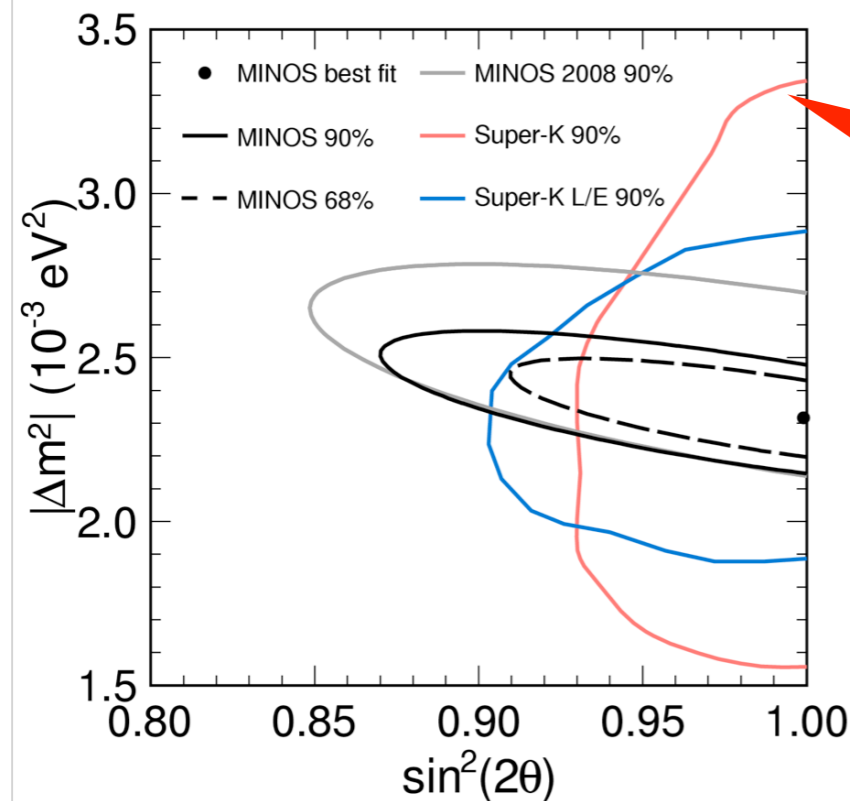


<http://www-nuui.fnl.gov/PublicInfo/forscientists.html>

Fit Ratio



Extract Physics!



But, another (harder) problem!

- The above equations were for a disappearance measurement. For appearance measurements:

$$\frac{R_{\alpha \rightarrow \beta}(\text{Far}) L^2}{R_{\alpha \rightarrow \beta}(\text{Near})} = \frac{N_{\text{far}} \Phi_{\alpha} \tilde{\sigma}_{\beta} P(\nu_{\alpha} \rightarrow \nu_{\beta})}{N_{\text{near}} \Phi_{\alpha} \tilde{\sigma}_{\alpha}}$$
$$\frac{R_{\alpha \rightarrow \beta}(\text{Far}) L^2}{R_{\alpha \rightarrow \beta}(\text{Near})} = \frac{N_{\text{far}}}{N_{\text{near}}} \frac{\tilde{\sigma}_{\beta}}{\tilde{\sigma}_{\alpha}} P(\nu_{\alpha} \rightarrow \nu_{\beta})$$

- It is even worse than that, because the efficiencies and the cross sections will both change for antineutrinos.

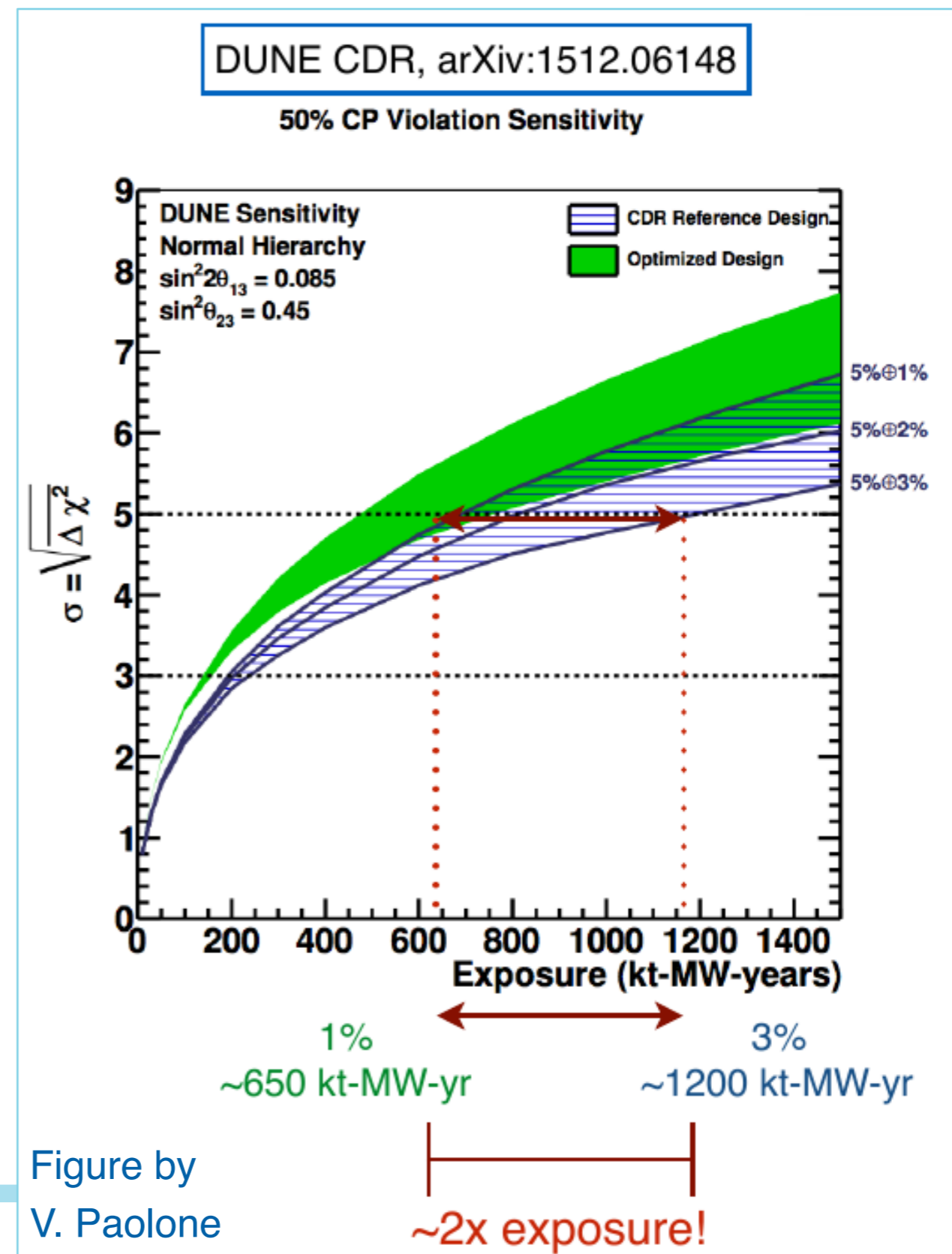
But wait - because our beams aren't pure, we're saved, right?

- The oscillated flux is different for a given species, so energy-dependent effects never really cancel.
- You need independent, external measurements to constrain your cross section model. Using internal measurements only introduces a degree of circularity.
- If you are truly able to build identical near and far detectors, projecting your near detector measurement to the far detector to compute an expected spectrum is a powerful technique. But...
 - It is hard to build identical near and far detectors!
 - If nothing else, you typically want your far detector to be much, much larger than your near detector. Even assuming perfect calibration this has important consequences for the **acceptance** in both detectors.
 - Scaling the same technology in the same way is also challenging. Typically technologies that scale well for both small and large detectors involve making granularity sacrifices in the near detector.

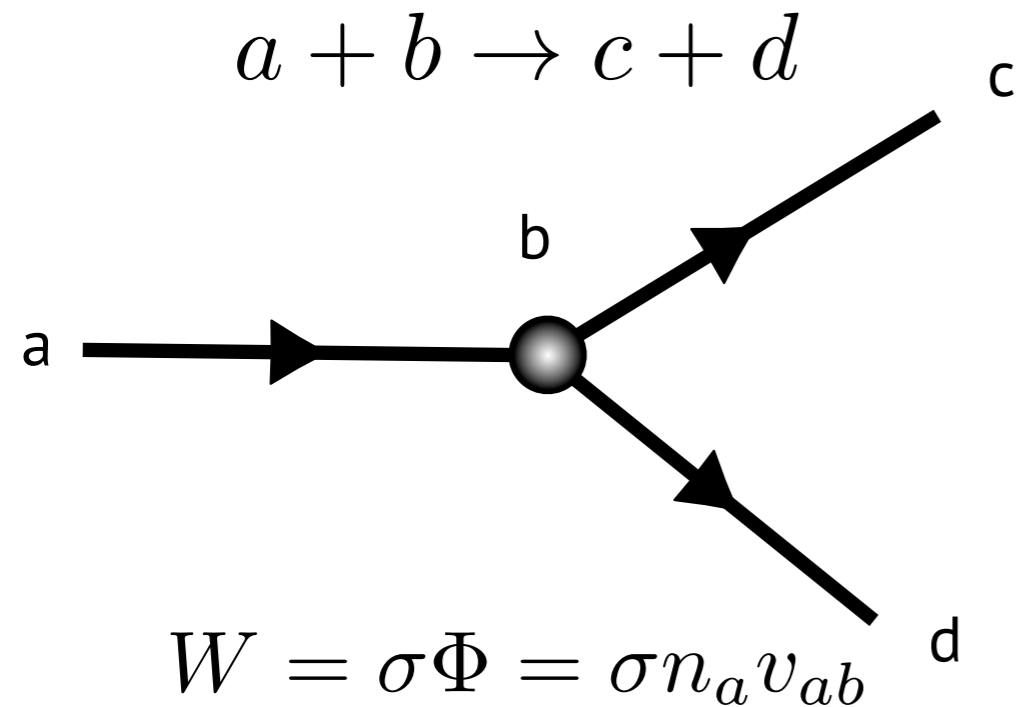
The challenge and the reward

- Current and future neutrino oscillation experiments have a very ambitious program.
- U. Mosel, NuInt 2017: **DUNE is “an impossible” experiment:**
 - Flux not fully specified,
 - Beamline is over 1,000 km, diameter is over 1 km at Far Detector,
 - Cross sections are tiny (10^{-11} mb) and plagued by numerous theory and experimental uncertainties,
 - Somehow we need to extract evidence of physics beyond the Standard Model!
- **Control of cross section systematics is a critical piece** - requires a multi-pronged effort involving theorists, experimenters, and and Monté Carlo authors all **working together**.
 - No single measurement or calculation will solve it all!

See more from S. Bolognesi in this school!



Basic Formalism



Fermi's Second Golden Rule

$$W = \frac{1}{h} |M_{if}|^2 \rho_f$$

M is the "Matrix Element"

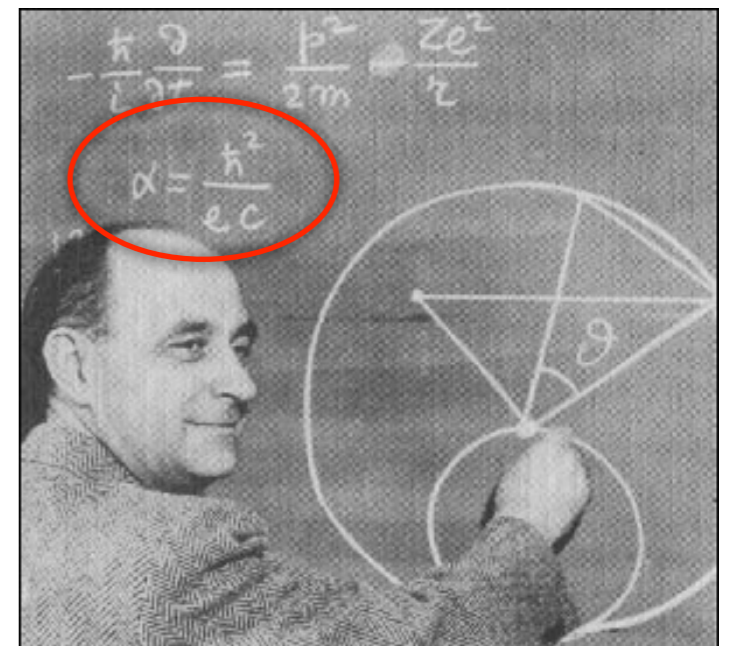
Perturbation Theory: $M_{if} = \int \psi_f^* \mathcal{H} \psi_i d\tau$

ρ_f is the density of states (phase space factor).

$$\sigma(a + b \rightarrow c + d) \propto |M_{if}|^2 \rho_f$$

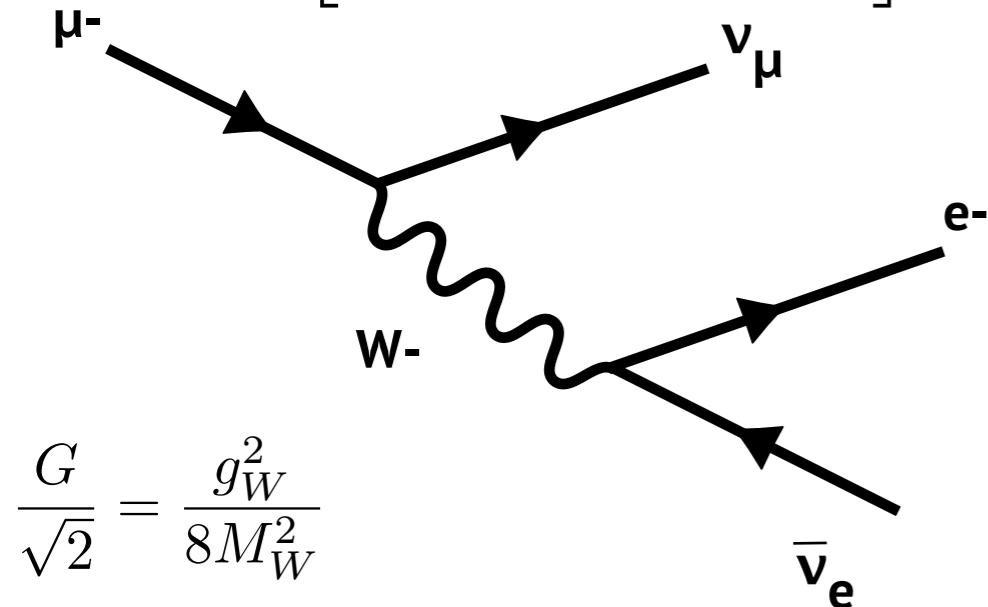


Fermi makes the rules.



Weak Interactions in the Standard Model

$$M_{\mu-decay} = \left[\frac{g_W}{\sqrt{2}} \bar{u}_\nu \gamma^\sigma \frac{(1 - \gamma^5)}{2} u_\mu \right] \left(\frac{1}{M_W^2 - q^2} \right) \left[\frac{g_W}{\sqrt{2}} \bar{u}_e \gamma^\sigma \frac{(1 - \gamma^5)}{2} u_{\bar{\nu}_e} \right]$$



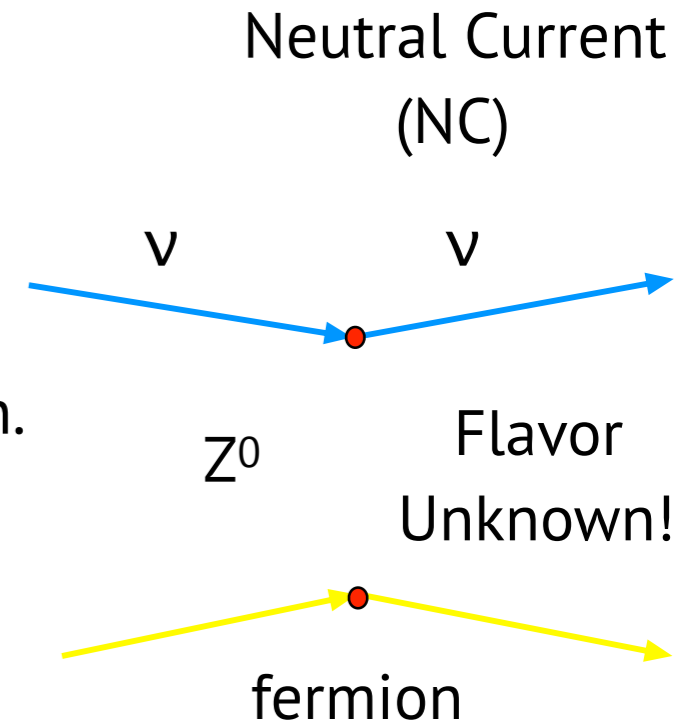
$$\frac{G}{\sqrt{2}} = \frac{g_W^2}{8M_W^2}$$

Lepton Number Conservation*

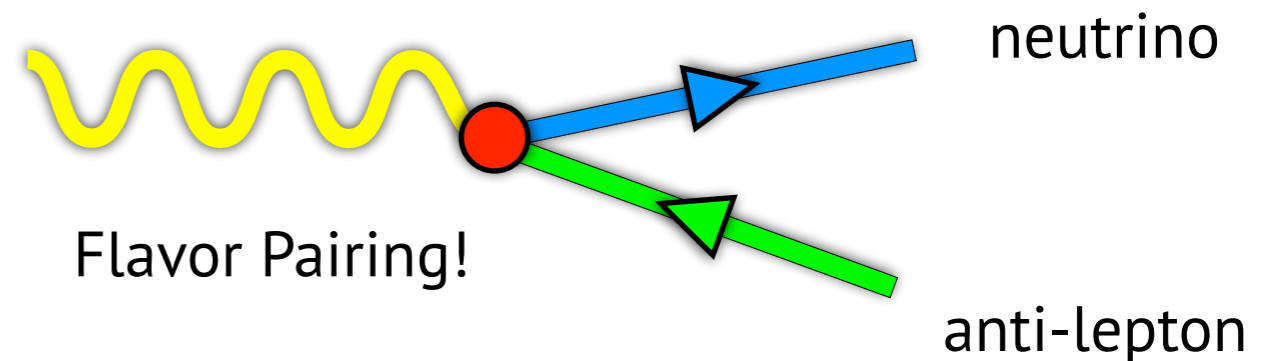
Particle	e^-	e^+	ν_e	$\bar{\nu}_e$
l_e	+1	-1	+1	-1

Massive Propagator!

Parity Violation.



Charged Current (CC) W^\pm

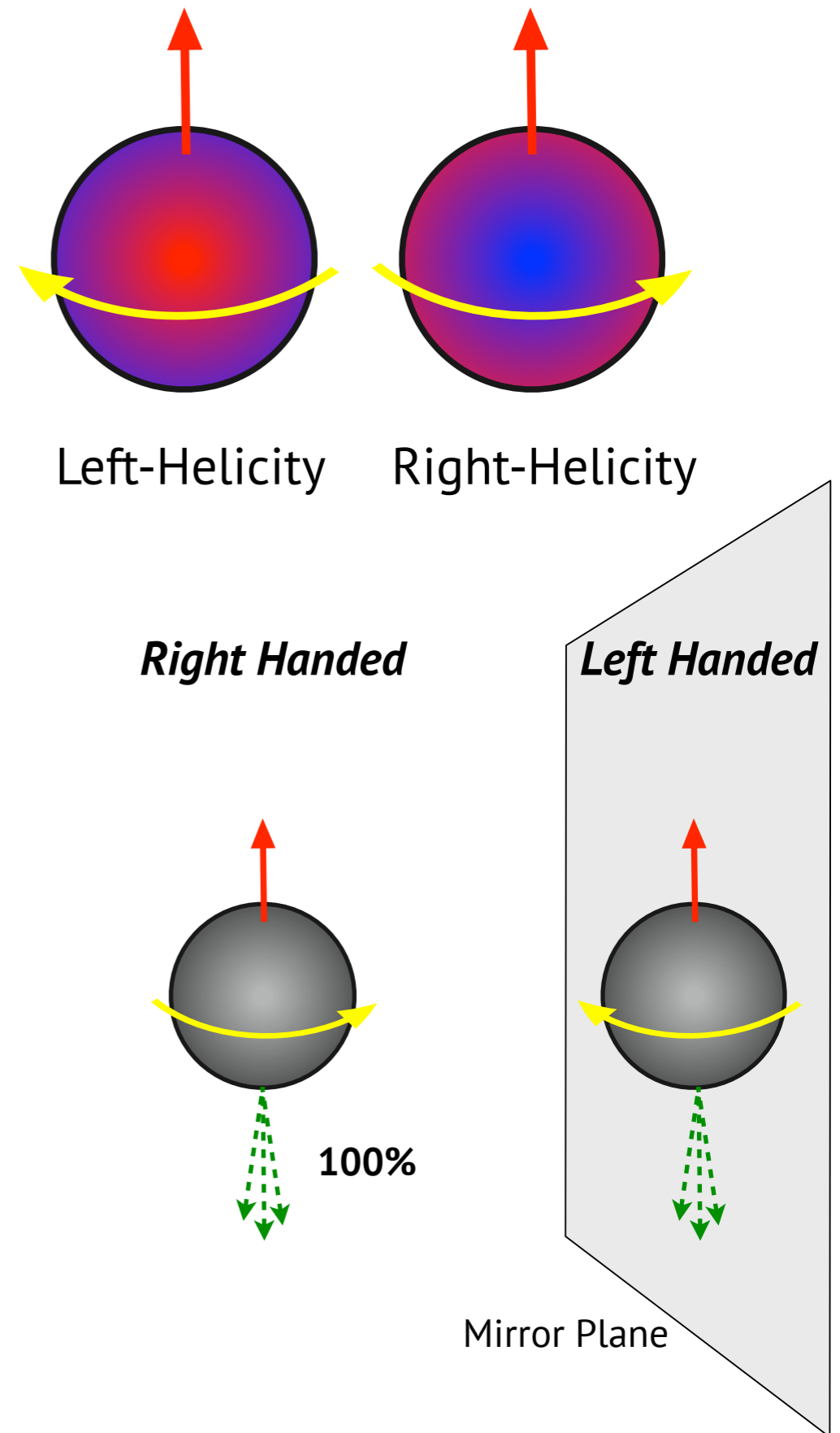


*Actually, "hiding" behind Parity violation. Hmmm...

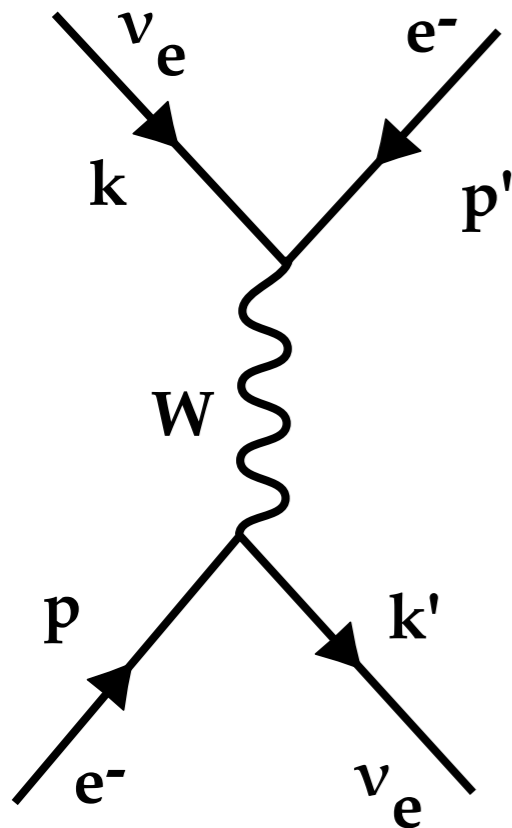
Maximal Parity Violation

- ***The Weak force is left-handed.***
 - Put simply, the Weak force couples to *left-handed stuff* and *right-handed anti-stuff*.
 - Handedness is frame dependent for massive particles.
 - To the extent neutrinos are massless, the Weak force couples to left-handed neutrinos and right-handed anti-neutrinos only.

$$\frac{1}{2} (1 - \gamma^5) \psi = \psi_L$$



Neutrino-Electron Scattering



Assume: $m_e = 0$ & $s = (k + p)^2 = 2k \cdot p = 2k' \cdot p'$

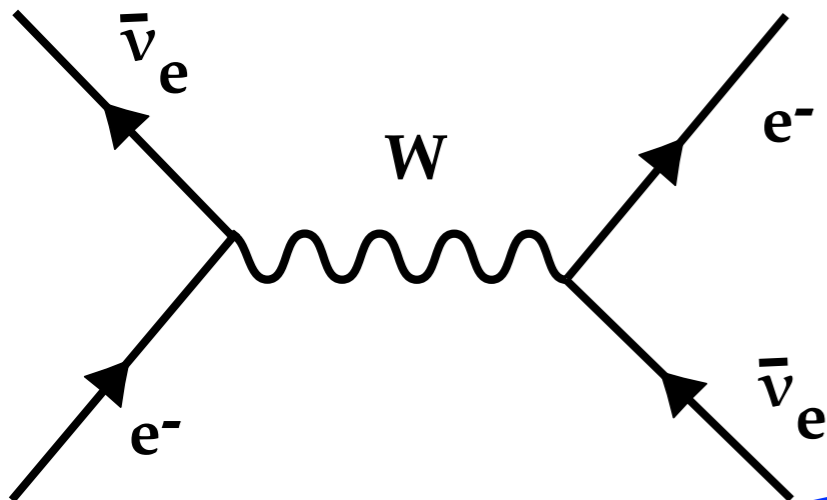
$$\frac{1}{2} \sum_{spins} |\mathcal{M}|^2 = 64G_F^2 (k \cdot p) (k' \cdot p')$$

$$= 16G_F^2 s^2$$

Skip a lot of steps! See: *Halzen & Martin Quarks & Leptons* or *Griffiths Intro. to Elementary Particles*.

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} |\mathcal{M}|^2 = \frac{G_F^2 s}{4\pi^2} \implies \sigma = \frac{G_F^2 s}{\pi}$$

Anti-Neutrino-Electron Scattering



By crossing the neutrinos of previous diagram, we have the result for antineutrinos, replacing s with t :

$$\frac{1}{2} \sum_{spins} |\mathcal{M}|^2 = 16G_F^2 t^2$$

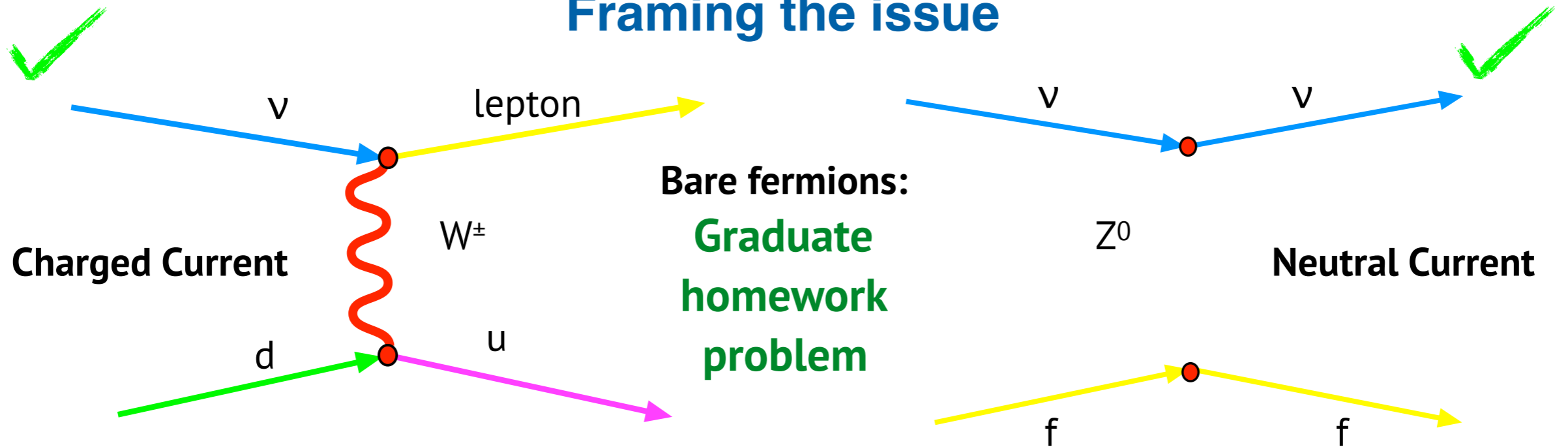
$$= 4G_F^2 s^2 (1 - \cos \theta)^2$$

Integrating over angles, we have:

$$\frac{d\sigma}{d\Omega} = \frac{G_F^2 s}{16\pi^2} (1 - \cos \theta)^2 \implies \sigma = \frac{G_F^2 s}{3\pi}$$

**DONE!,
right?...**

Framing the issue



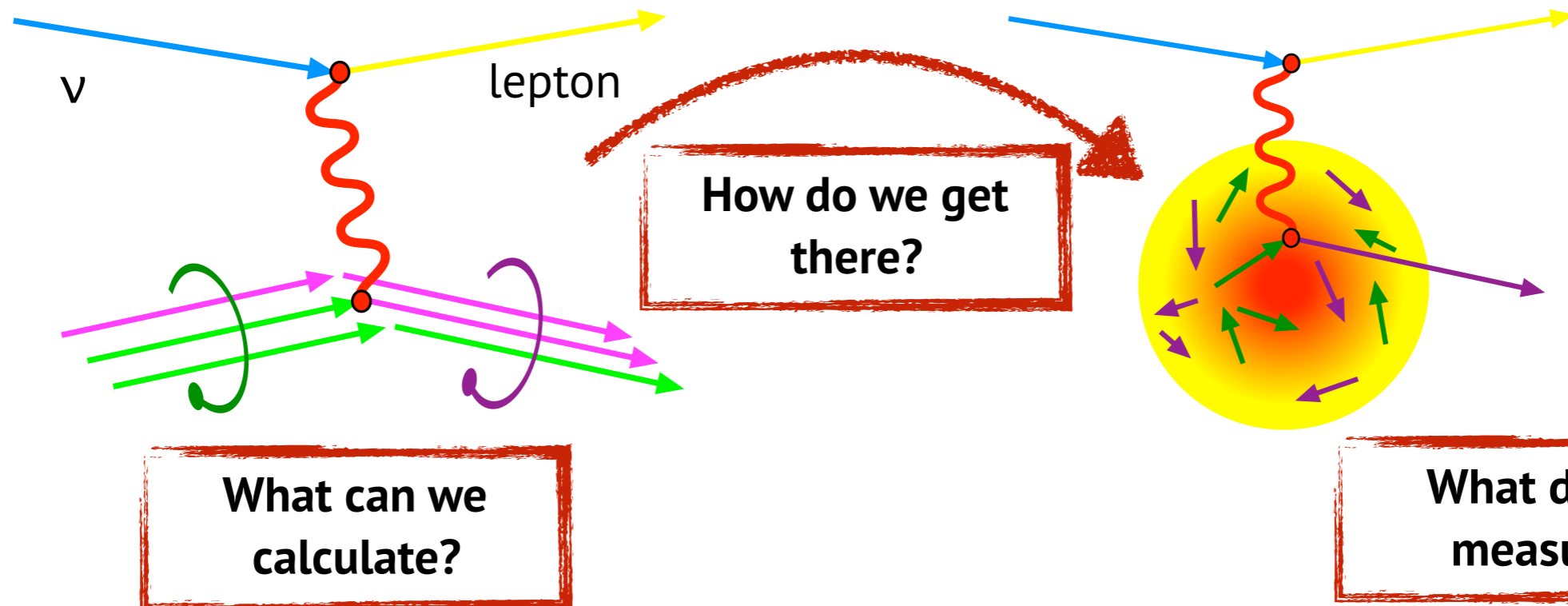
Bare fermions:
Graduate homework problem

Free Nucleon:
Parameterize
w/ Form Factors...

Nucleus:
What is the initial state?
What escapes the nucleus?

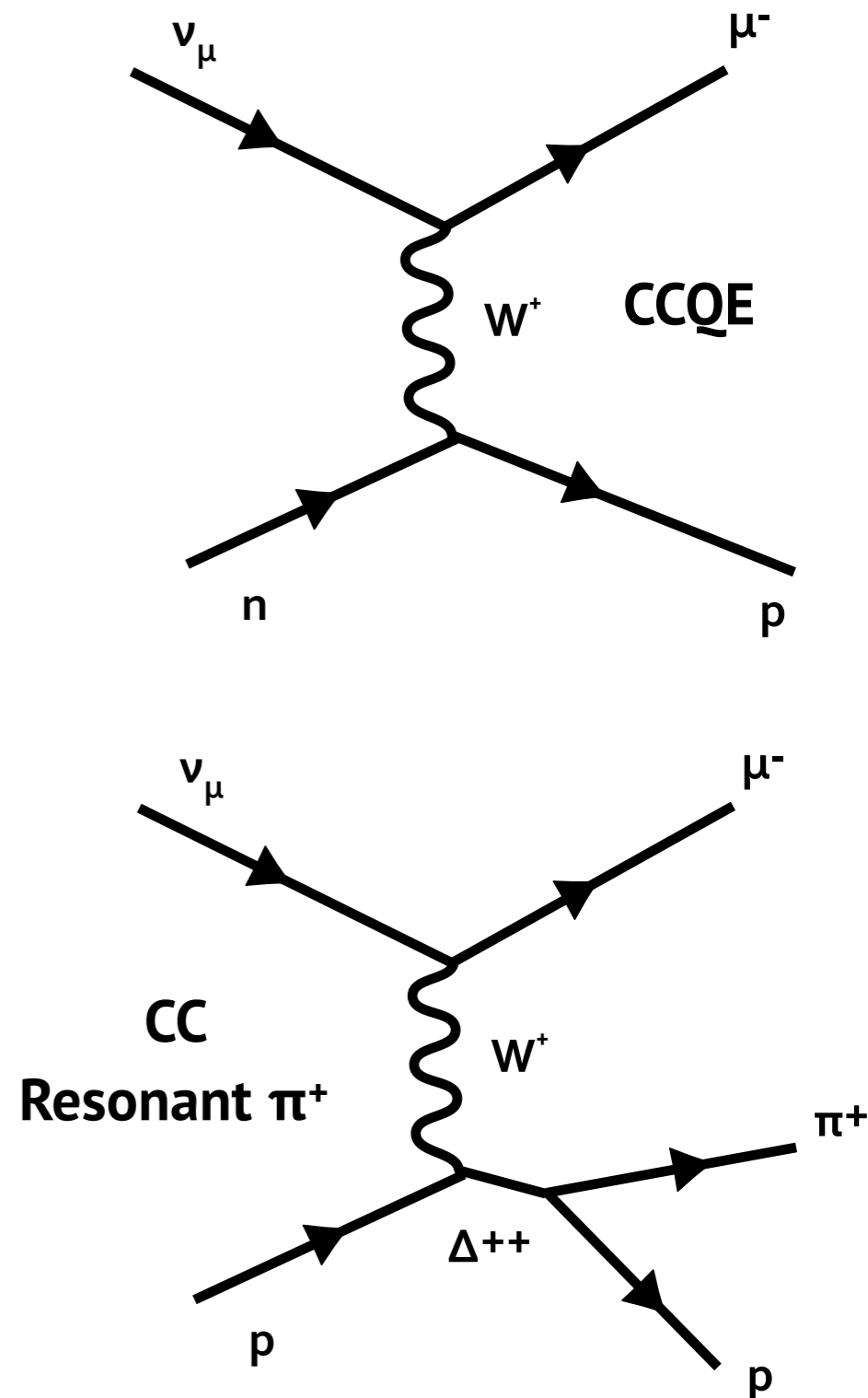
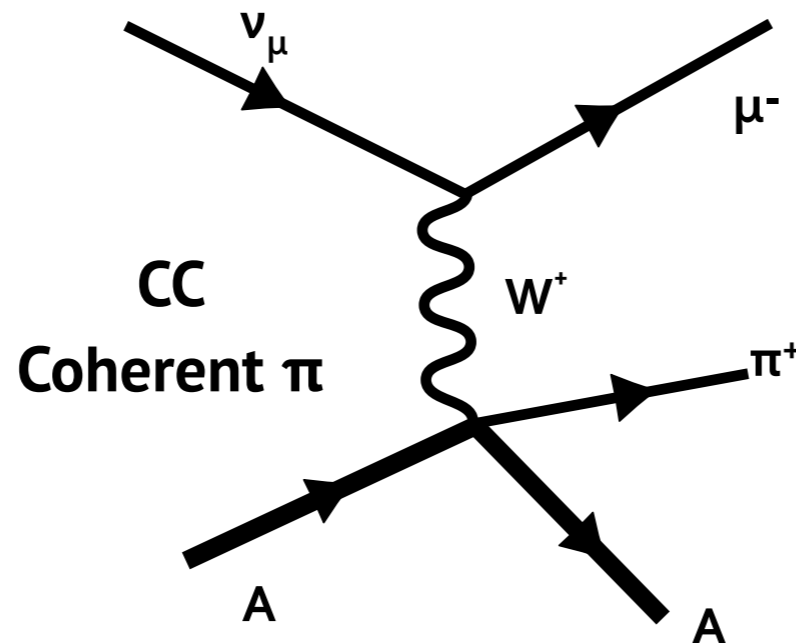
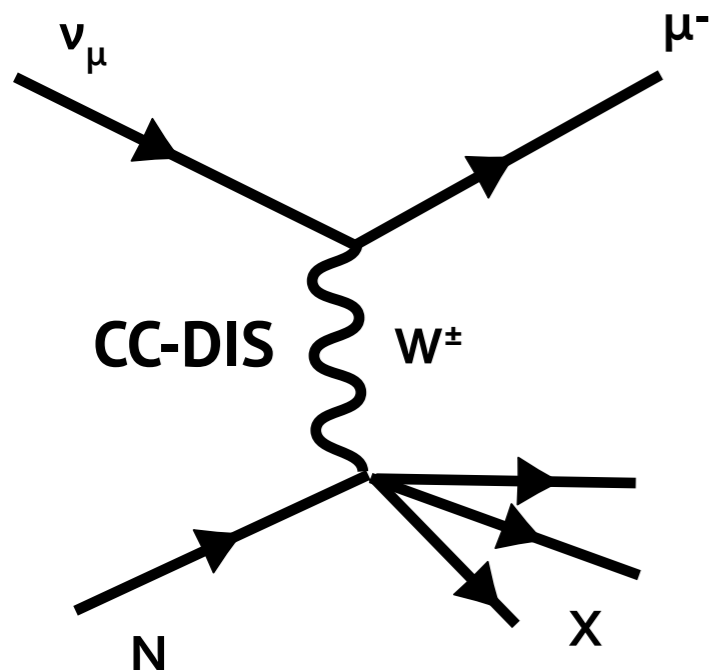
?

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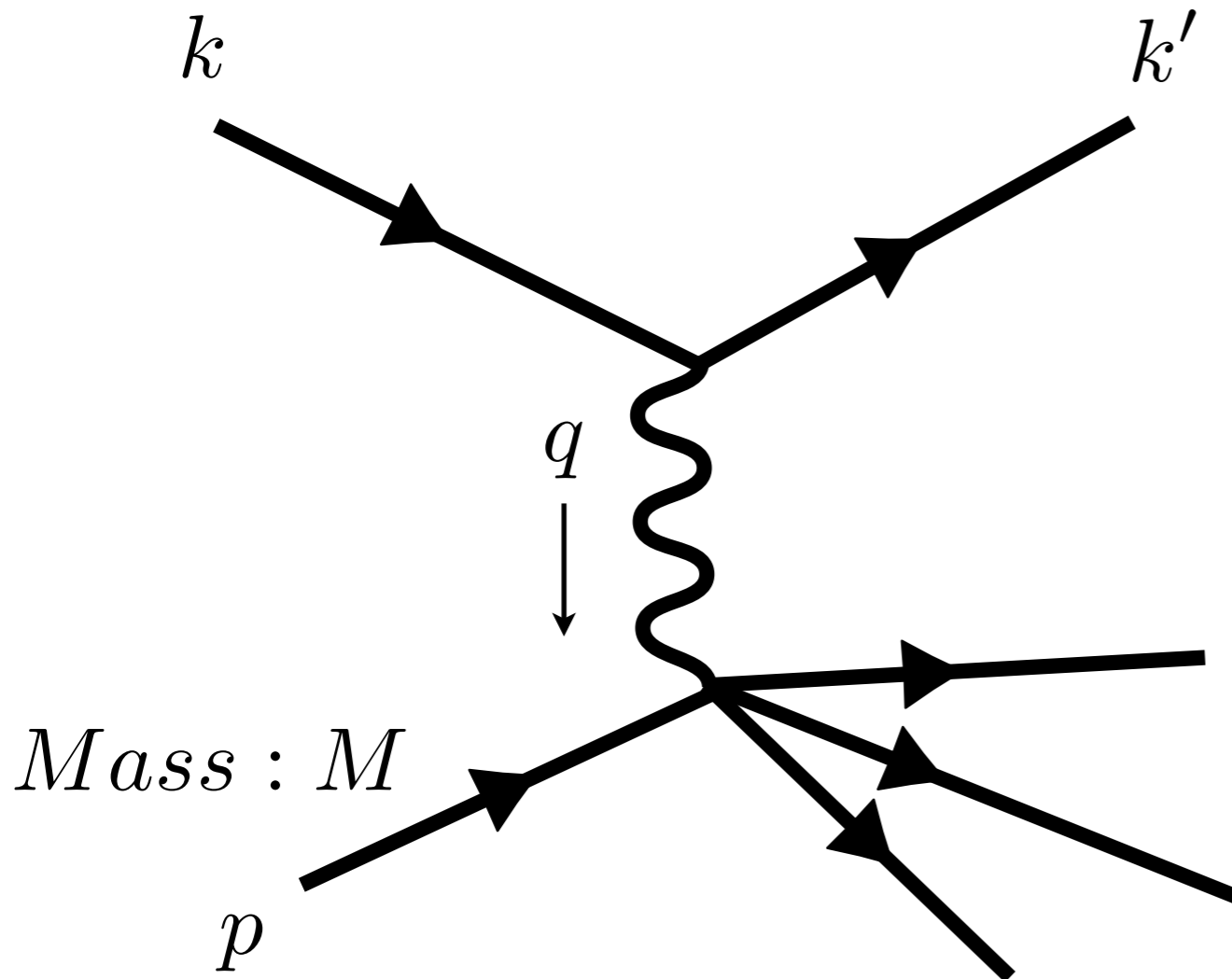


Reaction Channel Menagerie: A Glossary

- Charged current: exchange a W boson; neutral current: exchange a Z (not shown) - no charged lepton in the final state for NC.
 - CCQE : Charged-Current Quasi-Elastic
 - CC π^\pm , π^0
 - Coherent (no break-up) & Resonance Production
 - Deep Inelastic Scattering (DIS - scatter on a parton)
- Our descriptive language is something of a historical accident. These terms are really only proper when discussing scattering on **free nucleons**.
 - When scattering on nuclei, final state interactions (FSI) mix up the particles leaving the nucleus, making this sort of assignment impossible.
 - We should prefer **specification by visible particles in the final state** (but we still use these terms anyway).



Neutrino Kinematics Jargon : Technical



$$q^2 \equiv (p' - p)^2 = -Q^2$$

(Momentum Transfer)²

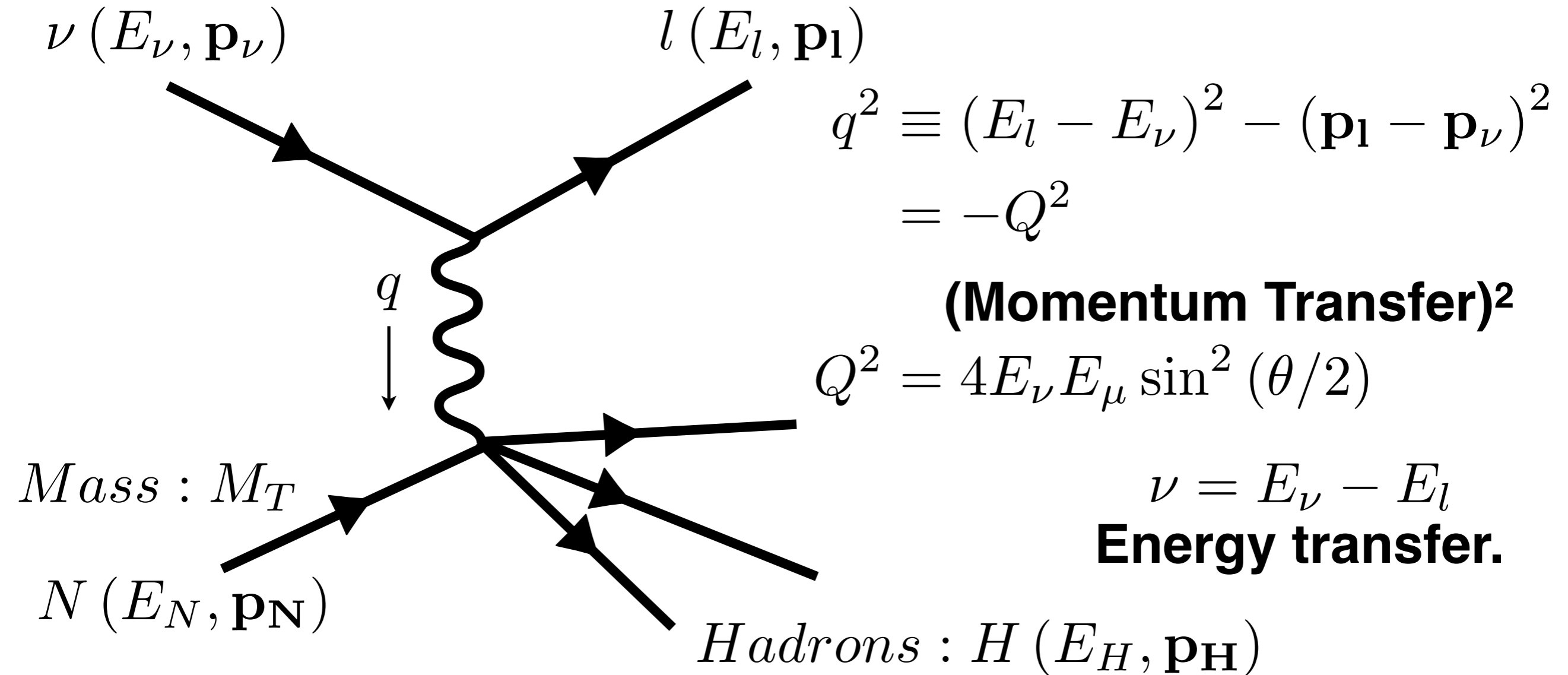
Energy transfer.

$$\nu \equiv \frac{p \cdot q}{M}$$

$$W^2 \equiv (p + q)^2 = E_H^2 - \mathbf{p}_H^2 \text{ **(Hadronic Invariant Mass)²**}$$

$$y = \frac{p \cdot q}{p \cdot k} \text{ **Inelasticity** } \quad x = \frac{-q^2}{2p \cdot q} = \frac{-q^2}{2M\nu} \text{ **Parton Momentum Fraction**}$$

Neutrino Kinematics Jargon : Practical



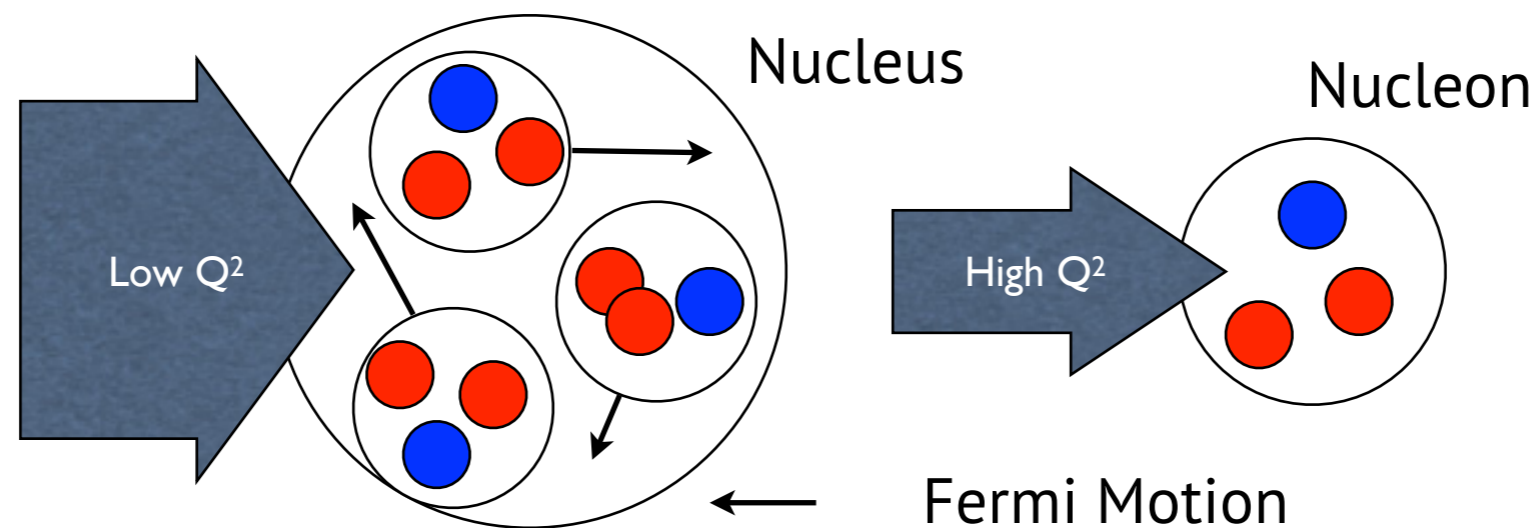
$$W^2 = M_T^2 + 2M_T E_H - Q^2 \text{ **(Hadronic Invariant Mass)²**}$$

$$y = 1 - \frac{E_l}{E_\nu} \text{ **Inelasticity**}$$

$$x_{Bj} = \frac{Q^2}{2M_T \nu} \text{ **Parton Momentum Fraction**}$$

How to think about neutrino interactions

- W/Z boson exchange with target is complex.
- We slide around in Q^2 , W , x , and y with reasonably hard divisions between coherent, elastic, and inelastic reactions, but the target is very messy - do we resolve partons? nucleons? correlated groups of nucleons? the whole nucleus? It depends on the kinematics.
- The produced particles can vary a lot for a given set of kinematics and the inverse is also true - many different kinematic configurations can produce the same set of produced particles.
- Then everything gets smeared by final state interactions.

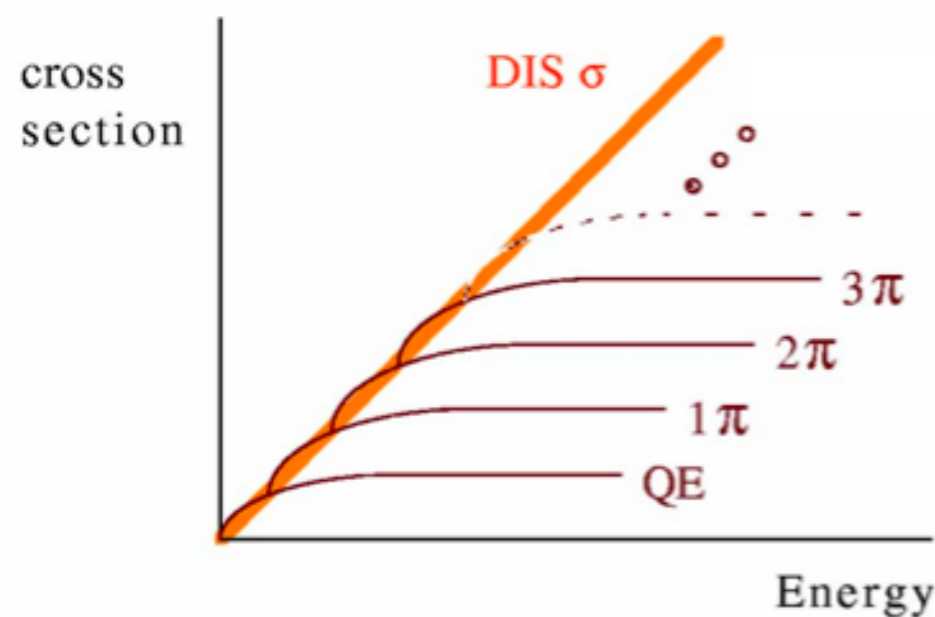


How to think about neutrino interactions

- Generically though, at very low energy transfer to the nucleus (t), reactions are coherent. The nucleus remains in the ground state and we produce a pion and a lepton.
- As we move up in energy transfer and four momentum transfer, we start to resolve more of the inside of the nucleus and scatter of nucleons and correlated groups of nucleons (all bound, of course). We generally call these reactions quasielastic, elastic, or “2p2h” depending on current and scattering target.
- As we go up we may produce resonances and then mesons through non-resonant processes. In this smeary ("transition") region of energy and momentum transfer we are no longer “elastic”.
- We pass through a complex transition region until we begin resolving partons. This is the domain of deep inelastic scattering (high Q^2 , high W , very messy and busy final states).

Features

- Cross-sections scale \sim linearly with the number of targets.
- Experiments often report cross-sections per:
 - Isoscalar nucleon (sum of protons and neutrons)
 - Atom (e.g. per ^{12}C , etc.)
 - Per proton / neutron (typically for anti- ν / ν)



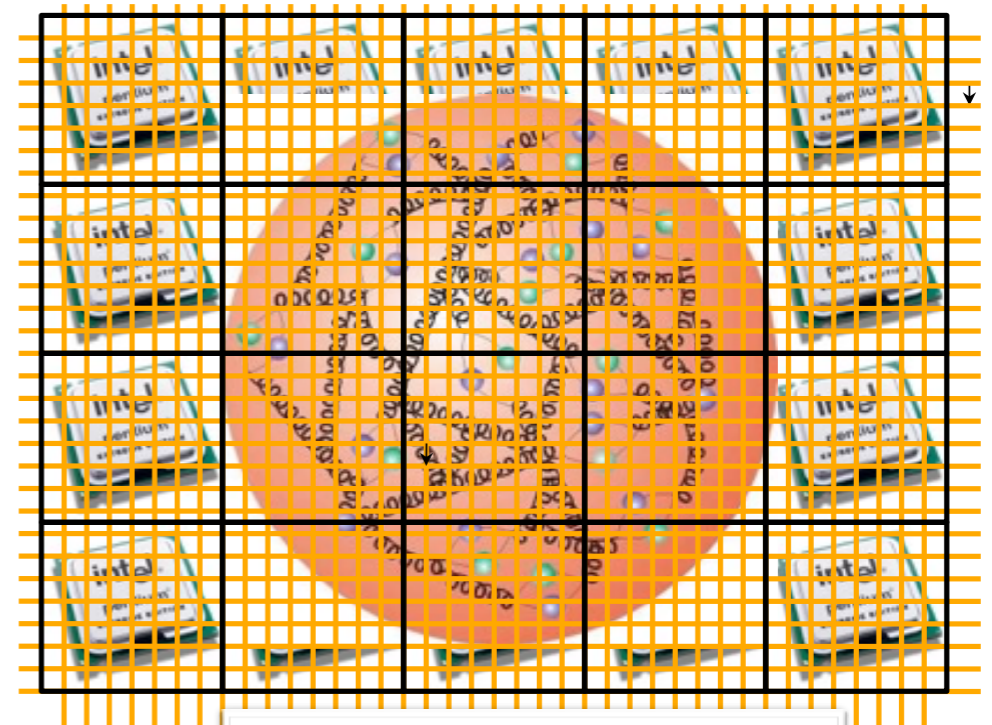
The total cross-section increases linearly with energy!

QCD and the nuclear Hamiltonian

- Owing to its non-abelian character, QCD is strongly nonperturbative at “large” distances.

- Lattice-QCD is the most reliable way of “solving” QCD in the low-energy regime, and it promises to provide a solid foundation for the structure of nuclei directly from QCD

- The applicability of Lattice-QCD is limited to few body systems, ($A < 4$), and to a nuclear physics in which the pion mass must be kept much higher than the physical one.



Courtesy of M. Savage

- Quark and gluons do not exist in the physical spectrum as asymptotic states

We must use effective theories (differing approaches to the use of relativity) - connecting the physics of scattering on nucleons to scattering on nuclei is particularly tricky, important.

Alessandro Lovato

NuINT 2017

Ask him about this at NuSTEC 2017!

Neutrino-nucleus interaction

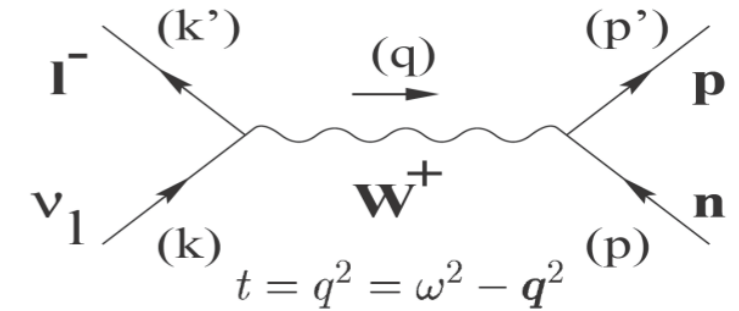
$$\mathcal{L}_W = \frac{G_F}{\sqrt{2}} \cos(\theta_C) l_\mu h^\mu$$

lepton

$$\langle k', s' | l_\mu | k, s \rangle = e^{-iqx} \bar{u}(k', s') [\gamma_\mu (1 - \gamma_5)] u(k, s)$$

hadron

$$\langle p', s' | h_\mu^+ | p, s \rangle = e^{iqx} \bar{u}(p', s') \left[\underbrace{F_1(t) \gamma^\mu + F_2(t) \sigma^{\mu\nu} \frac{i q_\nu}{2M_N}}_{\text{Vector}} + \underbrace{G_A(t) \gamma^\mu \gamma_5 + G_P(t) \gamma_5 \frac{q^\mu}{2M_N}}_{\text{Axial}} \right] \tau_+ u(p, s)$$



Cross section:

$$\frac{\partial^2 \sigma}{\partial \Omega_{k'} \partial k'} = \frac{G_F^2 \cos^2 \theta_C k'^2}{2\pi^2} \cos^2 \frac{\theta}{2} \left[\underbrace{G_E^2}_{\text{blue}} \left(1 - \frac{\omega^2}{q^2}\right)^2 \underbrace{R_C}_{\text{red}} + \underbrace{G_A^2}_{\text{blue}} \frac{(M_\Delta - M_N)^2}{q^2} \underbrace{R_L}_{\text{red}} \right. \\ \left. + \left(\underbrace{G_M^2}_{\text{blue}} \frac{\omega^2}{q^2} + \underbrace{G_A^2}_{\text{blue}} \right) \left(1 - \frac{\omega^2}{q^2} + 2 \tan^2 \frac{\theta}{2}\right) \underbrace{R_T}_{\text{red}} \pm \underbrace{G_A}_{\text{blue}} \underbrace{G_M}_{\text{blue}} 2 \frac{k + k'}{M_N} \tan^2 \frac{\theta}{2} \underbrace{R_T}_{\text{red}} \right]$$

Nucleon properties → Form factors: Electric G_E , Magnetic G_M , Axial G_A

Nuclear dynamics → Nuclear Response Functions:

Phys. Rev. C 80 065501 (2009)
Phys. Rev. C 81 045502 (2010)
Phys. Rev. C 84 055502 (2011)
Phys. Rev. D 85 093012 (2012)
Phys. Rev. D 87 013009 (2013)
Phys. Rev. C 87 065501 (2013)

Charge $R_C(\tau)$, Isospin Spin-Longitudinal $R_L(\tau \sigma \cdot q)$, Isospin Spin Transverse $R_T(\tau \sigma \times q)$

M. Martini, INT Seattle

http://www.int.washington.edu/talks/WorkShops/int_13_54W/People/Martini_M/Martini.pdf

Neutrino vs Antineutrino-nucleus cross-section

The asymmetry between neutrinos and antineutrinos interactions is important for the investigation of CP violation effects.

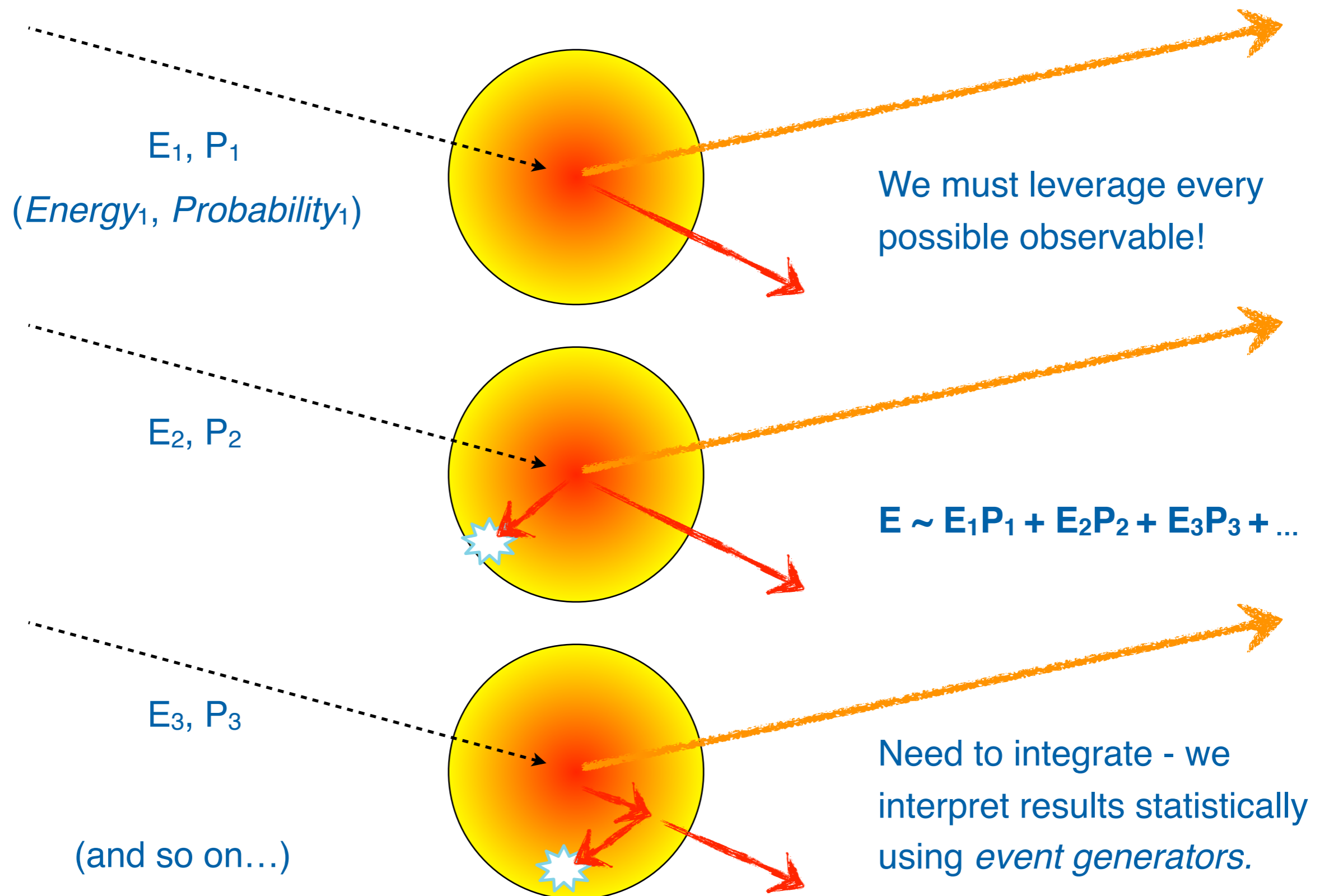
Nuclear effects generate an additional asymmetry due to **interference term**

$$\begin{aligned}
 \frac{\partial^2 \sigma}{\partial \Omega \partial k'} = & \frac{G_F^2 \cos^2 \theta_c (\mathbf{k}')^2}{2 \pi^2} \cos^2 \frac{\theta}{2} \left[G_E^2 \left(\frac{q_\mu^2}{\mathbf{q}^2} \right)^2 \boxed{R_\tau^{NN}} \text{isovector nuclear response} \right. \\
 & + G_A^2 \frac{(M_\Delta - M_N)^2}{2 \mathbf{q}^2} \boxed{R_{\sigma\tau(L)}} \text{isospin spin-longitudinal} \\
 & + \left(G_M^2 \frac{\omega^2}{\mathbf{q}^2} + G_A^2 \right) \left(-\frac{q_\mu^2}{\mathbf{q}^2} + 2 \tan^2 \frac{\theta}{2} \right) \boxed{R_{\sigma\tau(T)}} \text{isospin spin-transverse} \\
 & \left. \left\{ \begin{array}{l} + \quad (\nu) \\ - \quad (\bar{\nu}) \end{array} \right\} \boxed{\pm} 2 G_A G_M \frac{k + k'}{M_N} \tan^2 \frac{\theta}{2} \boxed{R_{\sigma\tau(T)}} \right] \text{interference V-A}
 \end{aligned}$$

M. Martini, INT Seattle

http://www.int.washington.edu/talks/WorkShops/int_13_54W/People/Martini_M/Martini.pdf

The Basic Problem: we must interpret with *models*



What do we measure?

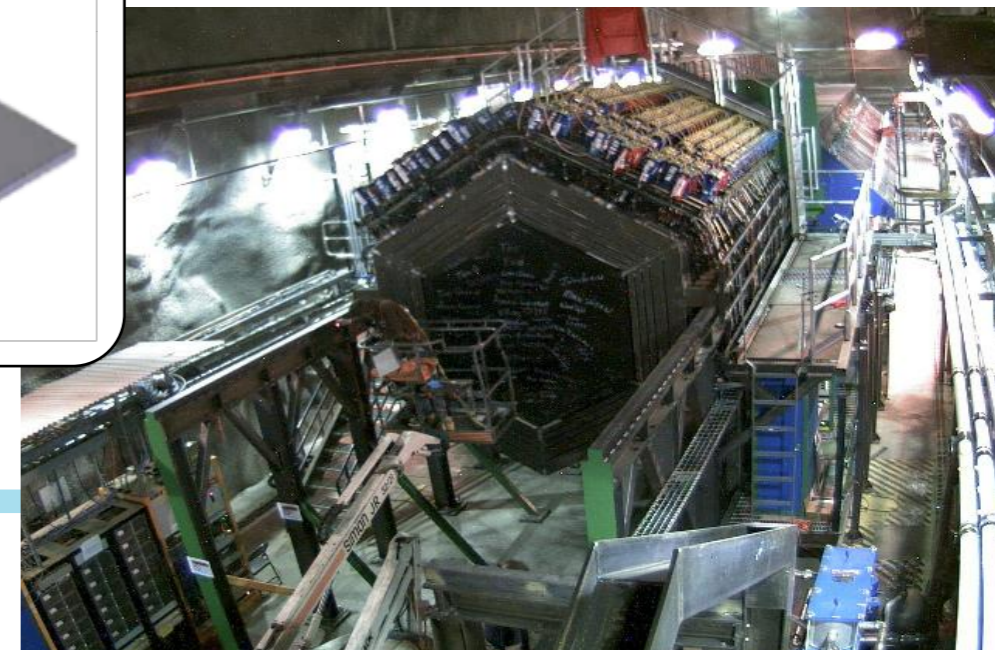
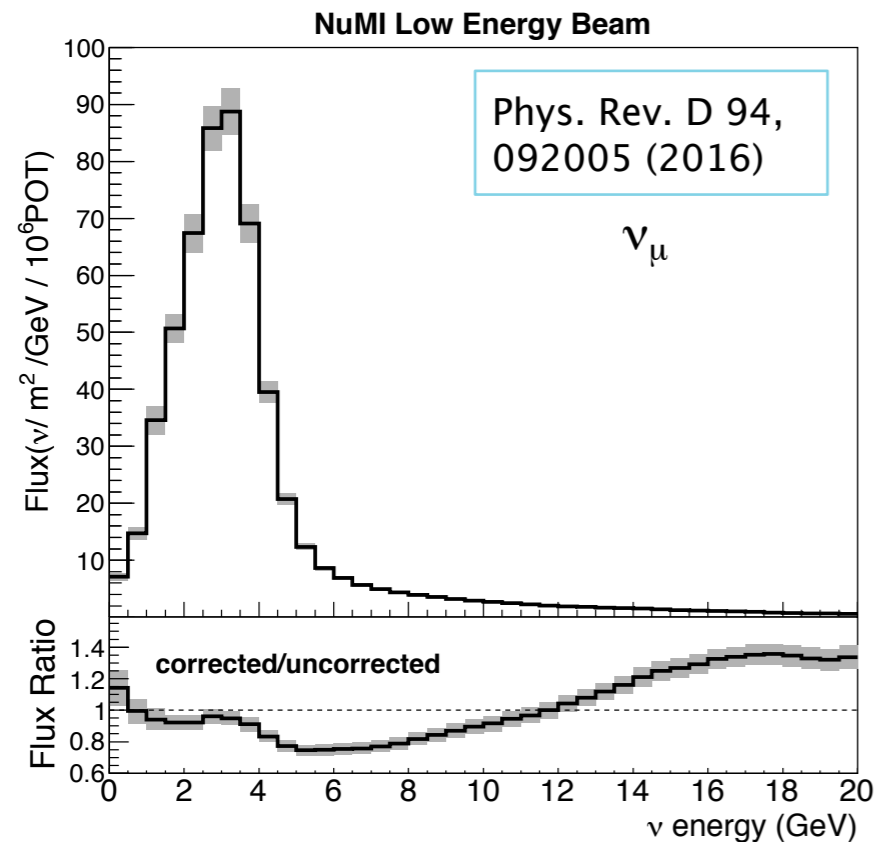
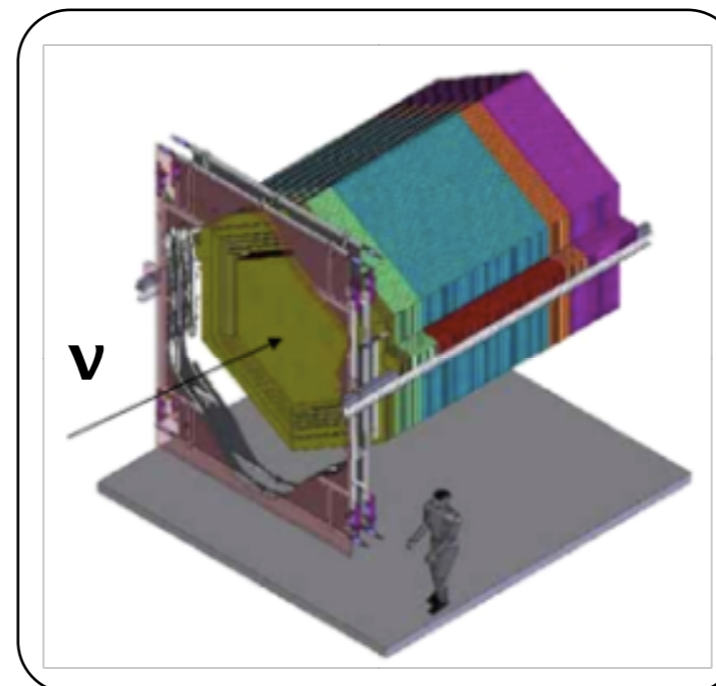
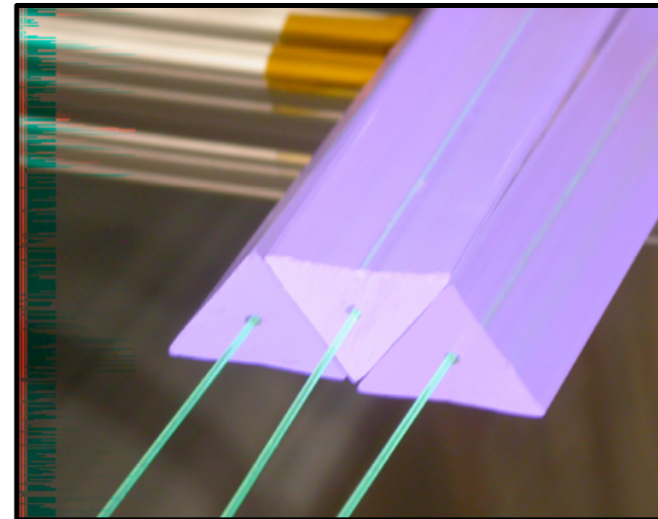
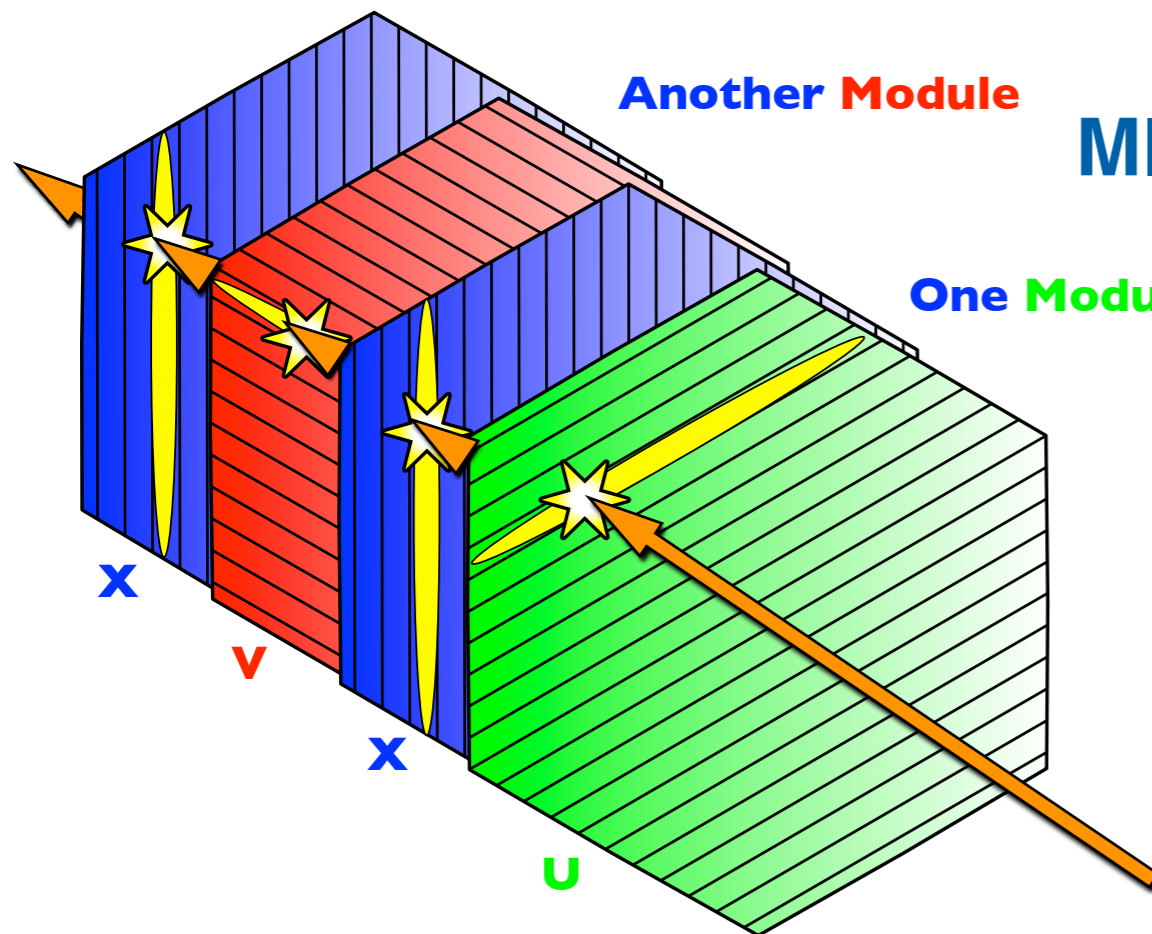
- It is important to understand observables, and to align what we measure with what we calculate.
- This is particularly important with neutrinos.
- Recall, we do not observe the incoming neutrino!
- Final states are "degenerate" - we do not have complete information about the final state - we must interpret observables with probabilistic models that integrate over different possible initial states for an observed final state.
- It is very dangerous to report results that can only be computed with a model - if you do (sometimes we can't help ourselves), we should also always provide cross sections of the input observables.
- More and better dialog is needed about what we mean even when we say we are measuring "simple" particle observables.
 - Are we sure we all agree on the definition of a particle?



MINERvA - Neutrino Scattering

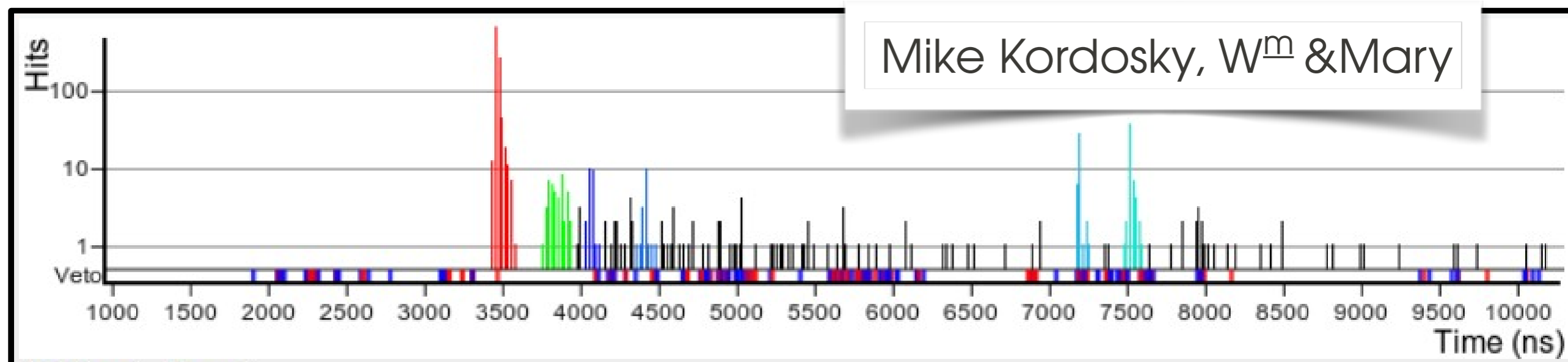
- Fine-grained, **high-resolution scintillator tracker** for detailed kinematic reconstruction of neutrino-nucleus interactions.
- **Cross-section** program.
- Nuclear effects with a **variety of target materials** ranging from Helium to Lead.

@minervaexpt

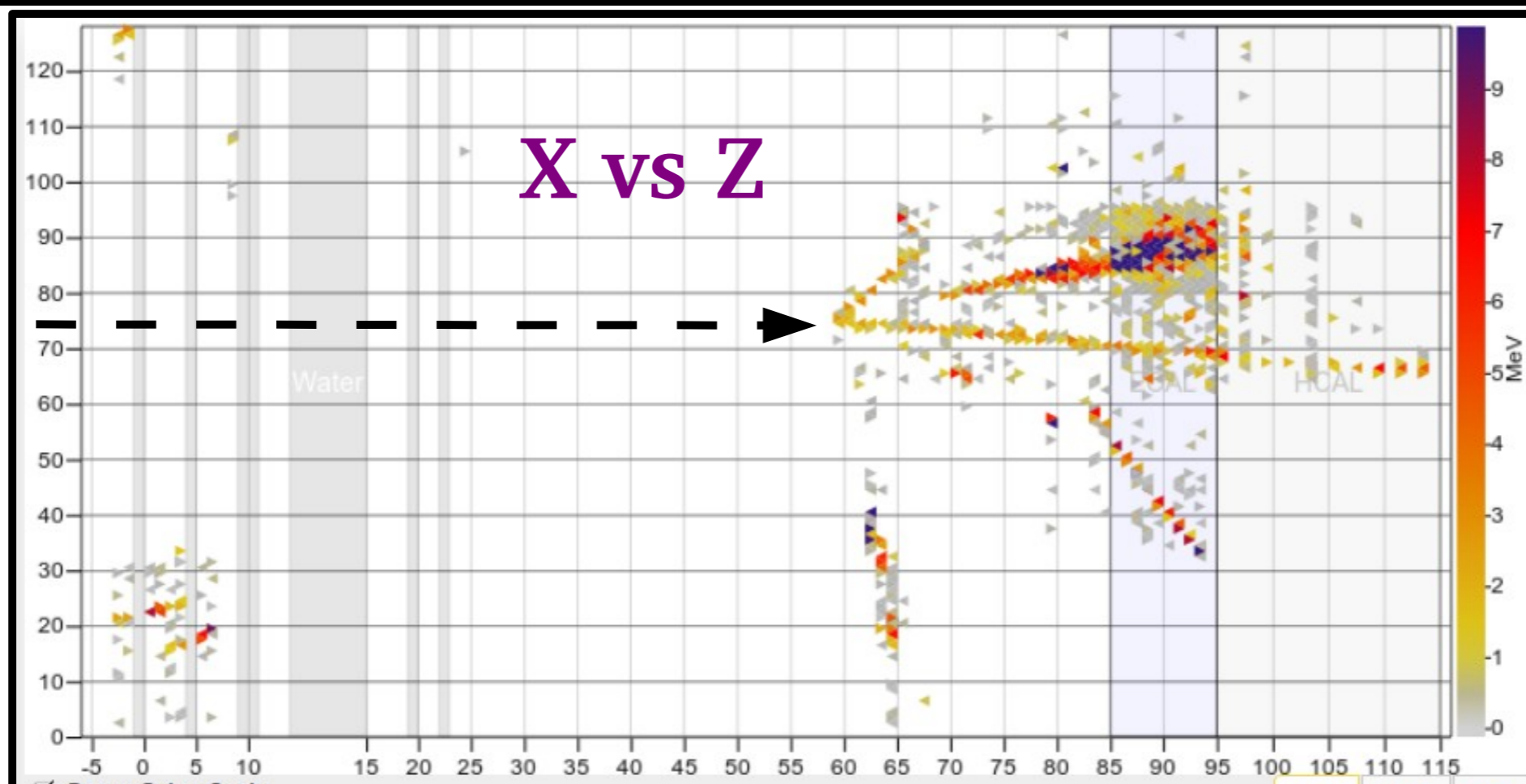




Modern neutrino detectors are *imaging* detectors.



ν_μ

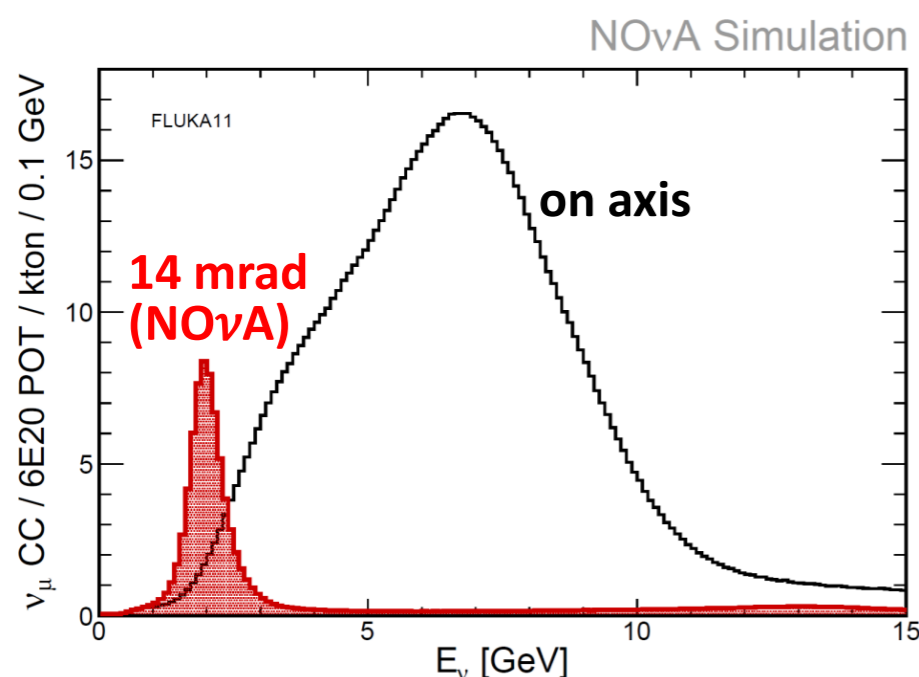
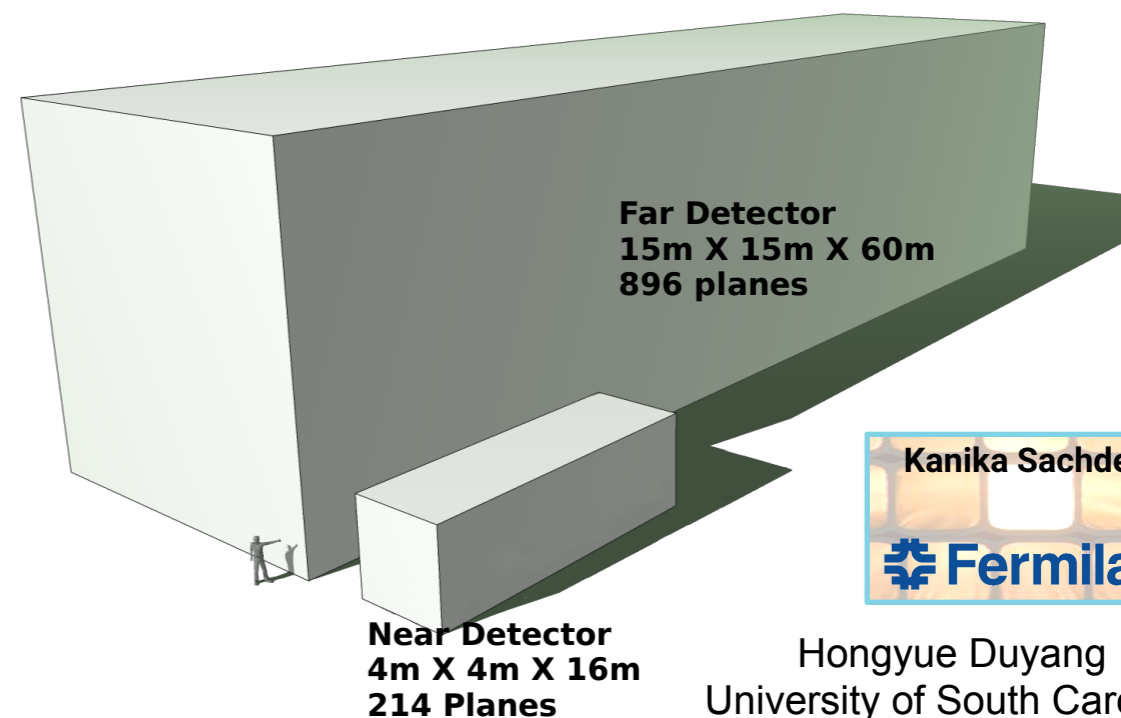
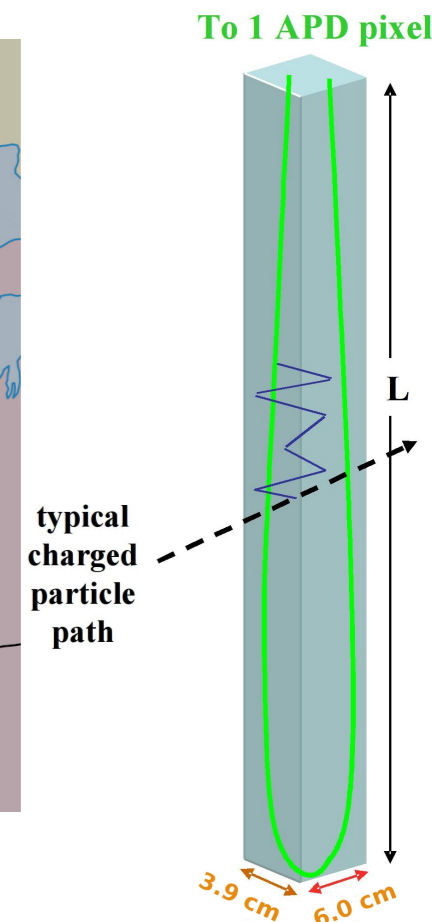




NOvA

@novaexperiment

- * NOvA (NuMI Off-axis ν_e Appearance) is a neutrino oscillation experiment
 - * Baseline of 810 km
 - * NuMI, beam of mostly ν_μ
 - * 14 mrad off-axis from the beam
 - * Two functionally identical detectors
- * Oscillation channels accessible to NOvA:
 - * $\nu_\mu(\bar{\nu}_\mu)$ to $\nu_e(\bar{\nu}_e)$ (appearance)
 - * $\nu_\mu(\bar{\nu}_\mu)$ to $\nu_\mu(\bar{\nu}_\mu)$ (disappearance)

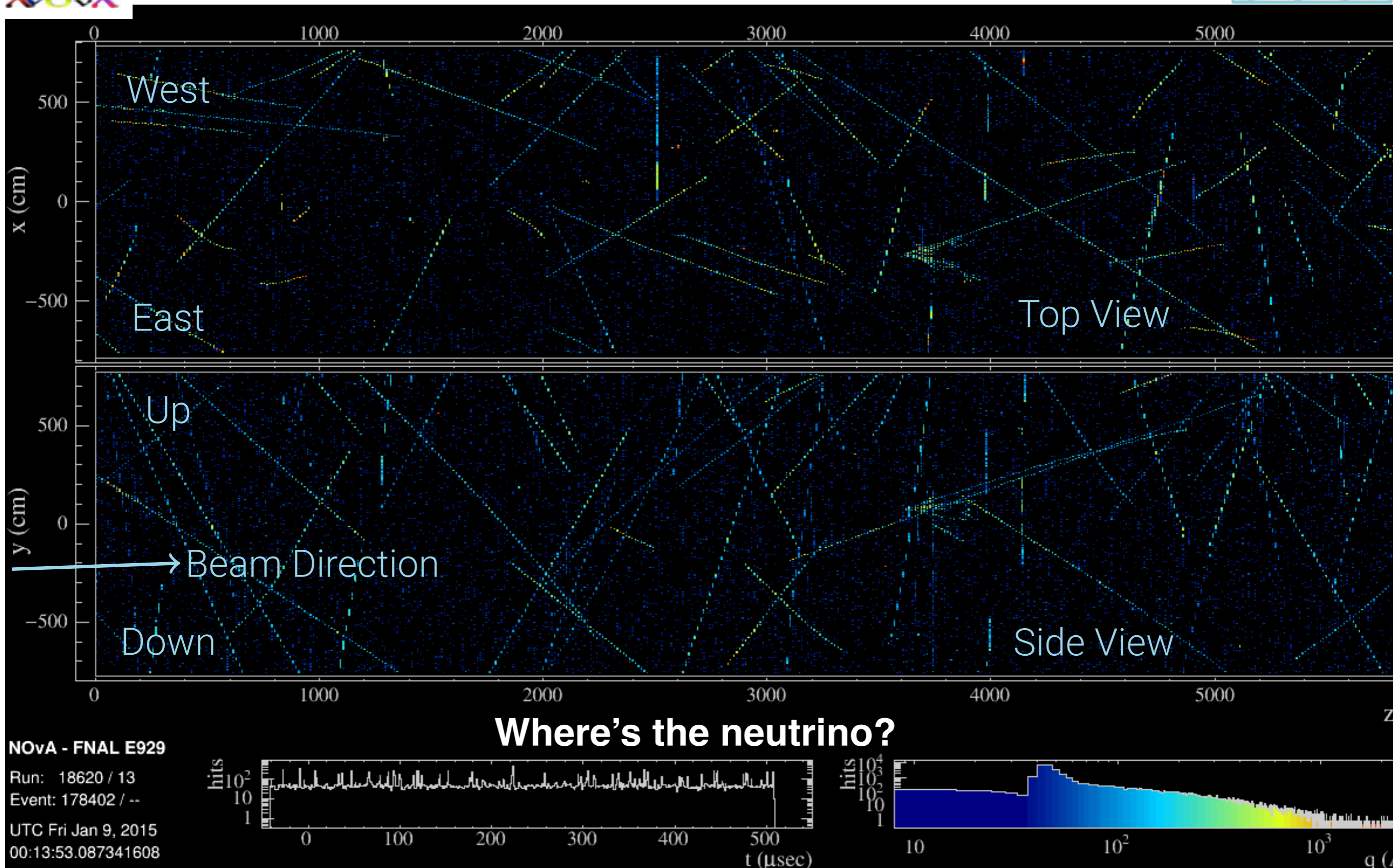


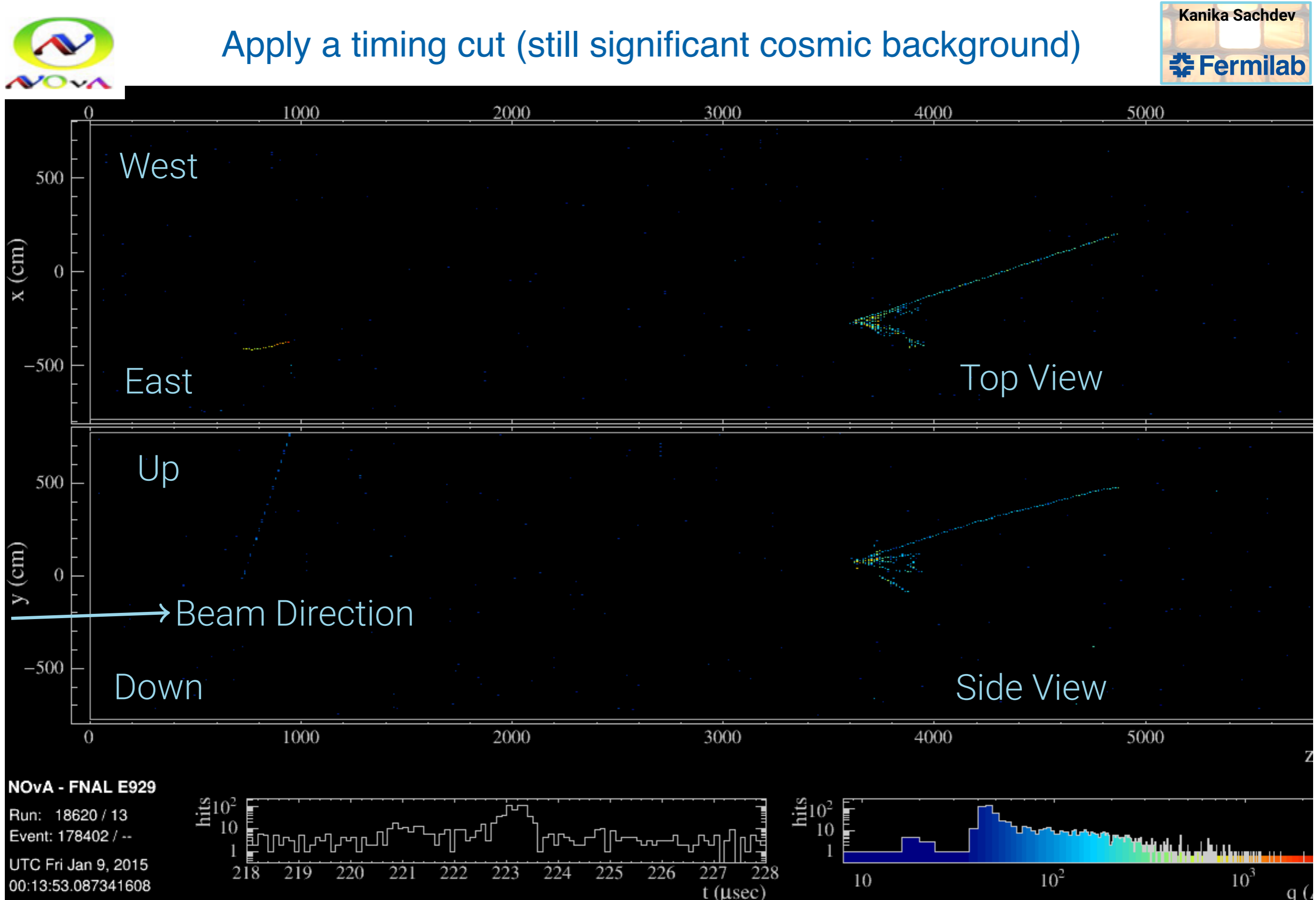
Far Detector (FD)

- * 14 kt, \approx 344,000 channels
- * On surface
- * 810 km from source

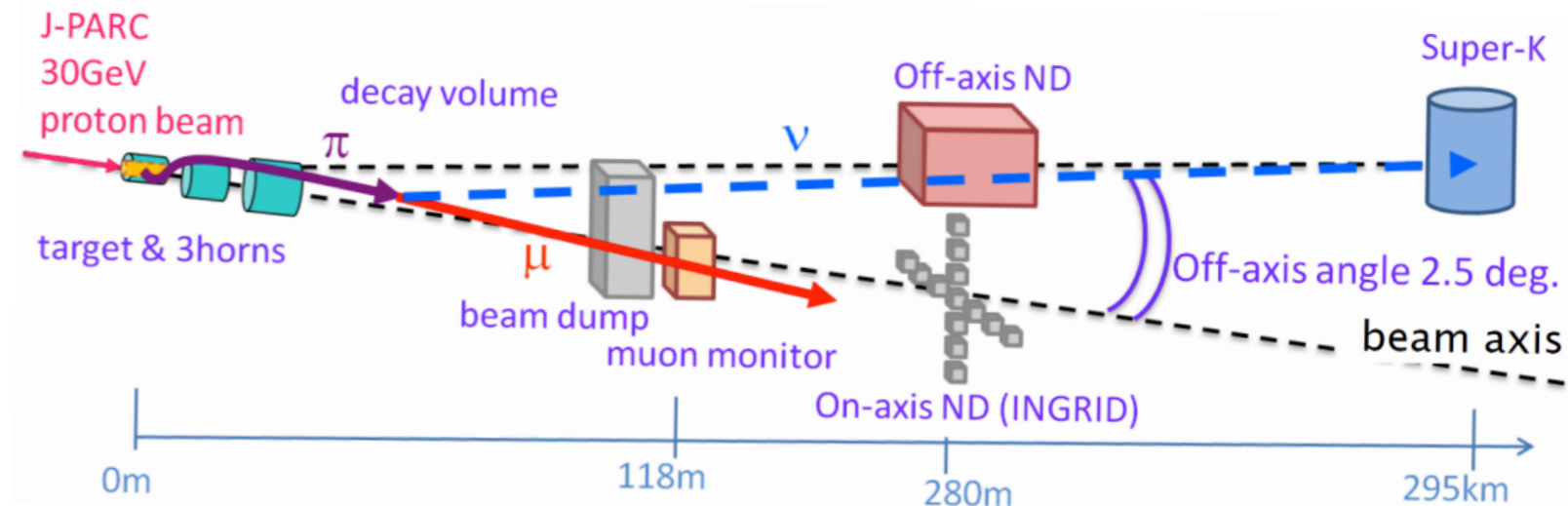
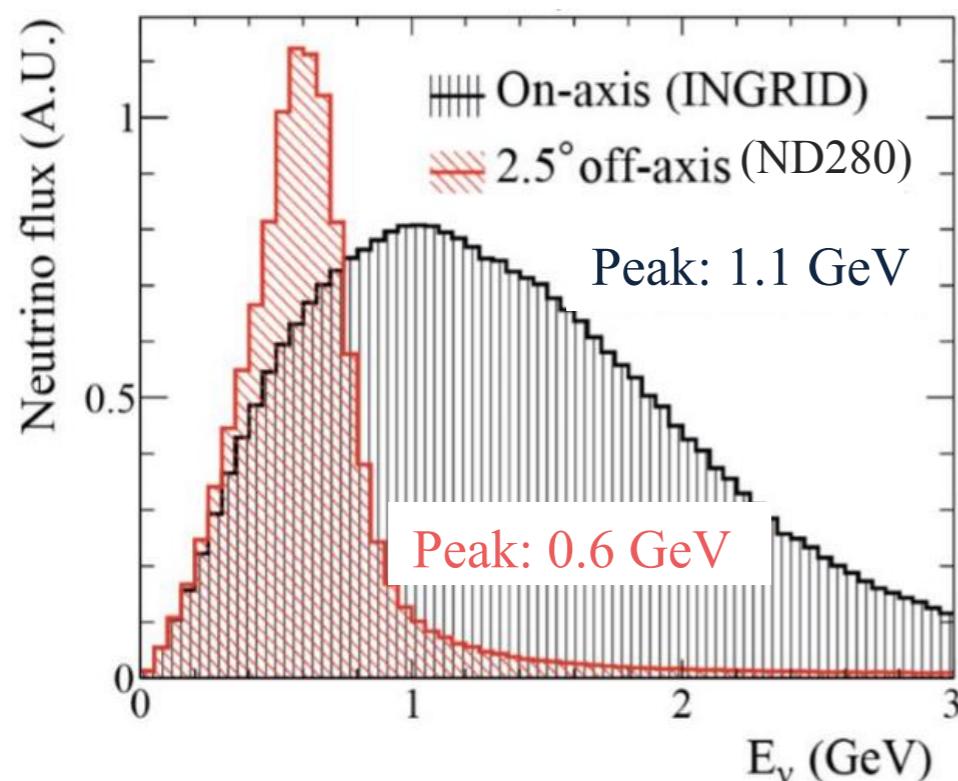
Near Detector (ND)

- * 0.3 kt, \approx 20,000 channels
- * 100 m below surface
- * 1 km from the NuMI





- Oscillation experiment (far detector Superkamiokande) with high granularity, **magnetized** near detector complex for comprehensive cross section program on Carbon, Oxygen.
- **POD** contains water layers, scintillator, and absorbers.
- **TPC** and **segmented scintillator** modules.



ND280 (off axis near detector)

On Axis ~ 1.1 GeV

Peak E_ν

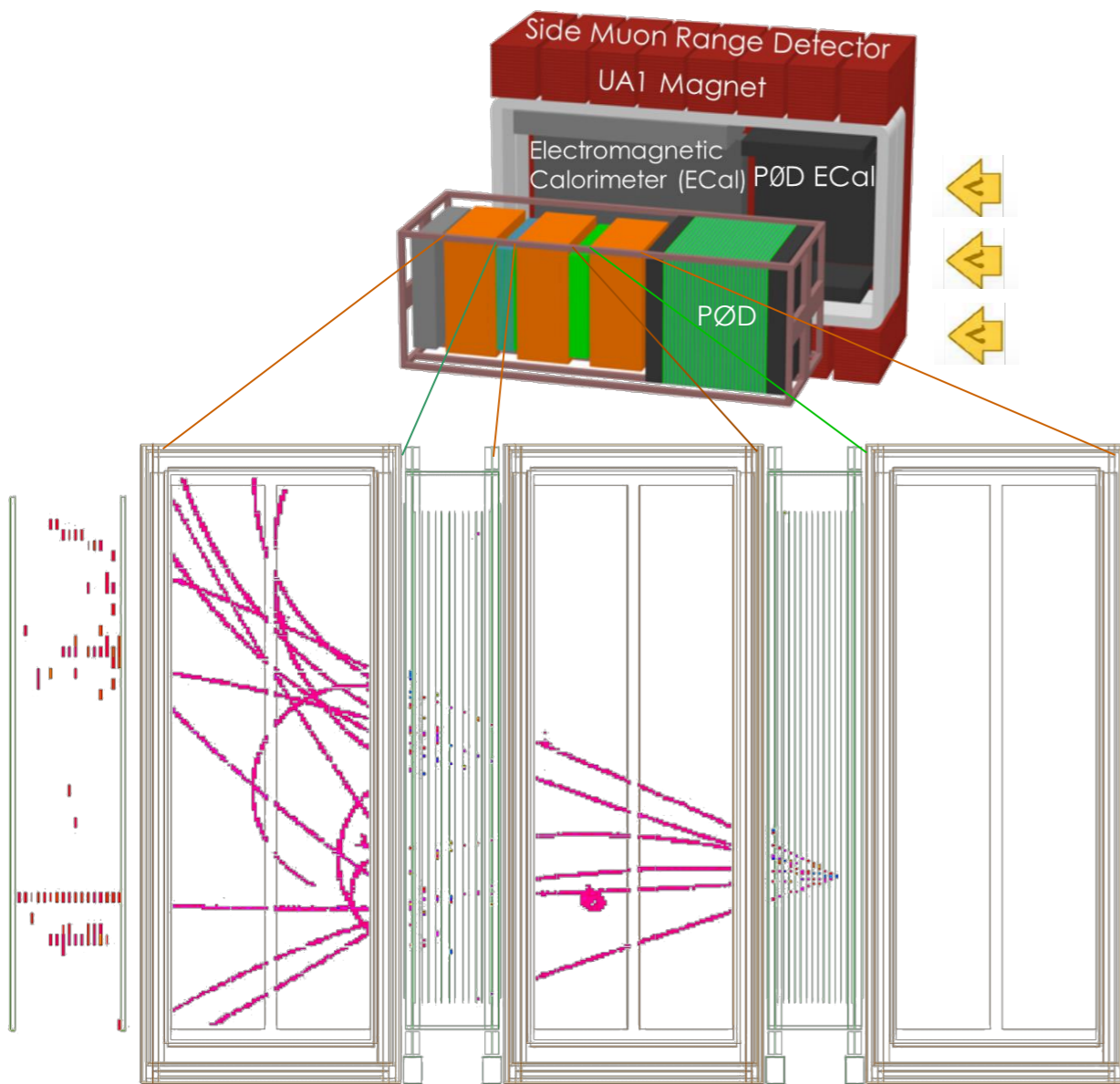
Off Axis ~ 0.6 GeV

π^0 detector (PØD): Interwoven heavy targets, scintillator and drainable water bags affords water subtraction measurements.

UA1 Magnet: Provides 0.2 T field.

Fine-Grained Detectors (FGD 1/2): Polycarbonate scintillator bars provide tracking & target mass. FGD 2 also contains water target layers.

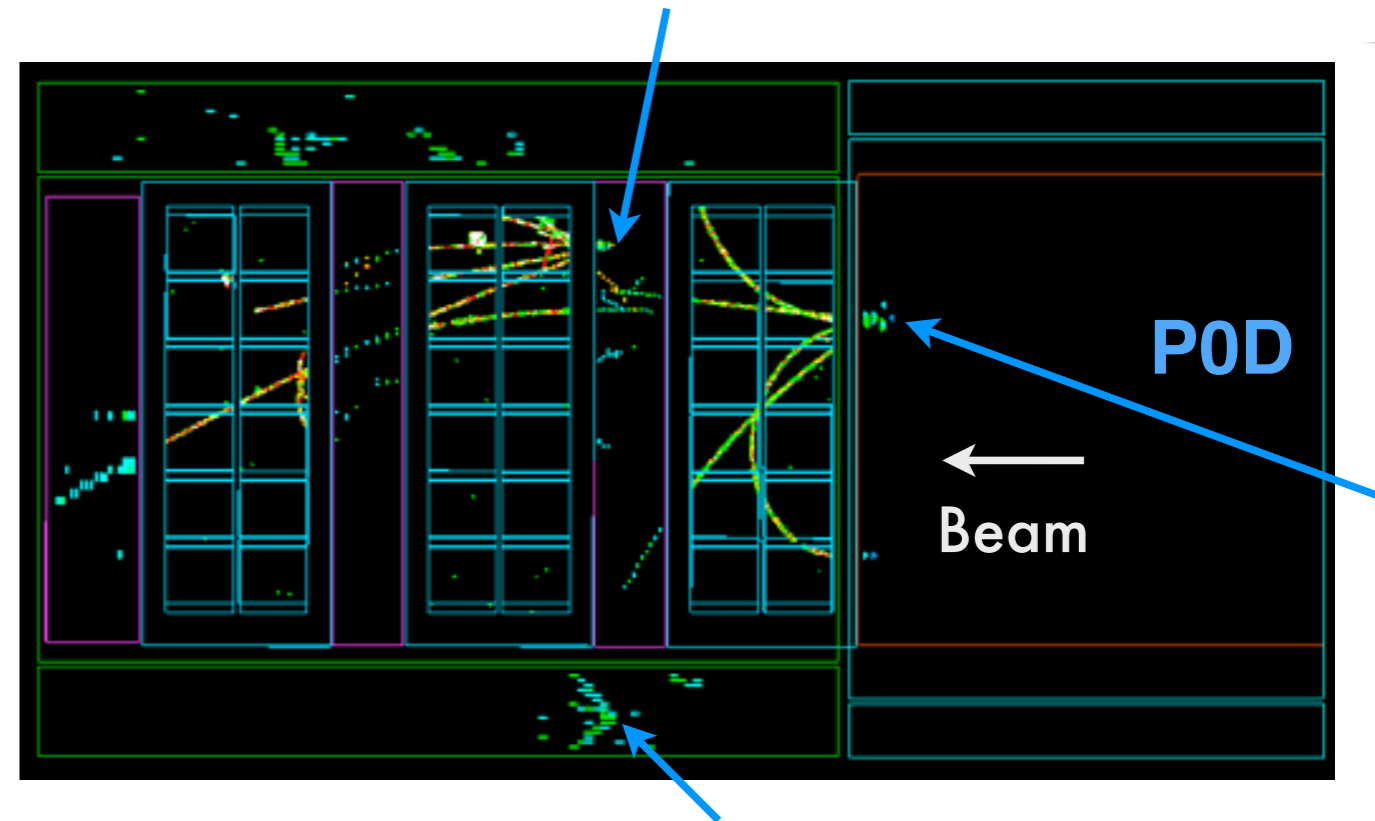
Time Projection Chambers (TPC): Excellent tracking allows high-resolution charged-particle momenta and accurate particle ID.



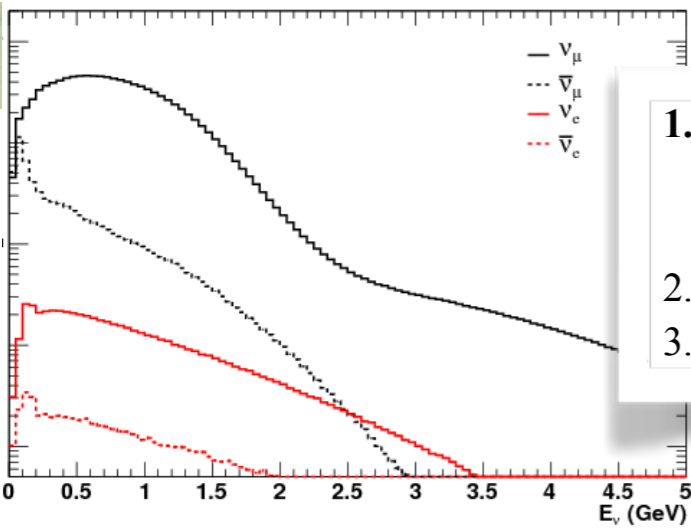
Primary Interaction Material: Carbon
 Secondary Interaction Materials:
 Oxygen, Lead, Brass, Argon

Asher Kaboth

Interaction in FGD1



Interaction in ECal

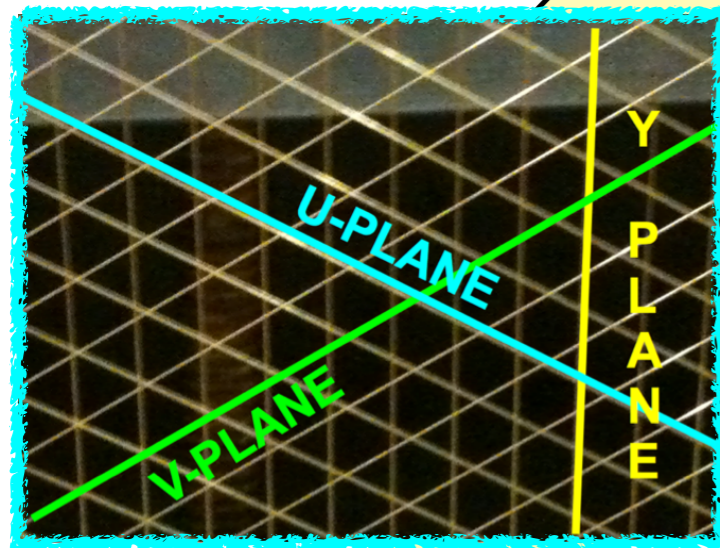


1. Charged particles interact in Ar

- Ionize argon
- Produce scintillation light

2. Ionization e- drift toward anode
3. Wire planes detect drift e-

Three Wire Planes



170 tons of liquid argon
 — 50% inside the TPC

cosmic rate $\sim 200 \text{ m}^{-2}\text{s}^{-1}$: ~ 8 muons
 per drift time

Cathode @ 70 kV
 (plate)

Electric Field
 $\sim 270 \text{ V/cm}$

Anode
 (wire plane)

TPC drift time $\sim 2\text{ms}$

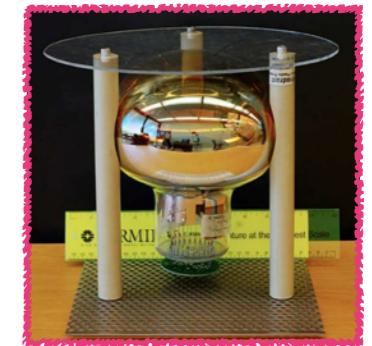
Drift Time = X position

X = 2.5 m

Y = 2.3 m

Z = 10.4 m

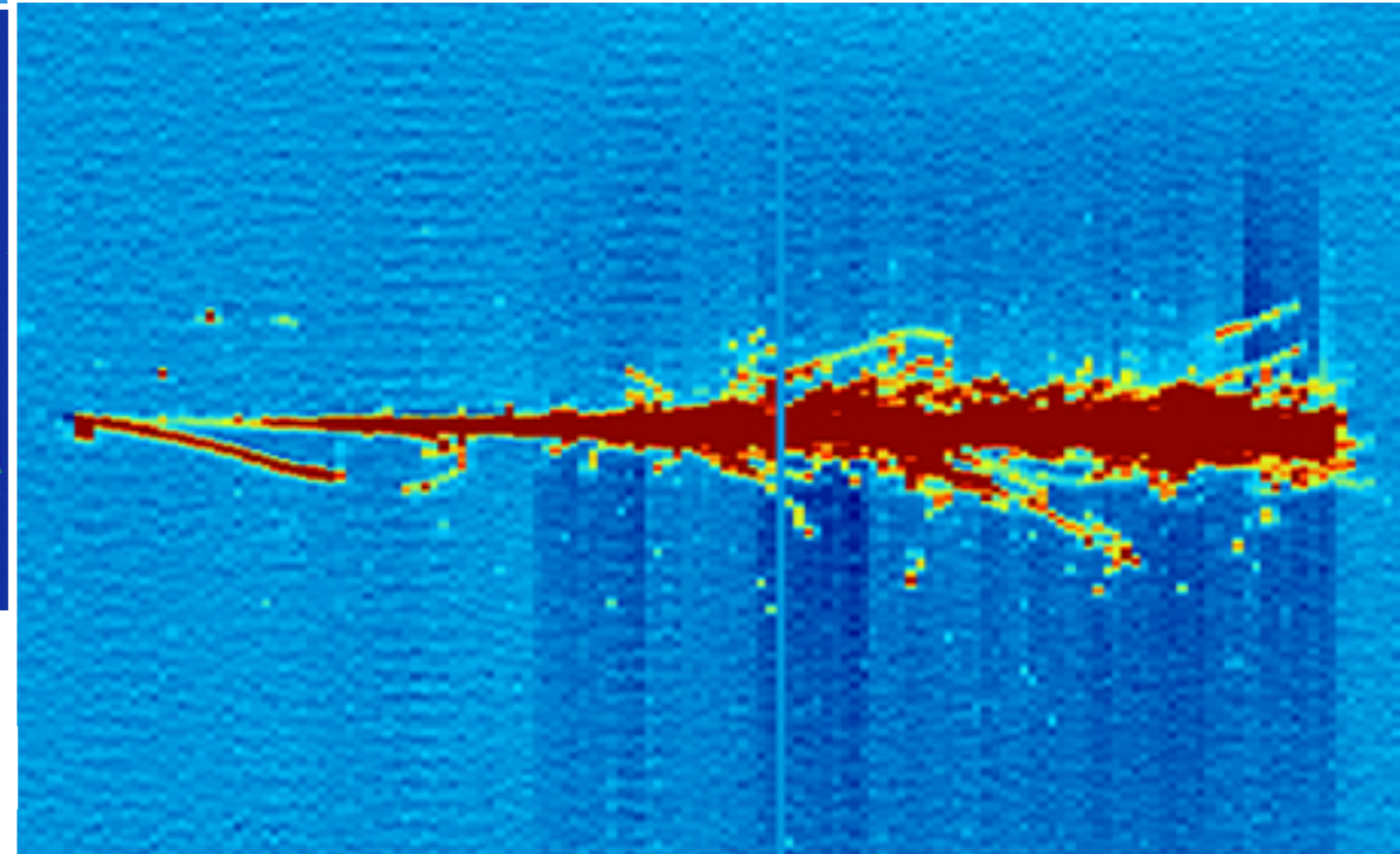
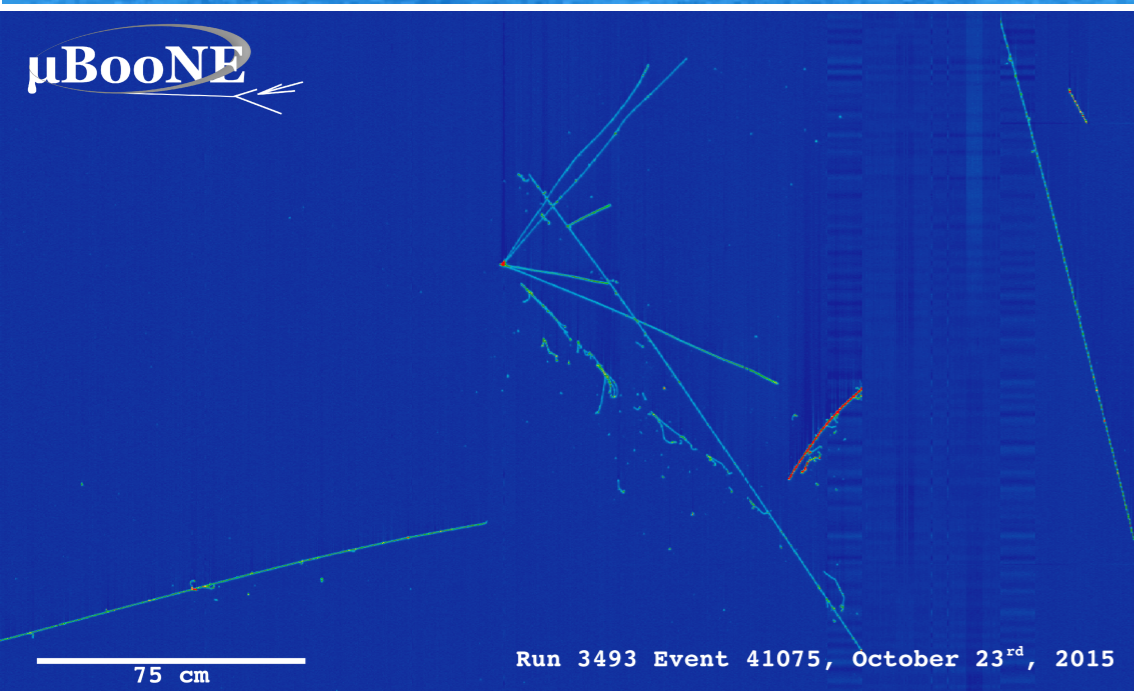
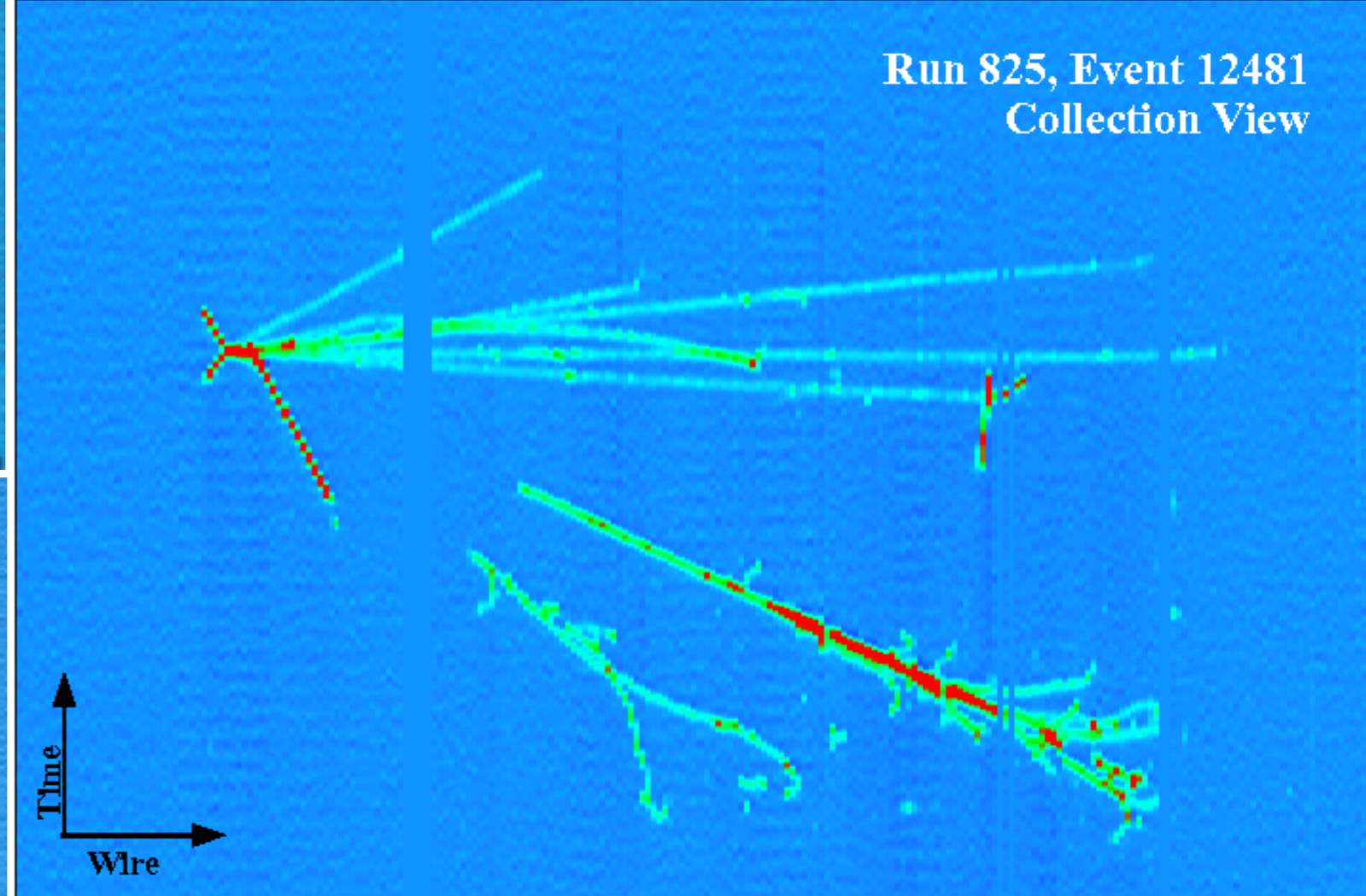
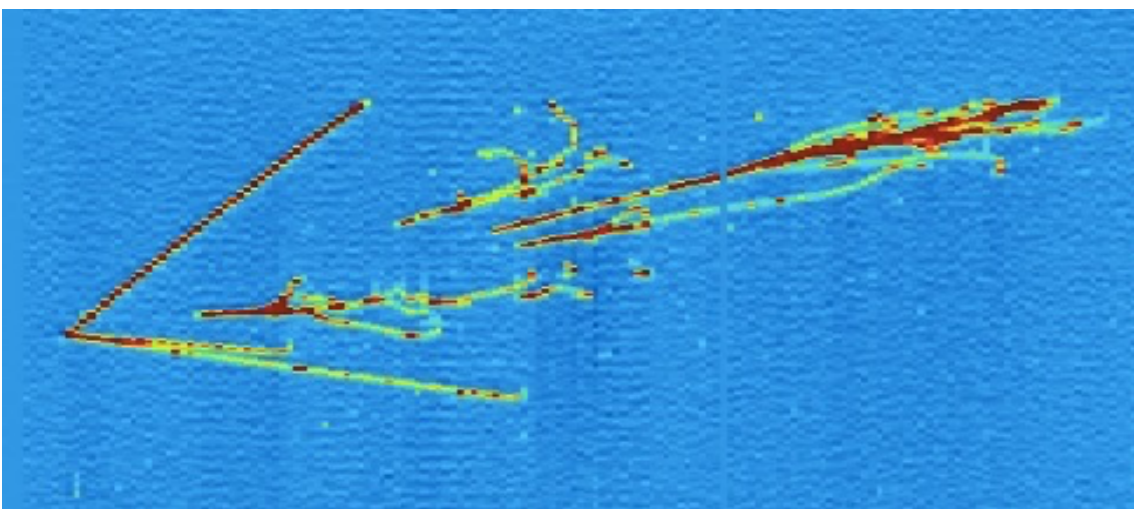
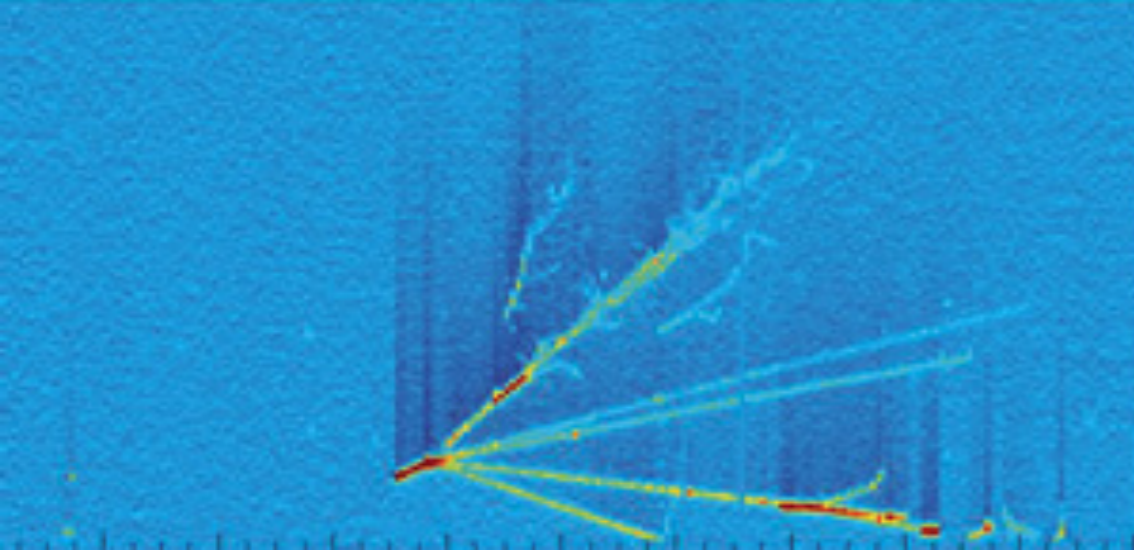
Charge collected
 by wire plane



Scintillation Light
 detected by PMTs

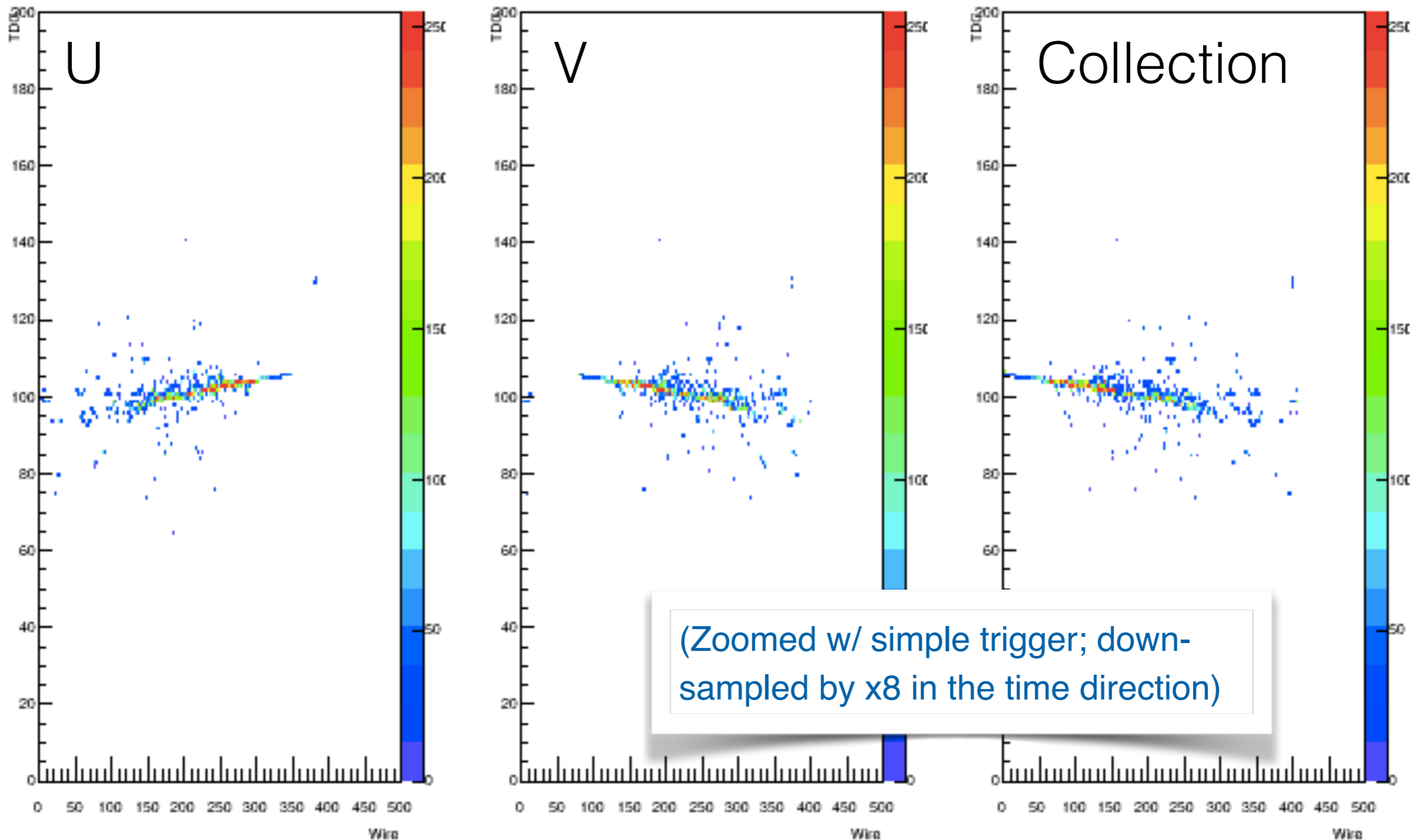
32 eight-inch PMTs for
 scintillation light (fast)

Figures: Kazuhiro Terao, SLAC &
 Andy Furmanski, Manchester



DUNE TPC Images

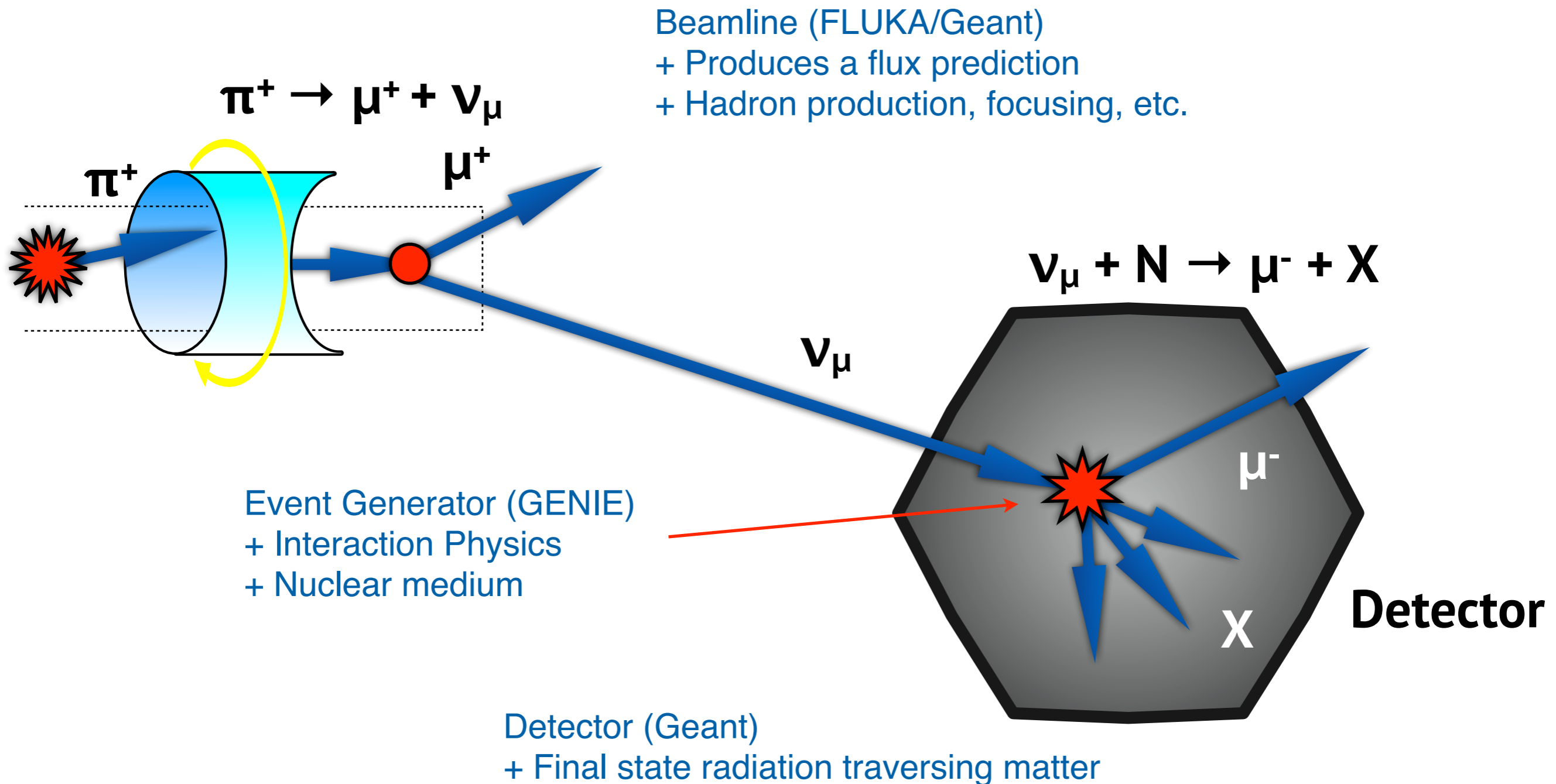
Sparsity (40,000 tons of LAr) is a major challenge (but no cosmic background).



May not be able to downsample for supernova burst or proton decay triggers.

Alexander Radovic
College of William and Mary

Neutrino Simulations: A Three-Part Software Stack



Neutrino MC Event Generators



- The generator must simulate all the types and momenta of every particle that appears in the final state.
- Some generators (MadGraph, Pythia, etc.) are computation aids for theorists, but most neutrino event generators are not (GiBUU is somewhat different).
- This is because we lack a theoretical framework that is both *complete* and *consistent*.
- The ideal input theory would be internally consistent and provide fully-differential cross sections in the kinematics of *every* final state particle over all reaction mechanisms, energies, and targets.
 - But the experiments must go on! So we must *stitch together* an ensemble that is consistent with all the data.

What else do neutrino event generators provide?



- Interfaces to geometry engines for modeling complex detectors.
- Flux drivers for computing exposure (atmospheric/solar sources) or normalizing responses to accelerator beams.
- Event re-weighting engines for studying systematic uncertainties and performing error propagation.
- Databases of electron, hadron, and neutrino scattering experiments with applications for comparing simulation and data.
 - Electron and hadron scattering event generator functionality.
- Nucleon decay generators.
- Libraries of pre-computed cross sections.

Pieces (Usually)



- Vertex selection
 - Simple nuclear density model
- Initial state nuclear model
 - Removal energy and momentum
 - RFG with Bodek-Ritchie tails.
 - New: Local Fermi Gas
 - New: Effective Spectral Function
 - Almost there: "Benhar" spectral function
 - Just started: Correlated Fermi Gas (MIT)

GROUND STATE

- Hard scattering process
 - Differential cross section formula to get event kinematics (x, y, Q², W, t, etc.)
- Lepton kinematics
- Hadronic system

INITIAL STATE

- Propagation/transport (default is an "effective cascade")
 - Fast and re-weightable

FINAL STATE

Pieces (Usually)

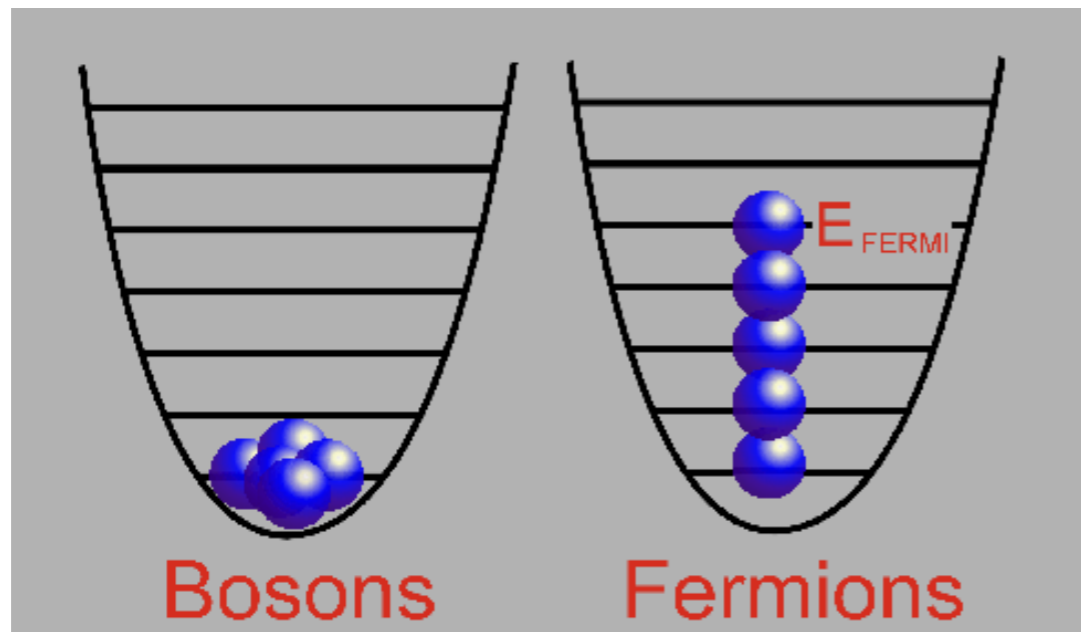


- Decays before and after propagation
- Remnant decay
 - Just started caring about this, really...
 - Current model is very simple

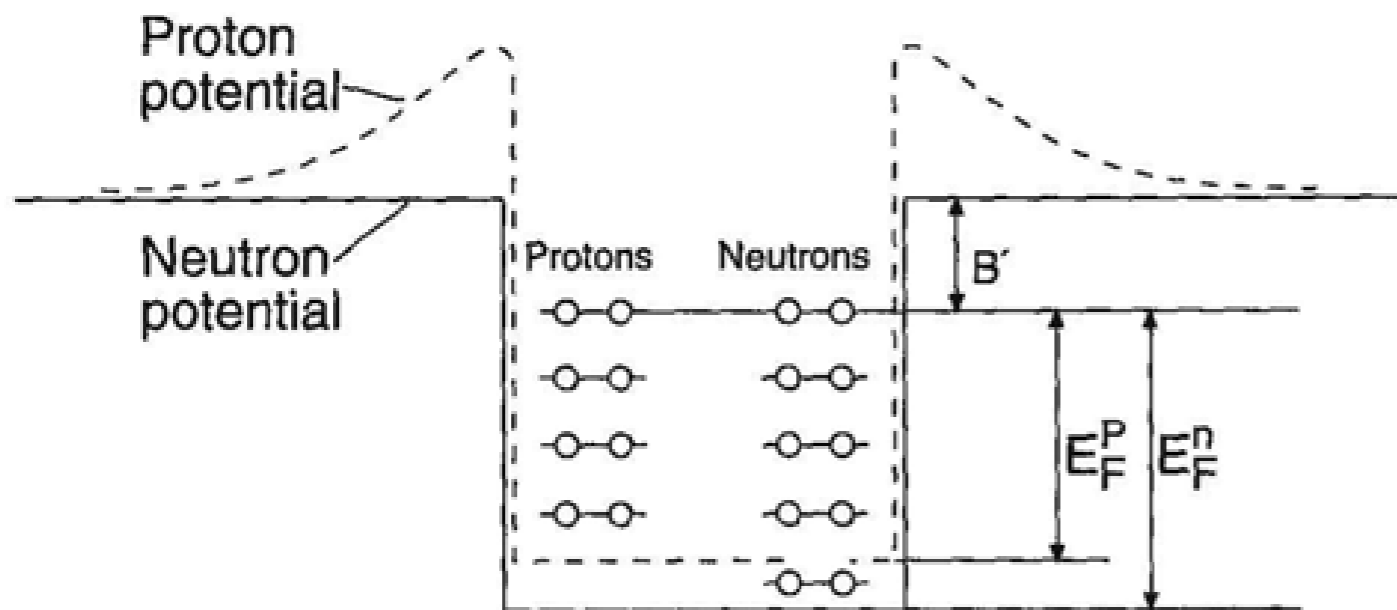
REMNANT STATE

- Working on adopting other codes (Geant4, INCL++, possibly GiBUU) to handle clustering, de-excitation, evaporation
 - May be a bridge to more sophisticated transport codes
-
- Sometimes models can't work this way - e.g., discovering we can't separate lepton and hadron kinematics into separate modules for QE events (can't compute cross section in Q2 and then compute lepton and hadron kinematics, need to flip the procedure and then accept-reject based on Q2), etc.
 - (Actually, we should do all events this way - but the code runs much slower and so we're working on ways to make that process fast enough to be more widely used.)

Nucleon not at rest: Fermi Gas Model



- **Impulse approximation:** scatter off independent single nucleons summed (incoherently) over the nucleus.
- In the FGM, all the nucleons are non-interacting and all states are filled up to k_F .
- The IA becomes problematic when the momentum transfer is *smaller* than ~ 300 MeV (think about the de Broglie wavelength and remember $1 \text{ fm} = 1/200 \text{ MeV}$).



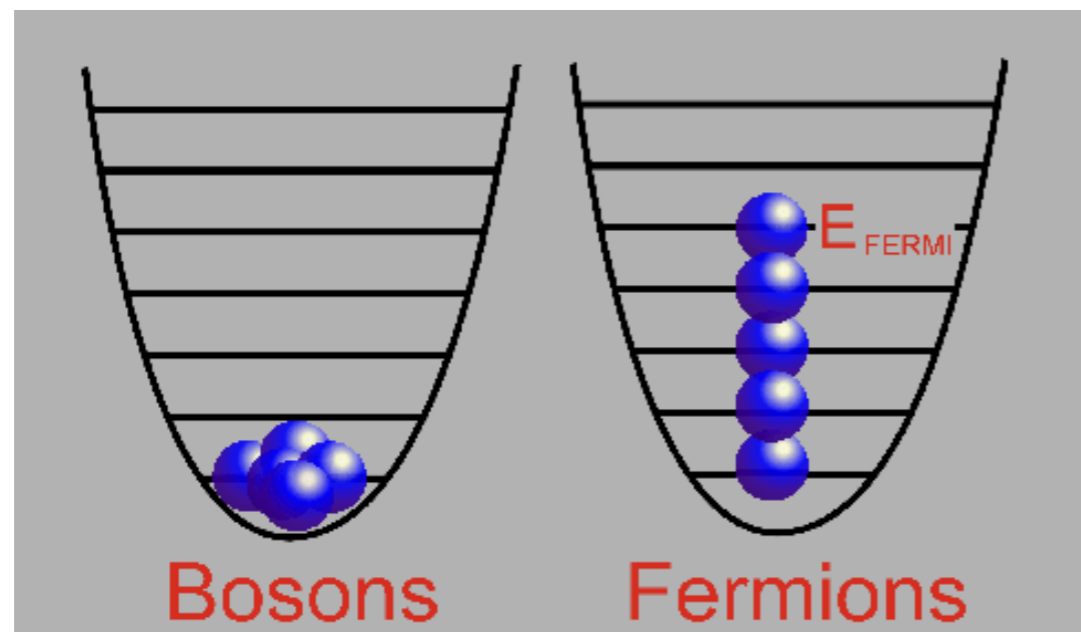
B. Povh et al, *Particles and Nuclei*, Springer 2002

^{12}C	$E_B = 25 \text{ MeV}$	$p_F = 220 \text{ MeV}/c$
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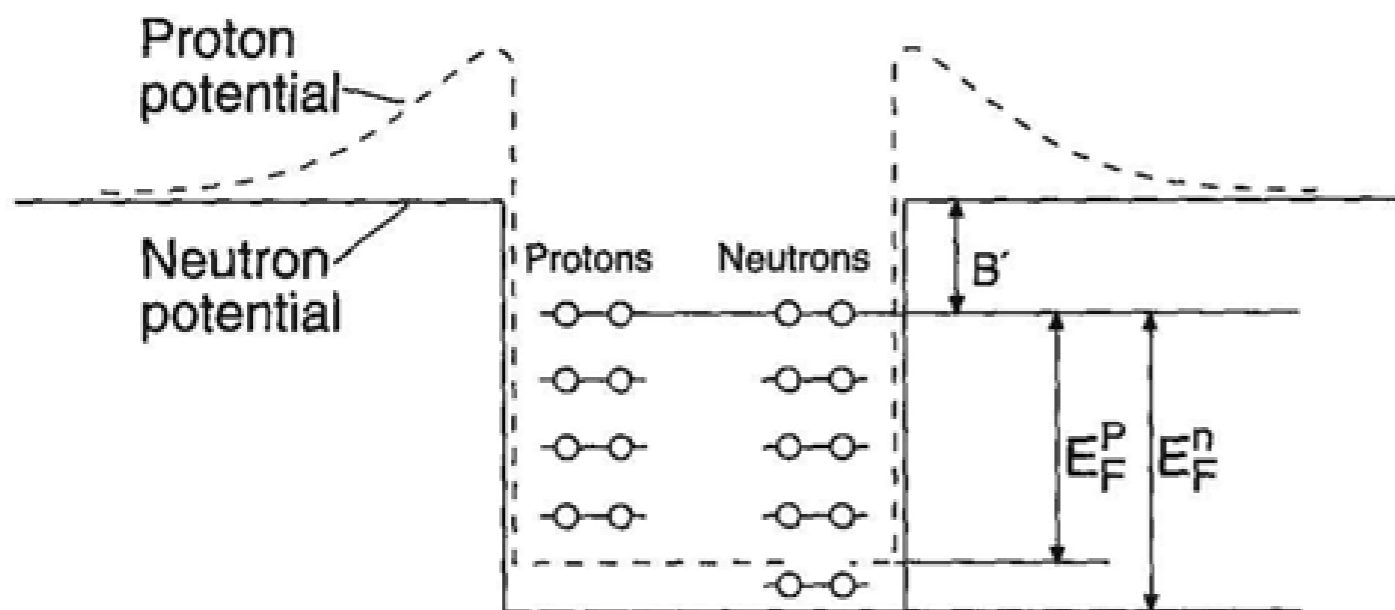


It is nice to see this problem getting high-level attention.

Nucleon not at rest: Fermi Gas Model



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- In the FGM, all the nucleons are non-interacting and all states are filled up to k_F .
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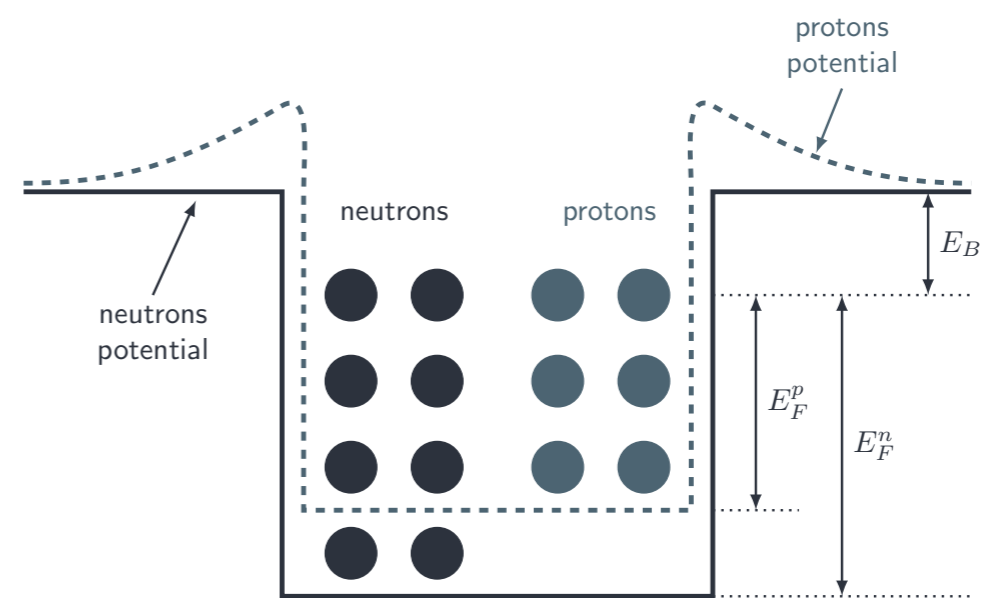
B. Povh et al, *Particles and Nuclei*, Springer 2002

^{12}C	$E_B = 25 \text{ MeV}$	$p_F = 220 \text{ MeV}/c$
-----------------	------------------------	---------------------------

You can't use the Fermi Gas Model anymore!



Nucleons move freely within the nuclear volume in constant binding potential.



Global Fermi Gas

Local Fermi Gas

$$p_F = \frac{\hbar}{r_0} \left(\frac{9\pi N}{4A} \right)^{1/3}$$

$$p_F(r) = \hbar \left(3\pi^2 \rho(r) \frac{N}{A} \right)^{1/3}$$

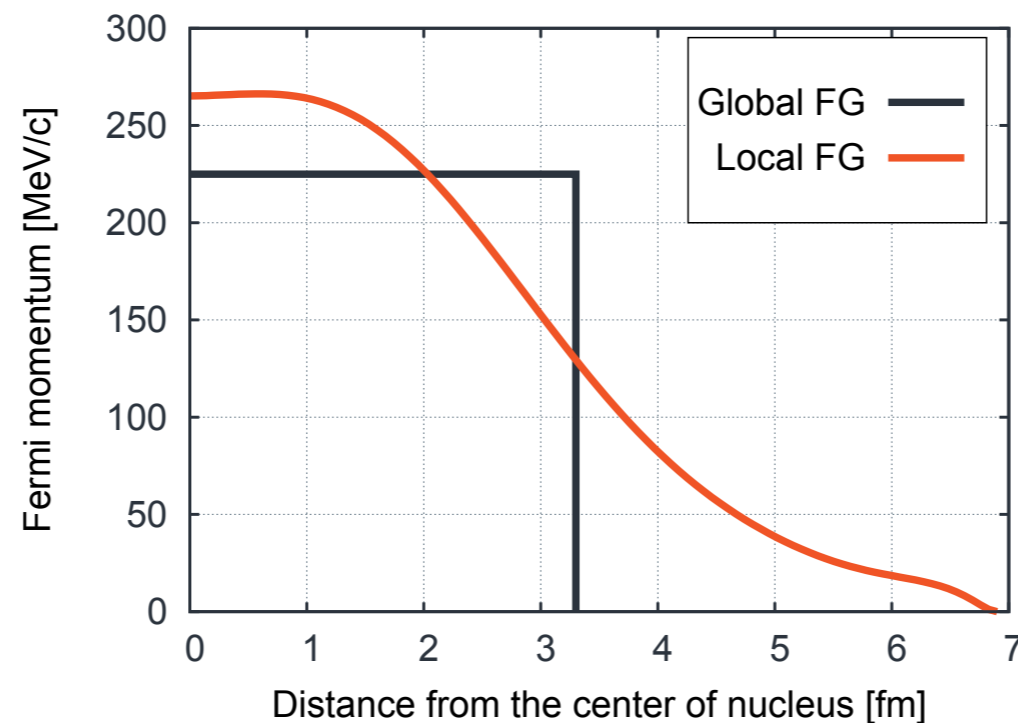


Figure by T. Golan

QE Cross Section

ν Cross Section:
$$\frac{d\sigma}{dQ^2} = \frac{M^2 G_F^2 \cos^2 \theta_c}{8\pi E_\nu^2} \left[A(Q^2) \pm B(Q^2) \frac{s-u}{M^2} + C(Q^2) \frac{(s-u)^2}{M^4} \right]$$

- Early formalism by Llewellyn Smith.
- Vector and Axial-Vector Components.
 - Vector piece can be lifted from (“easier”) electron scattering data.
 - We have to measure the Axial piece.
- Q^2 is the 4-momentum transfer ($-q^2$).
- s and u are Mandelstam variables.
- The lepton vertex is known; the nucleon structure is parameterized with 2 vector (F_1, F_2) and 1 axial-vector (F_A) form factors.
 - Form factors are $f(Q^2)$ and encoded in A, B , and C .

C. H. Llewellyn Smith, Phys. Rept. 3 261 (1972).

Form Factors

$$\begin{aligned} A &\simeq \frac{t}{M^2} \left(|f_{1V}|^2 - |f_A|^2 \right) + \frac{t^2}{4M^2} \left(|f_{1V}|^2 + \xi^2 |f_{2V}|^2 + |f_A|^2 + 4\xi \operatorname{Re}(f_{1V} f_{2V}^*) \right) \\ &\quad + \frac{t^3 \xi^2}{16M^6} |f_{2V}|^2 \\ B &\simeq \frac{1}{M^2} \left(\operatorname{Re}(f_{1V} f_A^*) + \xi \operatorname{Re}(f_{2V} f_A^*) \right) t \\ C &= \frac{1}{4} \left(|f_{1V}|^2 + |f_A|^2 - \frac{\xi^2 |f_{2V}|^2}{4M^2} t \right) \end{aligned}$$

$$f_A(q^2) = \frac{f_A(0)}{\left(1 - \frac{q^2}{M_A^2}\right)^2}$$

f_A is the axial-vector form factor. We must measure this in V-scattering. The dipole has been dominant, but that is changing...

The **form factors** (f) contain parameterized information about the target (general shape of the form factors comes from symmetry arguments).

Not calculable from first principles, instead we measure them experimentally.

Lattice QCD and neutrino-nucleus scattering

- New neutrino-nucleon data will be hard to come by, making lattice contributions potentially critical.
- We have **started to phase out the dipole form factor in favor of the model-independent z-expansion** (lattice program ongoing to compute the elements of the z-expansion):

z-expansion: conformal mapping taking kinematically allowed region ($t = -Q^2$) to $|z| < 1$.

PHYSICAL REVIEW D **84**, 073006 (2011)
Model-independent determination of the axial mass parameter in quasielastic neutrino-nucleon scattering

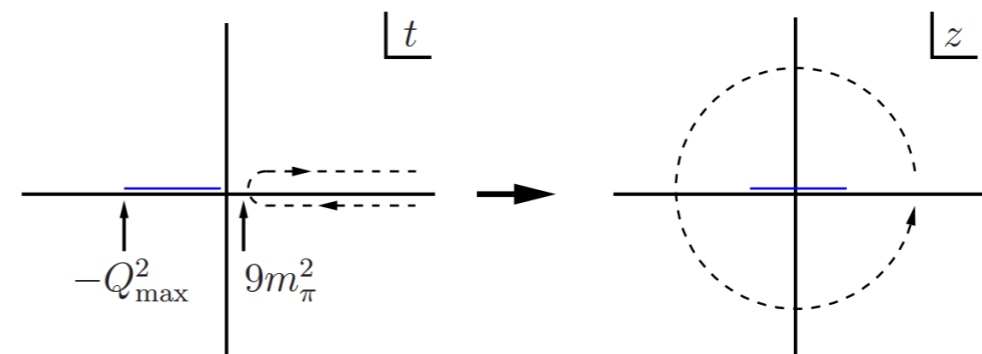
Bhubanjyoti Bhattacharya, Richard J. Hill, and Gil Paz

$$F_A^{\text{dipole}}(Q^2) = \frac{g_A}{\left(1 + \frac{Q^2}{m_A^2}\right)^2}$$

(Llewellyn-Smith, 1972)

[Phys.Rept.3 (1972),261]

$$z(t; t_0, t_c) = \frac{\sqrt{t_c - t} - \sqrt{t_c - t_0}}{\sqrt{t_c - t} + \sqrt{t_c - t_0}} \quad F_A(z) = \sum_{n=0}^{\infty} a_n z^n \quad t_c = 9m_\pi^2$$



Aaron S. Meyer (asmeyer2012@uchicago.edu)

University of Chicago/Fermilab

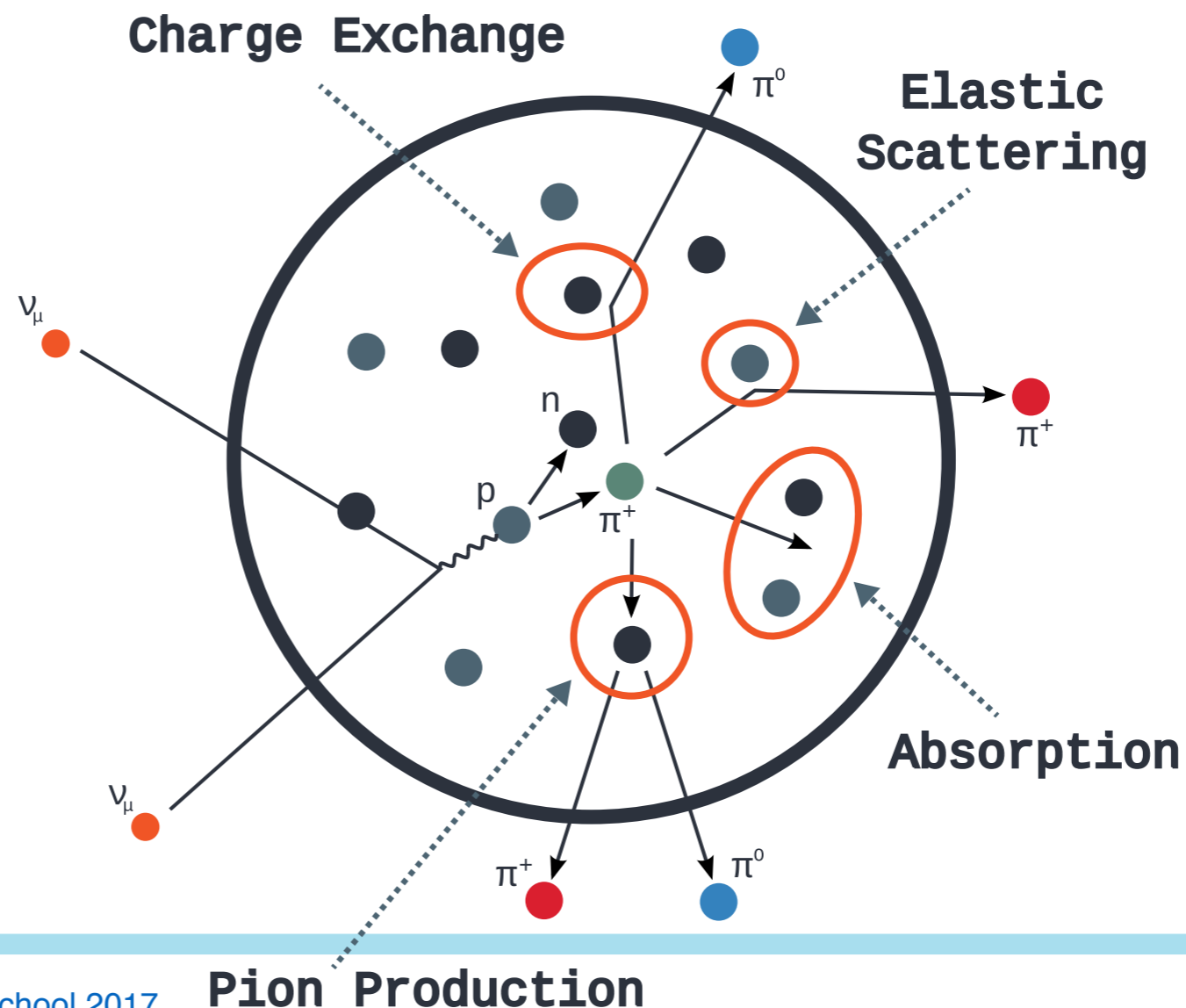
Radiative Corrections at the
Intensity Frontier of Particle Physics

Final State Interactions

- Hadrons produced at the hard-scattering vertex must propagate out of the nucleus - very complex process (everything is an off-shell, many-bodied, non-perturbative, strongly coupled mess).
- Three ways of handling it on the market: transport theory (GiBUU - <http://gibuu.hepforge.org> - best theory), intranuclear cascade (“billiard balls”), parameterized cascade.

Figure by T. Golan

Plus, much more from T. Golan on generators later in this school...



NuWro

- ▶ NuWro is not an official MC in any experiment and serves as a laboratory for new developments.
- ▶ New (or relatively new) ingredients:
 - ▶ Berger-Sehgal coherent pion production
 - ▶ π momentum distribution from Δ decay
 - ▶ effective density and momentum dependent potential for CCQE (C. Juszczak, J. Nowak, J. Sobczyk)
- ▶ eWro - electron scattering module (a work in progress) C. Juszczak, K. Graczyk, JTS, J. Zmuda

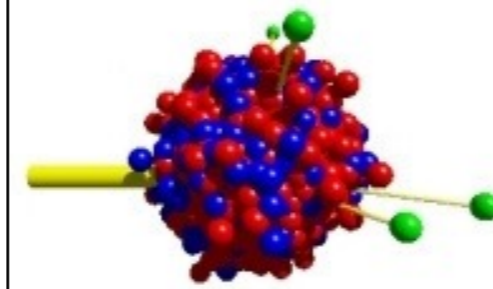
Jarek Nowak, Lancaster University

IPPP/NuSTEC topical meeting on
Neutrino-Nucleus scattering

- <http://school.genie-mc.org> (lecture by T. Golan)
 - Plus, this school!
- <https://github.com/NuWro/nuwro>
- <https://nuwro.github.io/user-guide/>



“Nature”



GiBUU

The Giessen Boltzmann-Uehling-Uhlenbeck Project

- <http://gibuu.hepforge.org>
- Strives to use the “best possible theory” in all cases.

PLUS, new in 2017 (just last week!)... see their site.

■ *Initial interactions:*

- Mean field potential with local Fermigas momentum distribution, nucleons are bound (not so in generators!)
- Initial interactions calculated by summing over interactions with all bound, Fermi-moving nucleons
- 2p2h from electron phenomenology

■ *Final state interaction:*

- propagates outgoing particles through the nucleus using *quantum-kinetic transport theory*, fully relativistic (off-shell transport possible).

Initial and final interactions come from the same Hamiltonian.

CONSISTENCY of inclusive and semi-inclusive X-sections

Ulrich Mosel

New in 2016:

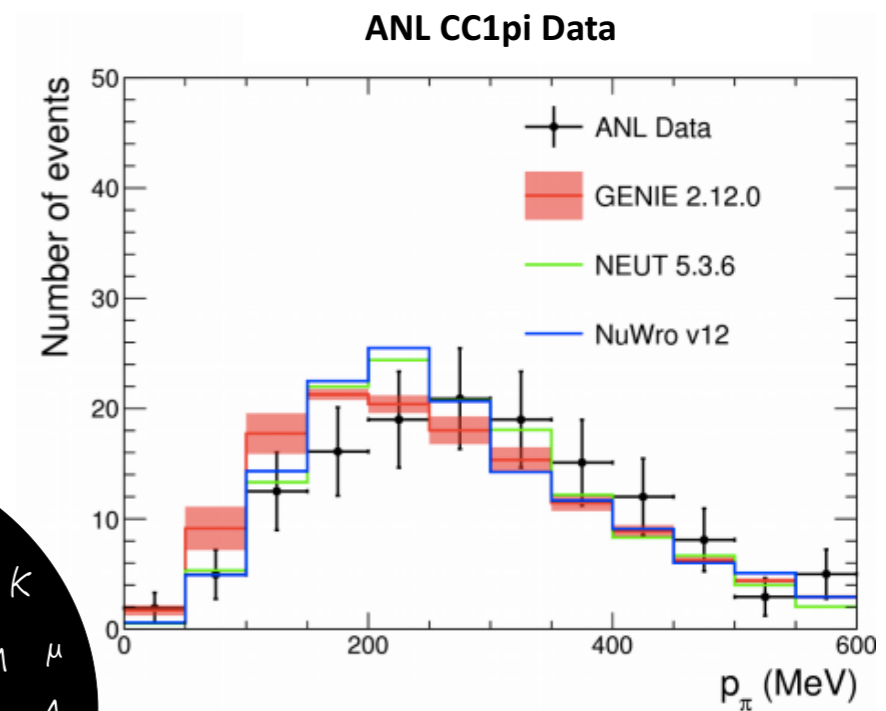
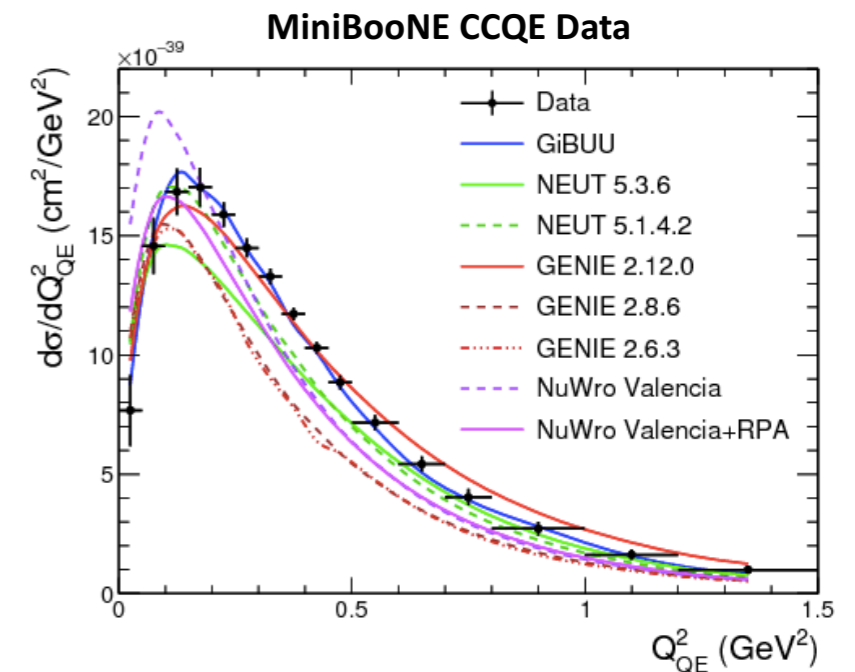
IPPP/NuSTEC (Durham) 2017

- Stable groundstate implemented -> improved hole spectral functions
- 2p2h structure function for all kinematics, fitted to e-scattering, is used for neutrinos as well

K. Gallmeister at this school...

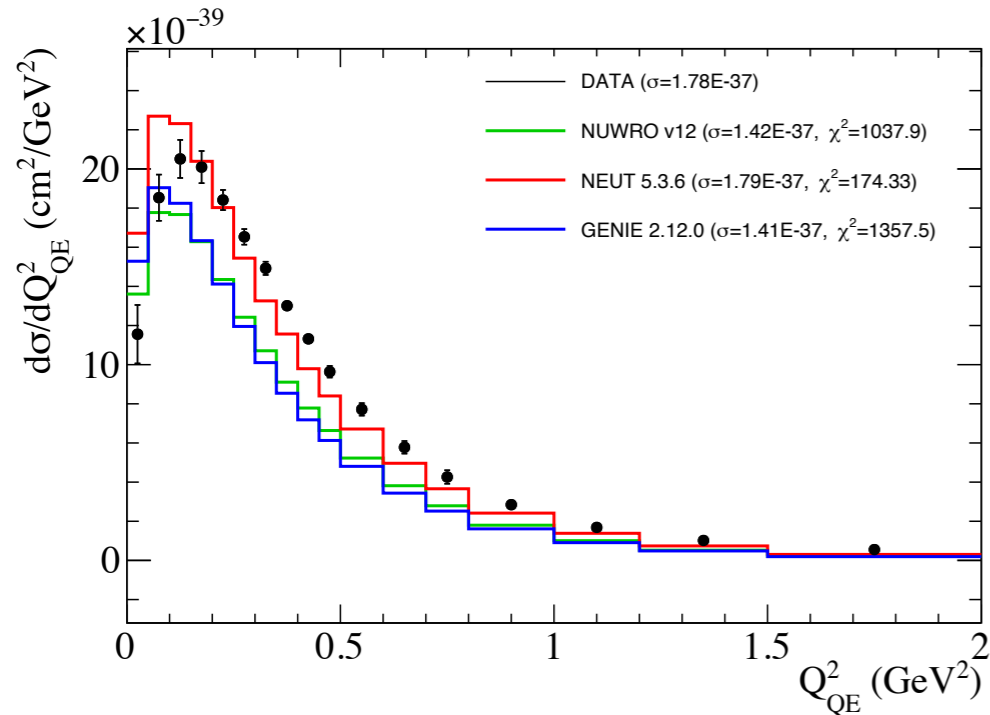
NUISANCE

- Open source generator tuning framework.
 - Tools for comparison plots, re-weighting (for NEUT, NuWro, and GENIE), fitting.
 - Interfaces to re-weighting tools in generators; can add ad-hoc weights as well.
 - Tuning mechanisms include support for priors via penalties in the likelihood.
 - Migrad & Bayesian tuning.
 - Reproducible results via job cards.
- <http://nuisance.hepforge.org>
 - Plus P. Stowell at this school!

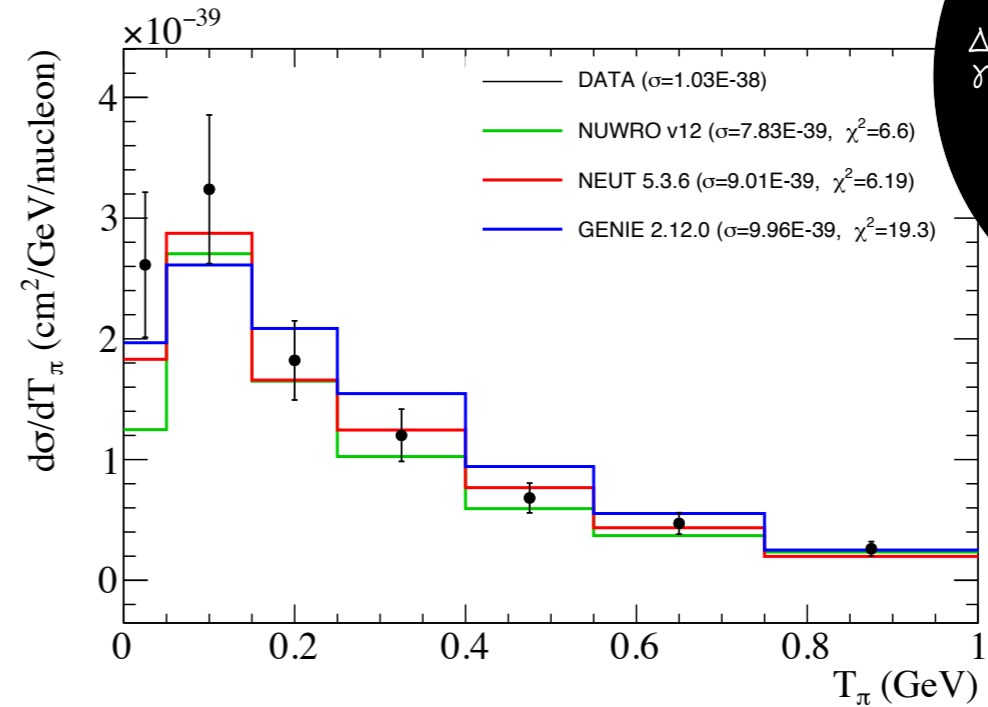




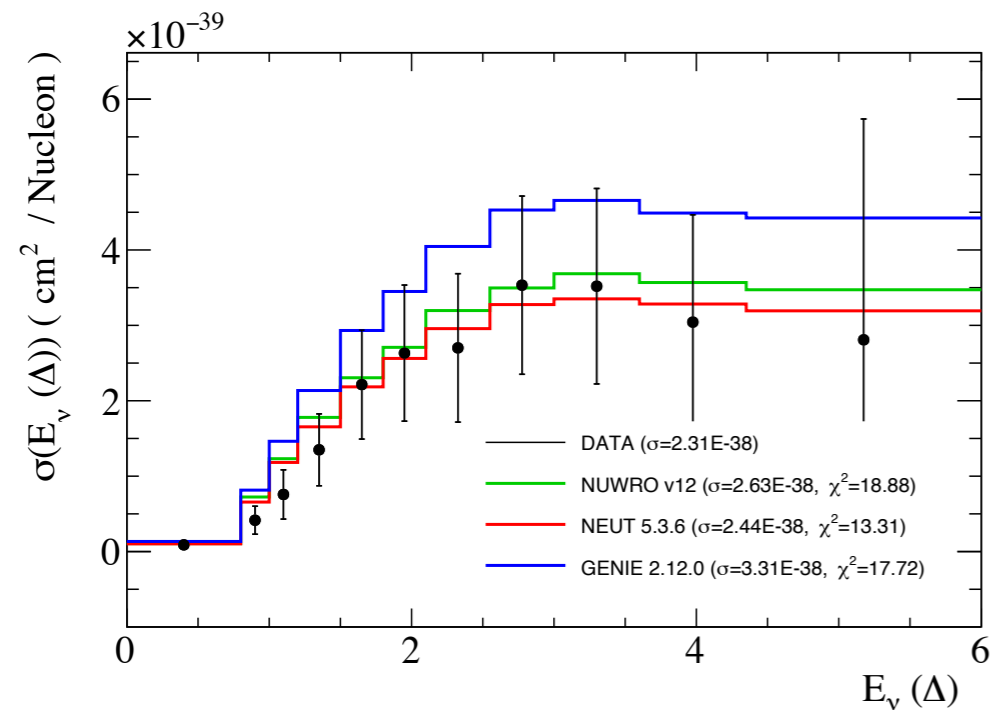
MiniBooNE CCQElike XSec 1DQ² nu data



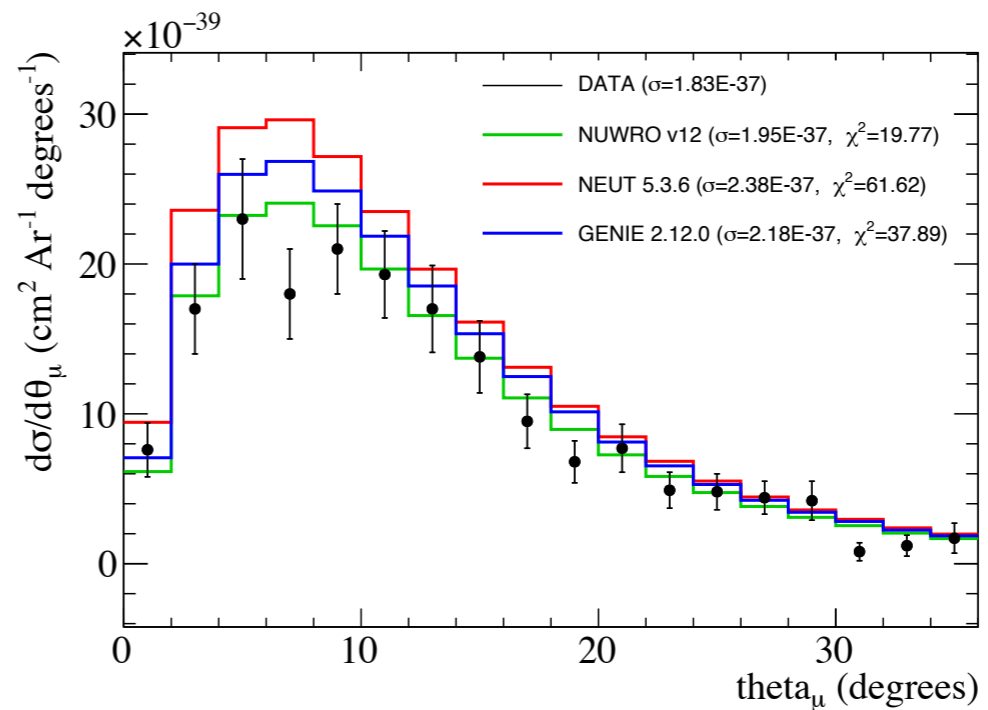
MINERvA CC1pi0 XSec 1DTpi0 antinu 2016 data



T2K CC1pip H2O XSec 1DEnuDelta nu data



ArgoNeuT CCInc XSec 1Dthetamu antinu data



http://nuisance.hepforge.org/files/validation/nuisancevalidation_v1r0_280217/nuisance_v1r0_validation_280217.pdf

The stage is set...

- The title of this talk was "The Practical Beauty of Neutrino-Nucleus Interactions".
- Hopefully everyone is convinced of the "practical" part, but the "beautiful?"



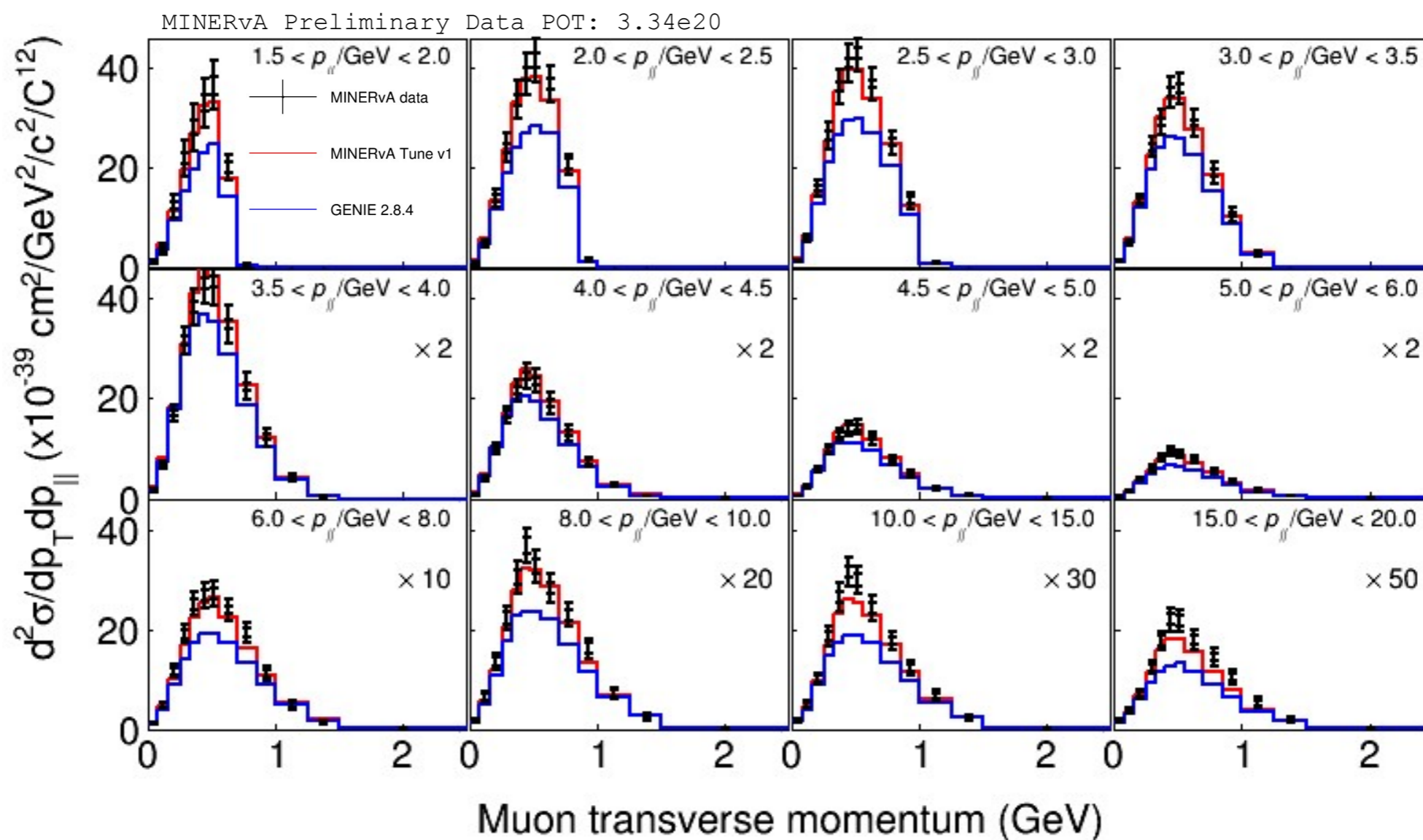
MINERvA double-differential CCQE-like cross sections



Fiducial, QE-like:

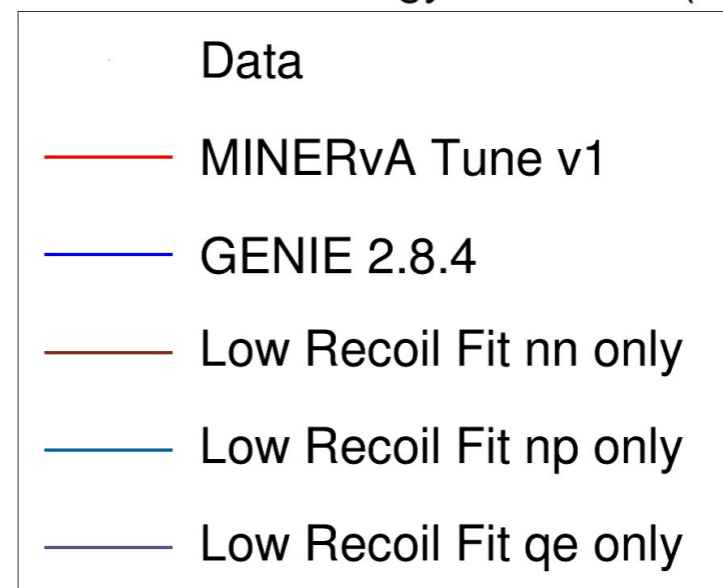
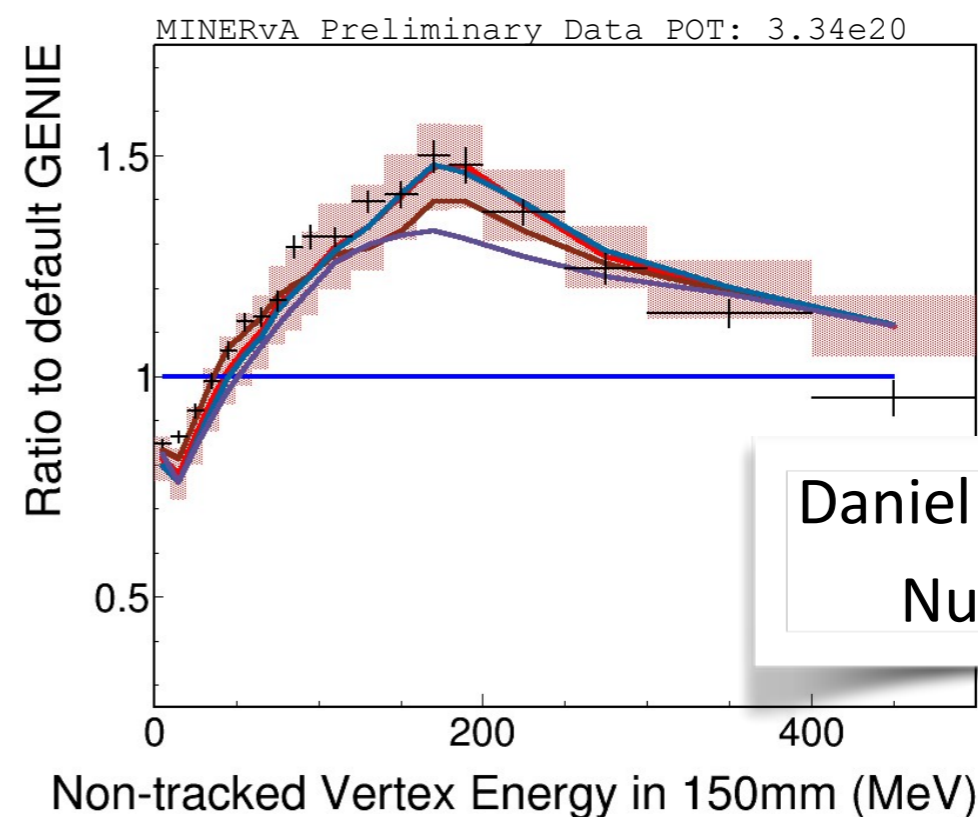
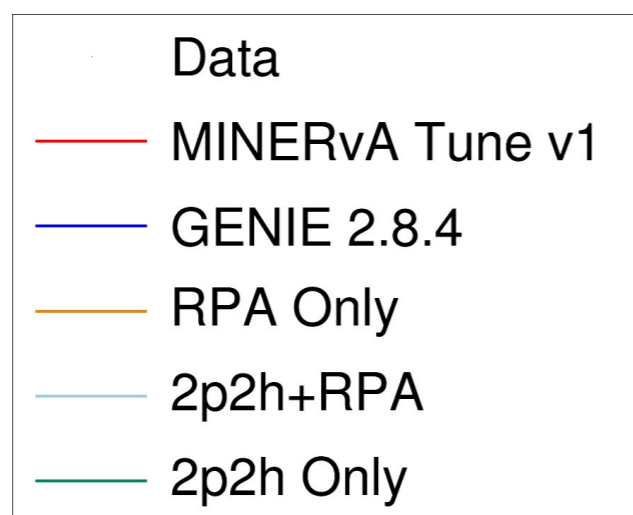
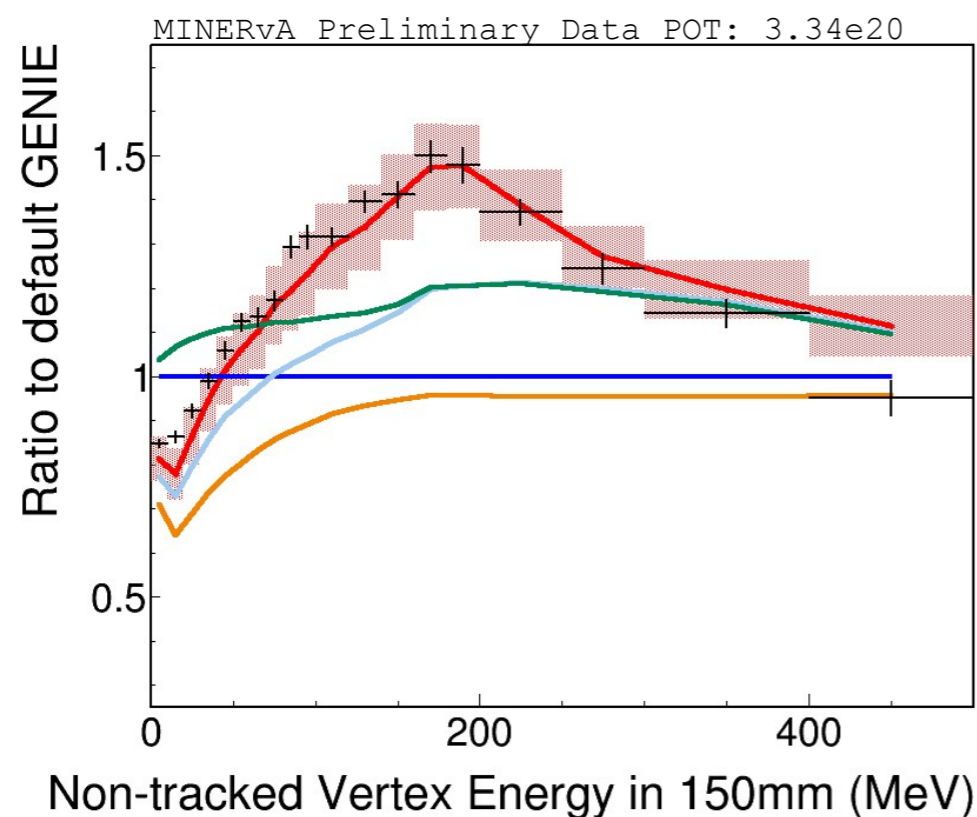
ν -Result

Daniel Ruterbories
NuInt 2017



Similar results available for anti-neutrinos, see C. Patrick FNAL JETP, 2016 June 17

Double-differential CCQE-like vertex E: model evolution



Daniel Ruterbories
NuInt 2017

Valverde, Amaro, Nieves PLB 638 (2006) 325 with unpub. followup by F. Sanchez plus **muon capture uncertainty** and implementation R. Gran, arXiv:1705.02932

Nieves, Ruiz Simo, Vicente Vacas PRC83 (2011) 045501

Gran, Nieves, Sanchez, Vicente Vacas PRD 88 (2013) 113007

Rodrigues, Demgen, Miltenberger et al. [MINERvA] PRL 116 071802

Organizing the challenges - NuSTEC

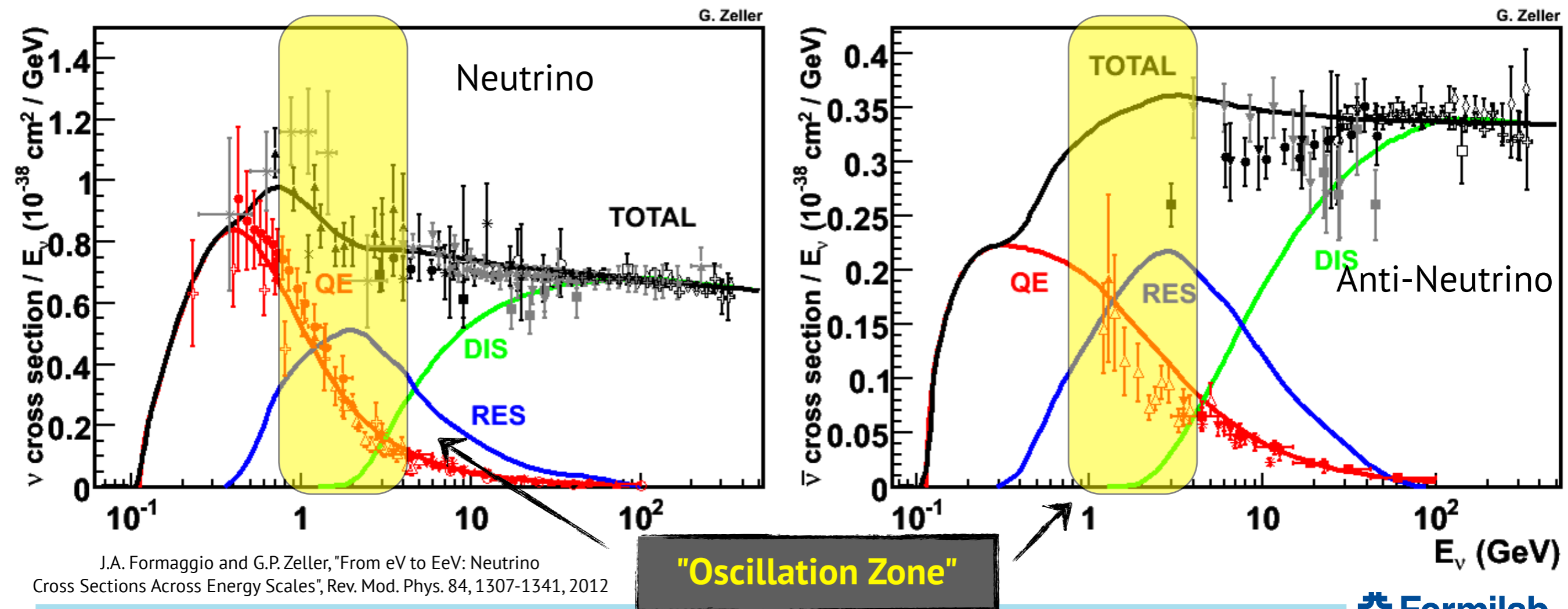
- New paper from NuSTEC (<http://nustec.fnal.gov>) outlines the **current challenges** facing the field of neutrino-nucleus scattering
 - <https://arxiv.org/abs/1706.03621>
- The paper summarizes
 - the impact of **interaction uncertainties on oscillation physics**,
 - the role of **event generators** in accelerator-based neutrino experiments,
 - how **electron-nucleus scattering** experiments inform our understanding of neutrino-nucleus scattering,
 - our **current understanding of the various interaction channels** (ranging from the elastic regime through deep inelastic scattering).
- After you're done with the school, give the paper a read!

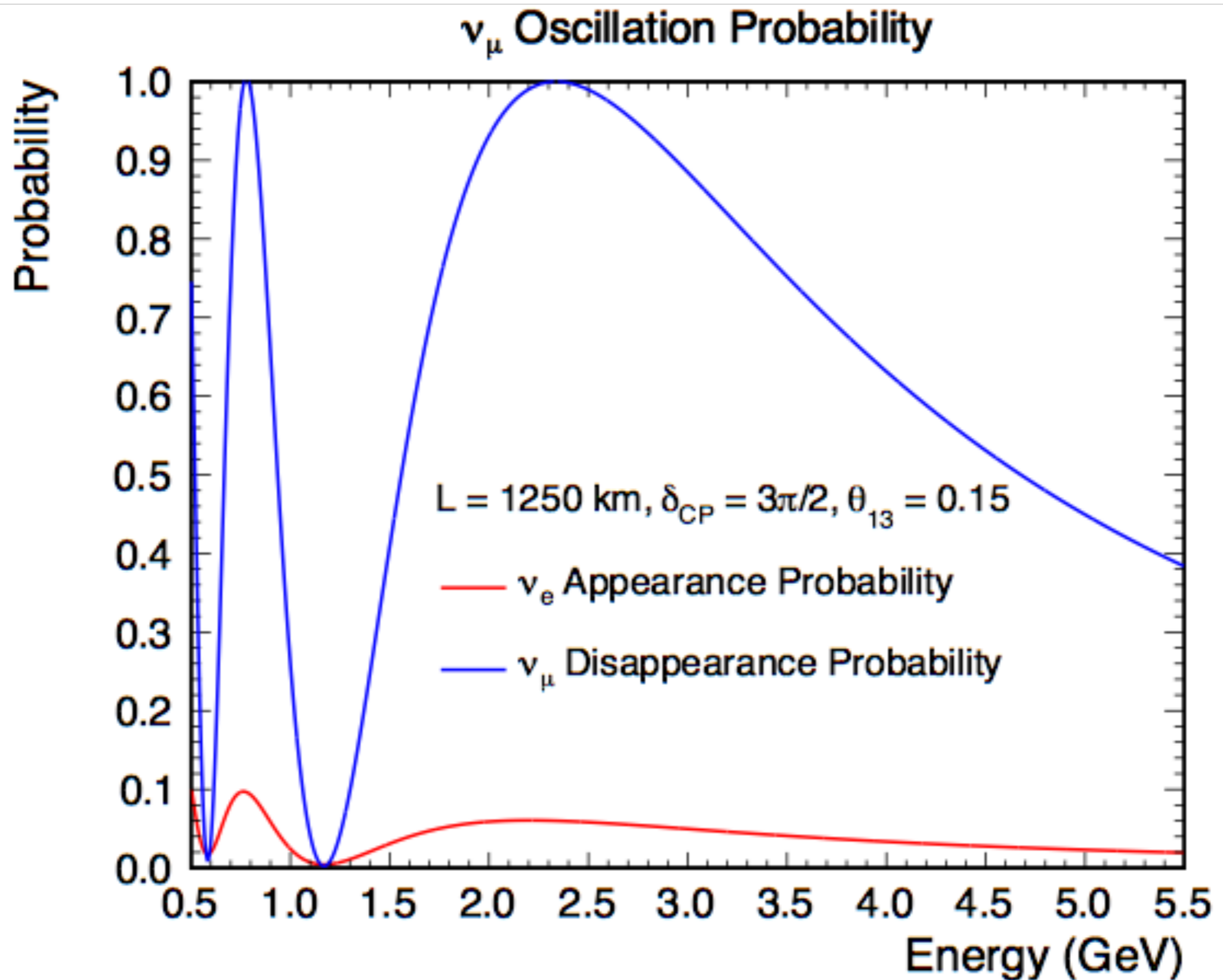
Thank you!

Backup

Embedded Assumptions

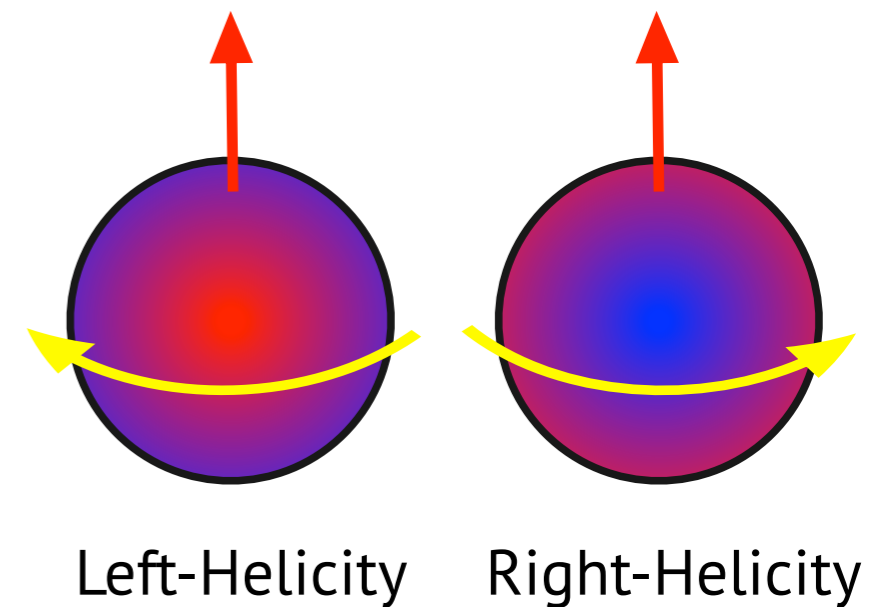
- There are a few facts that are often buried in the details of discussions of neutrino interactions:
 - Your knowledge of the flux is typically only good to 10-20% and you have *no information event-by-event*.
 - Kinematic distributions are always integrated over a *specific* (barely known) flux.
 - Measurements are always convolutions of flux, cross section, nuclear effects, and detector efficiencies.





And remember, we need to do it all over again for antineutrinos!

Helicity, Chirality, & Parody, oops, Parity!



- *The Weak force is left-handed.*
 - $(1-\gamma^5)$ projects onto **left-handed states** for **massless fermions** and **right-handed states** for **massless anti-fermions**.

$$\frac{1}{2} (1 - \gamma^5) \psi = \psi_L$$

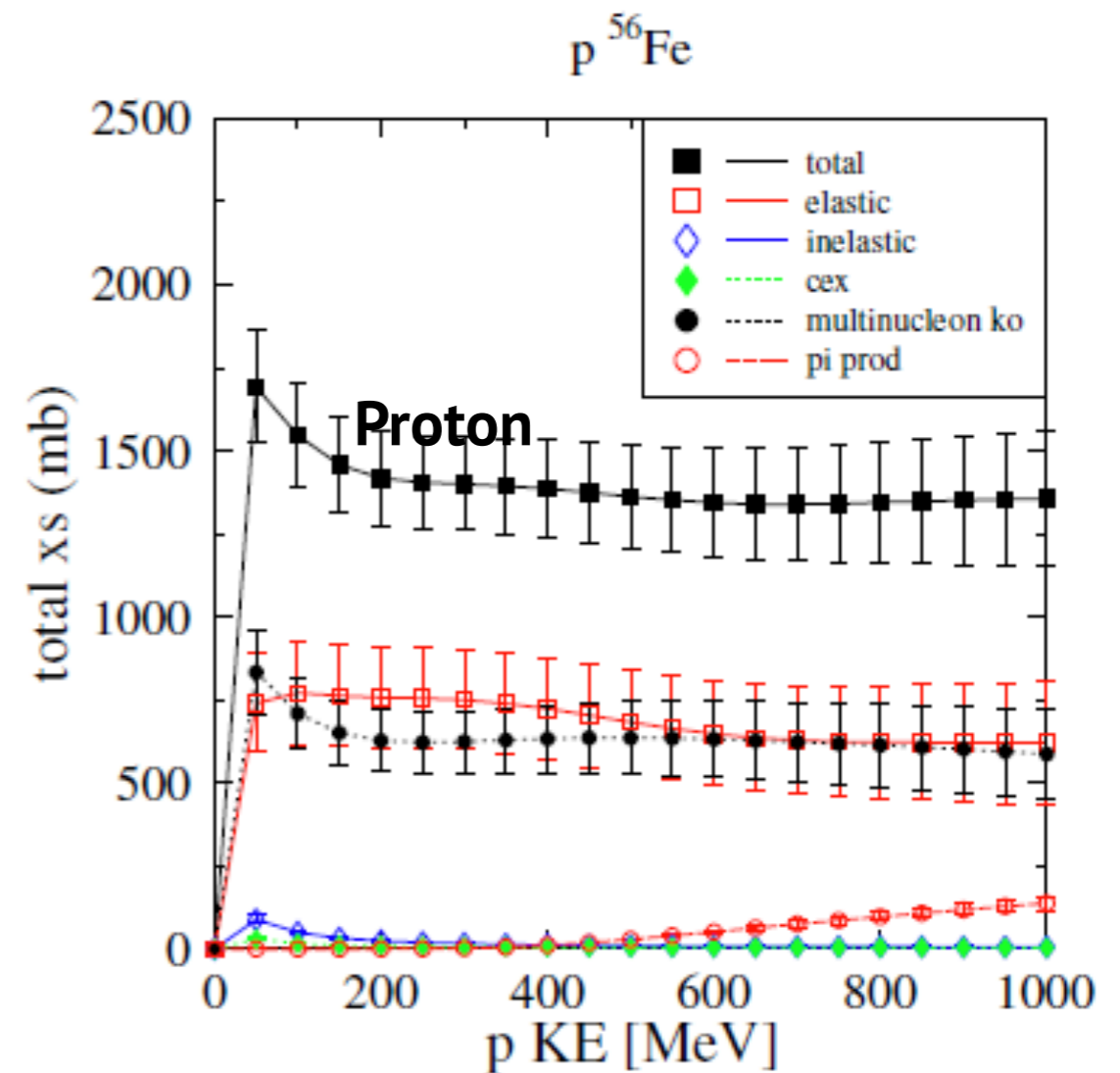
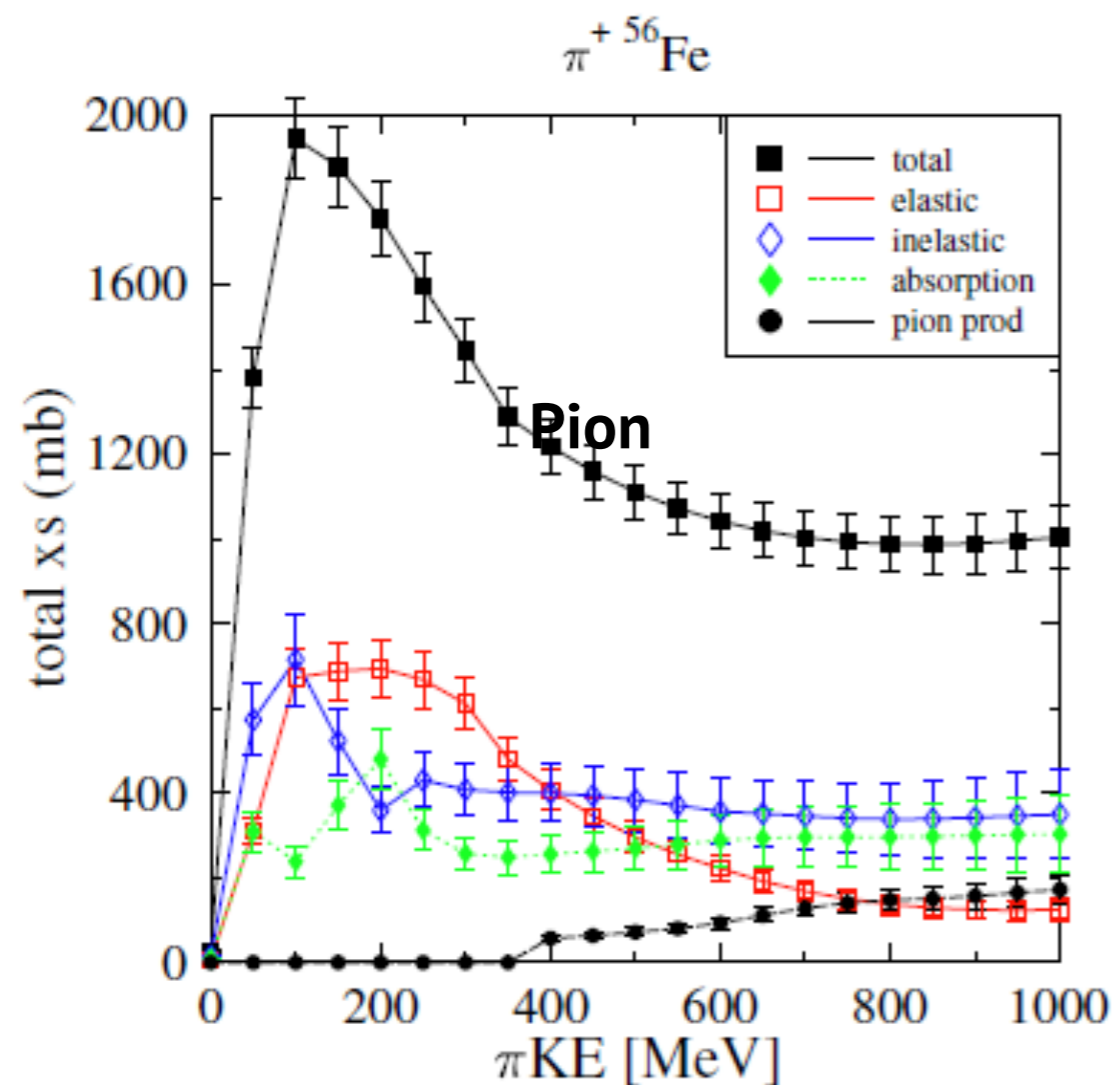
- **Helicity**
 - Projection of spin along a particle's momentum vector.
 - **Frame-dependent for massive particles.**

- **Chirality**

- Lorentz invariant version of helicity (= helicity for massless particles).
- It is determined by whether the particle transforms in a right or left-handed representation of the Poincaré group. Some representations (e.g. Dirac spinors) have right and left-handed components. We define projection operators that project out either the right or left hand components.

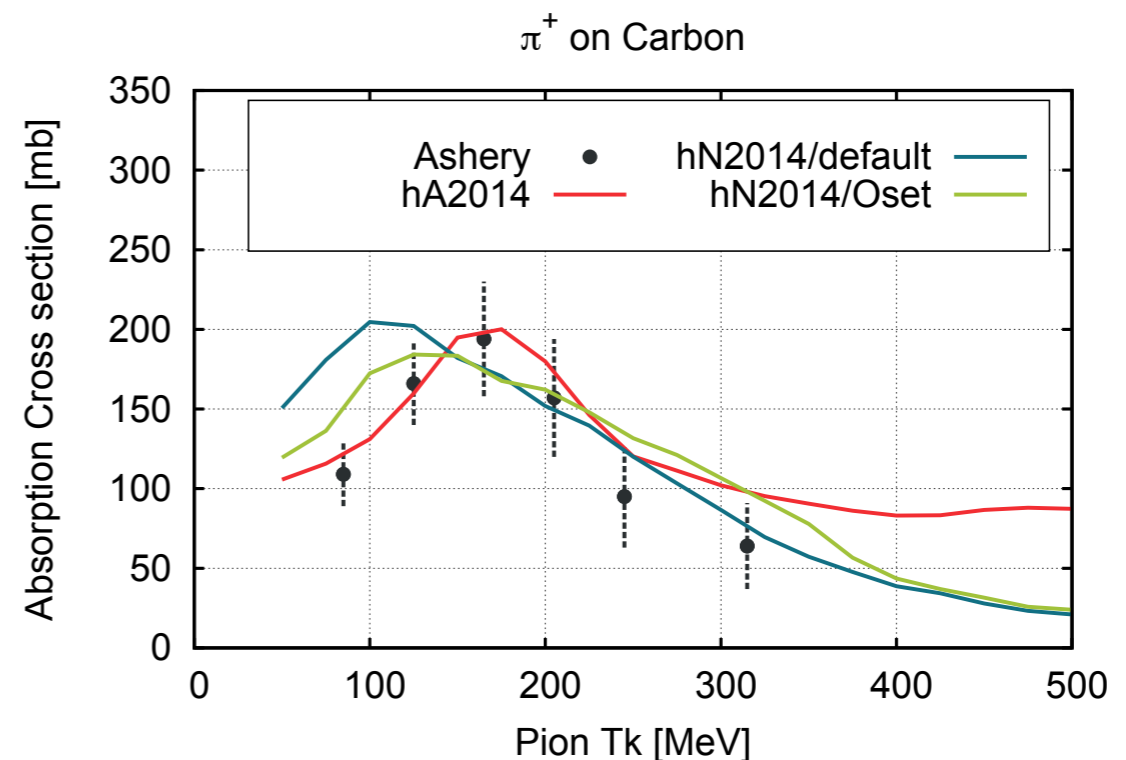
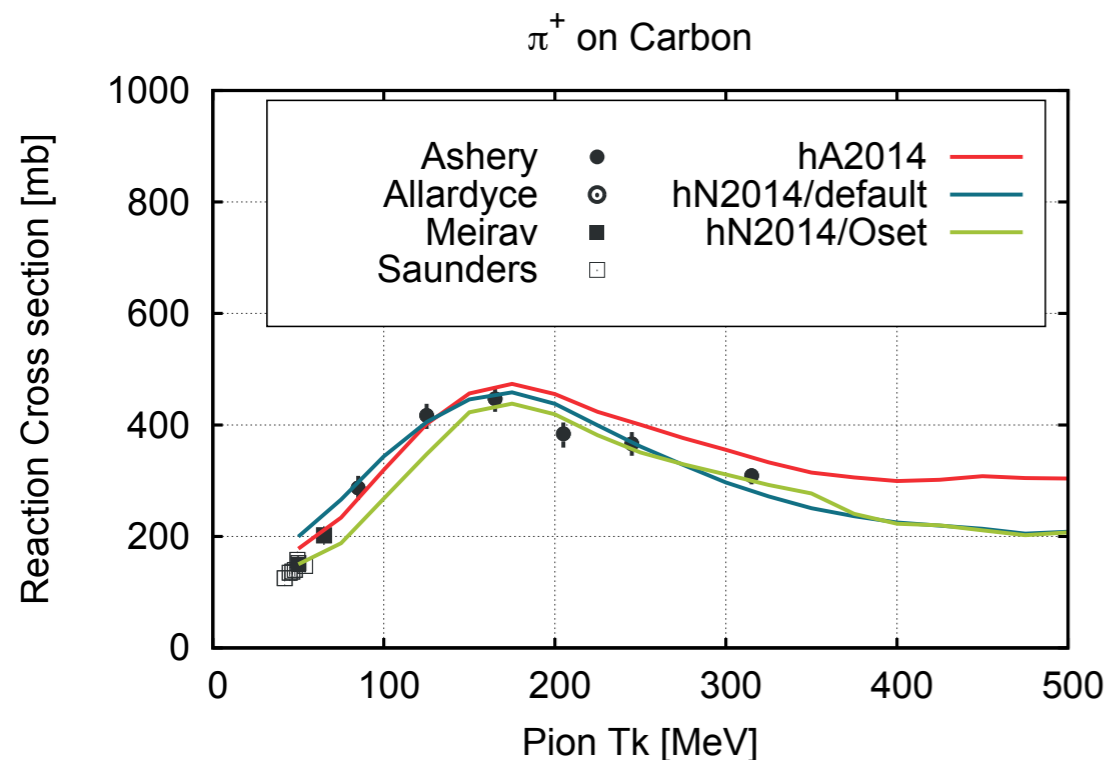
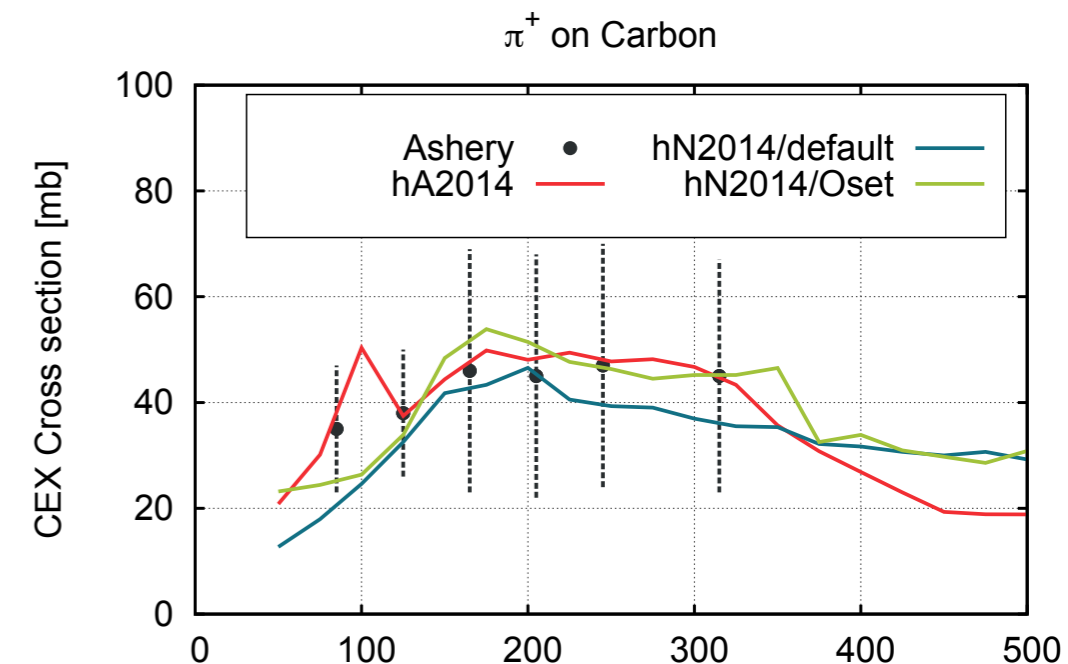
FSI Models

- GENIE: "hA" (default) - use iron reaction cross section data, isospin symmetry, and $A^{2/3}$ scaling to predict the FSI reaction rates.
- Individual particle energies and angles use data templates or sample from the allowed phase space.



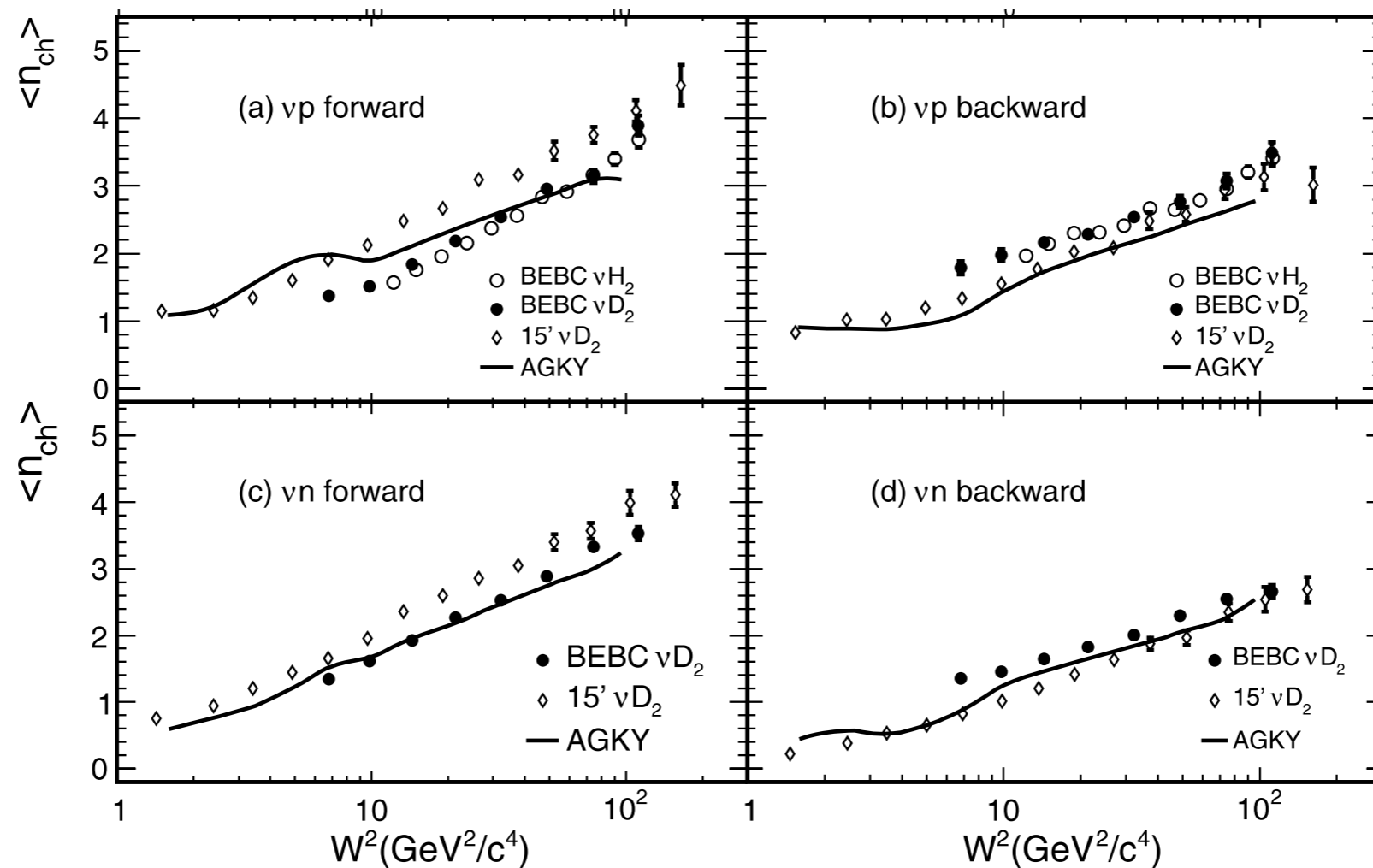
FSI Models

- GENIE "hN" is our cascade model.
- New to hN are: Oset et al, Nucl. Phys. A468 (1987), Oset et al, Nucl. Phys. A484 (1998)
- Model describes low energy (kinetic E around Delta peak, 85 MeV - 350 MeV) pion interactions inside nuclear matter.
 - Nuclear effects are implemented as modifications of the Delta width.
- Introduced here as a modification of the GENIE cascade model (hN). Modifications not yet filtered down into the parameterized (hA, default) model.



Modeling Nuclear Effects

- What about hadronization in the nuclear medium?
- We use Pythia (currently version 6, migration to 8 is on-going).
- GENIE does reasonably well, but the validation uses deuterium or hydrogen - little influence from nuclear effects.



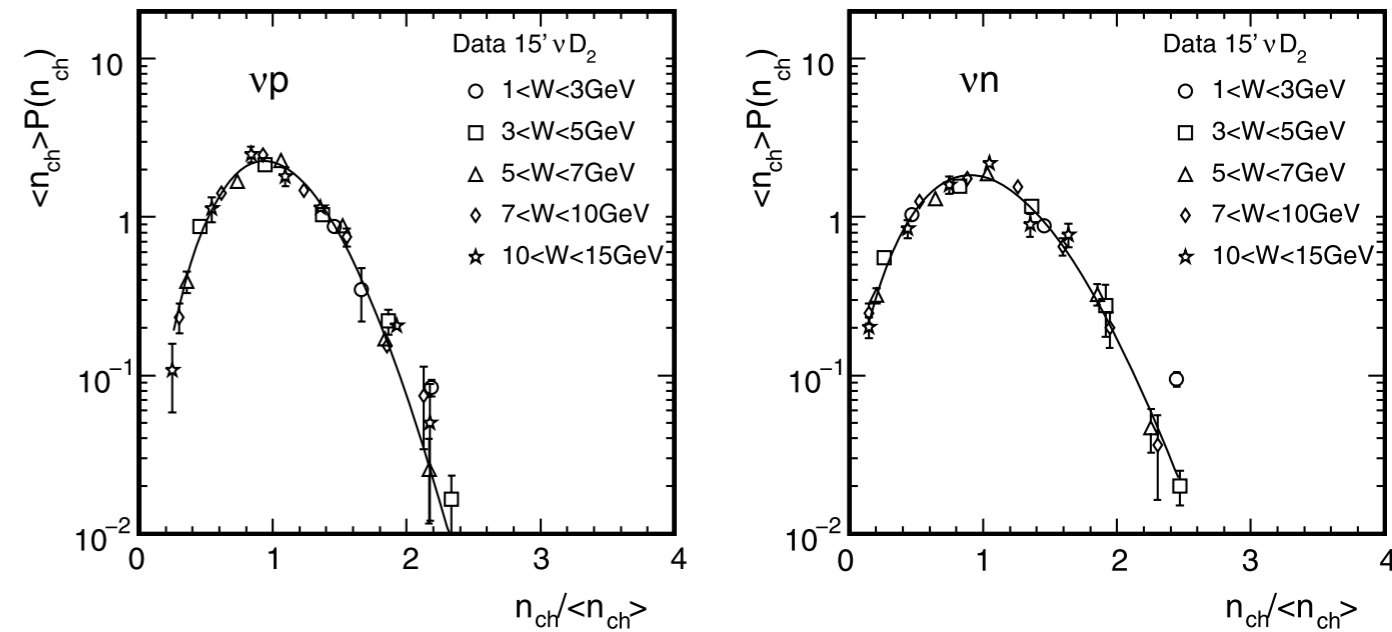
T. Yang et al, Eur. Phys. J C (2009) 63:1-10

AGKY Hadronization

The AGKY model, which is now the default hadronization model in the neutrino Monte Carlo generators NEUGEN [9] and GENIE-2.0.0 [10], includes a phenomenological description of the low invariant mass region based on Koba–Nielsen–Olesen (KNO) scaling [11], while at higher masses it gradually switches over to the PYTHIA/JETSET model. The transition from the KNO-based model to the PYTHIA/JETSET model takes place gradually, at an intermediate invariant mass region, ensuring the continuity of all simulated observables as a function of the invariant mass. This is accomplished by using a transition window $[W_{\min}^{\text{tr}}, W_{\max}^{\text{tr}}]$ over which we linearly increase the fraction of neutrino events for which the hadronization is performed by the PYTHIA/JETSET model from 0% at W_{\min}^{tr} to 100% at W_{\max}^{tr} . The default values used in the AGKY model are

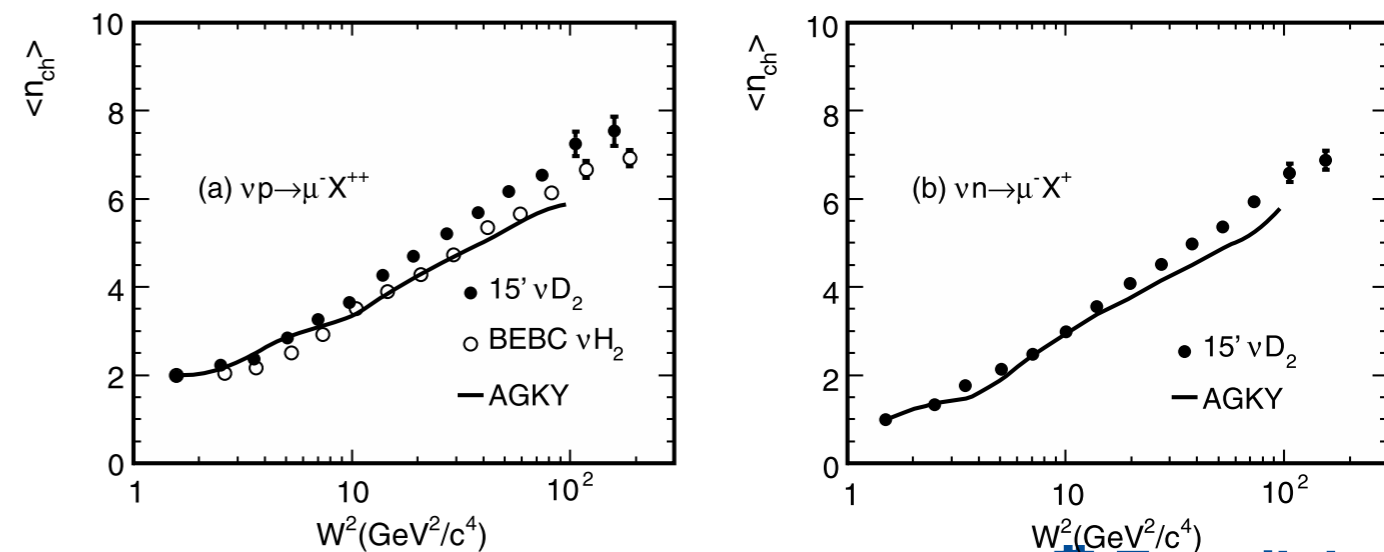
$$W_{\min}^{\text{tr}} = 2.3 \text{ GeV}/c^2, \quad W_{\max}^{\text{tr}} = 3.0 \text{ GeV}/c^2.$$

Fig. 1 KNO scaling distributions for νp (left) and νn interactions. The curve represents a fit to the Levy function. Data points are taken from [7]



T. Yang et al, Eur. Phys. J C (2009) 63:1-10

Fig. 3 Average charged-hadron multiplicity $\langle n_{\text{ch}} \rangle$ as a function of W^2 . (a) νp events. (b) νn events. Data points are taken from [7, 20]



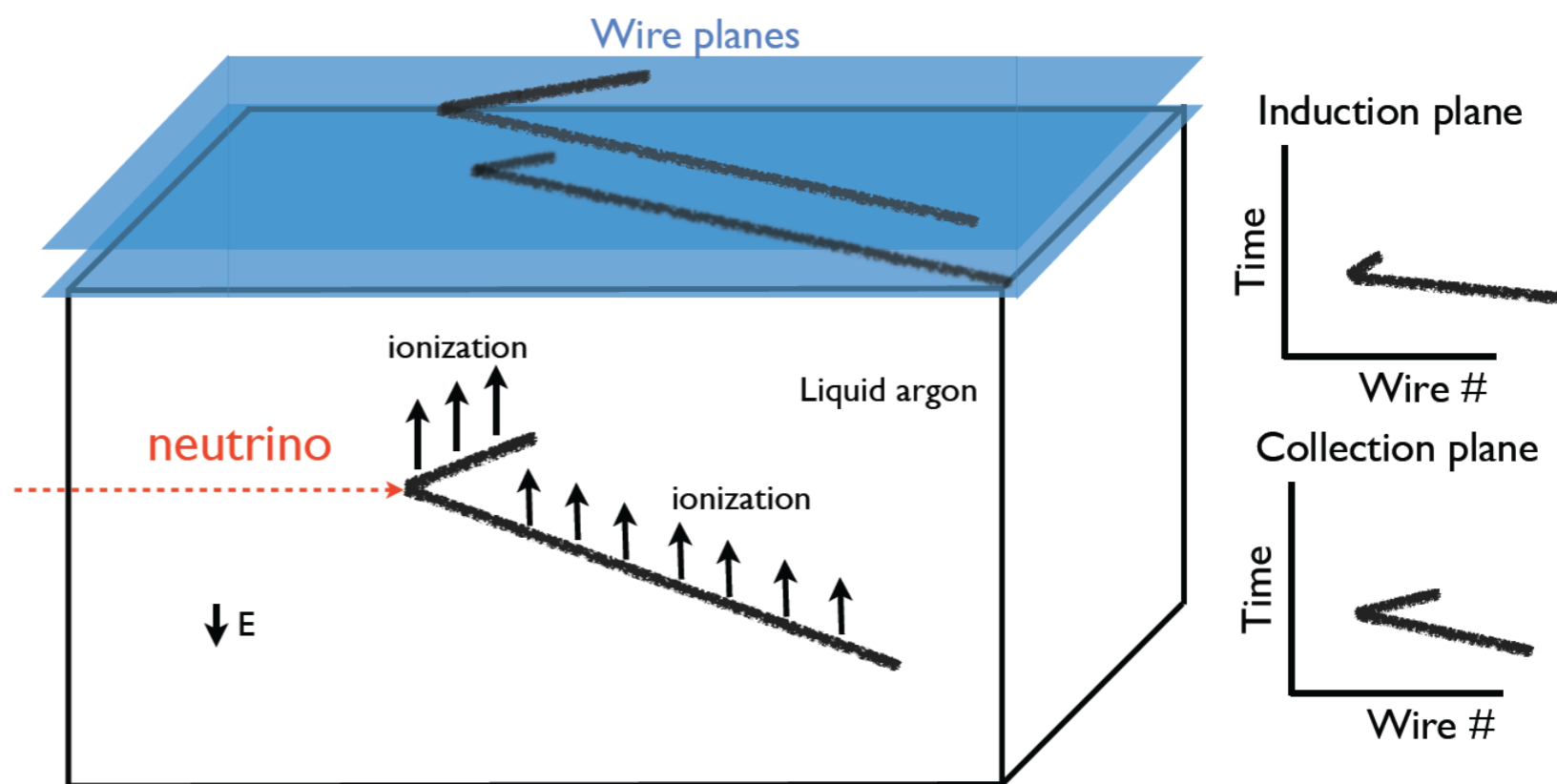


ArgoNeuT

- **175L Liquid Argon Time Projection Chamber (TPC).**
- **First step** in the US liquid argon program (MicroBooNE, LBNE) & first LArTPC in a low-energy neutrino beam.
- Physics run in the **NuMI Beam** June '09 ⊕ Sept. '09 - Feb. '10.
 - Located between MINOS ND and MINERvA & utilized MINOS for muon momentum and charge sign. (NuMI "LE" beam.)

TPC / Cryostat Volume	175 / 500 L
# of Electronics Channels*	480
Wire Pitch	4 mm
Max Drift Length	0.5 m (330 μ s)
Electric Field	500 V/cm

*Two readout planes: Induction & Collection
Each Channel: 2048 Samples / 400 μ s



J. Spitz, arXiv: 1009.2515v1

Understanding the theory of neutrino-nucleus interactions

- The **particle theory community** has begun to really **engage with neutrino interactions** (the field has long been of interest to nuclear theorists).
- We've begun to organize problems more clearly around **leptonic, nucleon, and nuclear effects** in the full picture, with particle theorists beginning to work more vigorously on the first two.
 - Our model involves going from quarks to nucleons and again to nuclei. For precision, we “**need to control both form factors and nuclear effects**” and we must **properly separate them** (G. Paz, NuInt 2017).
 - e.g., M_A from the dipole parameterization of the vector axial form factor is often presented with inflated uncertainties to cover *nuclear* modeling effects. **It is time to do better** than that and recent work shows us how.
 - Our understanding of *proper* nucleon level uncertainties has leapt forward, but **understanding how to fully leverage this information in a nuclear context** is important and will help direct all of our efforts (R. Hill, Radiative Corrections at the IF, Perimeter Inst. 2017).
 - **Nuclear modeling must first succeed with electrons:** good progress here!

Precise nucleon form factors

- **Nucleon inputs** will play an important role in assessing the overall nuclear uncertainties, whether they come from calculations or measurement.
 - For another calculation, see e.g. Meyer, Hill, Kronfeld, Li and Simone, arXiv 1610.04593
 - Also, new (this week!) nucleon vector form factors from Ye, Arrington, Hill and Lee in arXiv 1707.09063
- Re-analyzing existing **deuterium data using the z-expansion** from above is important for properly specifying the axial-vector form factor and its uncertainties.

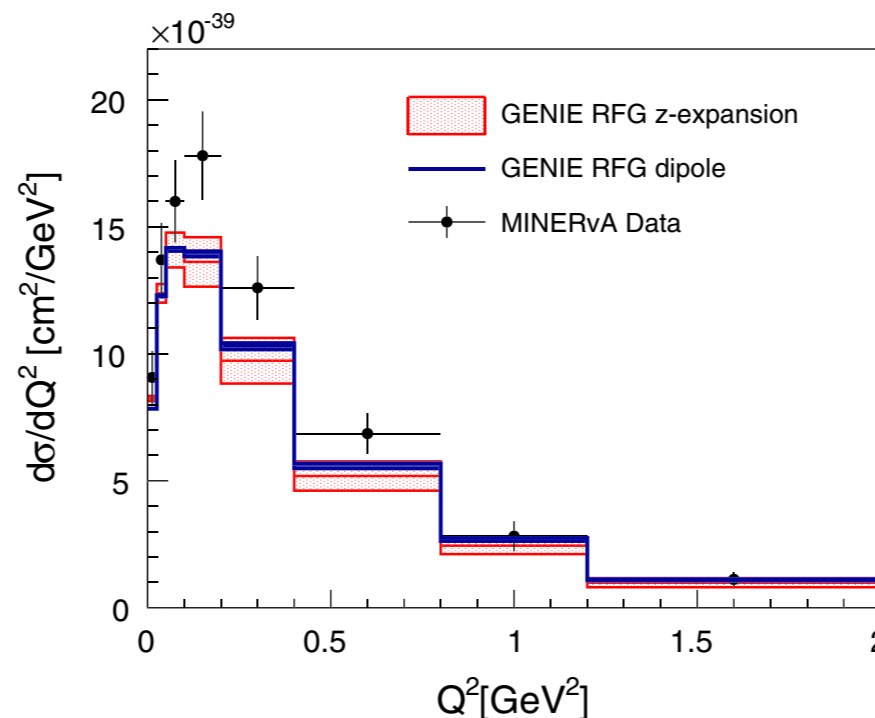
$$F_A(q^2) = \sum_{k=0}^{k_{\max}} a_k z(q^2)^k,$$

(Form factor fully specified in the paper, etc.)

Here, Minerva for **illustration** - fit was to **deuterium data**.

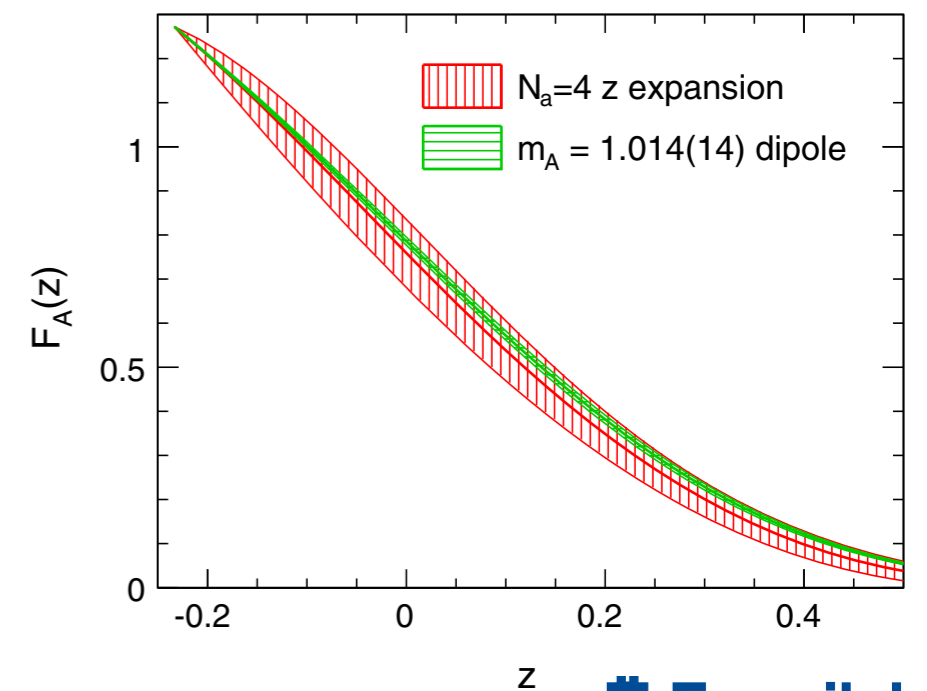
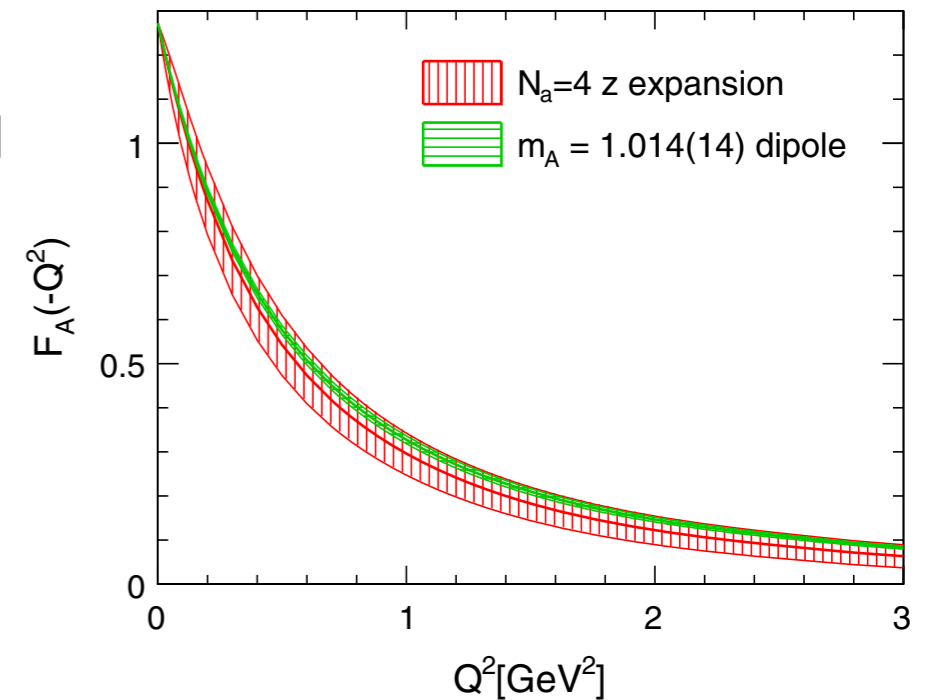
Nucleon physics

PHYSICAL REVIEW D **93**, 113015 (2016)

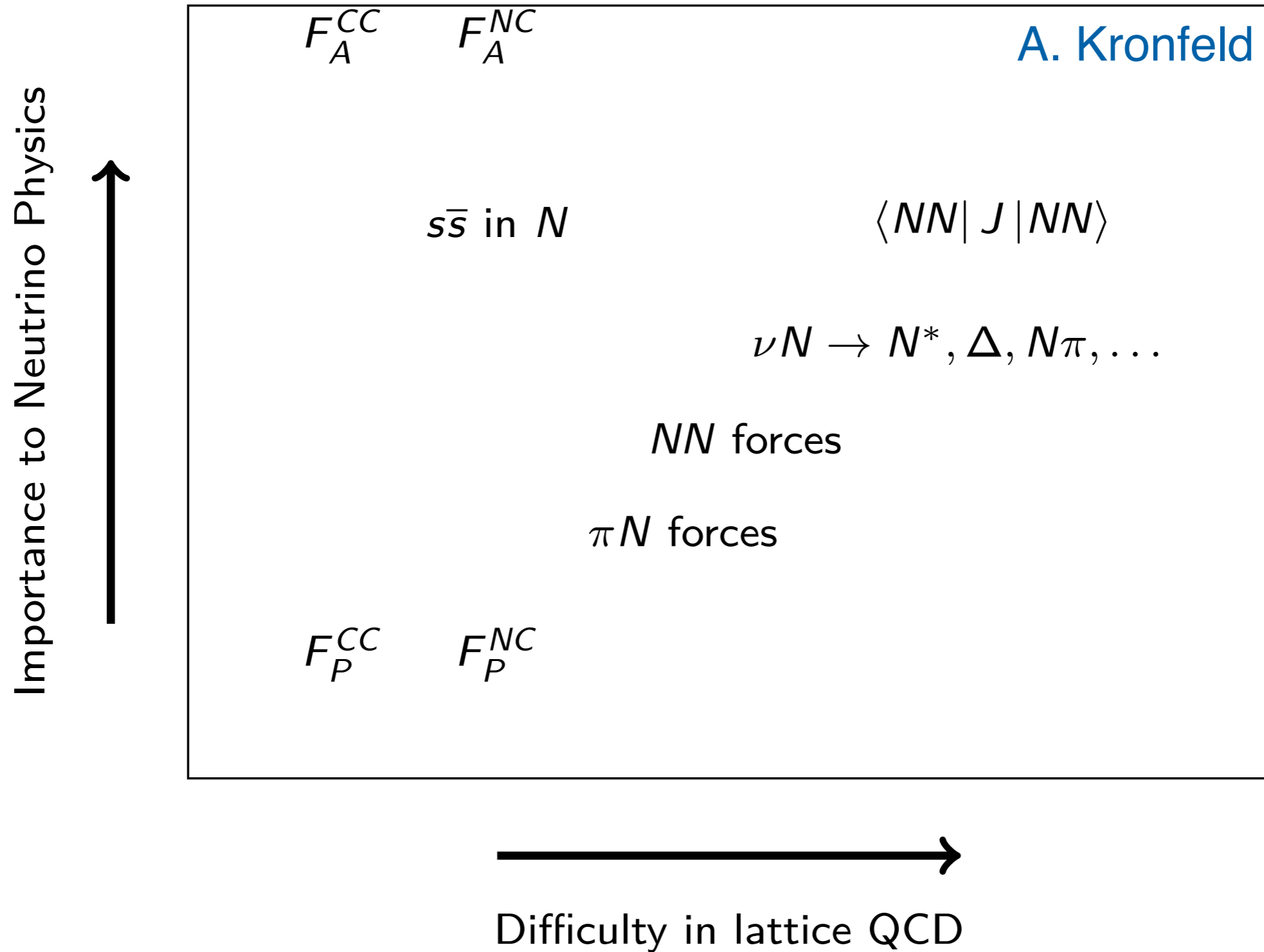


PHYSICAL REVIEW D **93**, 113015 (2016)

Meyer, Betancourt, Gran, Hill (2016)



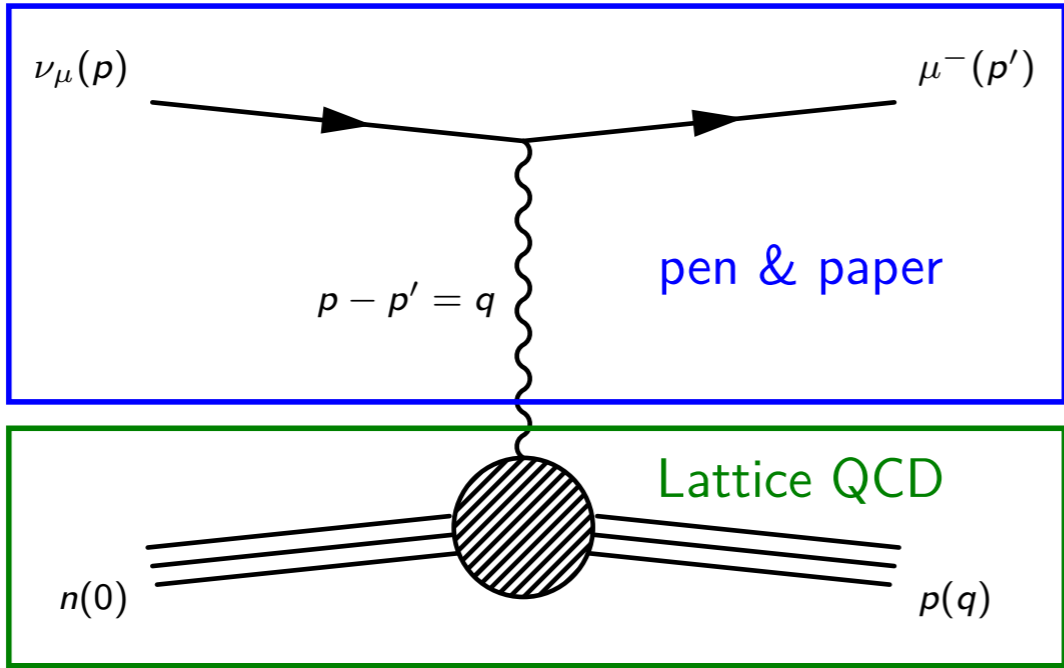
Lattice prospects



How Does Lattice Help?

Lattice is well suited to compute matrix elements:

$$\mathcal{M}_{\nu_\mu n \rightarrow \mu p}(p,p') = \langle \mu(p') | (V_\mu - A_\mu) | \nu(p) \rangle \langle p(q) | (V_\mu - A_\mu) | n(0) \rangle$$



Aaron S. Meyer (asmeyer2012@uchicago.edu)

University of Chicago/Fermilab

Radiative Corrections at the
Intensity Frontier of Particle Physics

Nucleon Axial FFs

$$\frac{d\sigma}{dQ^2} = \frac{G_f^2 M^2 \cos^2 \theta_C}{8\pi E_\nu^2} \left[A \mp \frac{(s-u)}{M^2} B + \frac{(s-u)^2}{M^4} C \right]$$

$$A = \frac{(m^2 + Q^2)}{M^2} [(1 + \tau) G_A^2 - (1 - \tau) F_1^2 + \tau(1 - \tau) F_2^2 + 4\tau F_1 F_2 - \frac{m^2}{4M^2} \left((F_1 + F_2)^2 + (G_A + 2G_P)^2 - \left(\frac{Q^2}{M^2} + 4 \right) G_P^2 \right)]$$

$$B = \frac{Q^2}{M^2} G_A (F_1 + F_2)$$

$$C = \frac{1}{4} (G_A^2 + F_1^2 + \tau F_2^2)$$

- G_A
- dominant contribution
 - largest uncertainty

- $F_{1,2}$
- Well-determined from electron scattering expts
- G_P
- can be related to G_A by pion pole dominance

Phiala Shanahan
MIT

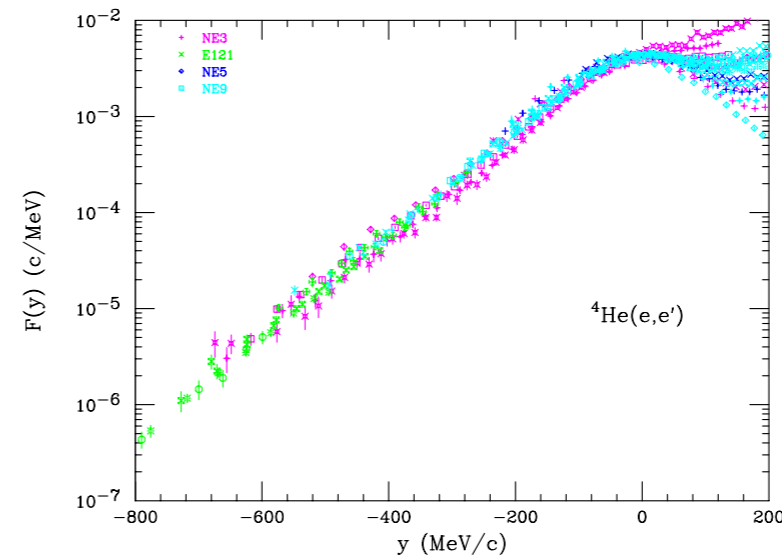
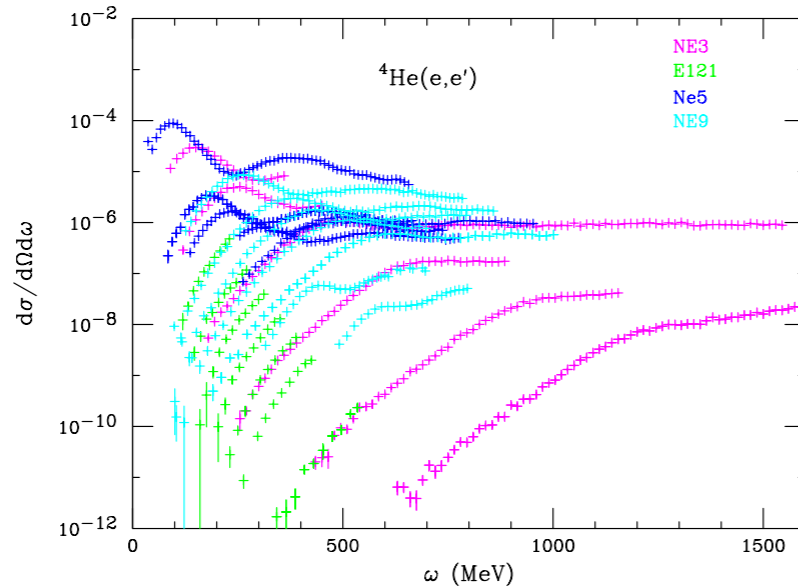
Quick refresher - scaling

- Scaling of the first kind occurs when the electron-nucleus cross section or longitudinal/transverse response functions (divided by a function describing free nucleon physics) no longer depend on two variables (e.g. energy transfer and the absolute value of the 3-momentum transfer), but only on a specific function of them, which defines the *scaling variable*.
- Scaling of the second kind takes place when there is no dependence on the nuclear species.
- The simultaneous occurrence of both kinds of scaling is called *superscaling*.
- Scaling of the zeroth kind occurs when the scaling function is the same for the longitudinal and transverse responses.

Experimental scaling function:

$$F(q, y) = \frac{[d\sigma/d\omega d\Omega']_{exp}}{\bar{\sigma}_{eN}(q, \omega; p = -y, \varepsilon = 0)}$$

$$\bar{\sigma}_{eN}(q, \omega; p, \varepsilon) \equiv \frac{1}{2\pi} \int d\phi_N \frac{E_N}{q} [Z\sigma_{ep}(q, \omega; p, \varepsilon, \phi_N) + N\sigma_{en}(q, \omega; p, \varepsilon, \phi_N)]$$



Scaling of the first kind: $q \rightarrow \infty \Rightarrow F(q, y) \longrightarrow F(y) \equiv F(\infty, y)$

$$f(q, \psi) \equiv k_F \frac{[d\sigma/d\omega d\Omega_e]}{\sigma_M [v_L G^L + v_T G^T]}, \quad f^L(q, \psi) \equiv k_F \frac{R^L(q, \omega)}{G^L}, \quad f^T(q, \psi) \equiv k_F \frac{R^T(q, \omega)}{G^T}$$

- Scaling of the first kind: $f_{exp}(q, \psi) \xrightarrow{q \rightarrow \infty} f_{exp}(\psi)$; $\psi \approx y/k_F$ – *superscaling variable*
- Scaling of the second kind: $f_{exp}(\psi)$ – *independence on the nuclear system*

SUPERSCALING

- Scaling of the zeroth kind: $f_{exp}(q, \psi) = f_{exp}^L(q, \psi) = f_{exp}^T(q, \psi)$

J. Caballero, Seville
NuInt 2017

Inelastic Reactions

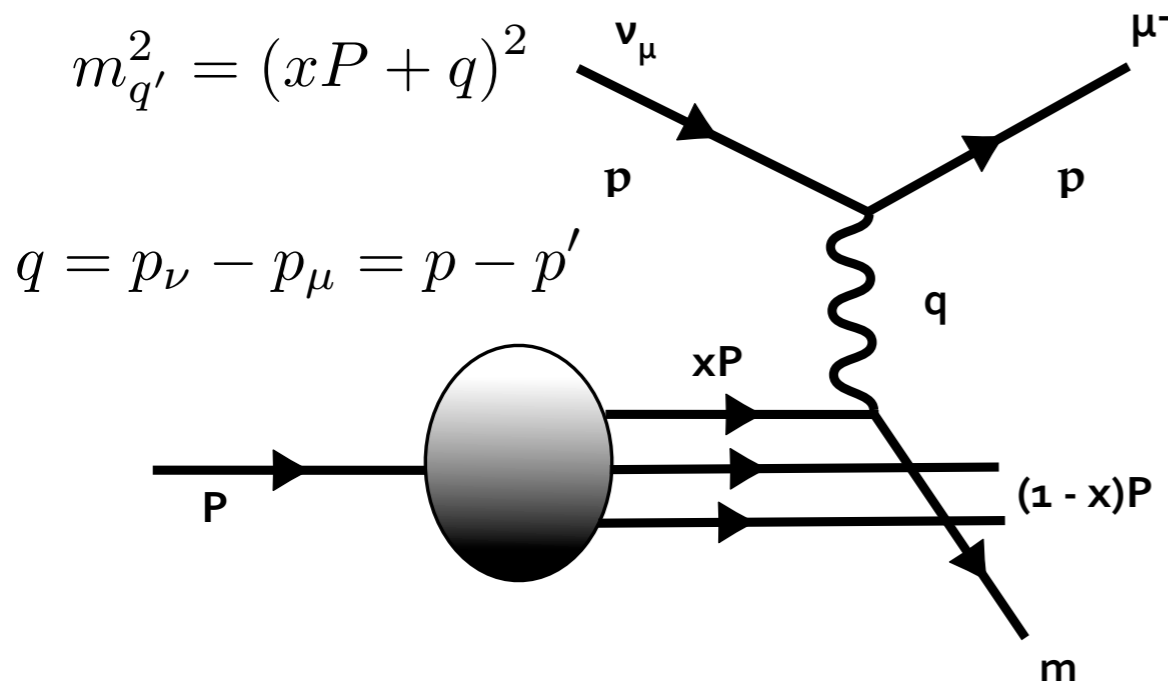
- “Real” scattering involves very complicated targets. Electroweak theory does not provide couplings for composite particles (e.g. nucleons).
- We assume *massless* leptons in the following section...

In DIS, the neutrino scatters against an individual parton, carrying momentum fraction x , inside the nucleon.

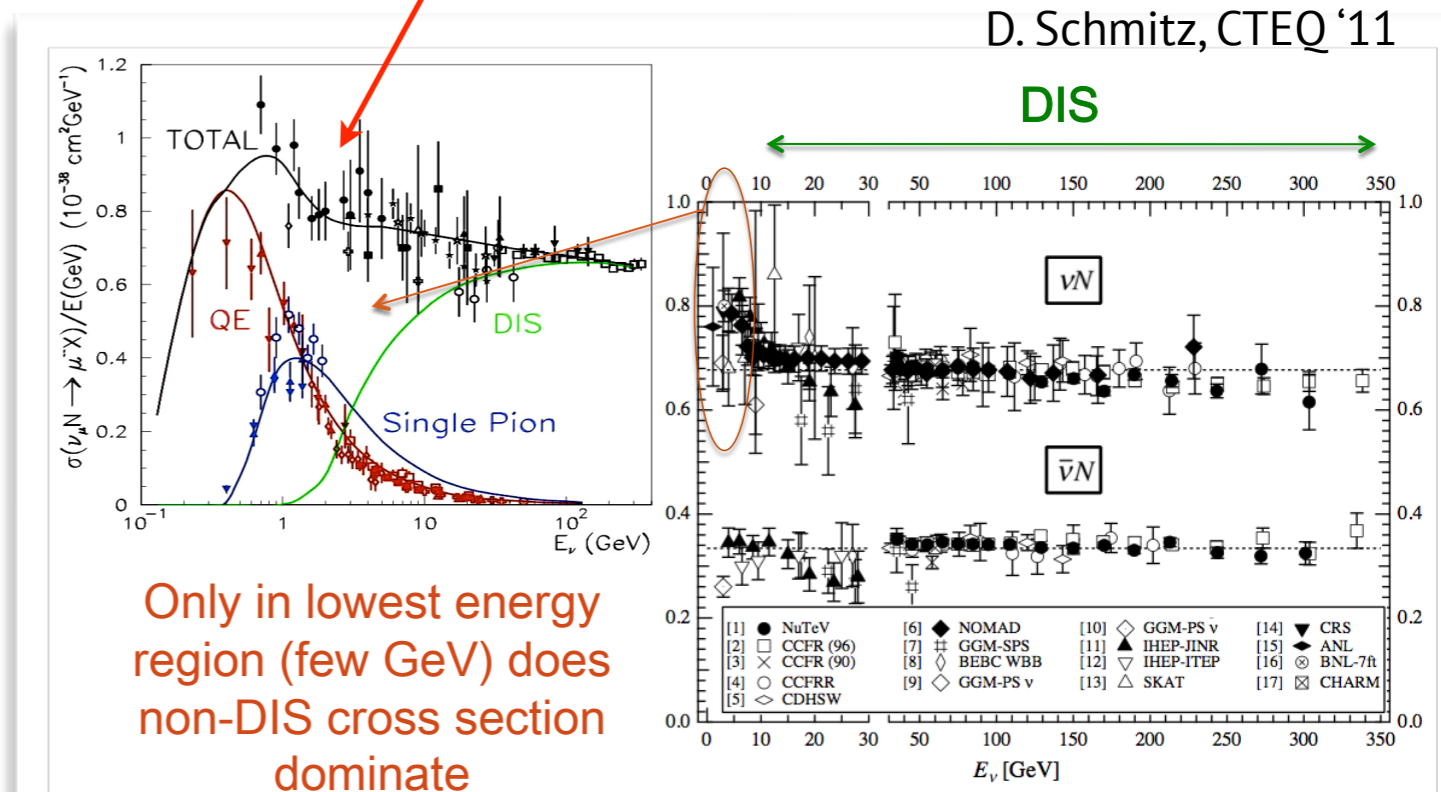
$$m_q^2 = x^2 P^2 = x^2 M_T^2$$

$$m_{q'}^2 = (xP + q)^2$$

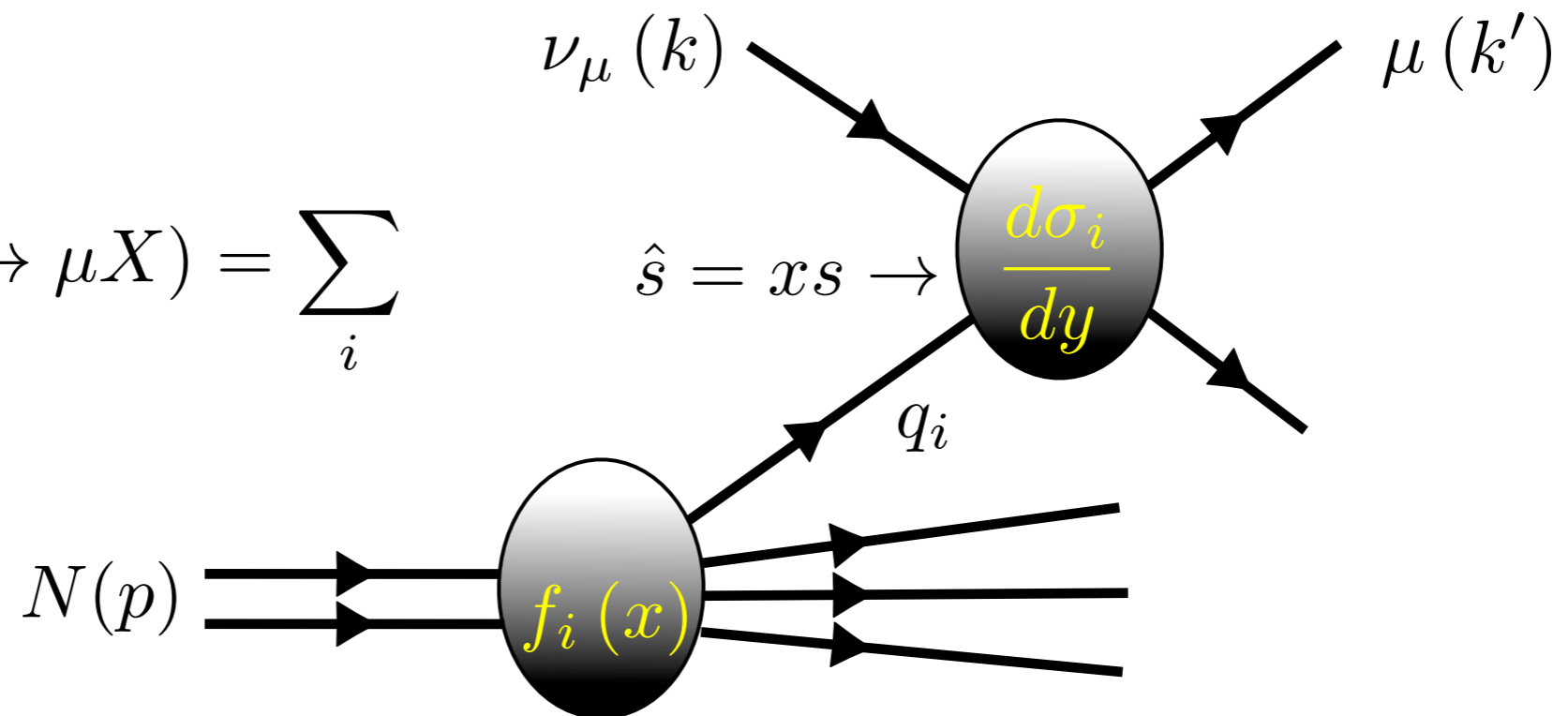
$$q = p_\nu - p_\mu = p - p'$$



Transition Region - Messy Final States, but not scattering cleanly off partons.



$$\frac{d^2\sigma}{dx dy} (\nu N \rightarrow \mu X) = \sum_i$$

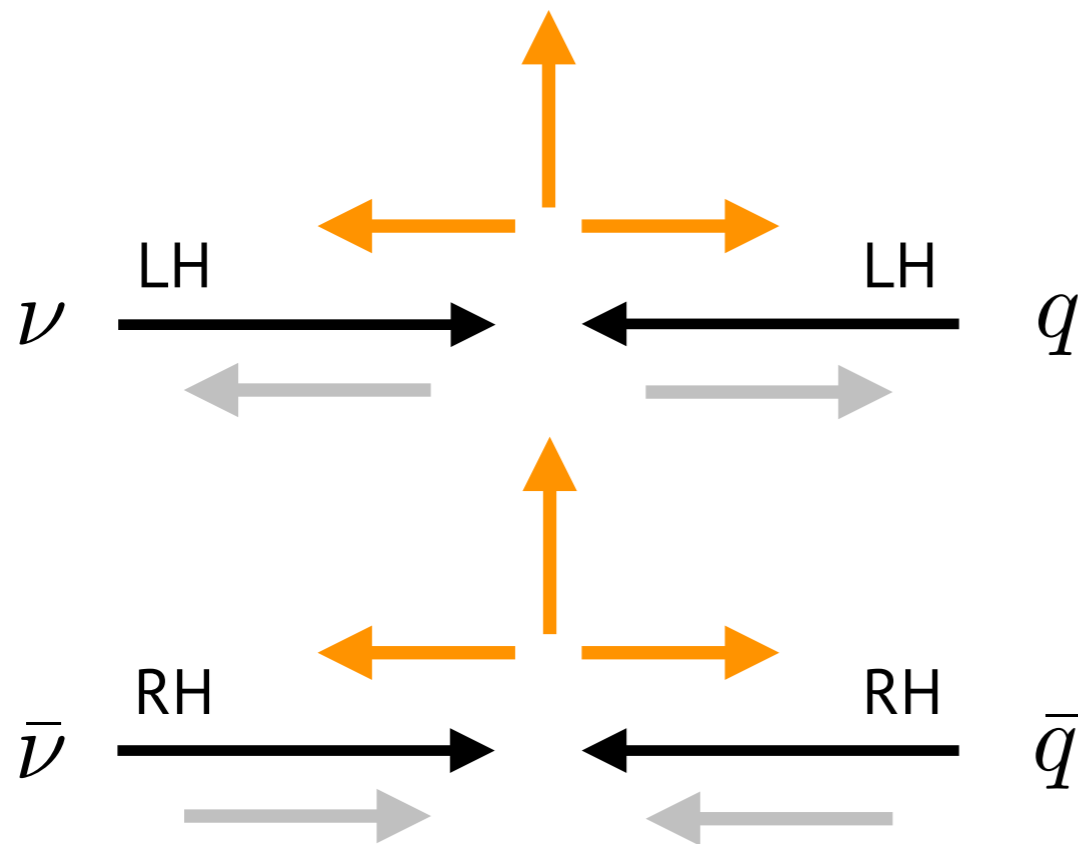


$$= \sum_i f_i(x) \left(\frac{d\sigma_i}{dy} \right)_{\hat{s}=sx}$$

$$1 - y \equiv \frac{p \cdot k'}{p \cdot k} = \frac{1}{2} (1 + \cos \theta) \quad \& \text{ Center-of-Mass Energy} = xs$$

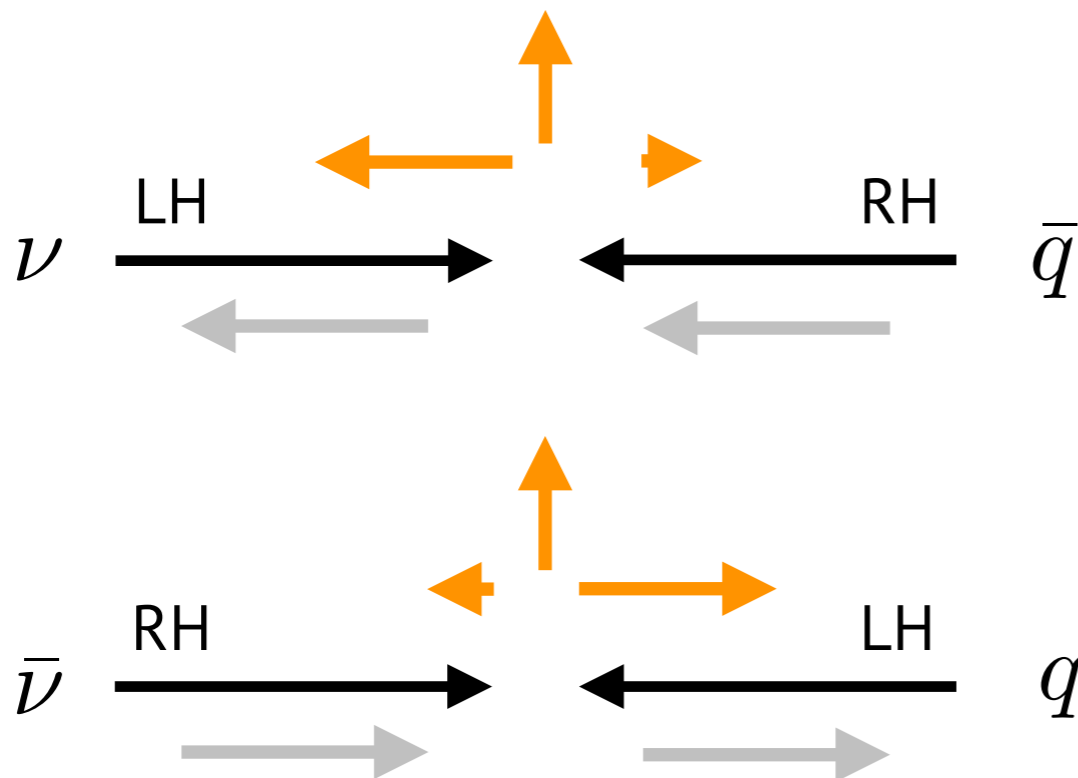
➔

$$\frac{d\sigma}{dy} (\nu_\mu d \rightarrow \mu^- u) = \frac{G_F^2 xs}{\pi} \quad \frac{d\sigma}{dy} (\bar{\nu}_\mu u \rightarrow \mu^+ d) = \frac{G_F^2 xs}{\pi} (1 - y)^2$$



neutrino + quark
anti-neutrino + anti-quark

$$\frac{d\sigma}{dy}(\nu q) = \frac{d\sigma}{dy}(\bar{\nu} \bar{q}) = \frac{G_F^2}{\pi} s x$$



neutrino + anti-quark
anti-neutrino + quark

$$\frac{d\sigma}{dy}(\bar{\nu} q) = \frac{d\sigma}{dy}(\nu \bar{q}) = \frac{G_F^2}{\pi} s x (1 - y)^2$$

$$1 - y \simeq \frac{1}{2} (1 + \cos \theta)$$

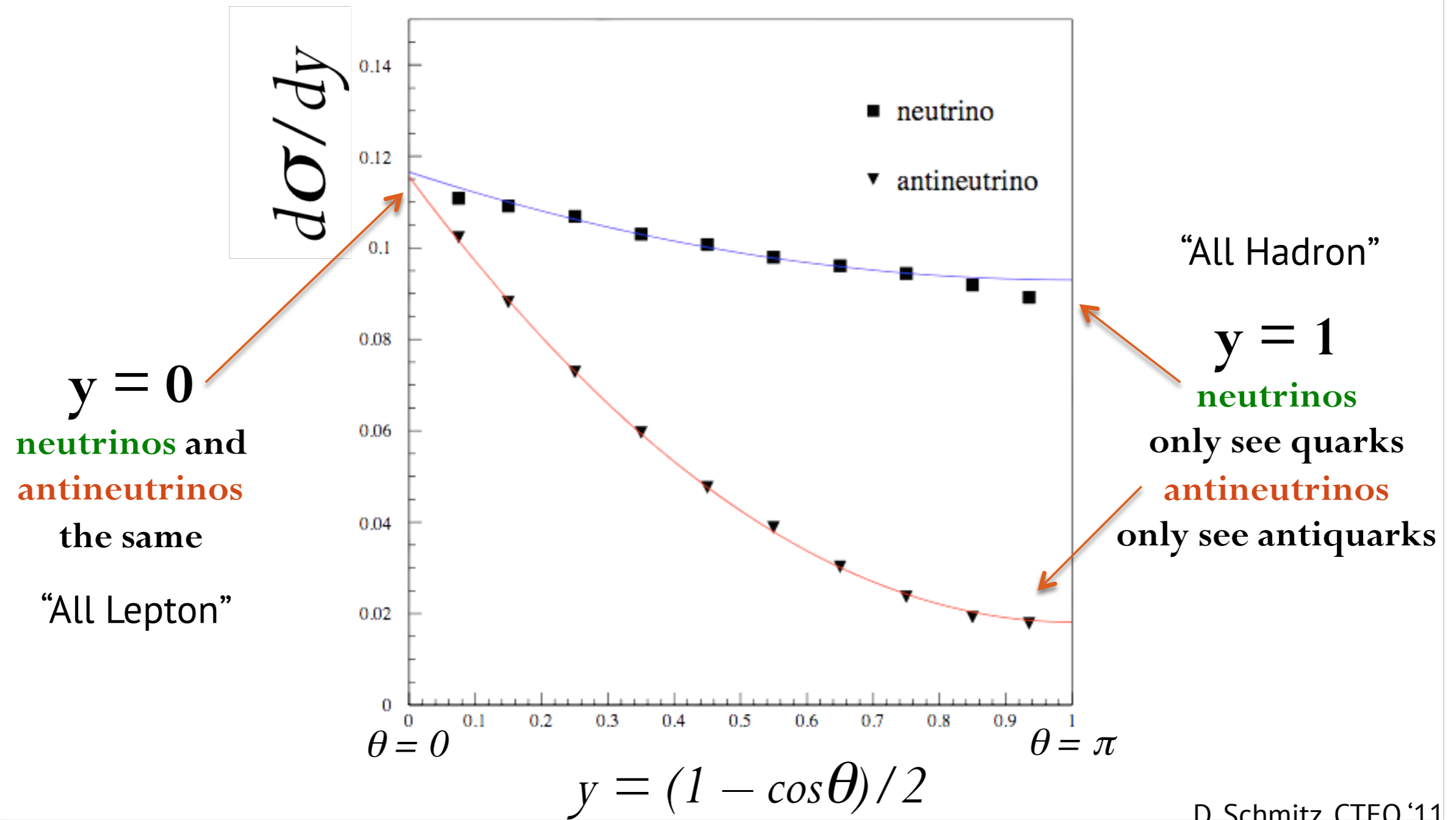
Parton Distribution Functions $q(x)$: Charge and Helicity

- Neutrinos and anti-neutrinos “taste” different quark flavors!
 - Neutrinos: d , s , u -bar, c -bar *ONLY*
 - Anti-neutrinos: u , c , d -bar, s -bar *ONLY*
- Scattering is *not* from free quarks though! We must use *parton distribution functions*!
 - We cannot calculate these with QCD, but we do know they are universal:

$$\frac{d^2\sigma}{dx dy}(\nu + \text{proton}) = \frac{G_F^2 s}{\pi} x \left[d(x) + s(x) + [\bar{u}(x) + \bar{c}(x)] (1-y)^2 \right]$$
$$\frac{d^2\sigma}{dx dy}(\bar{\nu} + \text{proton}) = \frac{G_F^2 s}{\pi} x \left[\bar{d}(x) + \bar{s}(x) + [u(x) + c(x)] (1-y)^2 \right]$$

$$y = 1 - \frac{E_l}{E_\nu} \quad \text{Inelasticity}$$

Neutrino CC DIS cross section vs. y



D. Schmitz, CTEQ '11

Nucleon Structure Functions

- We may write the ν -N cross-sections in a model-independent way using three nucleon structure functions: F_1 , F_2 , xF_3 :

$$\frac{d^2\sigma^{\nu,\bar{\nu}}}{dx dy} = \frac{G_F^2 M_T E}{\pi} \left[xy^2 F_1(x, Q^2) + \left(1 - y - \frac{xyM_T}{2E}\right) F_2(x, Q^2) \pm y \left(1 - \frac{1}{2}y\right) xF_3(x, Q^2) \right]$$

- We may invoke Callan-Gross ($2xF_1 = F_2$) to simplify. *Deviations:*

$$R \equiv \left(1 + \frac{4M_T^2 x^2}{Q^2}\right) \frac{F_2}{2xF_1} - 1$$

- The functions $F_2(x, Q^2)$, $xF_3(x, Q^2)$, and $R(x, Q^2)$ may now be experimentally charted from the measured DIS cross-section, $d\sigma/dy$, in bins of x and Q^2 .

Nucleon Structure Functions

neutrino... (top)

$$\frac{d^2\sigma^{\nu A}}{dx dy} \propto [F_2^{\nu A}(x, Q^2) + xF_3^{\nu A}(x, Q^2)] + (1-y)^2 [F_2^{\nu A}(x, Q^2) - xF_3^{\nu A}(x, Q^2)] + f(R)$$

$$\frac{d^2\sigma^{\bar{\nu} A}}{dx dy} \propto [F_2^{\bar{\nu} A}(x, Q^2) - xF_3^{\bar{\nu} A}(x, Q^2)] + (1-y)^2 [F_2^{\bar{\nu} A}(x, Q^2) + xF_3^{\bar{\nu} A}(x, Q^2)] + f(R)$$

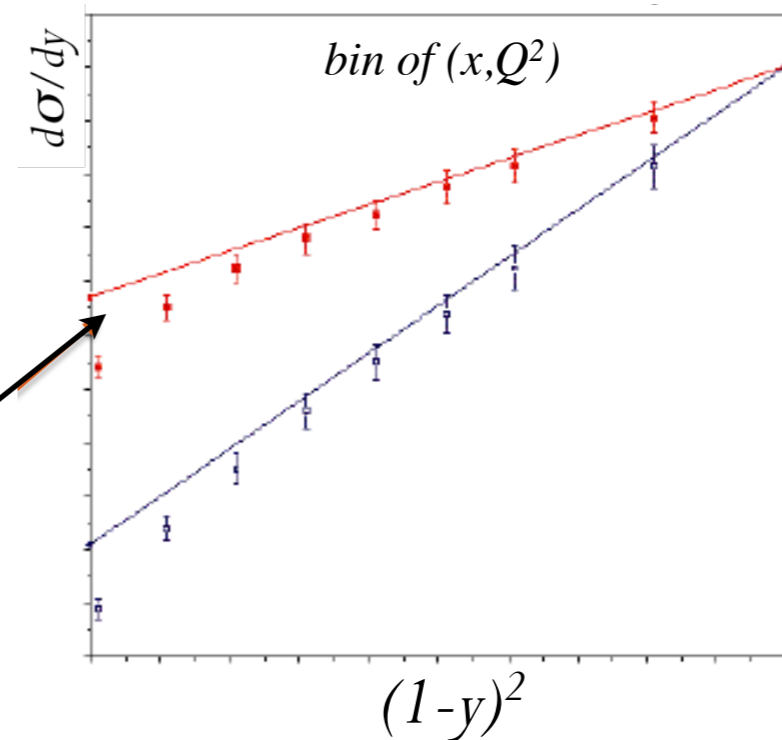
anti-neutrino... (bottom)

Equations of lines!

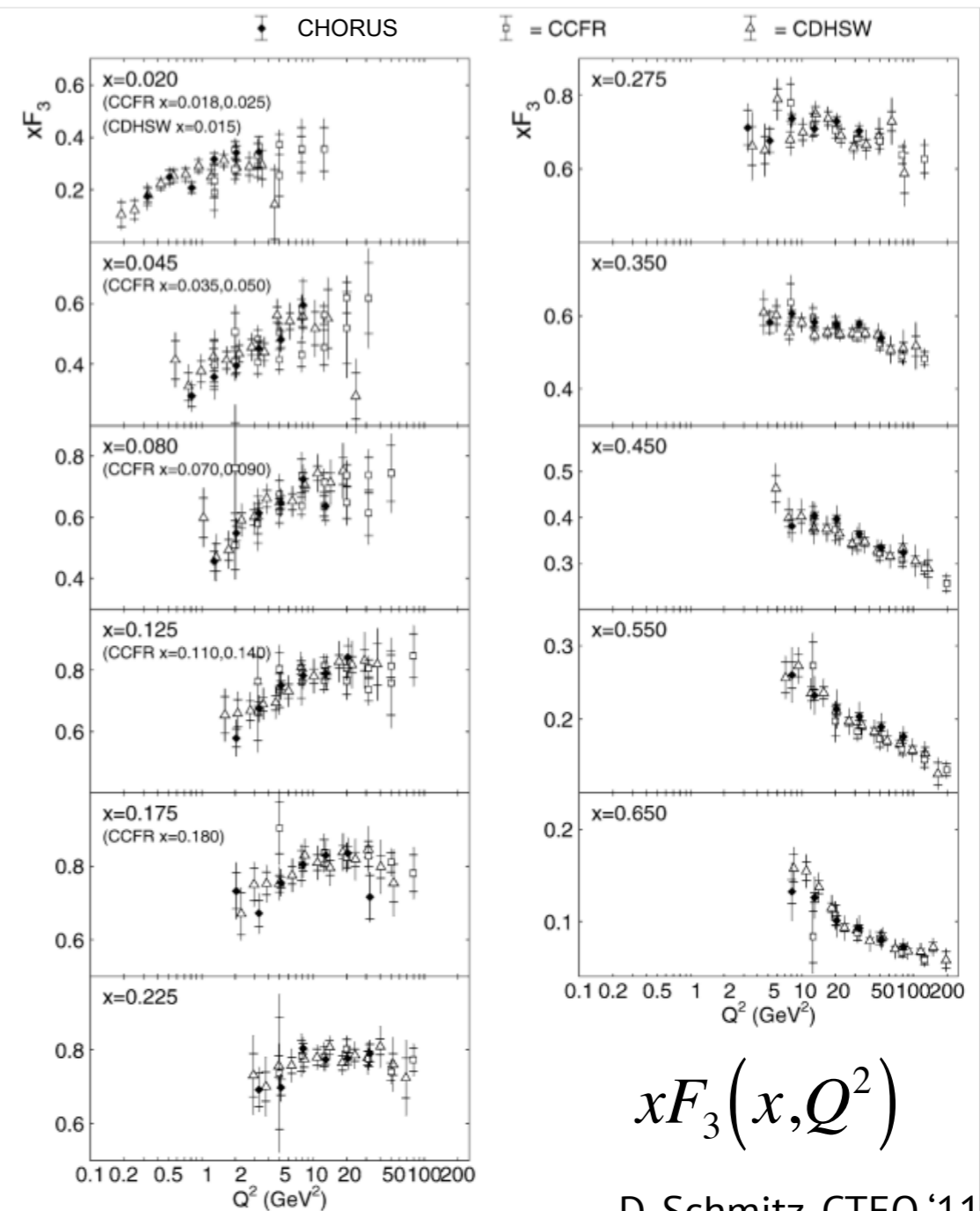
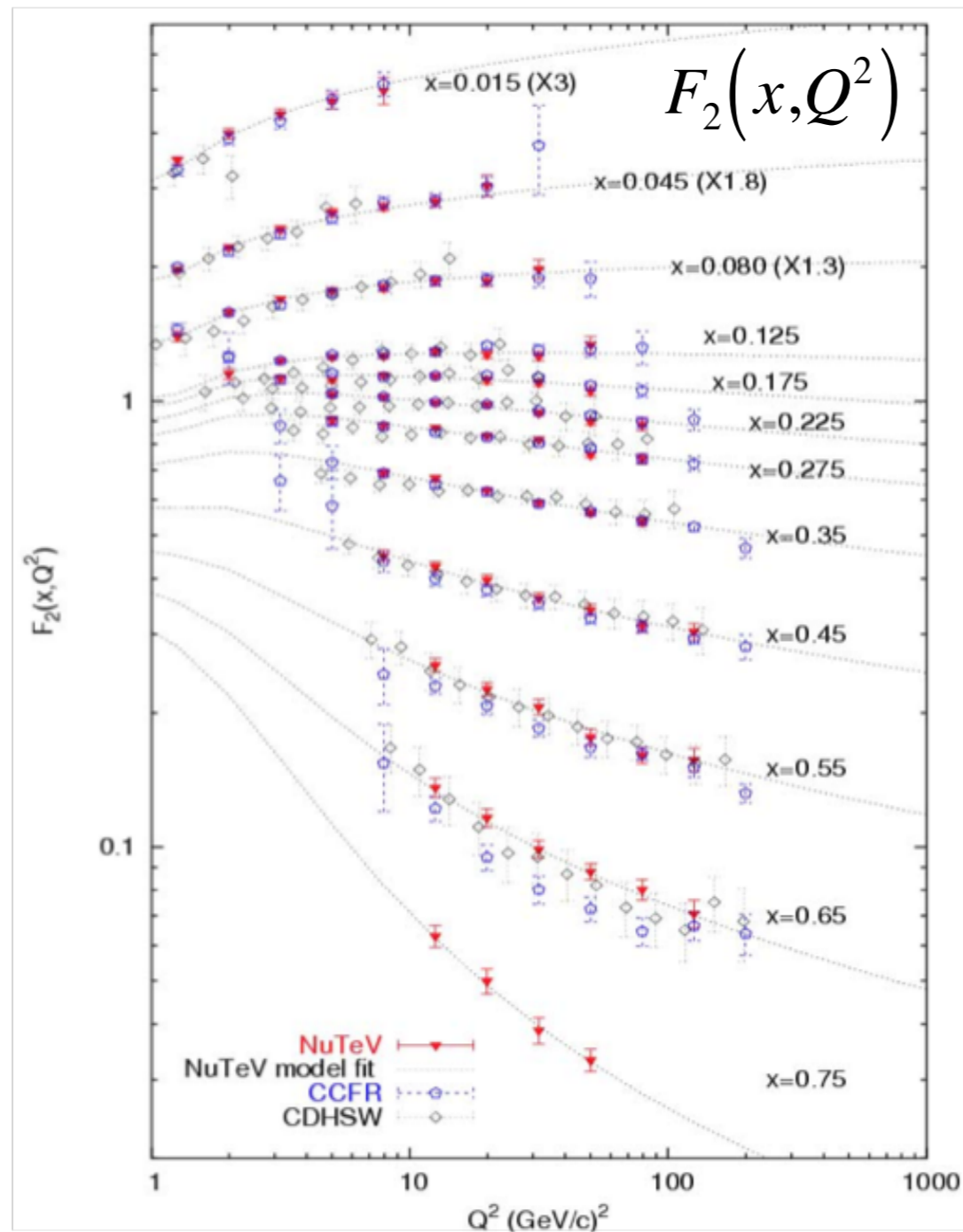
$$y \propto m \times x + b$$

Fit for F_2, xF_3 in bins of (x, Q^2) .

C.G. R is related to excursions from a straight-line slope.



Nucleon Structure Functions



D. Schmitz, CTEQ '11

Structure Functions & PDFs (Charged Current)

Leading order expressions to relate SFs to PDFs:

$$F_2^{\nu N}(x, Q^2) = x [u + \bar{u} + d + \bar{d} + 2s + 2\bar{c}]$$

$$F_2^{\bar{\nu} N}(x, Q^2) = x [u + \bar{u} + d + \bar{d} + 2\bar{s} + 2c]$$

$$xF_3^{\nu N}(x, Q^2) = x [u - \bar{u} + d - \bar{d} + 2s - 2\bar{c}]$$

$$xF_3^{\bar{\nu} N}(x, Q^2) = x [u - \bar{u} + d - \bar{d} - 2\bar{s} + 2c]$$

Assuming $c = \bar{c}$ & $s = \bar{s}$:

$$F_2^{\nu} - xF_3^{\nu} = 2(\bar{u} + \bar{d} + 2\bar{c}) = 2U + 4\bar{c}$$

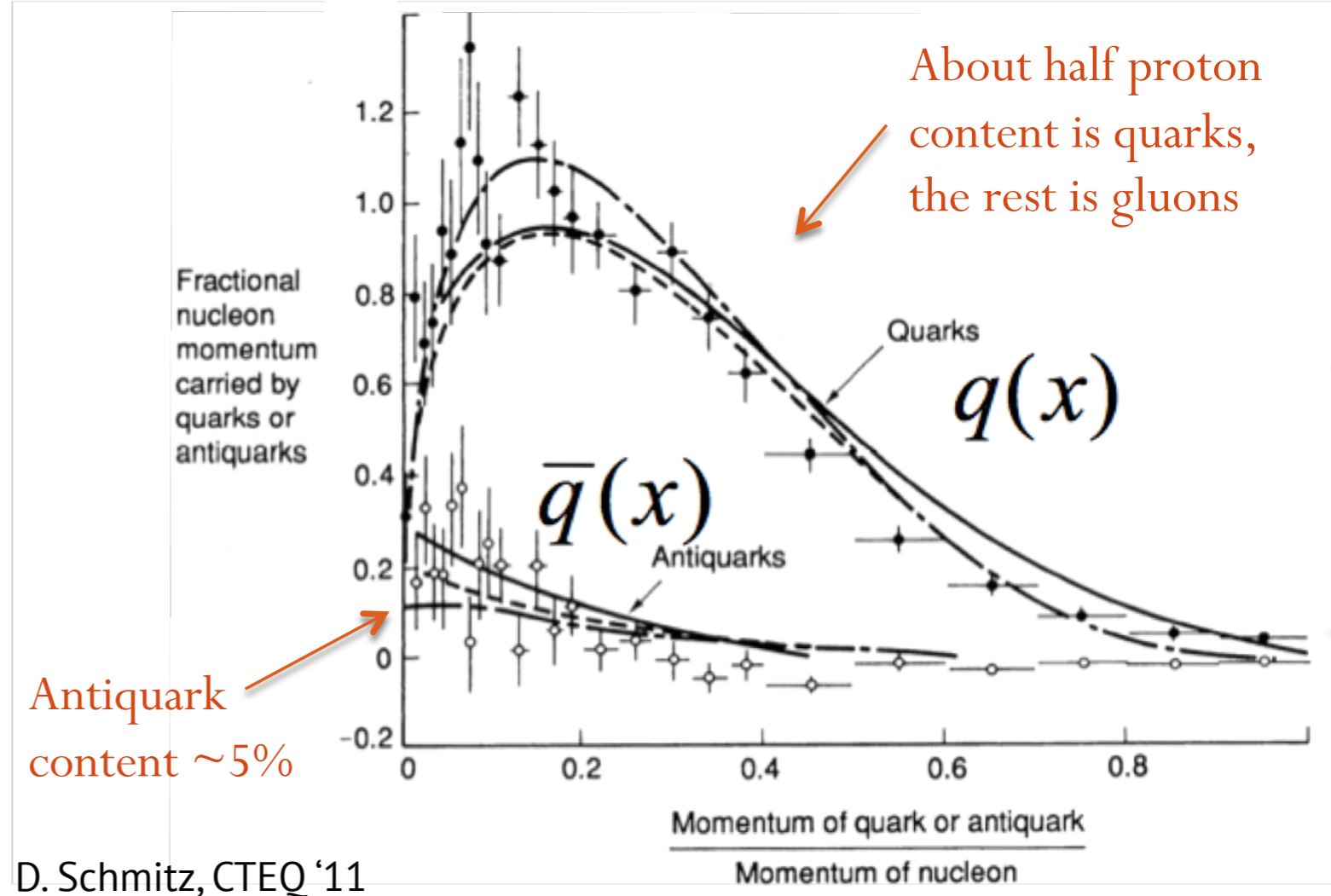
$$F_2^{\bar{\nu}} - xF_3^{\bar{\nu}} = 2(\bar{u} + \bar{d} + 2\bar{s}) = 2U + 4\bar{s}$$

$$xF_3^{\nu} - xF_3^{\bar{\nu}} = 2[(s + \bar{s}) - (c + \bar{c})] = 4\bar{s} - 4\bar{c}$$

Parton Distribution Functions

If there were no valence quarks
($Q\text{-bar} = 0$):

$$\frac{\sigma(\bar{\nu})}{\sigma(\nu)} = \frac{\int_0^1 dy (1-y)^2}{\int_0^1 dy} = \frac{1}{3}$$



$$\frac{d^2\sigma}{dx dy}(\nu + \text{proton}) = \frac{G_F^2 s x}{2\pi} \left[Q(x) + (1-y)^2 \bar{Q}(x) \right]$$

$$\frac{d^2\sigma}{dx dy}(\bar{\nu} + \text{proton}) = \frac{G_F^2 s x}{2\pi} \left[\bar{Q}(x) + (1-y)^2 Q(x) \right]$$