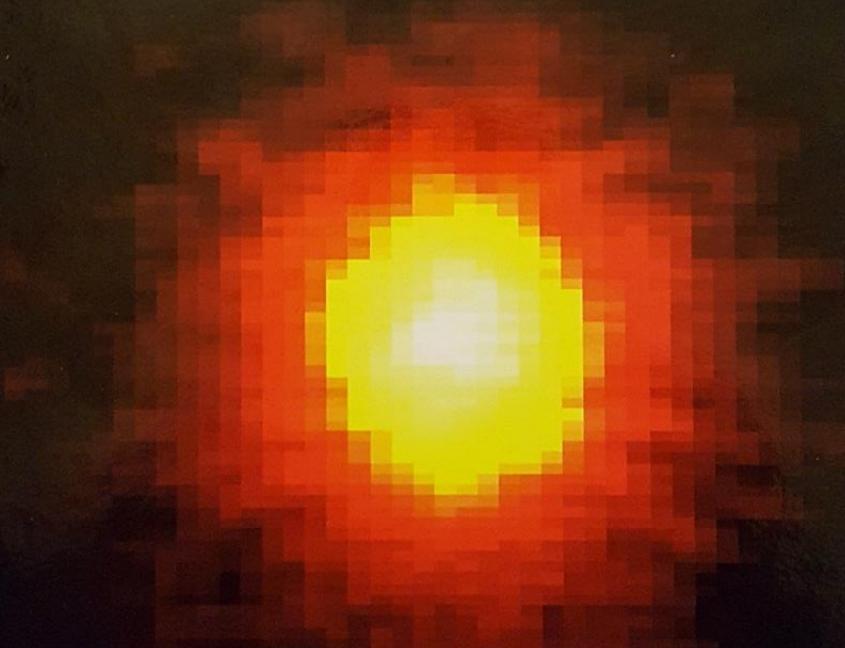


Introduction to Neutrino- Nucleus Scattering

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Old Dominion University
and
Jefferson Lab

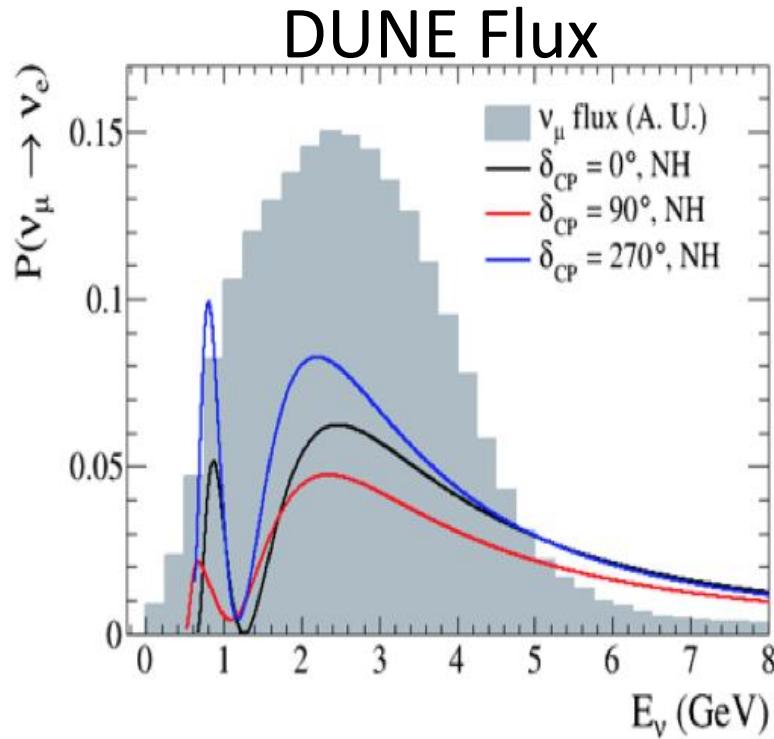


FOUNDATIONS OF NUCLEAR AND PARTICLE PHYSICS



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B. R. Holstein R. G. Milner B. Surrow

The Problem



The width of the energy-distribution implies that kinematics exist where the energy and momentum transfers to the nucleus are small, through the quasielastic, and Delta regions to Deep Inelastic Scattering where the four-momentum transfer is large.

The dynamics of neutrino-nuclear scattering must be understood to a sufficient level of accuracy to allow the initial neutrino energy to be determined.

It is not clear at this point what level of accuracy needs to be attained.

It is also not clear what level of accuracy can be obtained by theoretical calculations of neutrino scattering reactions on the nuclei used in the detectors.

All models must reproduce electron scattering data.

For larger four-momentum transfers, models of quasielastic scattering must at least incorporate relativistic kinematics.

The Basic Model of Nuclear Physics

This model is:

- nonrelativistic,
- assumes that the explicit degrees of freedom are nucleons,
- and is described by that Hamiltonian operator

$$\hat{H} = \sum_i^A T_i + \sum_{i < j}^A \hat{V}_{ij} + \sum_{i < j < k}^A \hat{V}_{ijk} + \dots$$

1 2 3

The two-body potential is obtained

- Phenomenologically
- Using one-boson exchange models
- Using χ EFT

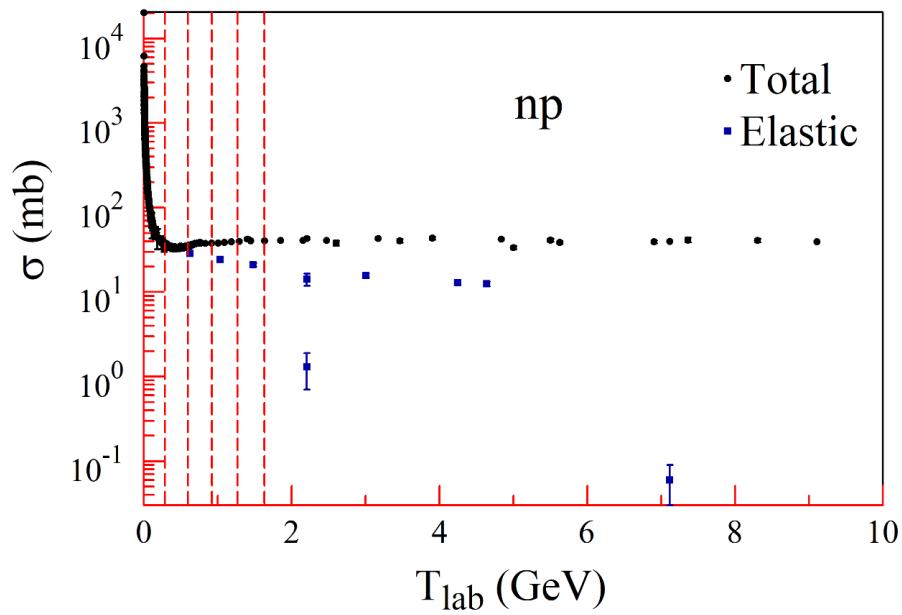
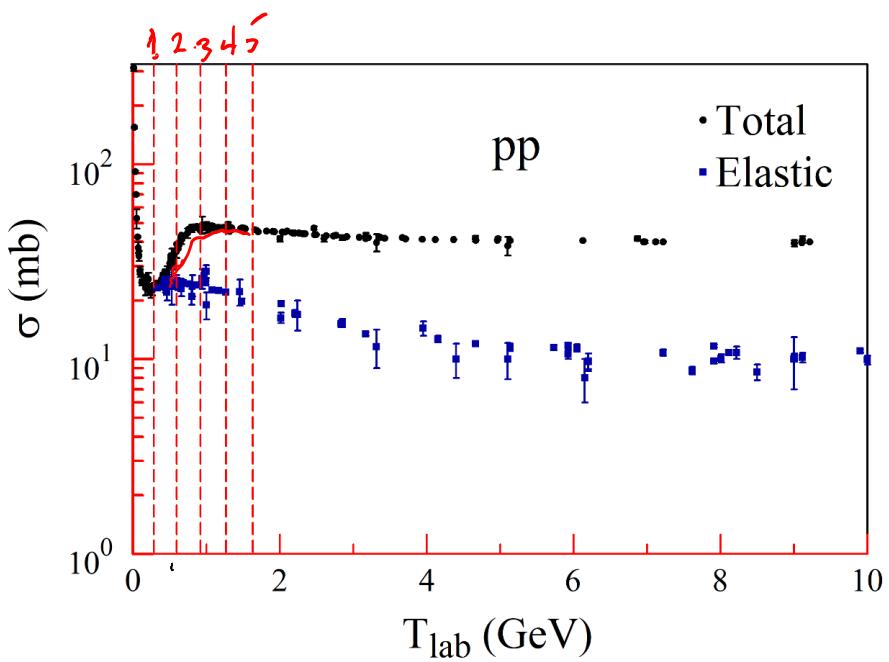
In all cases, the long-range part of the potential is given by 1-pion exchange.

Model parameters are fit to NN scattering for incident kinetic energies < 350 MeV.

The three-body potential is phenomenological or from χ EFT.

Parameters obtained by fitting few-body energies.

NN Total Cross Sections



The two-body potentials use angular momentum operators of the form

$$\left\{ 1, \ L \cdot S, \ \sigma_1 \cdot \sigma_2, \ S_{12}, \ L^2, \ (L \cdot S)^2, \ L^2 \sigma_1 \cdot \sigma_2 \right\}$$

and isospin operators of the form

$$\left\{ 1, \ \tau_1 \cdot \tau_2, \right\}$$

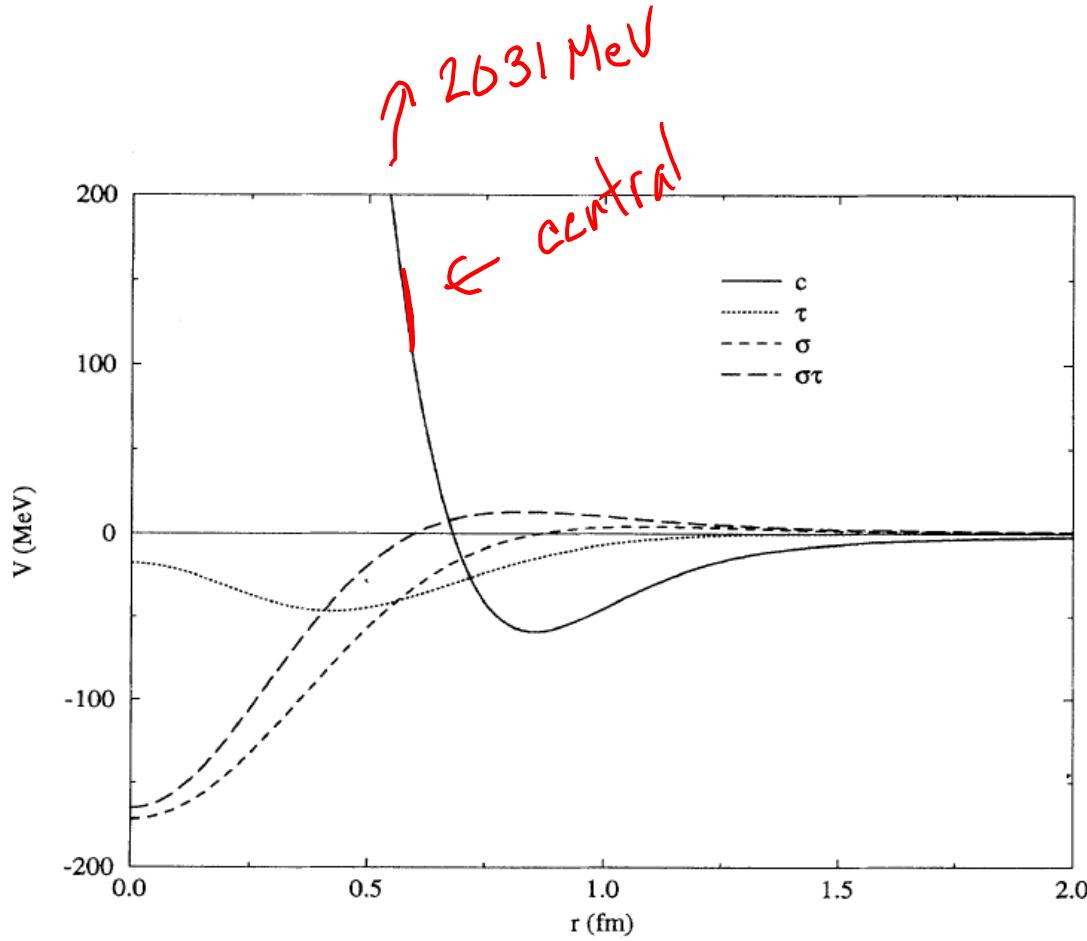
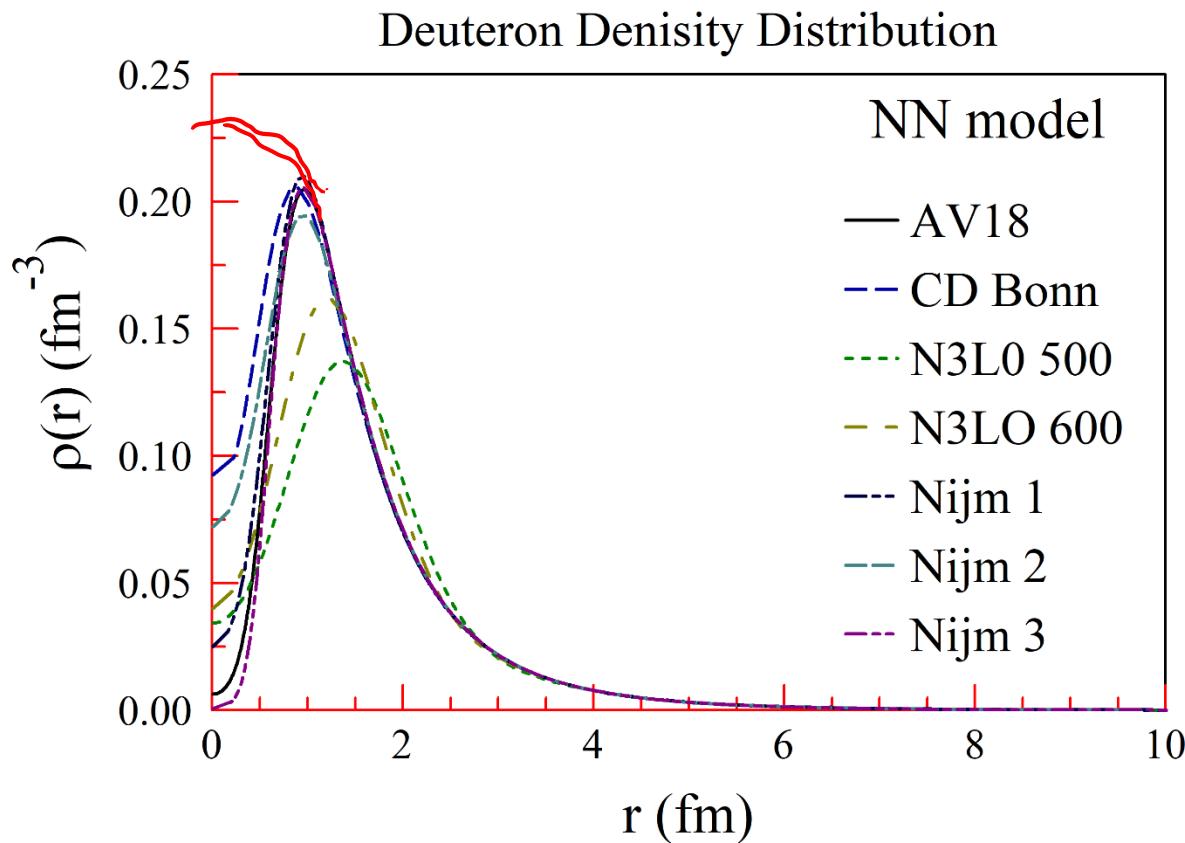


FIG. 6. Central, isospin, spin, and spin-isospin components of the potential. The central potential has a peak value of 2031 MeV at $r = 0$.

For the deuteron J=1, S=1, T=0.

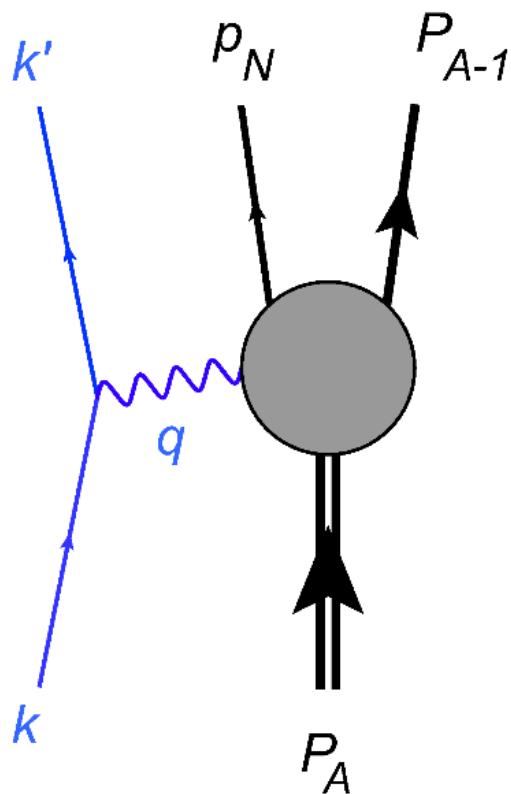
$$\psi_{M_J}(r) = R_{01}(r)\mathcal{Y}_{011}^{M_J}(\Omega) + R_{21}(r)\mathcal{Y}_{211}^{M_J}(\Omega)$$



Independent Particle Model (Simple Shell Model)

$$\hat{H} = \underbrace{\sum_i \hat{T}_i^{(1)} + U_i^{(1)}}_{\hat{H}^{(1)}} + \sum_{i < j} V_{ij}^{(2)} - U_i^{(1)}$$
$$+ \underbrace{\hat{H}^{(2')}}_{\cdot}$$

Electron Scattering



$$k = (\varepsilon, \vec{k}) \cong (|\vec{k}|, \vec{k})$$
$$\vec{k}' = (\varepsilon', \vec{k}') \cong (|\vec{k}'|, \vec{k}')$$

$$q = k - k'$$

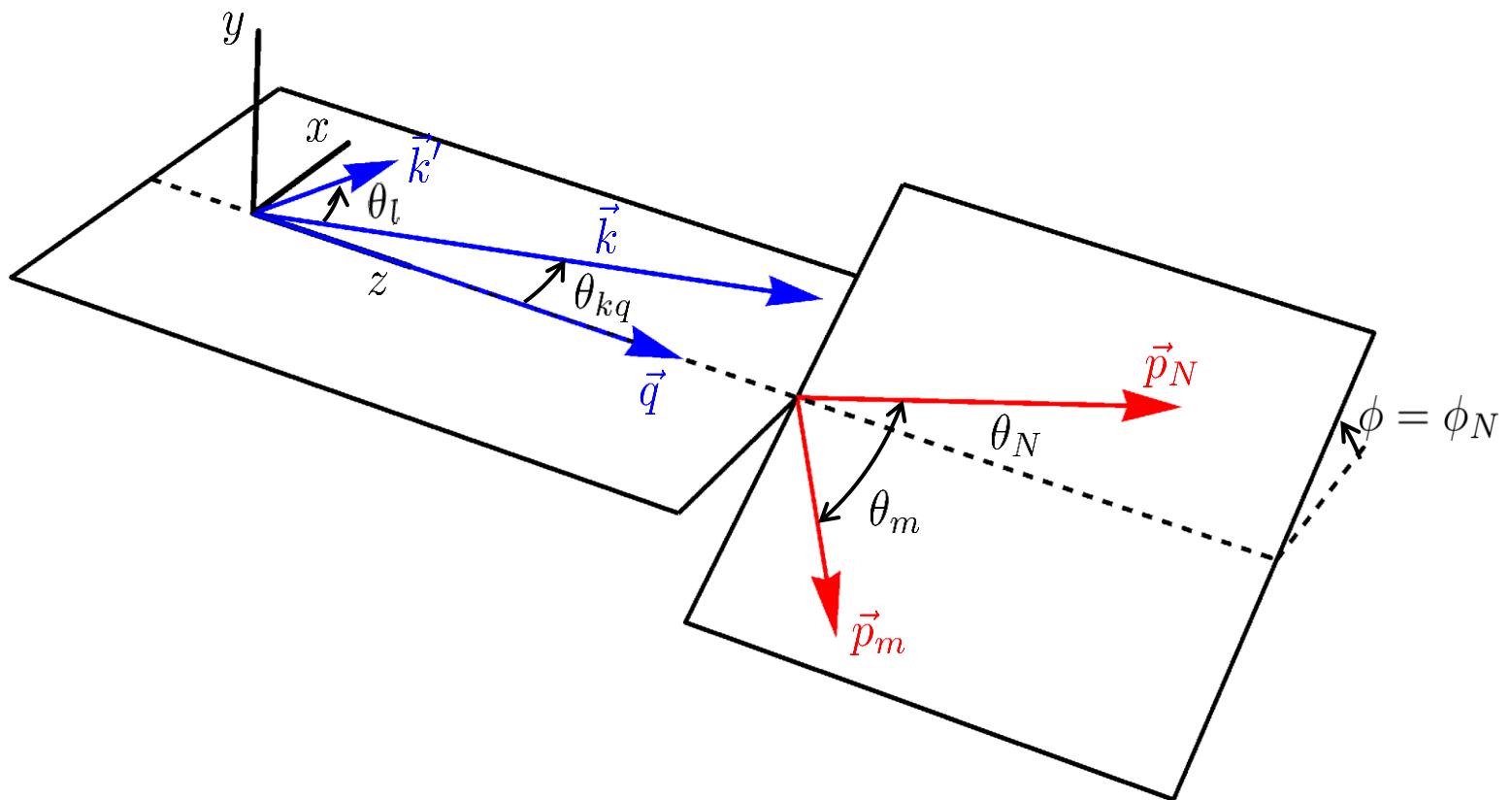
$$\vec{P}_A = (M_A, \vec{0})$$

$$p_N = (\sqrt{\vec{p}_N^2 + m_N^2}, \vec{p}_N)$$

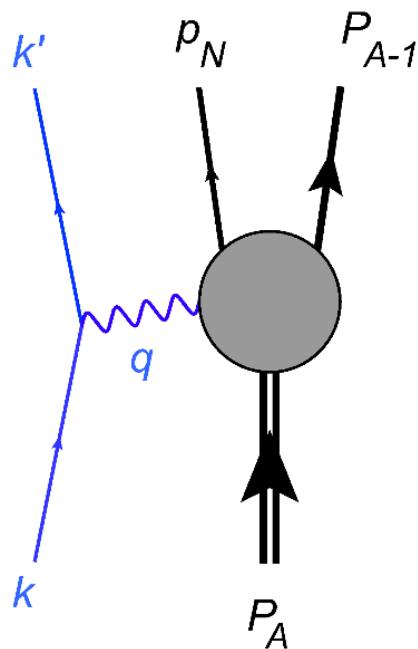
$$P_{A-1} = (\sqrt{\vec{P}_m^2 + W_{A-1}^2}, \vec{P}_m)$$

Kinematics

Fixed q frame variables



The Number of Free Kinematical Variables



5 four-momenta	+20
On-shell conditions	-5
Four-momentum conservation	-4
Choose z-axis	-2
Choose scattering plane	-1
Choose rest frame	-3
Independent degrees of freedom	5

Cross Sections

$$\mathcal{M} = \underbrace{\bar{u}(\vec{k}, \epsilon') \gamma^\mu u(\vec{k}, s)}_{\frac{1}{Q^2}} \Delta_{\mu\nu}(Q^2) \underbrace{\langle f | \hat{J}^\nu(g) | i \rangle}_{Q^2 = \vec{q}^2 - \omega^2}$$

$$\tilde{\mathcal{M}} \propto \eta_{\mu\nu} W^{\mu\nu} \frac{e^4}{Q^4} -$$

$\underbrace{(4\pi\alpha)^2}_{(4\pi\alpha)^2}$

Conserved Current

$$\vec{\nabla} \cdot \hat{j} + \frac{\partial \hat{p}}{\partial t} = 0 \Rightarrow \vec{\nabla} \cdot \hat{j} = i [\hat{p}, \hat{H}]$$

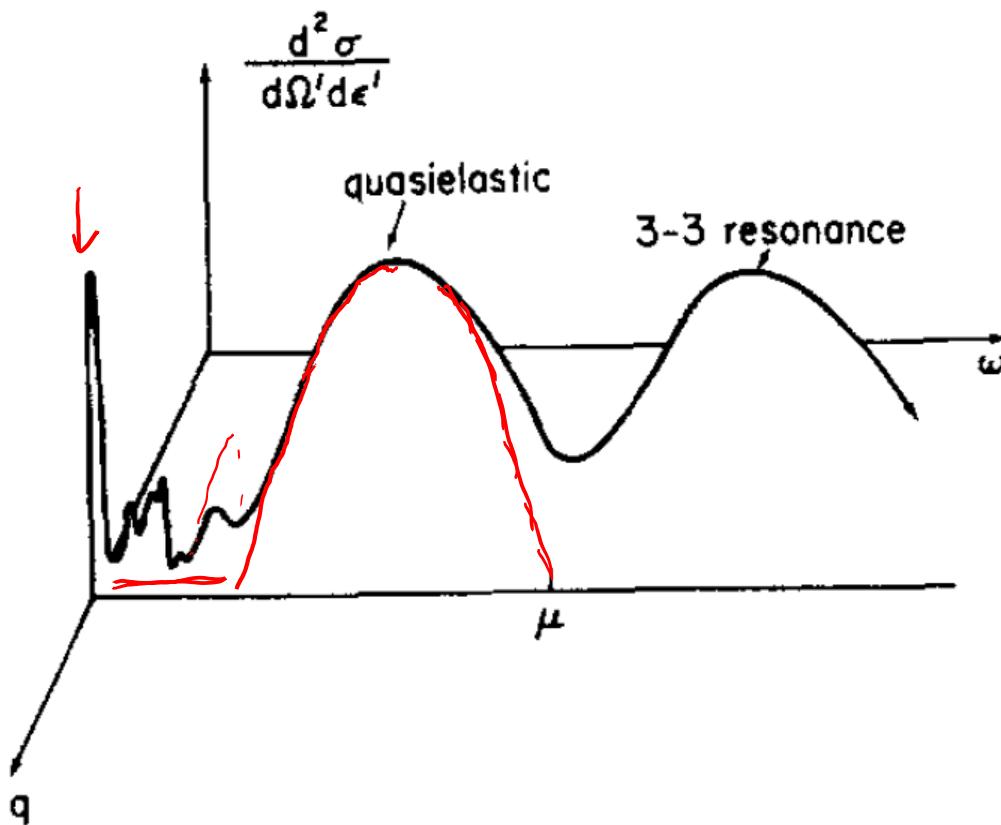
Inclusive Cross Section

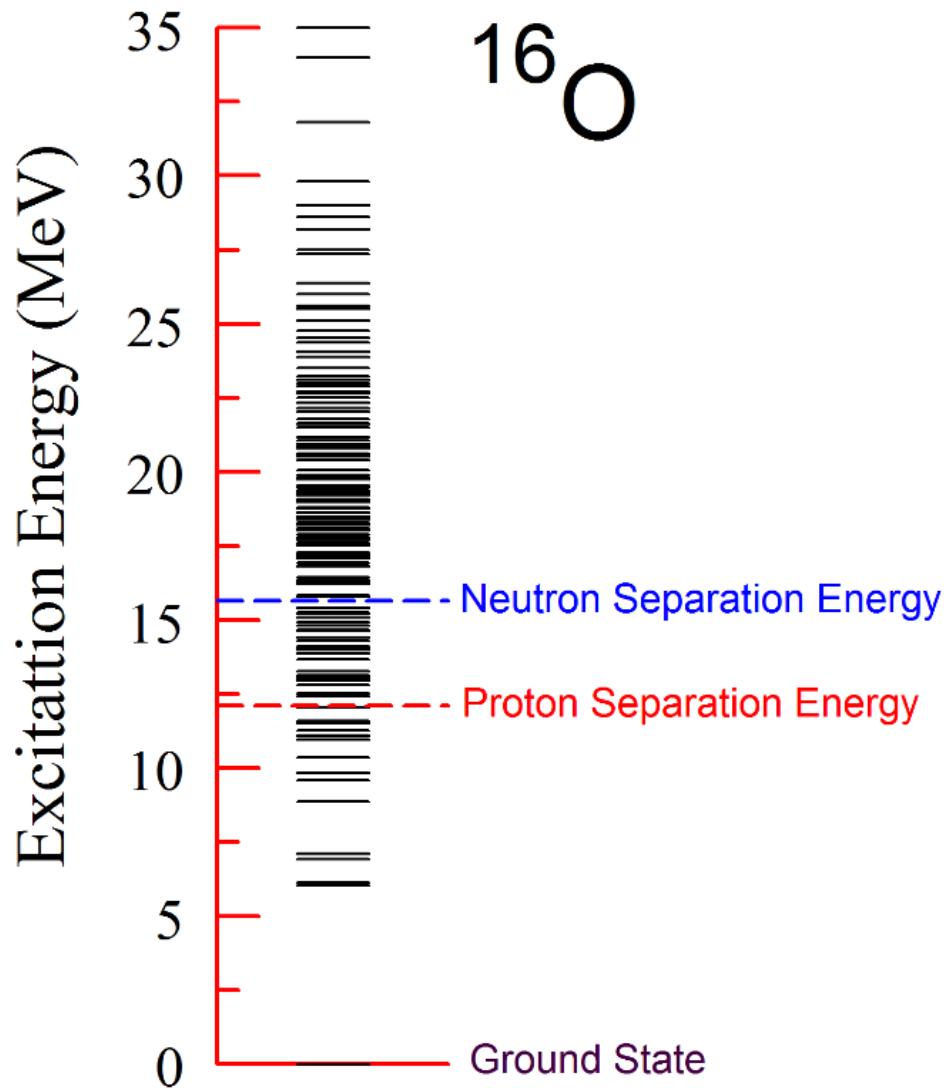
$$\frac{d\sigma^2}{dk'd\Omega_{k'}} = \frac{m_N}{4\pi^2} \sigma_{Mott} (v_L(R_L) + v_T(R_T))$$
$$\sigma_{Mott} = \frac{\alpha^2 \cos^2 \frac{\theta_l}{2}}{4k^2 \sin^4 \frac{\theta_l}{2}}$$

$v_L \quad = \quad \frac{Q^4}{\vec{q}^4}$
 $v_T \quad = \quad \frac{Q^2}{2\vec{q}^2} + \tan^2 \frac{\theta_l}{2}$

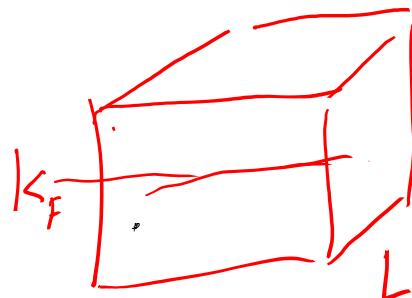
$\overset{\parallel}{W^{00}} \quad W^{11} + W^{22}$

Inclusive “Quasielastic” Scattering





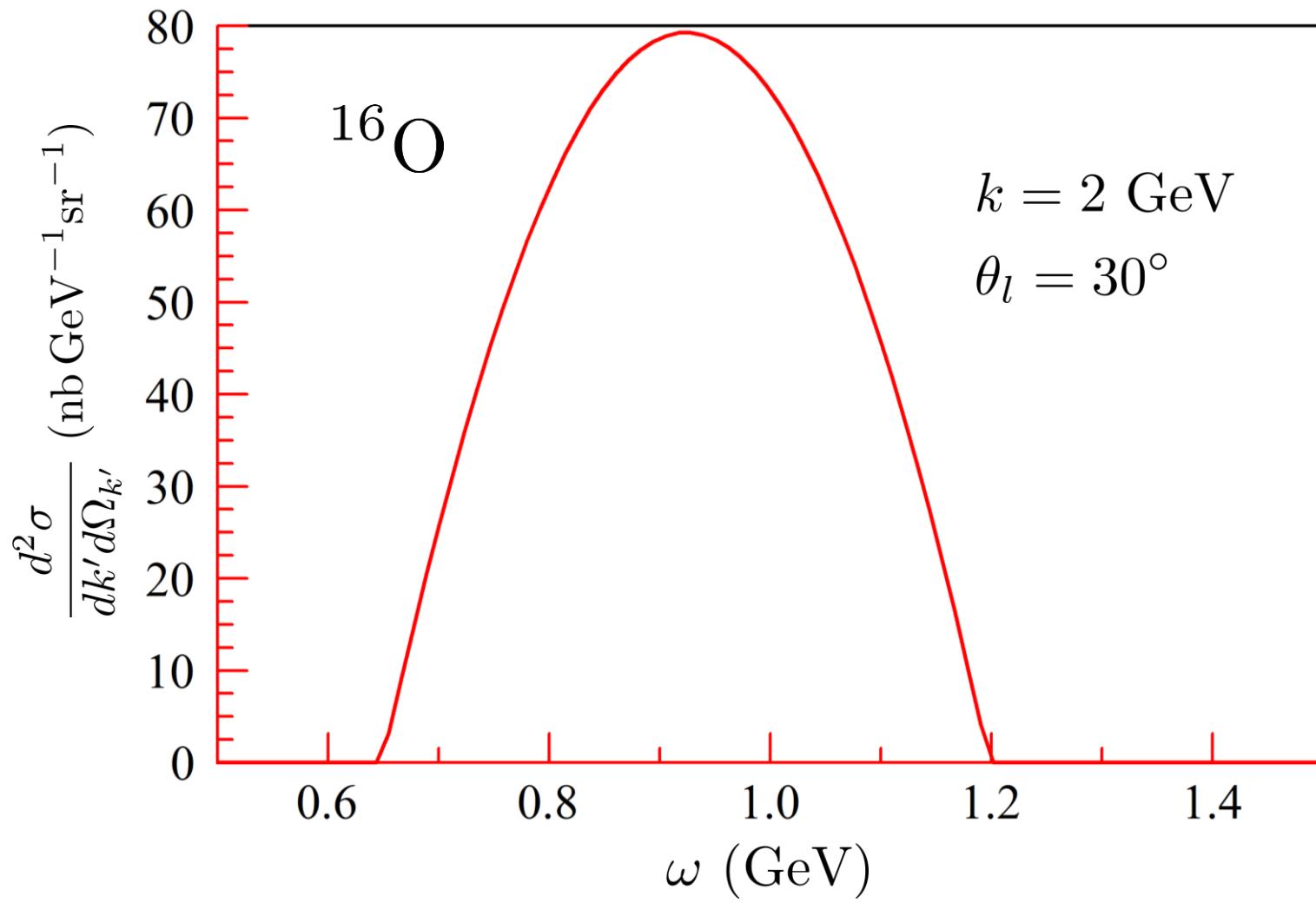
The Relativistic Fermi Gas Model

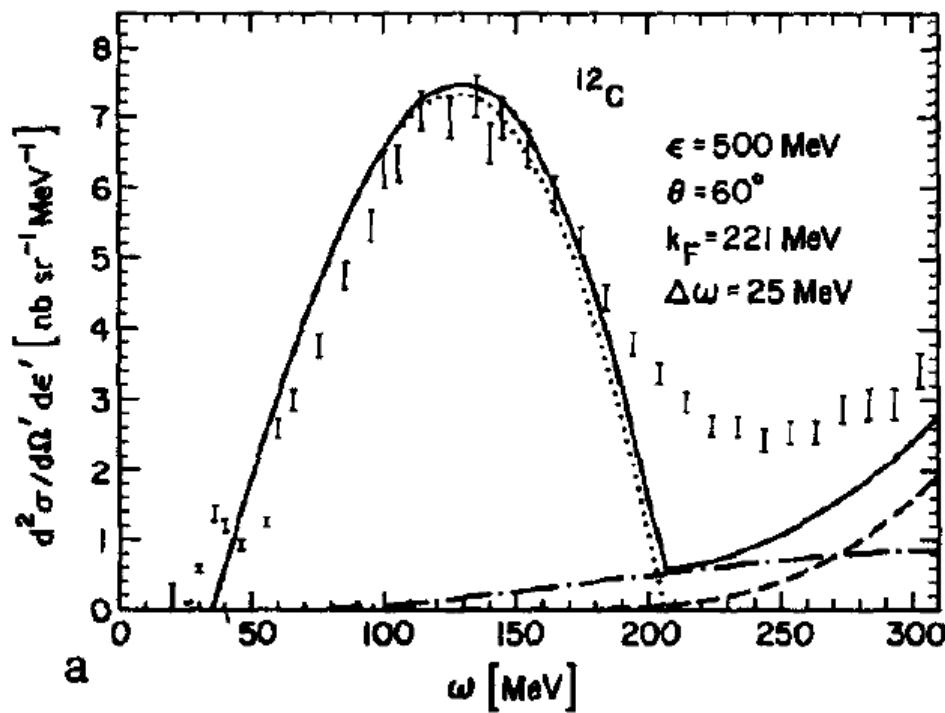


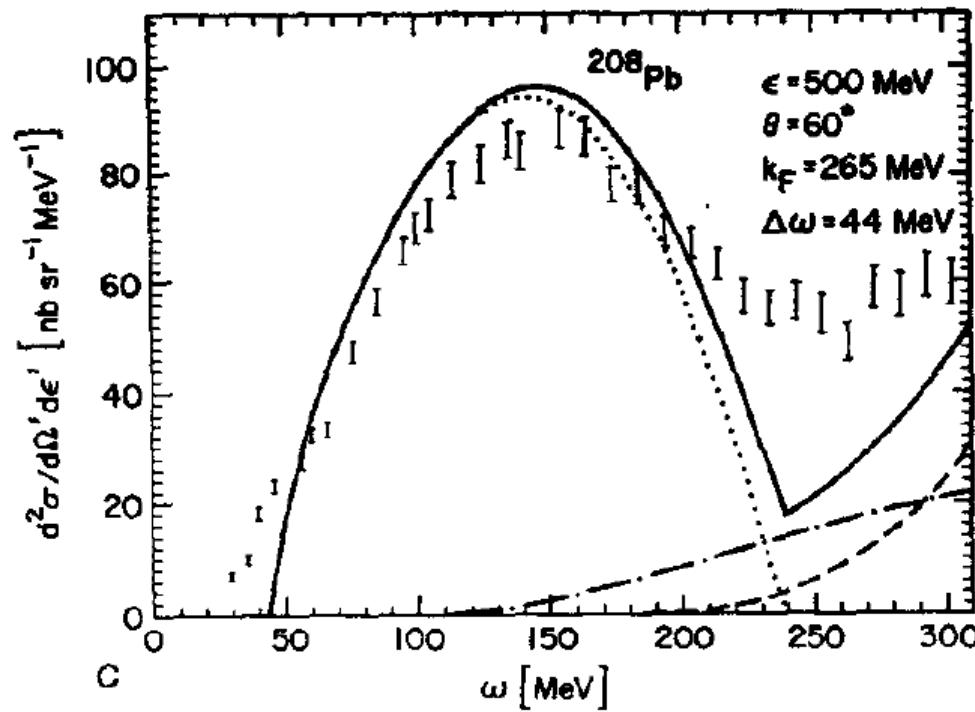
$$N = 2 \cdot 2V \int_0^{\infty} \frac{dp p^2}{(2\pi)^3} \Theta(k_F - |\vec{p}|)$$

$$N = \frac{4V}{(2\pi)^3} \frac{k_F^3}{3} \quad \underline{\frac{N}{V}} = \frac{4}{(2\pi)^3} \frac{k_F^3}{3}$$

$$\begin{aligned}
 W^{\mu\nu} \propto & \int \frac{d^3 p}{\sqrt{\vec{p}^2 + m_N^2}} \int \frac{d^3 p'}{\sqrt{\vec{p}'^2 + m_N^2}} W^{\mu\nu} \delta(\omega + \sqrt{\vec{p}^2 + m_N^2} - \sqrt{\vec{p}'^2 + m_N^2}) \\
 & \times \delta(\vec{p}' - \vec{p} - \vec{q}) \Theta(k_F - |\vec{p}|) \times \Theta(|\vec{p}'| - k_F) \\
 \rightarrow & \int \frac{d^3 p}{\sqrt{\vec{p}^2 + m_N^2}} \frac{1}{\sqrt{(\vec{p} + \vec{q})^2 + m_N^2}} \\
 & \times W^{\mu\nu}(\vec{p}, \vec{q}) \delta(\omega + \sqrt{\vec{p}^2 + m_N^2} - \sqrt{(\vec{p} + \vec{q})^2 + m_N^2}) \\
 & \times \Theta(k_F - |\vec{p}|) \Theta(|\vec{p} + \vec{q}|^2 - k_F^2)
 \end{aligned}$$







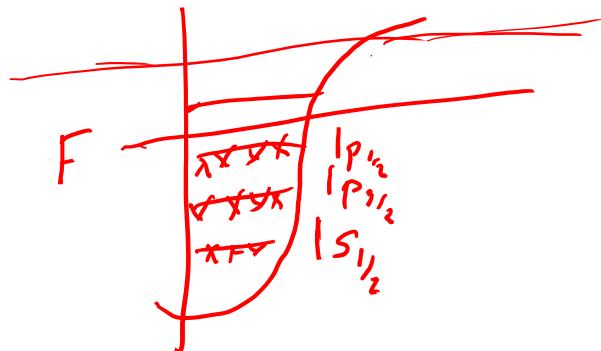
The Relativistic Mean Field Model

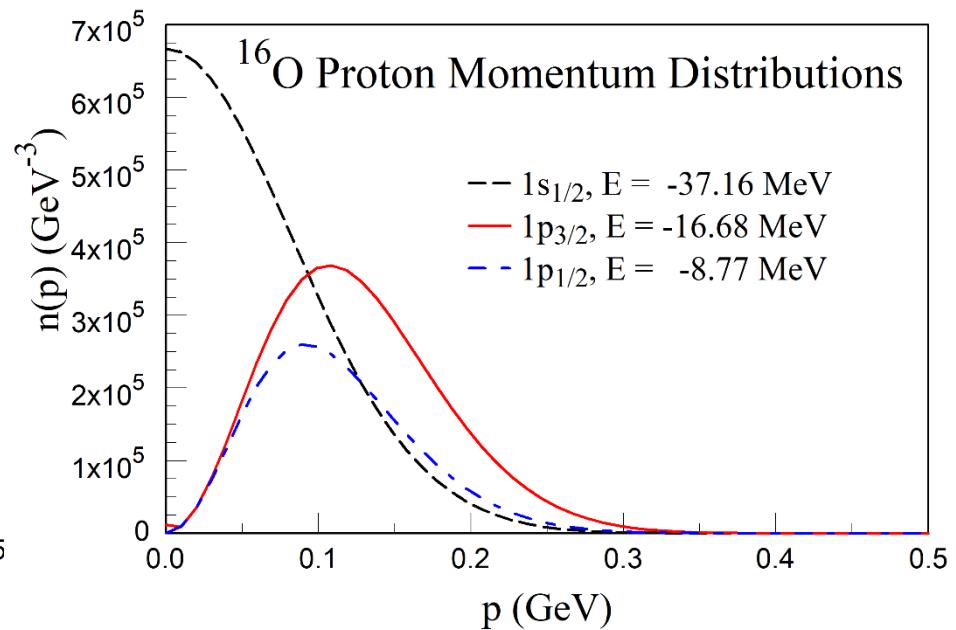
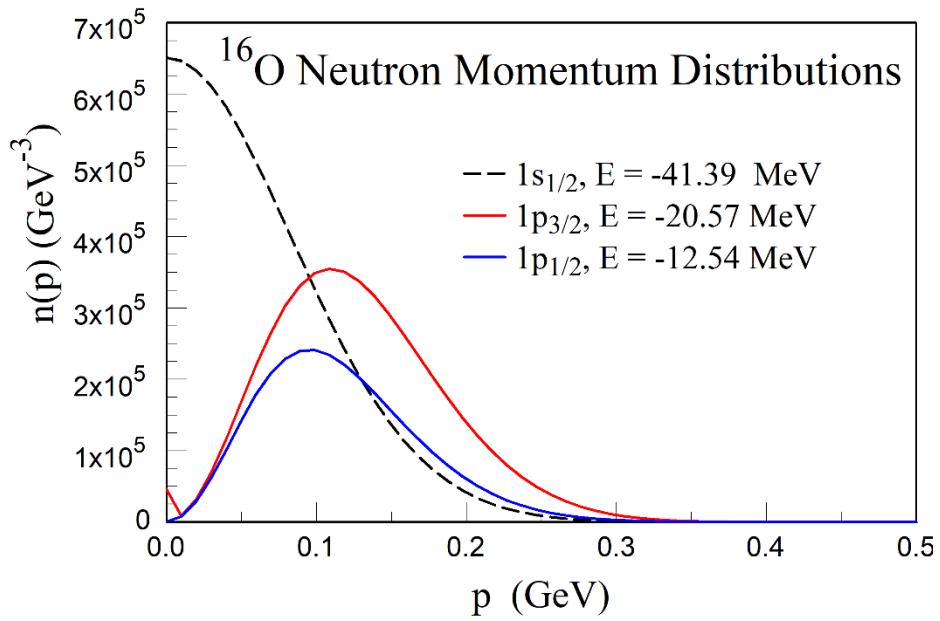
$$(Y^*E - \vec{\gamma} \cdot \hat{p} + m_n + V_s(r) + \gamma^0 V_v(r)) \psi(\vec{r}) = 0$$

$$(\vec{\gamma} \cdot \hat{p} - \gamma^0 m_n - \gamma^0 V_s(r) - V_v(r)) \psi(\vec{r}) = E \psi$$

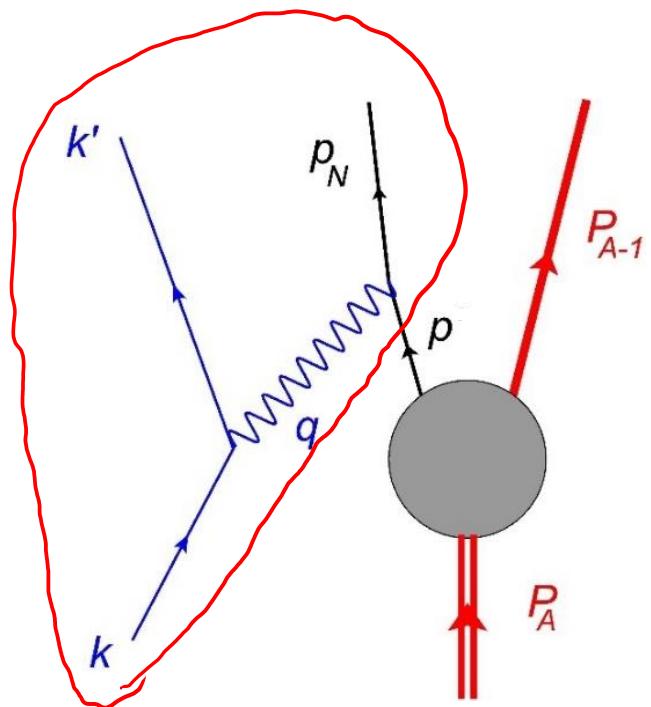
$\underbrace{\quad}_{\hat{H}}$

$$\psi(\vec{r}) = \begin{pmatrix} F_{n\ell j}(r) & Y_{\ell j}^m(\Omega) \\ iG_{n\ell j}(r) & Y_{\ell j}^{-m}(\Omega) \end{pmatrix} \quad \bar{l} = 2j - l$$

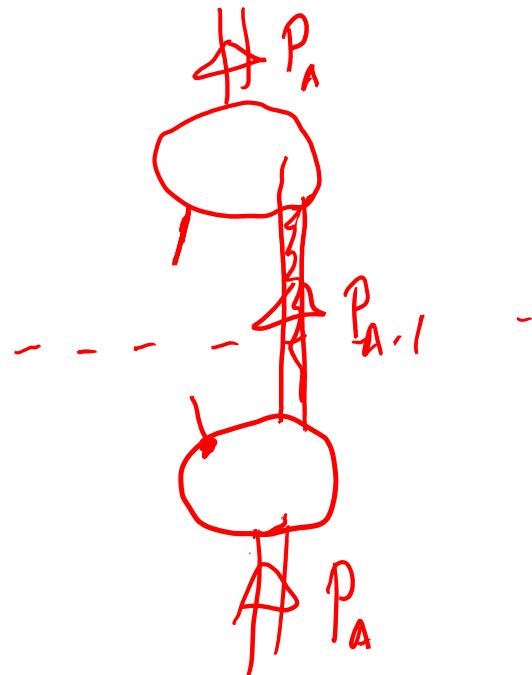




The Plane Wave Impulse Approximation



The Spectral Function

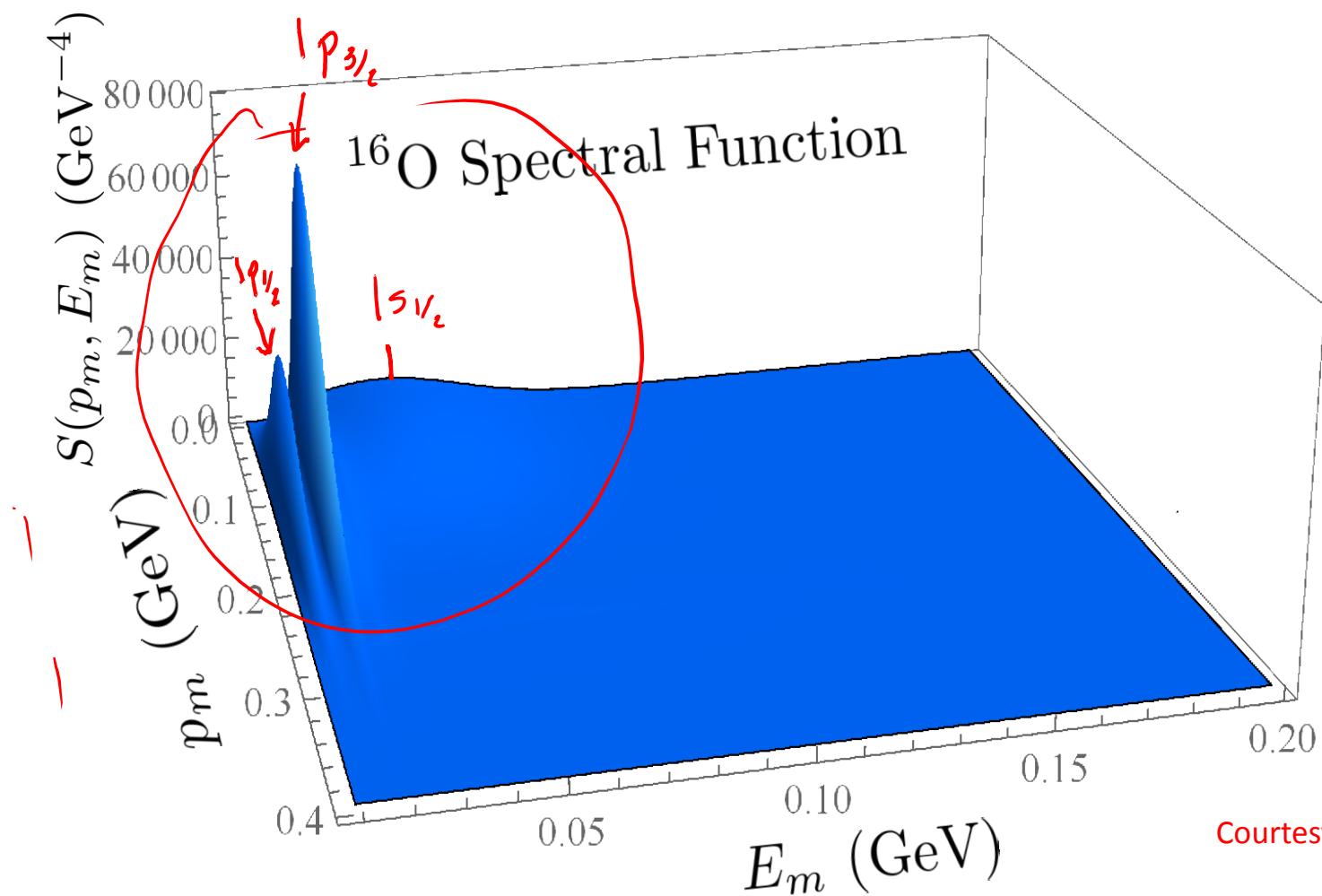


$$\begin{aligned}
W^{\mu\nu} &= \sum_{s_N} \sum_{s_A} \sum_{s_{A-1}} \sum_{s_m} \bar{u}(\mathbf{p}_N, s_N)_a J^\nu(q)_{ab} \Psi(P_{A-1}, s_{A-1}; P_A, s_A)_{bc} \\
&\quad \times \bar{\Psi}(P_{A-1}, s_{A-1}; P_A, s_A)_{cd} J^\mu(-q)_{de} u(\mathbf{p}_N, s_N)_e \\
&= \sum_{s_N} \bar{u}(\mathbf{p}_N, s_N)_a J^\nu(q)_{ab} \frac{1}{8\pi} \Lambda^+(\mathbf{p}_m)_{bd} S(p_m, E_m) J^\mu(-q)_{de} u(\mathbf{p}_N, s_N)_e \\
&= \frac{1}{8\pi} \text{Tr} [J^\mu(-q) \Lambda^+(\mathbf{p}_N) J^\nu(q) \Lambda^+(\mathbf{p}_m)] S(p_m, E_m) \\
&= \frac{1}{8\pi} w^{\mu\nu}(P_A - P_{A-1}, Q) S(p_m, E_m)
\end{aligned}$$

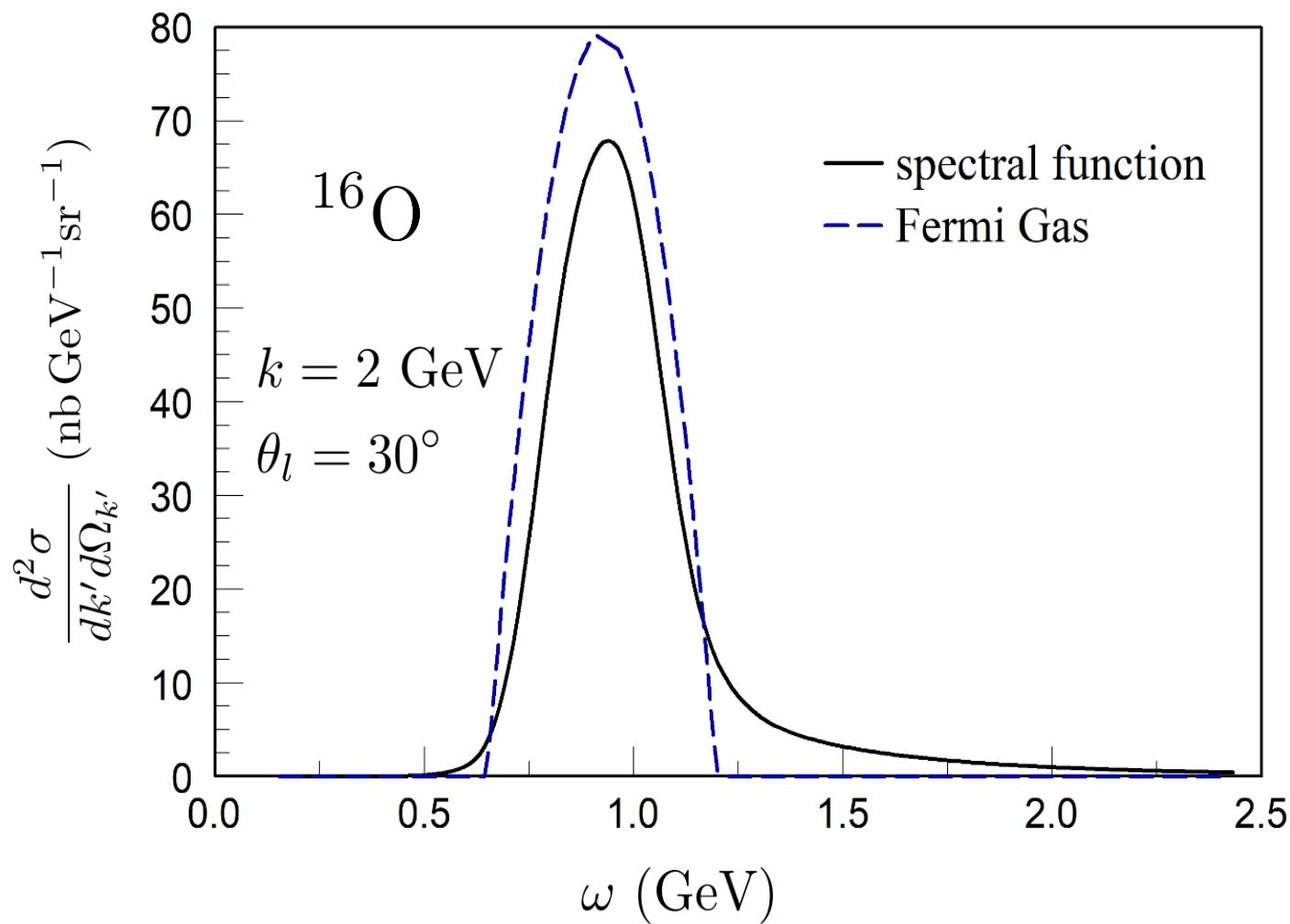
where $E_m = E_s + \mathcal{E}$ and $\mathcal{E} = \sqrt{p_m^2 + W_{A-1}^2} - \sqrt{p_m^2 + M_{A-1}^2}$

The Spectral Function is normalized such that:

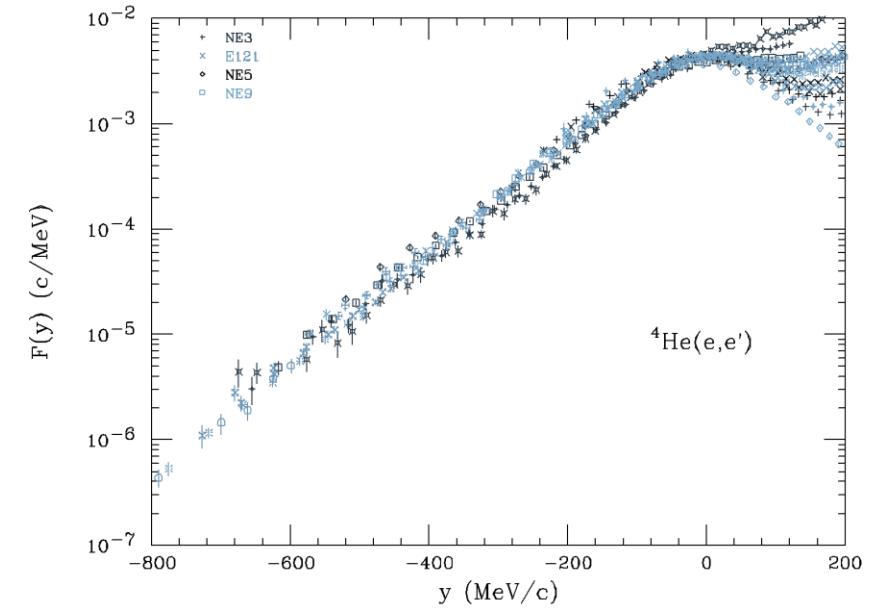
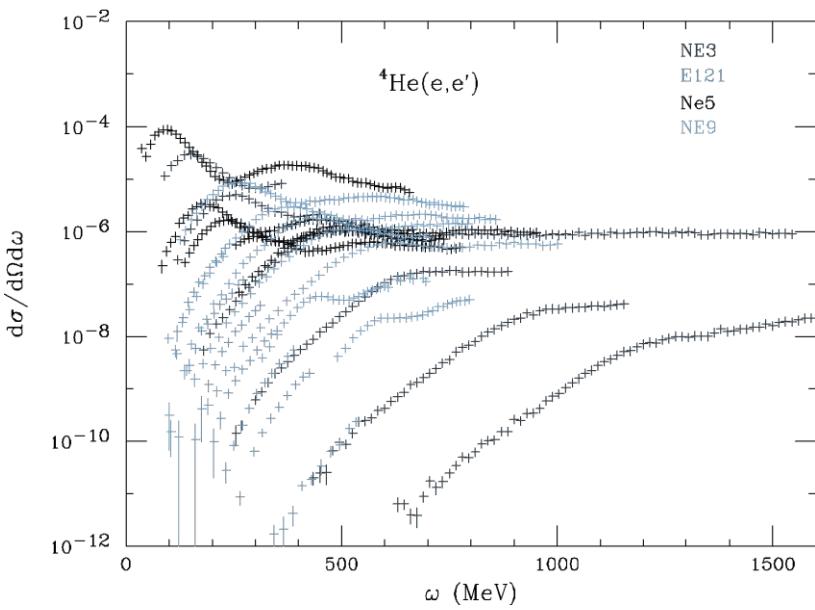
$$\int_0^\infty dE_m S(p_m, E_m) = n(p_m) \quad \frac{1}{(2\pi)^3} \int_0^\infty dp_m p_m^2 n(p_m) = N$$

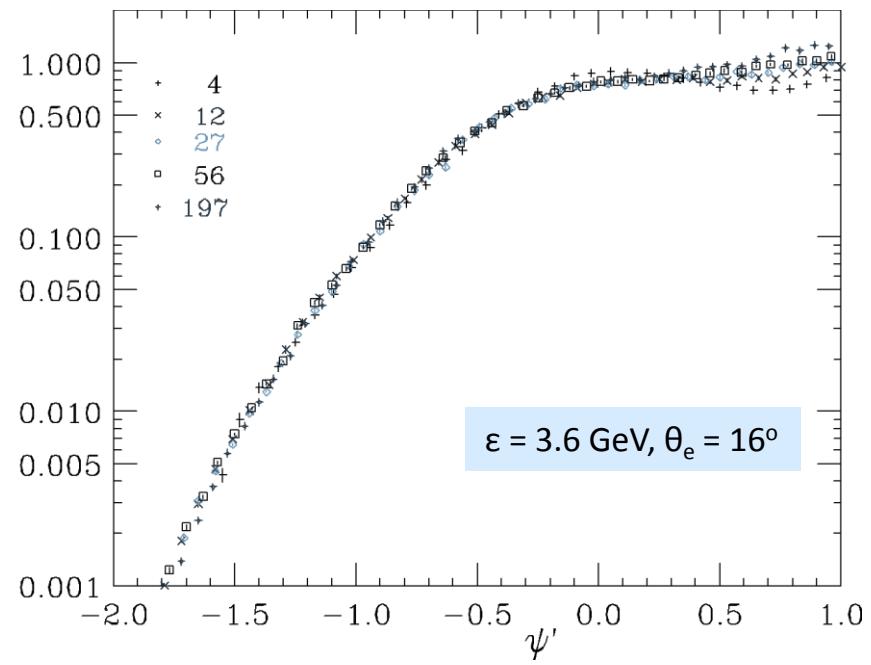
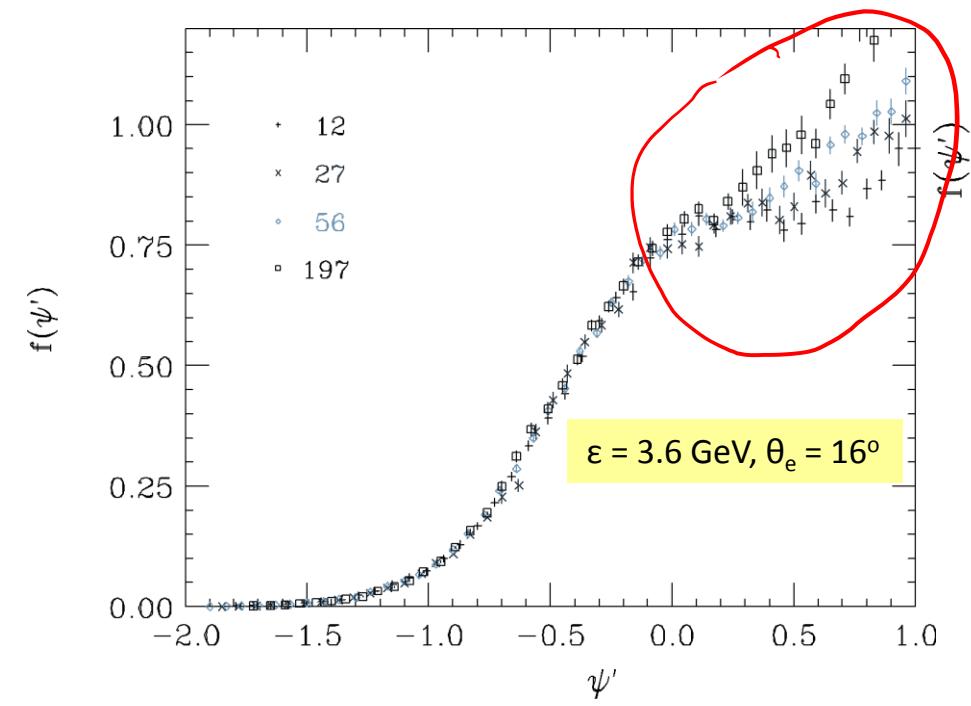


Courtesy of O. Benhar

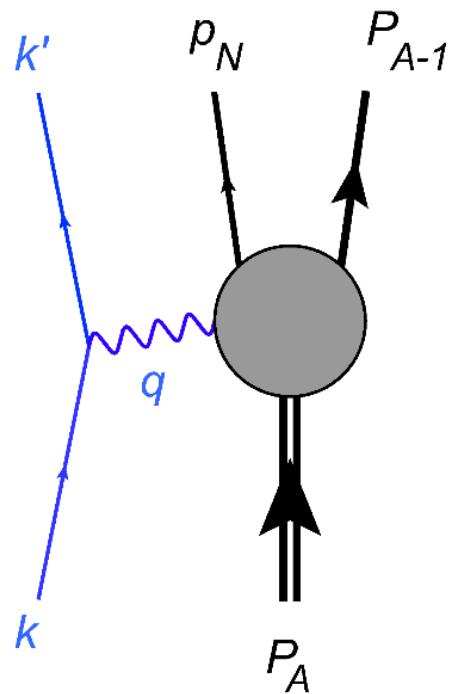


Scaling and Super Scaling





Semi-Inclusive Scattering



$$\left(\frac{d\sigma^4}{dk' d\Omega_{k'} dp_N d\Omega_N} \right)_h = \frac{m_N p_N^2}{(2\pi)^3 E_N} \sigma_{Mott} \left[\color{blue} v_L R_L^{(I)} + v_T R_T^{(I)} \right. \\ \left. + v_{TT} R_{TT}^{(I)} \cos 2\phi_N + v_{LT} R_{LT}^{(I)} \cos \phi_N \right. \\ \left. + h v_{LT'} R_{LT'}^{(II)} \sin \phi_N \right]$$

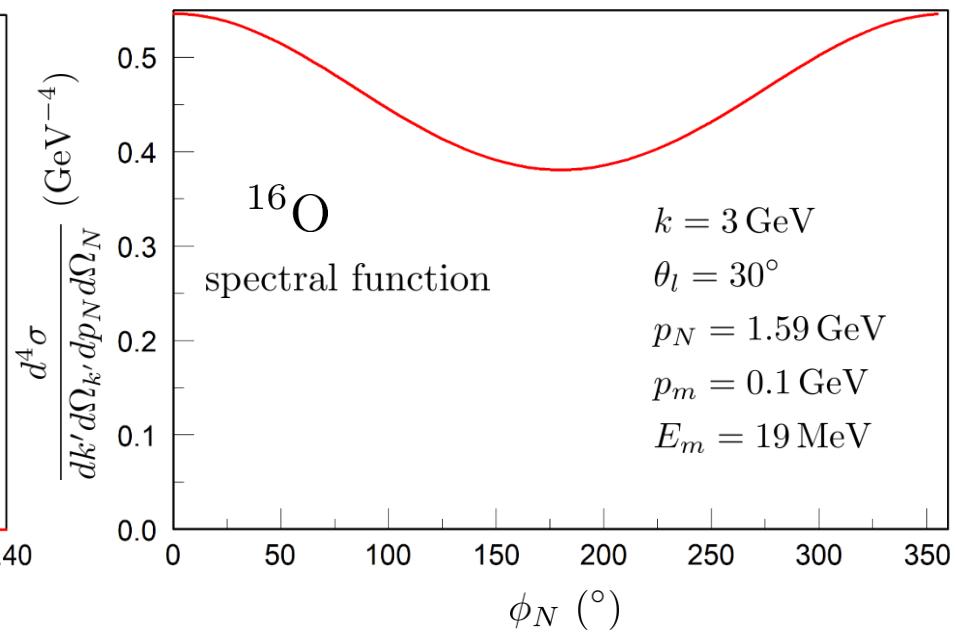
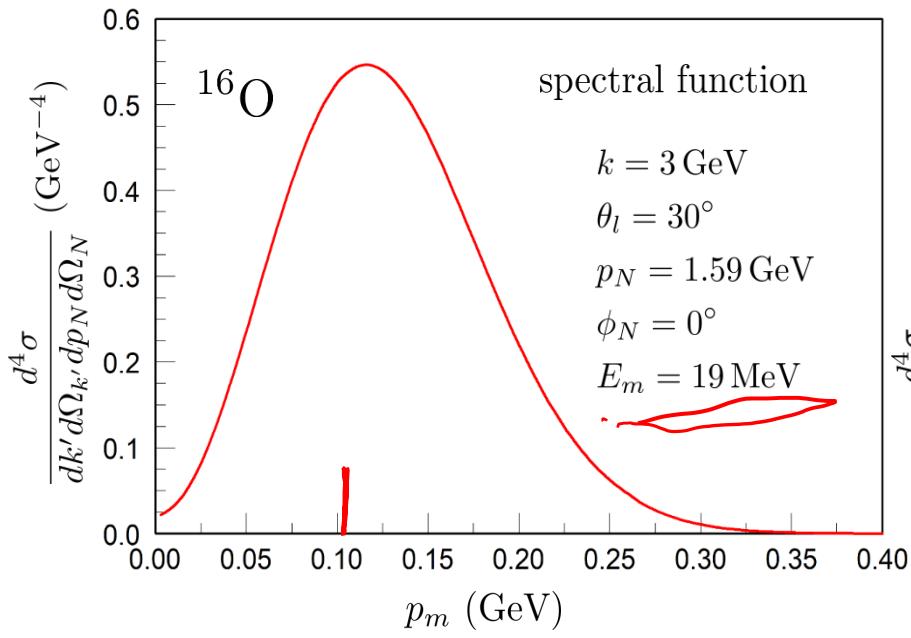
$$v_L = \frac{Q^4}{q^4}$$

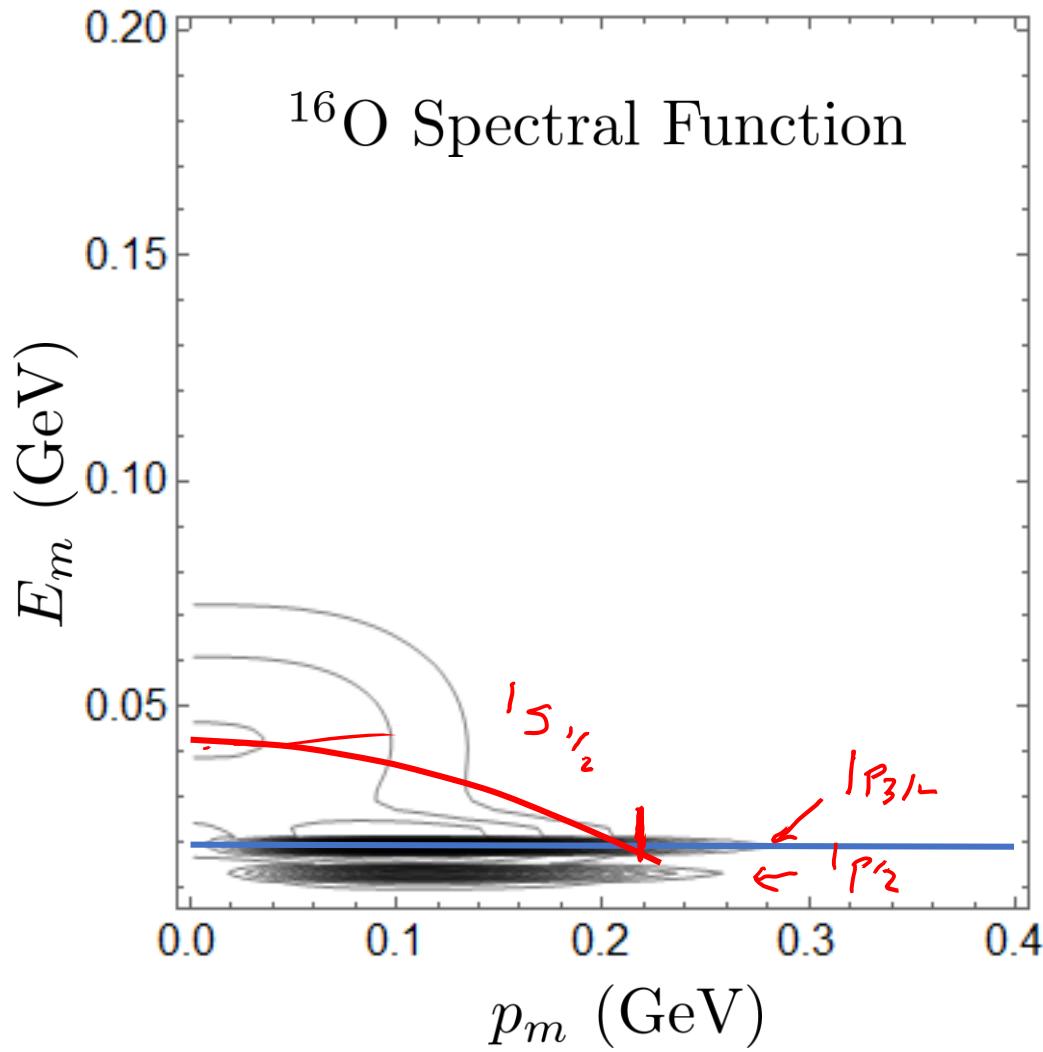
$$v_T = \frac{Q^2}{2q^2} + \tan^2 \frac{\theta_l}{2}$$

$$v_{TT} = - \frac{Q^2}{2q^2}$$

$$v_{LT} = - \frac{Q^2}{\sqrt{2}q^2} \sqrt{\frac{Q^2}{q^2} + \tan^2 \frac{\theta_l}{2}}$$

$$v_{LT'} = - \frac{Q^2}{\sqrt{2}q^2} \tan \frac{\theta_l}{2} \sqrt{\frac{Q^2}{q^2} + \tan^2 \frac{\theta_l}{2}}$$

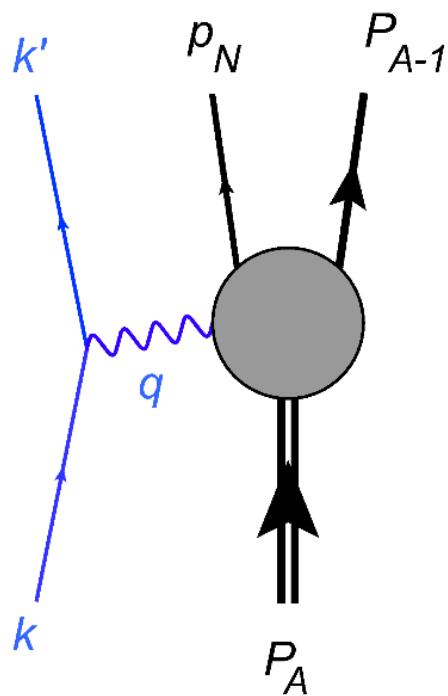




$$S_{RFG}(p_m, E_s + \mathcal{E}) = \frac{3(2\pi)^3 N}{k_F^3} \delta(\mathcal{E} - \sqrt{k_F^3 + m_n^2} + \sqrt{p_m^2 + m_n^2}) \theta(k_f - p_m)$$



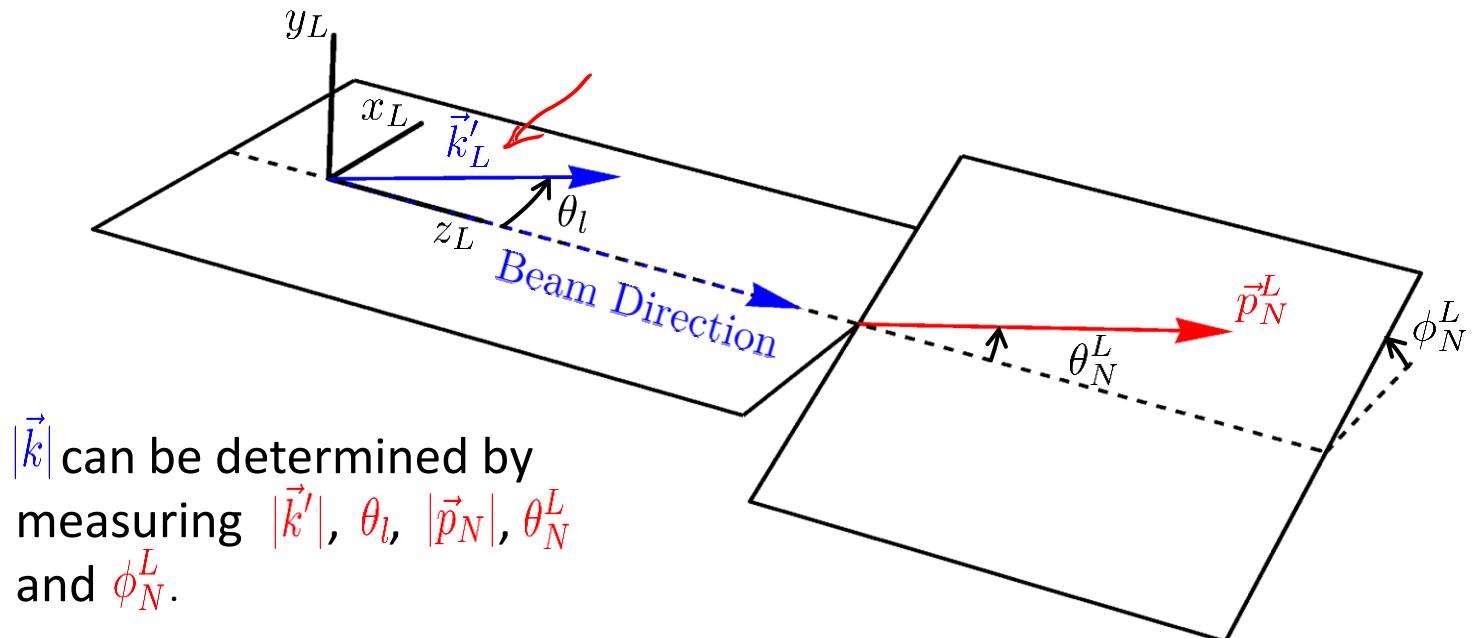
Charge-Changing Neutrino Scattering (CCv)



$$k = (\epsilon, \vec{k}) \quad \epsilon = \sqrt{\vec{k}^2 + m^2}$$
$$k' = (\epsilon', \vec{k}') \quad \epsilon' = \sqrt{\vec{k}'^2 + m'^2}$$

Kinematic Variables in the “Lab” Frame

Since the objective is to determine the incident neutrino energy to study neutrino oscillations and given that the beam direction is known but not the incident momentum, it is best to consider the frame.



Inclusive Cross Section

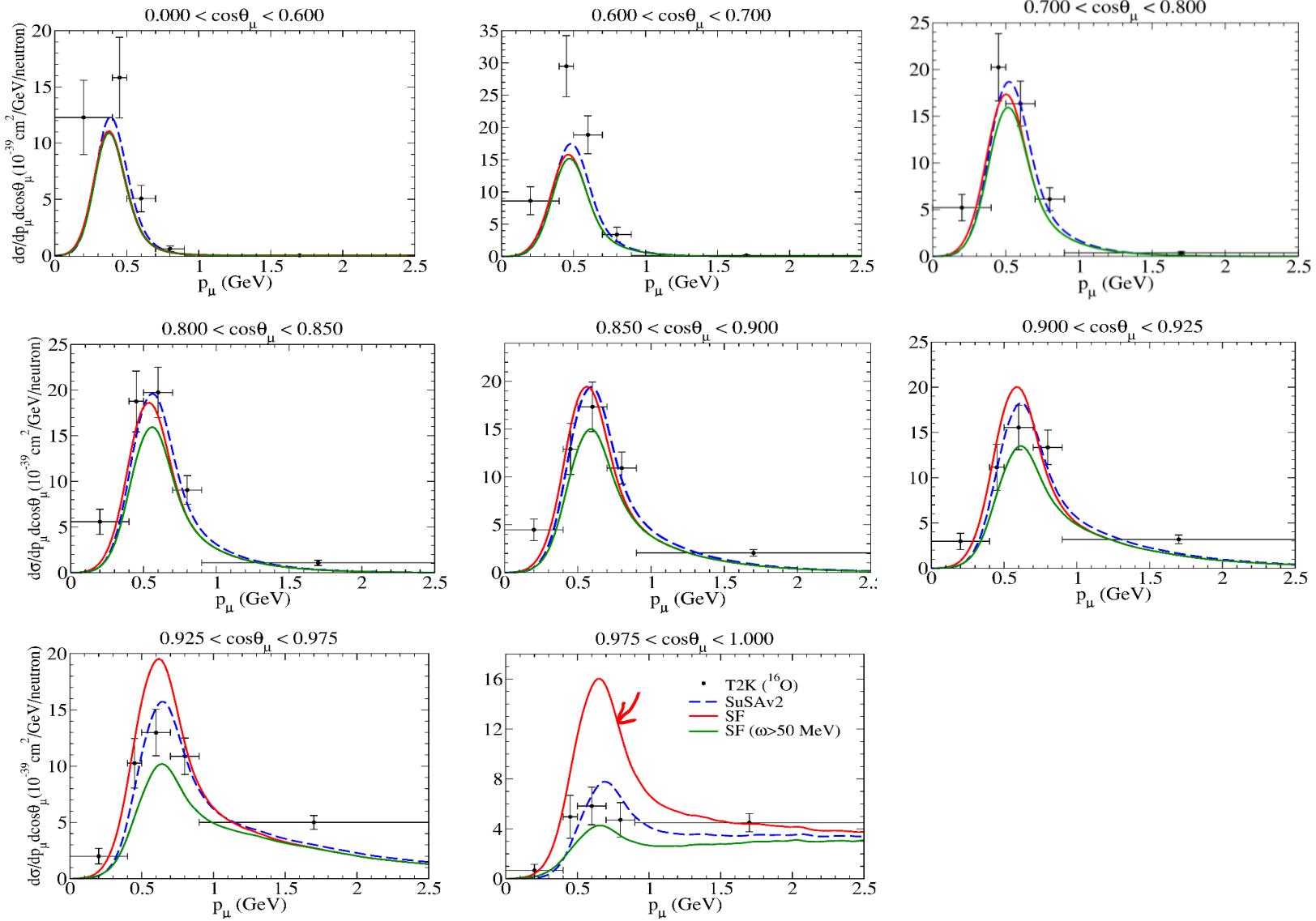
$$\left\langle \frac{d^4\sigma}{dk'd\Omega_{k'}} \right\rangle = \frac{G_F^2 \cos \theta_c m_N k'^2 v_0}{2(2\pi)^5 \varepsilon'} \int_{E_0}^{\infty} \frac{dk}{k} P(k) \left[\hat{V}_{CC} (w_{CC}^{VV(I)} + w_{CC}^{AA(I)}) \right.$$
$$+ 2\hat{V}_{CL} (w_{CL}^{VV(I)} + w_{CL}^{AA(I)}) + \hat{V}_{LL} (w_{LL}^{VV(I)} + w_{LL}^{AA(I)})$$
$$\left. + \hat{V}_T (w_T^{\cancel{VV}(I)} + w_T^{AA(I)}) + \chi \hat{V}_{T'} w_{T'}^{VA(I)} \right]$$

$$E_0 = \varepsilon' + M_{A-1} + m_N - M_A$$

$$\chi = \begin{cases} -1 & \text{for neutrinos} \\ 1 & \text{for antineutrinos} \end{cases}$$

$$v_0 \equiv (\varepsilon + \varepsilon')^2 - q^2$$

Comparison to Recent Data From T2K



Semi-Inclusive Cross Section

$$\begin{aligned}
\left\langle \frac{d^4\sigma}{dk'd\Omega_{k'}dp_Nd\Omega_N^L} \right\rangle &= \int_{M_{A-1}}^{\infty} dW_{A-1} \int_0^{\infty} dk \frac{G^2 \cos^2 \theta_c m_N k'^2 \varepsilon p_N^2 W_{A-1}}{2(2\pi)^5 k \varepsilon' E_N \sqrt{X_B^2 + m^2 a_B}} v_0 \mathcal{F}_\chi^2 \delta(k - k_0) P(k) \\
&= \int_{M_{A-1}}^{\infty} dW_{A-1} \frac{G^2 \cos^2 \theta_c m_N k'^2 \varepsilon_0 p_N^2 W_{A-1} v_0}{2(2\pi)^5 k_0 \varepsilon' E_N \sqrt{X_B^2 + m^2 a_B}} \mathcal{F}_\chi^2 P(k_0)
\end{aligned}$$

↑

$$E_B = \varepsilon' + E_N - M_A$$

$$\mathbf{p}_B = \mathbf{k}' + \mathbf{p}_N$$

$$X_B = \frac{1}{2} (p_B^2 - E_B^2 + W_{A-1}^2 - m^2)$$

$$a_B = p_B^2 \cos^2 \theta_B - E_B^2$$

$$k_0 = \frac{1}{a_B} \left(X_B p_B \cos \theta_B + E_B \sqrt{X_B^2 + m^2 a_B} \right)$$

$$\varepsilon_0 = \frac{1}{a_B} \left(E_B X_B + p_B \cos \theta_B \sqrt{X_B^2 + m^2 a_B} \right)$$

$$\begin{aligned}
\mathcal{F}_x^2 = & \hat{V}_{CC}(w_{CC}^{VV(I)} + w_{CC}^{AA(I)}) + 2\hat{V}_{CL}(w_{CL}^{VV(I)} + w_{CL}^{AA(I)}) + \hat{V}_{LL}(w_{LL}^{VV(I)} + w_{LL}^{AA(I)}) \\
& + \hat{V}_T(w_T^{VV(I)} + w_T^{AA(I)}) \\
& + \hat{V}_{TT} \left[(w_{TT}^{VV(I)} + w_{TT}^{AA(I)}) \cos 2\phi_N + (w_{TT}^{VV(II)} + w_{TT}^{AA(II)}) \sin 2\phi_N \right] \\
& + \hat{V}_{TC} \left[(w_{TC}^{VV(I)} + w_{TC}^{AA(I)}) \cos \phi_N + (w_{TC}^{VV(II)} + w_{TC}^{AA(II)}) \sin \phi_N \right] \\
& + \hat{V}_{TL} \left[(w_{TL}^{VV(I)} + w_{TL}^{AA(I)}) \cos \phi_N + (w_{TL}^{VV(II)} + w_{TL}^{AA(II)}) \sin \phi_N \right] \\
& + \chi \left[\hat{V}_{T'} w_{T'}^{VA(I)} + \hat{V}_{TC'} (w_{TC'}^{VA(I)} \sin \phi_N + w_{TC'}^{VA(II)} \cos \phi_N) \right. \\
& \left. + \hat{V}_{TL'} (w_{TL'}^{VA(I)} \sin \phi_N + w_{TL'}^{VA(II)} \cos \phi_N) \right]
\end{aligned}$$

Azimuthal angle
about \vec{q}

$$\cos \theta_N = \cos \theta_N^L \cos \theta_{kq} - \cos \phi_N^L \sin \theta_N^L \sin \theta_{kq}$$

$$\sin \theta_N = \sqrt{1 - \cos^2 \theta_N}$$

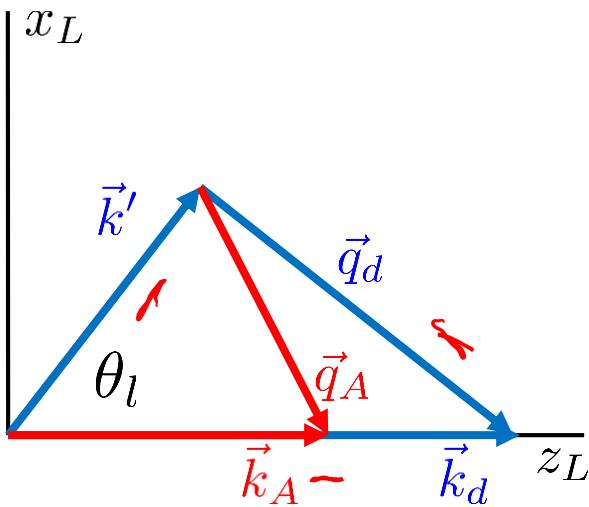
$$\cos \phi_N = \frac{\cos \phi_N^L \sin \theta_N^L \cos \theta_{kq} + \cos \theta_N^L \sin \theta_{kq}}{\sin \theta_N}$$

$$\sin \phi_N = \frac{\sin \phi_N^L \sin \theta_N^L}{\sin \theta_N}$$

Heavy Water

$^2\text{H}_2^{16}\text{O}$

Kinematics



Optimize kinematics for the deuteron

$$s_d = (M_d + \omega)^2 - \mathbf{q}^2$$

$$y = \frac{(M_d + \omega) \sqrt{s(s - 4m_N^2)}}{2s} - \frac{|\mathbf{q}|}{2}$$

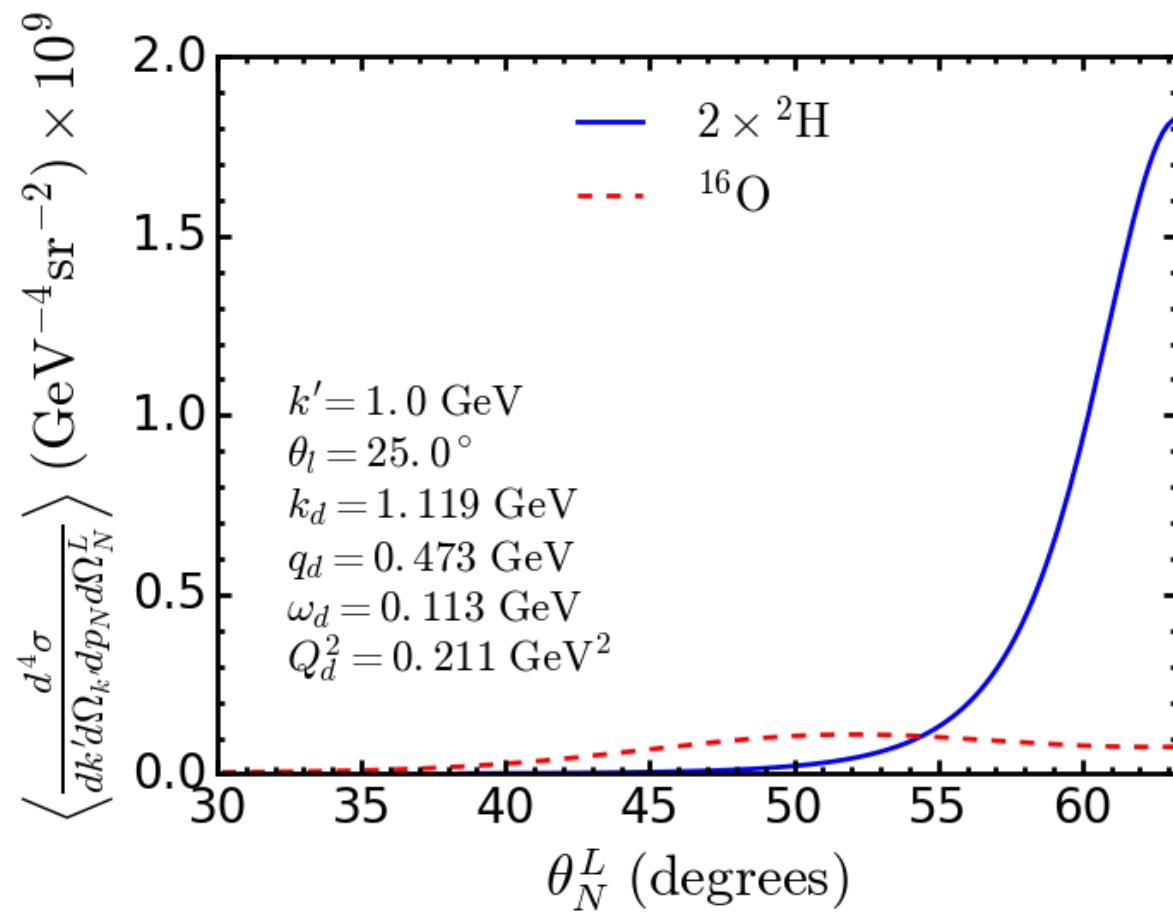
$$Y = y + |\mathbf{q}|$$

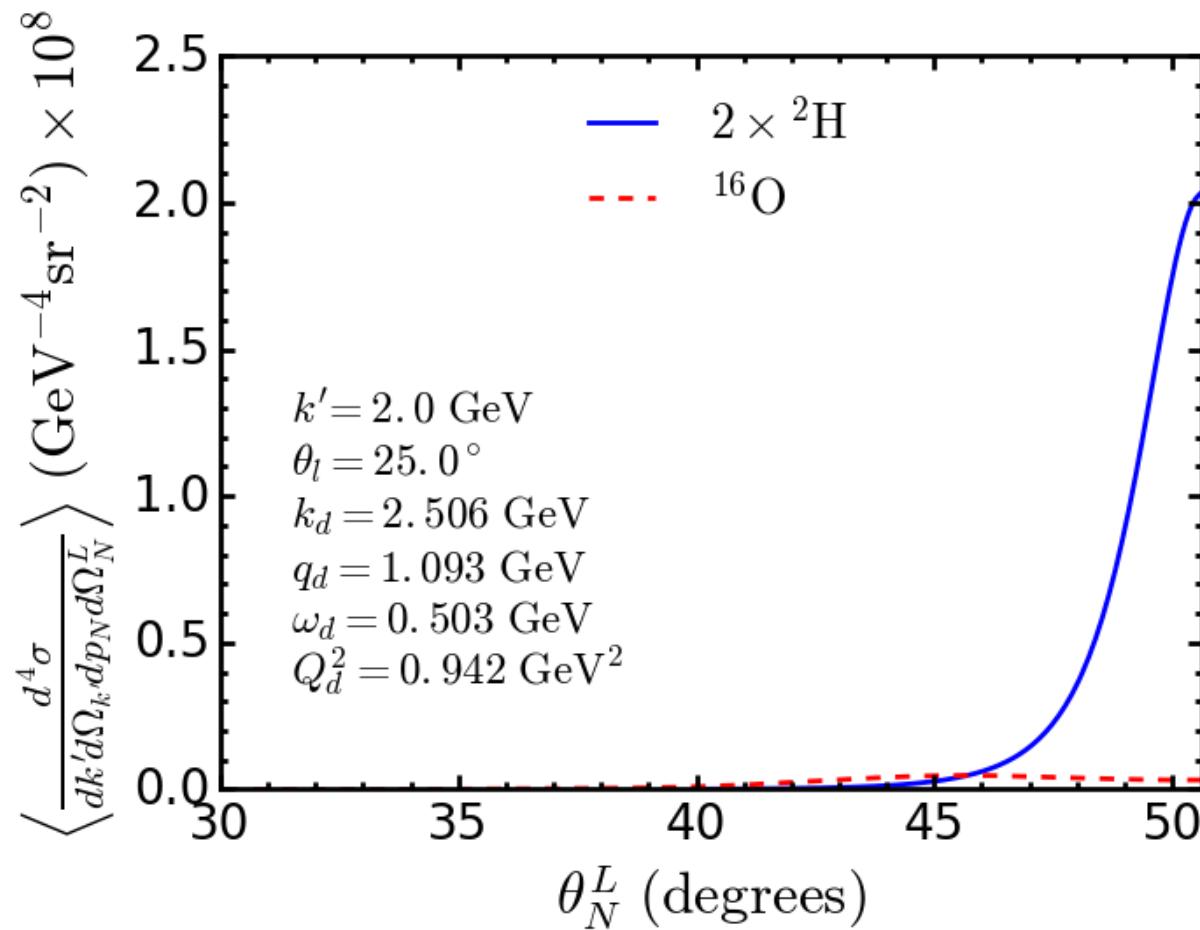
$$|y| \leq p \leq Y$$

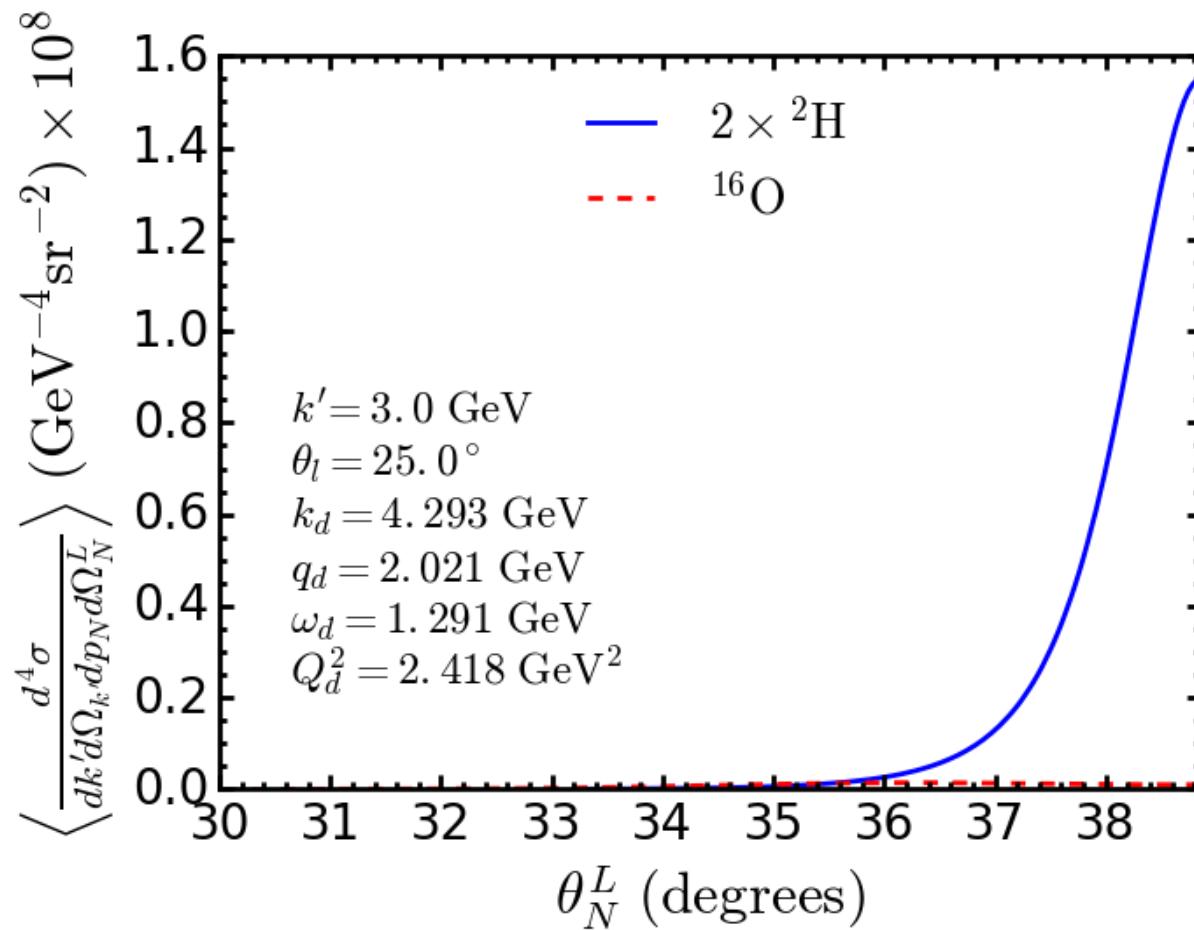
Given $|\mathbf{k}'|$ and θ_b choose $|\mathbf{k}_d|$ such that $y = 0$.

Then $|\mathbf{p}_N|$, θ_N^L and ϕ_N^L can be determined as functions of $|\mathbf{p}_N - \mathbf{q}_d|$

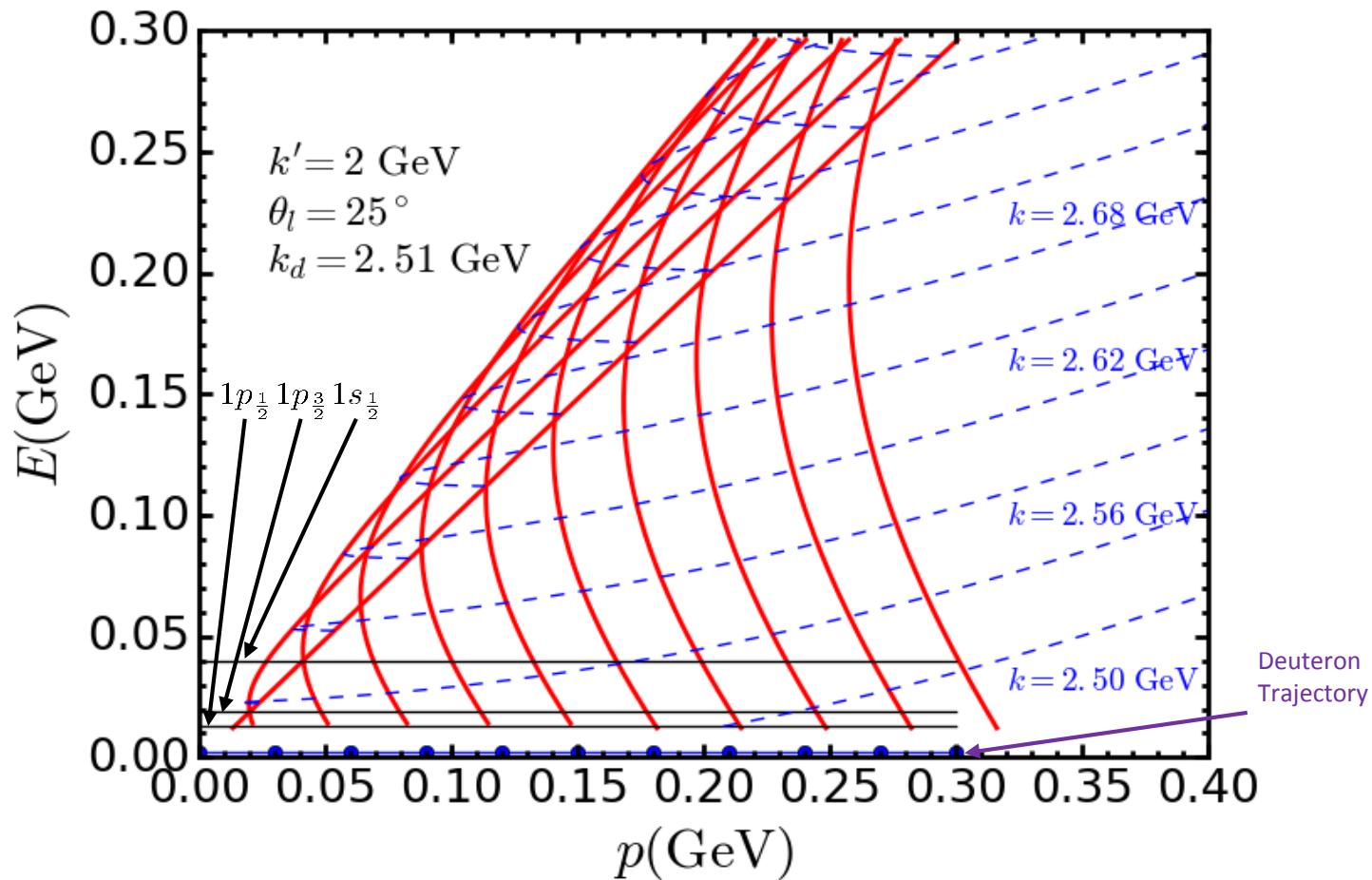
The cross section for semi-inclusive scattering from ^{16}O can then be determined as a function of $|\mathbf{k}_A|$ and $|\mathbf{p}_N - \mathbf{q}_d|$.







Trajectories in p and E



CCv Kinematic Factors

$$\Delta_1 = m^2 + m'^2$$

$$\Delta_2 = 2\varepsilon\varepsilon' - 2|\mathbf{k}||\mathbf{k}'|$$

$$\Delta_3 = 4\mathbf{k}^2\mathbf{k}'^2 - 4\varepsilon^2\varepsilon'^2$$

$$\Delta_4 = m'^2 - m^2$$

$$v_0 = (\varepsilon + \varepsilon')^2 - \mathbf{q}^2$$

$$= \Delta_2 + \Delta_1 + 4|\mathbf{k}||\mathbf{k}'| \cos^2 \frac{\theta_l}{2}$$

$$\kappa = \varepsilon + \varepsilon'$$

$$\hat{V}_{CC} = \left(1 + \frac{\Delta_1}{v_0} \right)$$

$$\hat{V}_{CL} = - \frac{1}{|\mathbf{q}|} \left(\omega + \frac{\Delta_4 \kappa}{v_0} \right)$$

$$\hat{V}_{LL} = \left(\frac{\omega^2}{\mathbf{q}^2} - \frac{\Delta_1}{v_0} + \frac{\Delta_4^2}{\mathbf{q}^2 v_0} + \frac{2\Delta_4 \kappa \omega}{\mathbf{q}^2 v_0} \right)$$

$$\hat{V}_T = \left[Q^2 \left(\frac{1}{2\mathbf{q}^2} + \frac{1}{v_0} \right) + \Delta_1 \left(\frac{1}{2\mathbf{q}^2} - \frac{1}{v_0} \right) - \frac{\Delta_1^2 - \Delta_3 + \Delta_1 Q^2}{2\mathbf{q}^2 v_0} \right]$$

$$\hat{V}_{TT} = - \left[\frac{\Delta_1 + Q^2}{2\mathbf{q}^2} \left(1 - \frac{\Delta_1}{v_0} \right) + \frac{\Delta_3}{2\mathbf{q}^2 v_0} \right]$$

$$\hat{V}_{TC} = - \frac{1}{\sqrt{2}v_0} \sqrt{1 + \frac{v_0}{\mathbf{q}^2}} \sqrt{\Delta_3 + (\Delta_1 + Q^2)(v_0 - \Delta_1)}$$

$$\hat{V}_{TL} = \frac{1}{\sqrt{2}\mathbf{q}^2 v_0} \sqrt{\Delta_3 + (\Delta_1 + Q^2)(v_0 - \Delta_1)} (\Delta_4 + \omega \kappa)$$

$$\hat{V}_{T'} = \frac{1}{v_0} \left(Q^2 \sqrt{1 + \frac{v_0}{\mathbf{q}^2}} - \frac{\Delta_4 \omega}{|\mathbf{q}|} \right)$$

$$\hat{V}_{TC'} = - \frac{1}{\sqrt{2}v_0} \sqrt{\Delta_3 + (\Delta_1 + Q^2)(v_0 - \Delta_1)}$$

$$\hat{V}_{TL'} = \frac{1}{\sqrt{2}|\mathbf{q}|v_0} \sqrt{\Delta_3 + (\Delta_1 + Q^2)(v_0 - \Delta_1)}$$