# $\nu$ Deeply Inelastic Scattering 

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## Lecture II - Beyond the Parton Model

- Higher order corrections
- Factorization schemes
- PDF scale dependence and DGLAP evolution equations
- QCD-improved parton model
- Global Fits for PDFs


## Higher Order QCD Corrections



- Lowest Order QCD gives the parton model result of scaling for the structure functions
- Radiative corrections due to gluon emission and quark-antiquark pair production will give rise to logarithmic dependences on $Q^{2}$
- We need to extend the previous results to include these effects
- Consider the QCD Compton subprocess $V(q) q(p) \rightarrow q^{\prime}\left(p^{\prime}\right) g(k)$
- The structure function at the parton level is given by

$$
W^{\mu \nu}=\frac{1}{8 \pi \eta} \int d(P S) \sum_{\text {spins }} h^{\mu \dagger} h^{\nu}
$$

where

$$
h^{\mu}=g T_{i j}^{a} \bar{u}\left(p^{\prime}\right)\left[\frac{\gamma^{\mu}\left(1-\gamma_{5}\right)(\not p-\not ้) \gamma^{\alpha}}{(p-k)^{2}}+\frac{\gamma^{\alpha}(\not p+\not q)\left(1-\gamma_{5}\right)}{(p+q)^{2}}\right] \epsilon_{\alpha}^{*}(k) u(p)
$$

and

$$
d(P S)=\frac{d^{3} k}{(2 \pi)^{3} 2 E_{k}} \frac{d^{3} p^{\prime}}{(2 \pi)^{3} 2 E^{\prime}}(2 \pi)^{4} \delta^{4}\left(q+p-k-p^{\prime}\right)
$$

- The tensor structure is now more complex than for the lowest order term
- A simplification can be made for the case of massless quarks since then the tensor given above satisfies the same current conservation relations that hold in the electromagnetic case:

$$
q_{\mu} W^{\mu \nu}=q_{\nu} W^{\mu \nu}=0
$$

Exercise: Show this
Then, the hadronic tensor can be written as

$$
\begin{aligned}
W^{\mu \nu}= & F_{1}\left(-g^{\mu \nu}+\frac{q^{\mu} q^{\nu}}{q^{2}}\right)+\frac{F_{2}}{M \nu}\left(P^{\mu}-\frac{P \cdot q q^{\mu}}{q^{2}}\right)\left(P^{\nu}-\frac{P \cdot q q^{\nu}}{q^{2}}\right) \\
& -i \frac{F_{3}}{2 M \nu} \epsilon^{\mu \nu \alpha \beta} P_{\alpha} q_{\beta}
\end{aligned}
$$

Exercise: Show that this expression satisfies the two relations given above.

- The simplification of the calculation follows from considering two contractions:

$$
C_{1}=g_{\mu \nu} W^{\mu \nu} \text { and } C_{2}=P_{\mu} P_{\nu} W^{\mu \nu}
$$

- The capital $P$ refers to the hadronic 4-vector with $p^{\mu}=\eta P^{\mu}$ Exercise: Show the following:

$$
\begin{aligned}
\frac{F_{2}}{x} & =\frac{12 x^{2}}{Q^{2}} C_{2}-C_{1} \\
F_{1} & =\frac{2 x^{2}}{Q^{2}} C_{2}-\frac{1}{2} C_{1}
\end{aligned}
$$

Filling in the Details

- Squaring the amplitude and taking the appropriate trace yields

$$
\begin{aligned}
& C_{1}=\frac{\alpha_{s}}{2 \eta} C_{F} \int d(P S) 16\left(\frac{\hat{s}}{\hat{t}}+\frac{\hat{t}}{\hat{s}}-2 \frac{\hat{u} Q^{2}}{\hat{s} \hat{t}}\right) \\
& C_{2}=\frac{\alpha_{s}}{2 \eta} C_{F} \int d(P S) \frac{-8 \hat{u}}{\eta^{2}}
\end{aligned}
$$

where
$\alpha_{s}=\frac{g^{2}}{4 \pi}$, the color factor $C_{F}=4 / 3$, and the parton Mandelstam variables are $\hat{s}=(p+q)^{2}, \quad \hat{t}=(p-k)^{2}, \quad \hat{u}=\left(q-p^{\prime}\right)^{2}$
Exercise: Derive the results for $C_{1}$ and $C_{2}$

The phase space factor can be easily evaluated in the parton center of mass frame and then expressed in terms of Lorentz scalars:

$$
d(P S)=\frac{d \cos \theta}{16 \pi}=\frac{-\hat{t}}{8 \pi\left(\hat{s}+Q^{2}\right)}
$$

Using the previous results for $C_{1}$ and $C_{2}$ and convoluting with a quark PDF yields

$$
\begin{gathered}
F_{1}=\frac{\alpha_{s}}{2 \pi} C_{F} \quad \int_{x}^{1} \quad \frac{d \eta}{\eta} q(\eta)\left[-\frac{\hat{s}}{\hat{t}}-\frac{\hat{t}}{\hat{s}}+2 \frac{\hat{u} Q^{2}}{\hat{s} \hat{t}}-2 \frac{x^{2} \hat{u}}{\eta^{2} Q^{2}}\right] \frac{d(-\hat{t})}{\left(\hat{s}+Q^{2}\right)} \\
F_{2}=\frac{\alpha_{s}}{2 \pi} C_{F} 2 x
\end{gathered} \int_{x}^{1} \quad \frac{d \eta}{\eta} q(\eta)\left[-\frac{\hat{s}}{\hat{t}}-\frac{\hat{t}}{\hat{s}}+2 \frac{\hat{u} Q^{2}}{\hat{s} \hat{t}}-6 \frac{x^{2} \hat{u}}{\eta^{2} Q^{2}}\right] \frac{d(-\hat{t})}{\left(\hat{s}+Q^{2}\right)}
$$

Note that the Callan-Gross relation is slightly broken by the last term in the square bracket for each structure function

- The results for $x F_{3}$ are easily obtained from the information provided since the tensor structure follows from taking a trace of four $\gamma$ matrices with a $\gamma_{5}$
- In what follows, I wish to focus attention on $F_{2}$ in order to see how the QCD radiative corrections give rise to a dependence on $Q^{2}$
- Let $z=\frac{x}{\eta}=\frac{Q^{2}}{\eta 2 P \cdot q}=\frac{Q^{2}}{2 p \cdot q}=\frac{Q^{2}}{\hat{s}+Q^{2}}$
- With these relations one gets also $\hat{s}=Q^{2} \frac{1-z}{z}$ and $\hat{u}=-\frac{Q^{2}}{z}-\hat{t}$
- Inserting into the expression for $F_{2}$ yields

$$
F_{2}=\frac{\alpha_{s}}{2 \pi} C_{F} 2 x \int \frac{d \eta}{\eta} q(\eta)\left[-\frac{1}{\hat{t}}\left(\frac{1+z^{2}}{1-z}+\cdots\right)\right] d(-\hat{t})
$$

- This shows a logarithmic divergence from the $\hat{t}$ integration since $0 \leq$ $-\hat{t} \leq Q^{2} / z$
- The ... indicate non-singular terms
- Temporarily, cut the lower limit off at $-\hat{t}=\mu^{2}$
- Also, change the integration variable to $z=x / \eta$ to get

$$
F_{2}=2 \frac{\alpha_{s}}{2 \pi} \int_{x}^{1} d z \frac{x}{z} q\left(\frac{x}{z}\right)\left[\tilde{P}_{q q}(z) \ln \frac{Q^{2}}{\mu^{2}}+\cdots\right]
$$

- The function $\tilde{P}_{q q}(z)=C_{F} \frac{1+z^{2}}{1-z}$ is one of four splitting functions that will be discussed shortly
- In order to interpret this correction, we need to put back the lowest order contribution which is just $2 x q(x)$
- This can be brought under the integral sign by a judicious use of a delta function

$$
F_{2}(x)=2 x \int_{x}^{1} \frac{d \eta}{\eta} q(\eta)\left[\delta\left(1-\frac{x}{\eta}\right)+\frac{\alpha_{s}}{2 \pi} \tilde{P}_{q q}\left(\frac{x}{\eta}\right) \ln \frac{Q^{2}}{\mu^{2}}+\cdots\right]
$$

- There are two problems with the expression on the preceding page

1. $\tilde{P}_{q q}\left(\frac{x}{\eta}\right)$ diverges logarithmically as $\eta \rightarrow x$
2. There is a logarithmic divergence as the cut-off $\mu^{2}$ goes to zero

- The first is an example of a soft singularity while the second is from a collinear singularity.
- Fortunately, the calculation is not yet complete - we have yet to include the 1-loop corrections to the lowest order diagram.
- These loop corrections have the same kinematics as the lowest order term - they contribute with a weight proportional to $q(x)$ or at $\eta=x$ under the integral
- Soft singularities associated with the loop corrections cancel those coming from the tree-level Compton process we have been calculating
- For our purposes, the main effect is to modify $\tilde{P}_{q q}$ and replace it with a modified splitting function given by

$$
P_{q q}(z)=C_{F}\left[\frac{1+z^{2}}{(1-z)_{+}}+\frac{3}{2} \delta(1-z)\right]
$$

- The so-called "plus" distribution is defined under the integral sign as

$$
\int_{0}^{1} d z f(z) \frac{1}{(1-z)_{+}}=\int_{0}^{1} d z \frac{f(z)-f(1)}{(1-z)}
$$

- With this form for the splitting function, the singularity at $z=1$ has been removed.
- This is an example of the idea of infrared safety whereby the soft singularities cancel between the loop and tree contributions for properly defined observables
- Next, consider the collinear singularity that has temporarily been regulated by the cut-off $\mu^{2}$
- This divergence came from the $\frac{1}{t}$ factor that originated in the propagator factor for the incoming quark line
- $\hat{t} \rightarrow 0$ corresponds to the the gluon being emitted collinear with the quark and the quark goes on-shell
- This is a long distance, i.e., low momentum transfer, effect associated with the incoming quark and is not part of the hard scattering process
- We can associate this with the bare quark distribution $q(x)$
- Suppose we split the $\log$ factor as $\ln \left(\frac{Q^{2}}{\mu^{2}}\right)=\ln \left(\frac{Q^{2}}{M_{f}^{2}}\right)+\ln \left(\frac{M_{f}^{2}}{\mu^{2}}\right)$
- Then we can define a new quark PDF by

$$
q\left(x, M_{f}^{2}\right)=q(x)+\frac{\alpha_{s}}{2 \pi} \int_{x}^{1} \frac{d y}{y} q(y) P_{q q}\left(\frac{x}{y}\right) \ln \left(\frac{M_{f}^{2}}{\mu^{2}}\right)
$$

- Here we have absorbed the $\mu$ dependence into the bare quark PDF and replaced it with one that depends on the new scale $M_{f}$ - this is called the factorization scale
- Physically, the factorization scale separates the long distance from the short distance physics.
- The expression for $F_{2}$ now takes the form

$$
F_{2}\left(x, Q^{2}\right)=2 x \int_{x}^{1} \frac{d \eta}{\eta} q\left(\eta, M_{f}^{2}\right)\left[\delta\left(1-\frac{x}{\eta}\right)+\frac{\alpha_{s}}{2 \pi}\left(P_{q q}\left(\frac{x}{\eta}\right) \ln \left(\frac{Q^{2}}{M_{f}^{2}}\right)+f\left(\frac{x}{\eta}\right)\right)\right]
$$

- Here the function $f$ contains all the finite non-logarithmic terms previously demoted by ...
- Finally, if we choose $M_{f}^{2}=Q^{2}$ we get a very simple expression

$$
F_{2}\left(x, Q^{2}\right)=2 x\left[q\left(x, Q^{2}\right)+\frac{\alpha_{s}}{2 \pi} \int_{x}^{1} \frac{d \eta}{\eta} q\left(\eta, Q^{2}\right) f\left(\frac{x}{\eta}\right)\right]
$$

## Comments

- The factorization scale $M_{f}$ has a simple interpretation. Its origin is the integration over the transverse momentum of the emitted gluon. Hence, it governs how much of the struck quark's transverse momentum distribution is integrated over.
- One often hears comments about the incoming quarks having no transverse momentum. That is false. We integrate out the transverse momentum thereby generating scale-dependent parton PDFs
- The initial partons are treated as if they have no transverse momentum when describing the initial state kinematics
- This is an approximation - what is really meant is that the dominant region of parton transverse momentum is much less than the characteristic hard scale for the process
- This approximation can be improved by going to higher order in perturbation theory


## DGLAP Equations

- It is all well and good to have a simple expression for $F_{2}$ in terms of scale-dependent PDFs, but where do the PDFs come from and how do you calculate their dependence on the scale?
- Refer back to the definition I introduced for the scale-dependent PDFs
- The scale entered through a logarithmic term as shown below

$$
q\left(x, M_{f}^{2}\right)=q(x)+\frac{\alpha_{s}}{2 \pi} \int_{x}^{1} d y q(y) P_{q q}\left(\frac{x}{y}\right) \ln \left(\frac{M_{f}^{2}}{\mu^{2}}\right)
$$

- The partial derivative of $q\left(x, M_{f}^{2}\right)$ with respect to $\ln M_{f}^{2}$ projects out the coefficient of the log term which is just the convolution of the splitting function and the appropriate PDF
- Note: The running coupling $\alpha_{s}$ also depends on a large scale which can be taken as $M_{f}^{2}$. The derivative of $\alpha_{s}\left(M_{f}^{2}\right)$ then gives rise to a term one order higher in $\alpha_{s}$ and is therefore dropped.
- Generalized to include gluons and all four possible parton splittings the result is known as the set of DGLAP (Dokshitzer-Gribov-Lipatov-Altarelli-Parisi) Equations
- They take the form

$$
\begin{aligned}
& \frac{\partial q(x, t)}{\partial t}=\frac{\alpha_{s}(t)}{2 \pi} \int_{x}^{1} \frac{d y}{y}\left[P_{q q}(y) q\left(\frac{x}{y}, t\right)+P_{q g}(y) g\left(\frac{x}{y}, t\right)\right] \\
& \frac{\partial g(x, t)}{\partial t}=\frac{\alpha_{s}(t)}{2 \pi} \int_{x}^{1} \frac{d y}{y}\left[P_{g q}(y) q\left(\frac{x}{y}, t\right)+P_{g g}(y) g\left(\frac{x}{y}, t\right)\right]
\end{aligned}
$$

- Here $t=\ln M_{f}^{2} / \Lambda^{2}$
- These coupled integro-differential equations may be solved iteratively by computer, given a set of initial boundary conditions at some scale
- The boundary conditions on the initial PDFs may be parametrized and then varied to fit a wide variety of data. This is the heart of the global fitting program for determining PDFs, about which more will be said later


## Splitting Functions

- The splitting functions, $P_{i j}$, can be expanded in a perturbative series
- The lowest order expressions are referred to as the one-loop splitting functions

$$
\begin{aligned}
P_{q q}^{(0)}(z) & =C_{F}\left[\frac{1+z^{2}}{(1-z)_{+}}+\frac{3}{2} \delta(1-z)\right] \\
P_{q g}^{(0)}(z) & =T_{R}\left[z^{2}+(1-z)^{2}\right] \\
P_{g q}^{(0)}(z) & =C_{F}\left[\frac{1+(1-z)^{2}}{z}\right]=P_{q q}^{(0)}(1-z), z<1 \\
p_{g g}^{(0)}(z) & =2 C_{A}\left[\frac{z}{(1-z)_{+}}+\frac{1-z}{z}+z(1-z)\right]+\delta(1-z) \frac{11 C_{A}-4 n_{f} T_{R}}{6}
\end{aligned}
$$

- For $\operatorname{SU}(3) C_{F}=4 / 3, C_{A}=3, T_{R}=1 / 2$ and $n_{f}$ denotes the number of active flavors.


## DGLAP Equations and Scaling Violations

Consider the average of $F_{2}$ with neutrino and antineutrino beams on an isoscalar target. Multiply the quark equation by $2 x$ and sum over all flavors. Using the lowest order expressions for $F_{2}$ one has

$$
\frac{\partial F_{2}\left(x, Q^{2}\right)}{\partial t}=\frac{\alpha_{s}\left(Q^{2}\right)}{2 \pi} \int_{x}^{1} d y\left[P_{q q}(y) F_{2}\left(\frac{x}{y}, Q^{2}\right)+2 n_{f} P_{q g}(y) \frac{x}{y} g\left(\frac{x}{y}, Q^{2}\right)\right]
$$

where $n_{f}$ is the number of active flavors. If $x \ll 1$ then the gluon PDF term dominates. Since

$$
P_{q g}(y)=\frac{1}{2}\left(y^{2}+(1-y)^{2}\right)
$$

is positive definite, we see that the slope in $\ln Q^{2}$ is positive

For large $x$ the first term dominates. Since

$$
P_{q q}(y)=C_{F}\left[\frac{1+y^{2}}{(1-y)_{+}}+\frac{3}{2} \delta(1-y)\right]
$$

we see the presence of $\left(1+y^{2}\right) F_{2}\left(\frac{x}{y}, Q^{2}\right)-2 F_{2}\left(x, Q^{2}\right)<1$, so the slope turns negative as $x \rightarrow 1$.



## Simple Interpretation

- In a hard collision quarks at high values of $x$ radiate gluons
- This depletes the high $x$ quark PDFs and builds them up at lower $x$
- The gluons can create $q \bar{q}$ pairs, thereby building up the quark PDFs at lower values of $x$

This explains the pattern of scaling violations seen in the data

## QCD Improved Parton Model

- The predictions of the parton model are justified by lowest order QCD predictions.
- For processes with one large scale - call it $Q^{2}$ - these can be improved upon by using the techniques discussed so far to sum corrections from large leading logarithms
- Three steps
- Replace $\alpha_{s}$ with the running coupling $\alpha_{s}\left(Q^{2}\right)$
- Replace PDFs with scale-dependent PDFs which are solutions of the DGLAP equations
- If fragmentation functions (FFs) contribute, replace the FFs with scale-dependent FFs which are solutions of their DGLAP equations
- These three steps constitute the leading logarithm approximation (today usually labeled as LO for lowest order, but note the scale-dependent functions involved.)


## Next Steps

For improved accuracy one can go to NLO calculations (Here's another three-step plan)

- Include the next-to-leading-order hard scattering parton cross sections
- Use the two-loop running coupling
- Use the two-loop splitting functions in the DGLAP equations for the PDFs and FFs
- We have seen how to include the next order hard scattering in one example. Extending the expressions for $\alpha_{s}$ and the splitting functions is straight forward


## Change of Pace

- So, suppose you had a set of PDFs - what could you do with them?

1. Needed for perturbative calculations of any hard scattering process with hadrons in the initial state
2. Precision PDFs needed for background estimates
3. Needed for understanding potential new signals as predicted in various models

- But, where do PDFs come from anyway?
- Various groups present evolved PDFs as functions of $\left(x, Q^{2}\right)$
- These PDFs are determined from Global Fits


## Global Fits - What are they?

Problem: We need a set of evolved PDFs in order to be able to calculate a particular hard-scattering process

Solution: Generate a set of PDF solutions using a parametrized functional form for the input PDFs. Repeatedly vary the parameters and evolve the PDFs again in order to obtain an optimal fit to a set of data for various hardscattering processes

Key points:

- Parametrized functional form for input PDFs at the scale $Q_{0}$
- Choice of data sets to use and the kinematic cuts to place on them
- Truncation of the perturbation series for the hard-scattering calculations and the PDF evolution (LO, NLO, NNLO)
- Definition of "optimal fit"
- Treatment of errors


## Useful PDF properties

For the specific case of the proton we know that

- The gluon distribution dominates at low values of $x$ and falls steeply as $x$ increases
- The antiquarks and quarks are comparable at low values of $x$ and the antiquarks fall off in $x$ even faster than the gluons
- the $u$ and $d$ PDFs dominate at large values of $x$ with $u>d$


The pattern is easily understood by studying the evolution equations.

- The $u$ and $d$ distributions dominate at large $x$ and radiate gluons as they interact in the hard-scattering process
- This causes the quark distributions to get steeper (they give up some of their momentum fraction) and the gluon distribution to get larger
- Gluons can also radiate gluons so the gluon distribution tends to also get steeper and builds up at low values of $x$
- Gluons can also create quark-antiquark pairs so the antiquarks increase at low values of $x$ and have a steeper distribution than the gluons
- Keep these ideas in mind as we look for ways to separate these distributions.


## Observables

Each observable involves a characteristic linear combination (or product) of PDFs. Thus, different observables can be used to constrain specific PDFs.

- Representative global fits today use data of the following types
- Deep inelastic scattering $\left(l^{ \pm} p, l^{-} d, \nu N, \bar{\nu} N\right)$
- Neutrino DIS dimuon production
- Vector boson production $\left(W^{ \pm}, Z^{0}, \gamma\right.$, lepton pair production)
- hadronic jet production
- Will look at representative data types in order to design a strategy for constraining individual PDFs using the parton model as a guide
- For purposes of illustration, consider just the LO expressions for the different observables.
- Basic pattern is not altered by NLO corrections


## Deep Inelastic Scattering

Lowest order $l^{ \pm} p-$

$$
F_{2}\left(x, Q^{2}\right)=x \sum_{i} e_{i}^{2}\left[q_{i}\left(x, Q^{2}\right)+\bar{q}_{i}\left(x, Q^{2}\right)\right]
$$

- Each flavor weighted by its squared charge
- Gluon doesn't enter in lowest order
- Quarks and antiquarks enter together
- The use of deuterium targets allows one to get information on neutron structure functions - isospin invariance results in the role of the $u$ and $d$ PDFs being interchanged
- Requires use of nuclear corrections (see Lecture III)

Move on to neutrino interactions - measure both $F_{2}$ and $x F_{3}$. As shown in Lecture I, the signs of the antiquark PDFs change for $x F_{3}$. Schematically,

$$
\begin{aligned}
F_{2}\left(x, Q^{2}\right) & =2 x \sum_{i}\left[q_{i}\left(x, Q^{2}\right)+\bar{q}_{i}\left(x, Q^{2}\right)\right] \\
x F_{3}\left(x, Q^{2}\right) & =2 x \sum_{i}\left[q_{i}\left(x, Q^{2}\right)-\bar{q}_{i}\left(x, Q^{2}\right)\right]
\end{aligned}
$$

- Here the sum over $i$ is determined by the beam and target
- Additional structure function allows the separation quarks and antiquarks, but not a complete flavor separation
- Can use charm production to constrain the $s, \bar{s}$ PDFs
- $\nu s \rightarrow \mu^{-} c$ followed by $c \rightarrow \mu^{+} \bar{s}$ and the charge conjugate process for antineutrinos
- Both processes result in opposite sign pairs of muons.
- A consideration for all the neutrino reactions is the need for nuclear corrections
- After correcting the neutrino and charged lepton cross sections to effective isoscalar targets $(\mathrm{N}=(p+n) / 2)$ and ignoring strange and charm contributions, one has

$$
\begin{gathered}
F_{2}^{l^{ \pm} N} \approx \frac{5}{18}[u+d+\bar{u}+\bar{d}] \\
F_{2}^{\nu N} \approx[u+d+\bar{u}+\bar{d}]
\end{gathered}
$$

- Hence $F_{2}^{l^{ \pm} N} \approx \frac{5}{18} F_{2}^{\nu N}$ so that similar information can, in principle, be obtained from either one.

But what about the gluon?

## Constraining the gluon in DIS

- The gluon does not contribute in lowest order to the DIS structure functions
- It does enter in next-to-leading order to all the structure functions
- Significant contribution to the longitudinal structure function $F_{L}$ starting at order $\alpha_{s}$, but the existing data have large errors
- Also enters through mixing in the evolution equations so the gluon contributes to the change of the structure functions as $Q^{2}$ increases


## Gluon PDF and Scaling Violations

- Consider the average of $F_{2}$ for neutrinos and antineutrinos on an isoscalar target

$$
F_{2}\left(x, Q^{2}\right)=\sum_{i=1}^{n_{f}} x\left[q_{i}\left(x, Q^{2}\right)+\bar{q}_{i}\left(x, Q^{2}\right)\right]
$$

- Keep just the dominant gluon term in the quark DGLAP equation for low values of $x$
- Form the combination of quarks which contribute to $F_{2}$

$$
Q^{2} \frac{d F_{2}}{d Q^{2}} \approx \frac{\alpha_{s}\left(Q^{2}\right)}{2 \pi} \int_{x}^{1} \frac{x d y}{y} 2 n_{f} P_{q g}(y) G\left(\frac{x}{y}, Q^{2}\right)
$$

- The $Q^{2}$ dependence at small- $x$ is driven directly by the gluon PDF


## Lepton Pair Production

- For $p p$ or $p d$ reactions lepton pair production involves the product of quark and antiquark PDFs

$$
\frac{d \sigma}{d Q^{2} d x_{f}} \propto \sum_{i} e_{i}^{2}\left[q_{i}\left(x_{a}, Q^{2}\right) \bar{q}_{i}\left(x_{b}, Q^{2}\right)+a \leftrightarrow b\right]
$$

- $x_{a}$ and $x_{b}$ are given by $x_{a, b}=\frac{ \pm x_{F}+\sqrt{x_{F}^{2}+4 Q^{2} / s}}{2}$
- For large $x_{F}$ one has $x_{a} \gg x_{b}$ and

$$
\begin{aligned}
\sigma^{p p} & \propto 4 u\left(x_{a}\right) \bar{u}\left(x_{b}\right)+d\left(x_{a}\right) \bar{d}\left(x_{b}\right) \\
\sigma^{p d} & \propto\left[4 u\left(x_{a}\right)+d\left(x_{a}\right)\right]\left[\bar{u}\left(x_{b}\right)+\bar{d}\left(x_{b}\right)\right]
\end{aligned}
$$

- Data sets for these processes can help determine the ratio $\bar{d} / \bar{u}$.

$$
W^{ \pm} \text {Lepton Asymmetry }
$$

- The dominant contributions to $W$ production at the TeVatron come from $u \bar{d}$ and $\bar{u} d$ collisions.
- But $G_{\bar{u} / \bar{p}}(x)=G_{u / p}$ and similarly for the $d$ quark.
- Hence, at the TeVatron one is sensitive to the product ud
- Define the $W$ asymmetry in rapidity $y$ as

$$
A_{W}(y)=\frac{d \sigma^{+} / d y-d \sigma^{-} / d y}{d \sigma^{+} / d y+d \sigma^{-} / d y}
$$

- For $p \bar{p}$ collisions one has

$$
A_{W}(y) \approx \frac{u\left(x_{a}\right) d\left(x_{b}\right)-d\left(x_{a}\right) u\left(x_{b}\right)}{u\left(x_{a}\right) d\left(x_{b}\right)+d\left(x_{a}\right) u\left(x_{b}\right)}
$$

- Here $x_{a, b}=x_{0} e^{ \pm y}$ with $x_{0}=M_{W} / \sqrt{s}$.
- Let $R_{d u}=d / u$ so that

$$
A_{W}(y)=\frac{R_{d u}\left(x_{b}\right)-R_{d u}\left(x_{a}\right)}{R_{d u}\left(x_{b}\right)+R_{d u}\left(x_{a}\right)}
$$

- For small $y$ one has $R_{d u}\left(x_{a}\right) \approx R_{d u}\left(x_{b}\right) \approx R_{d u}\left(x_{0}\right)$.
- Using a Taylor Series expansion one gets

$$
A_{W}(y) \approx-x_{0} y \frac{1}{R_{d u}\left(x_{0}\right)} \frac{d R_{d u}}{d x}\left(x_{0}\right)
$$

- The $W$ asymmetry thus yields information on the slope of the $d / u$ ratio.
- The same conclusion holds for the lepton asymmetry from the $W$ decay, but the effect is washed out somewhat by the decay.


## Hadronic Production of Jets

So far we have not obtained much information about the gluon distribution. Need a process where the gluon contributes in lowest order.

- Direct photon production is one candidate
- Hadronic jet production includes, in lowest order, $q q \rightarrow q q, q g \rightarrow q g$, and $g g \rightarrow g g$
- At high $x_{T}=2 p_{T} / \sqrt{s}$ one might expect the quark distributions to dominate since the relevant values of $x$ are of order $x_{T}$.

- The $q q$ subprocesses do dominate the high- $E_{T}$ region.
- But, there is enough contribution from the gluon that high- $E_{T}$ jet data can be used to constrain the large- $x$ gluon behavior.
- Combined with the low- $x$ data and the momentum sum rule (to be discussed later) one has strong constraints on the gluon distribution.


## Global Fits

Ok - so you think you are ready to do some global fits...
$\sqrt{ }$ Collected data for a representative set of processes
$\sqrt{ }$ Obtained an evolution program for the PDFs
$\sqrt{ }$ Written or obtained a set of programs to evaluate the various observables
$\sqrt{ }$ Interfaced a fitting package with the observable and evolution routines
$\sqrt{ }$ Ready to go - right?
Oh, but wait. Not so fast ...

There are just a few details left to address

- Parametrization and choice of parameters to vary
- Order of perturbation theory (LO, NLO, NNLO, ...)
- Scheme dependence (DIS, $\overline{\mathrm{MS}}, \ldots$ )
- Choices for scales in the hard scattering processes
- Target mass and higher twist effects
- Treatment of heavy quarks
- Effects due to choosing or deleting a given data set
- Choice of kinematic cuts
- Treatment of errors
- Error estimates on the PDFs
- The purpose of this brief description of Global Fits is so that you can see the role that neutrino DIS can play in studying nucleon structure and how it is related to the study of other processes
- Issues that must be addressed when using neutrino data

1. Nuclear corrections!
2. Target Mass Corrections
3. Higher Twist contributions

- These topics are being studied in the context of Global Fits by the CTEQ-Jefferson Lab (CJ) Collaboration
- These issues are the subject of the next lecture

