

# Strong and electroweak interactions in nuclei

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NuSTEC

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Thanks to Jorge and Luis

## Topics (3 hours)

- \* Two- and Three-nucleon Pion Exchange Interactions
- \* Realistic Models of Two- and Three-Nucleon Interactions
- \* Realistic Models of Many-Body Nuclear Electroweak Currents
- \* Short-range Structure of Nuclei, Nuclear Correlations, and Quasi-Elastic Scattering

# Reading Material

## \* On line material \*

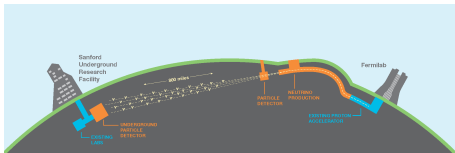
- \* Notes from Prof Rocco Schiavilla (for personal use only)  
<https://indico.fnal.gov/event/8047/material/0/0>
- \* Notes from Prof Luca Girlanda (for personal use only)  
[http://chimera.roma1.infn.it/OMAR/ECTSTAR\\_DTP/girlanda/lez1.pdf](http://chimera.roma1.infn.it/OMAR/ECTSTAR_DTP/girlanda/lez1.pdf)  
[http://chimera.roma1.infn.it/OMAR/ECTSTAR\\_DTP/girlanda/lez2.pdf](http://chimera.roma1.infn.it/OMAR/ECTSTAR_DTP/girlanda/lez2.pdf)  
[http://chimera.roma1.infn.it/OMAR/ECTSTAR\\_DTP/girlanda/lez3.pdf](http://chimera.roma1.infn.it/OMAR/ECTSTAR_DTP/girlanda/lez3.pdf)
- \* Review Articles on *Ab initio* calculations of electromagnetic properties of light nuclei
  - \* Carlson & Schiavilla - Rev.Mod.Phys. 70 (1998) 743-842: <http://inspirehep.net/record/40882>
  - \* Bacca & Pastore - J.Phys. G41 (2014) no.12, 123002: <http://inspirehep.net/record/1306337>
  - \* Marcucci & F. Gross & M.T. Pena & M. Piarulli & R. Schiavilla & I. Sick & A. Stadler & J.W. Van Orden & M. Viviani - J.Phys. G43 (2016) 023002: <https://inspirehep.net/record/1362209>

## \* Textbooks \*

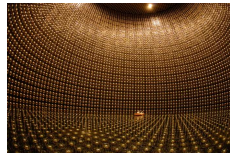
- \* *Pions and Nuclei* by Torleif Ericson and Wolfram Weise, Oxford University Press (October 6, 1988)
- \* *Theoretical Nuclear and Subnuclear Physics* by John Dirk Walecka, Oxford University Press (March 23, 1995)
- \* *Foundations of Nuclear and Particle Physics* by T. William Donnelly, Joseph A. Formaggio, Barry R. Holstein, Richard G. Milner, Bernd Surrow, Cambridge University Press; 1st edition (February 1, 2017) **new item!**
- \* *A Primer for Chiral Perturbation Theory* by Stefan Scherer and Matthias R. Schindler, Springer; 2012 edition (September 30, 2011) **(somewhat) new item!**

## Motivations (Why Nuclear Physics?)

# Fundamental Physics Quests: Accelerator Neutrinos



LBNF



T2K

neutrinos oscillate



they have tiny masses

=

BSM physics

Beyond the Standard Model

Simplified 2 flavors picture:

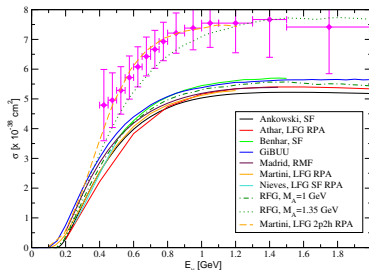
$$P(\nu_\mu \rightarrow \nu_e) = \sin^2 2\theta \sin^2 \left( \frac{\Delta m^2 L}{2E_\nu} \right)$$

\* Unknown \*

$\nu$ -mass hierarchy, CP-violation,  
accurate mixing angles

## Neutrino-Nucleus scattering

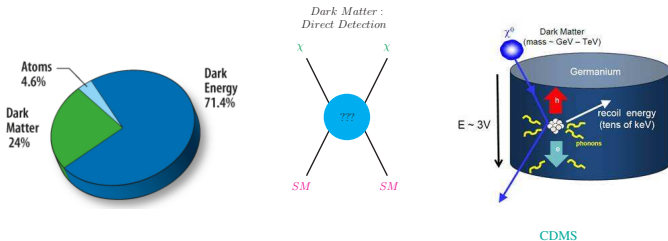
CCQE on  $^{12}\text{C}$



Alvarez-Ruso [arXiv:1012.3871](https://arxiv.org/abs/1012.3871)

DUNE, MiniBoone, T2K, Minerva ... active material \*  $^{12}\text{C}$ ,  $^{40}\text{Ar}$ ,  $^{16}\text{O}$ ,  $^{56}\text{Fe}$ , ... \*

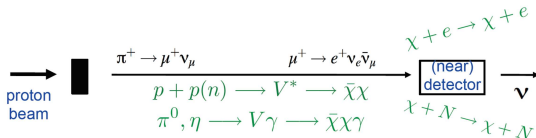
# Fundamental Physics Quests: Dark Matter Direct Detection



Dark Matter Beam Production and Direct detection:

$$\chi + A \rightarrow \chi + A$$

Dark Matter is detected via scattering on nuclei in the detector  
 Detection of Sub-GeV Dark Matter requires knowledge of nuclear responses



# Fundamental Physics Quests: Double Beta Decay

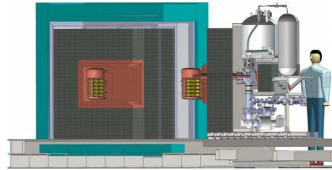
observation of  $0\nu\beta\beta$ -decay

→

lepton #  $L = l - \bar{l}$  not conserved

→

implications in  
matter-antimatter imbalance



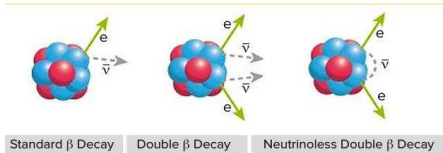
Majorana Demonstrator

\* detectors' active material  $^{76}\text{Ge}$  \*

$0\nu\beta\beta$ -decay  $\tau_{1/2} \gtrsim 10^{25}$  years (age of the universe  $1.4 \times 10^{10}$  years)

1 ton of material to see (if any)  $\sim 5$  decays per year

\* also, if nuclear m.e.'s are known, absolute  $\nu$ -masses can be extracted \*



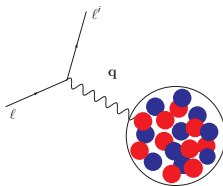
## Creating a Common Language

\* Interface Theory with Experiments and with Neutrino Generators and likewise \*



# The Basic Model

## aka Microscopic or *ab initio* Description of Nuclei



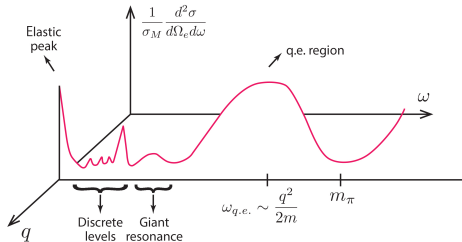
### GOAL

Develop a **comprehensive theory** that describes **quantitatively** and **predictably** **all** nuclear structure and reactions

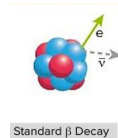
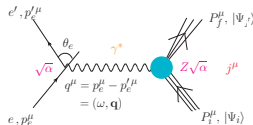
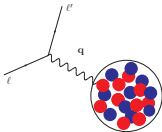
### \* The *ab initio* Approach \*

In the *ab initio* Approach one assumes that **all** nuclear phenomena can be explained in terms of (or emerge from) **interactions between nucleons**, and interactions between nucleons **and external electroweak probes** (electrons, photons, neutrinos, DM, ...)

# Electroweak Reactions



- \*  $\omega \sim 10^2$  MeV: Accelerator neutrinos
- \*  $\omega \sim 10^1$  MeV: EM decay,  $\beta$ -decay
- \*  $\omega \lesssim 10^1$  MeV: Nuclear Rates for Astrophysics

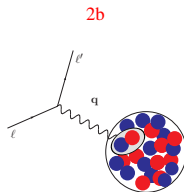
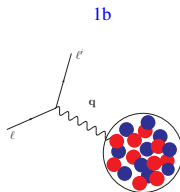


## The *ab initio* Approach

The nucleus is made of  $A$  interacting nucleons and its energy is

$$H = T + V = \sum_{i=1}^A t_i + \sum_{i<j} \textcolor{blue}{v}_{ij} + \sum_{i<j<k} \textcolor{red}{V}_{ijk} + \dots$$

where  $\textcolor{blue}{v}_{ij}$  and  $\textcolor{red}{V}_{ijk}$  are **two-** and **three-**nucleon operators based on EXPT data fitting and fitted parameters subsume underlying QCD



$$\rho = \sum_{i=1}^A \textcolor{blue}{\rho}_i + \sum_{i<j} \textcolor{red}{\rho}_{ij} + \dots,$$

$$\mathbf{j} = \sum_{i=1}^A \textcolor{blue}{j}_i + \sum_{i<j} \textcolor{red}{j}_{ij} + \dots$$

Two-body **2b** currents essential to satisfy current conservation

$$\mathbf{q} \cdot \mathbf{j} = [H, \rho] = [t_i + \textcolor{blue}{v}_{ij} + \textcolor{green}{V}_{ijk}, \rho]$$

\* “Longitudinal” component fixed by current conservation

\* “Transverse” component “model dependent”

# The Basic Model

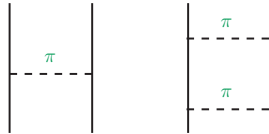
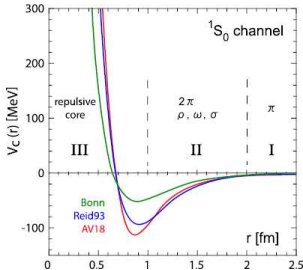
## Requirement 1: Nuclear Interactions

### DAY 1

$$H = T + V = \sum_{i=1}^A t_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \dots$$

Step 1. Construct two- and three-body interactions

- \* Chiral Effective Field Theory Interactions
- \* “Conventional” or “Phenomenological” Interactions

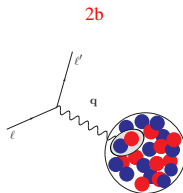
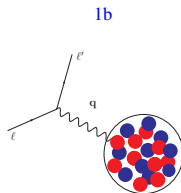


- \* One-pion-exchange: range  $\sim \frac{1}{m_\pi} \sim 1.4$  fm
- \* Two-pion-exchange: range  $\sim \frac{1}{2m_\pi} \sim 0.7$  fm

# The Basic Model

## Requirement 2: Nuclear Many-Body Currents

### DAY 2 and 3



$$\rho = \sum_{i=1}^A \rho_i + \sum_{i<j} \rho_{ij} + \dots,$$

$$\mathbf{j} = \sum_{i=1}^A \mathbf{j}_i + \sum_{i<j} \mathbf{j}_{ij} + \dots$$

**Step 2.** Understand how external probes (electrons, neutrinos, DM ...) interact with nucleons, nucleon pairs, nucleon triplets...

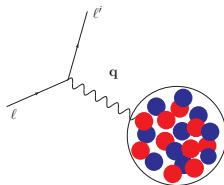
- \* Chiral Effective Field Theory Electroweak Many-Body Currents
- \* “Conventional” or “Phenomenological” Electroweak Many-Body Currents

**Step 2.a** Validate and then Use the model

- \* Validation of the theory against Electromagnetic observables in a wide range of energies
- \* Neutrino-Nucleus Observables from low to high energies and momenta

## The Basic Model

### Requirement 3: Solve the Many-Body Nuclear Problem



**Step 3.** Develop Computational Methods to solve (numerically) exactly or within approximations that are under control

$$H\Psi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_A, s_1, s_2, \dots, s_A, t_1, t_2, \dots, t_A) = E\Psi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_A, s_1, s_2, \dots, s_A, t_1, t_2, \dots, t_A)$$

\* Green's function Monte Carlo Method is a path-integral Monte Carlo Method

**pro** *Ab initio* Approach supported by GFMC Methods provides numerically exact results (*e.g.*, nuclear spectra, responses ...)

**con** The computational cost increases exponential with  $A$ , limited to  $A \leq 12$

For QMC Methods see Alessandro Lovato's Lectures

# History of Nuclear Force in one page

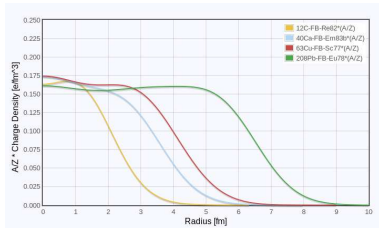


Fig. from [virginia.edu](#)

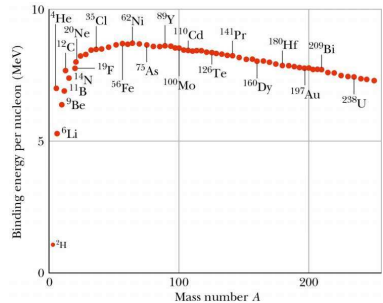


Fig. from [ohio.edu](#)

- \* Range  $\propto \frac{1}{m}$
- \* 1930s Yukawa Potential

$$v_Y \sim -\frac{e^{-mr}}{r}$$

- \* 1960-1990 Highly sophisticated “meson exchange” potentials
- \* 1990s to present - do it all over again but within Chiral Effective Field Theory (plus now we have better tools and data)

## Nuclear Force

### The Chiral Effective Field Theory Perspective

The nucleus is made of  $A$  interacting nucleons and its energy is

$$H = T + V = \sum_{i=1}^A t_i + \sum_{i < j} v_{ij} + \sum_{i < j < k} V_{ijk} + \dots$$

where  $v_{ij}$  and  $V_{ijk}$  are two- and three-nucleon operators based on EXPT data fitting and fitted parameters subsume underlying QCD



## Time-Ordered-Perturbation Theory

The relevant degrees of freedom of nuclear physics are bound states of QCD

- \* non relativistic nucleons  $N$
- \* pions  $\pi$  as mediators of the nucleon-nucleon interaction
- \* non relativistic Delta's  $\Delta$  with  $m_\Delta \sim m_N + 2m_\pi$

Transition amplitude in time-ordered perturbation theory

$$T_{fi} = \langle N'N' | H_1 \sum_{n=1}^{\infty} \left( \frac{1}{E_i - H_0 + i\eta} H_1 \right)^{n-1} | NN \rangle^*$$

$H_0$  = free  $\pi$ ,  $N$ ,  $\Delta$  Hamiltonians

$H_1$  = interacting  $\pi$ ,  $N$ ,  $\Delta$ , and external electroweak fields Hamiltonians

$$T_{fi} = \langle N'N' | T | NN \rangle \propto v_{ij}, \quad T_{fi} = \langle N'N' | T | NN; \gamma \rangle \propto (A^0 \rho_{ij}, \mathbf{A} \cdot \mathbf{j}_{ij})$$

\* Note no pions in the initial or final states, *i.e.*, pion-production not accounted in the theory

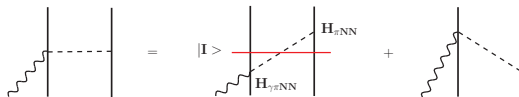
## Transition amplitude in time-ordered perturbation theory

Insert complete sets of eigenstates of  $H_0$  between successive terms of  $H_1$

$$T_{fi} = \langle N' N' | H_1 | NN; \gamma \rangle + \sum_{|I\rangle} \langle N' N' | H_1 | I \rangle \frac{1}{E_i - E_I} \langle I | H_1 | NN; \gamma \rangle + \dots$$

The contributions to the  $T_{fi}$  are represented by time ordered diagrams

Example: seagull pion exchange current



$N$  number of  $H_1$ 's (vertices)  $\rightarrow N!$  time-ordered diagrams

**Q1.** How do we construct  $H_1$  and how is it related to QCD ?

**A1.** Symmetries of QCD, Parity, Charge Conjugation, Isospin,  $\dots$ , and Chiral

**Richard Hill's lecture of this morning**

**Q2.** How do we know which dynamical process (diagram) is more relevant?

**A2.** We identify a (small) expansion parameter and do perturbation theory in terms of it

## Conceptual Perturbation Theory

$$\frac{1}{1-x} = \sum_{n=0} x^n = 1 + x + x^2 + x^3 \dots$$

- \*  $x$  is small expansion parameter
- \* one only needs to evaluate few terms in the expansion (if lucky)
- \* the error is given by the truncation in the expansion

\* Examples \*

- \* Chiral Effective Field Theory:  $x = Q$
- \* Large  $N_c$ :  $x = \frac{1}{N_c}$
- \* ...

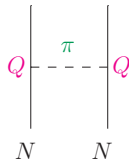
# Nuclear Chiral Effective Field Theory ( $\chi$ EFT) approach

S. Weinberg, Phys. Lett. **B251**, 288 (1990); Nucl. Phys. **B363**, 3 (1991); Phys. Lett. **B295**, 114 (1992)

- \*  $\chi$ EFT is a low-energy ( $Q \ll \Lambda_\chi \sim 1 \text{ GeV}$ ) approximation of QCD

- \* It provides effective Lagrangians describing  $\pi$ 's,  $N$ 's,  $\Delta$ 's, ... interactions that are expanded in powers  $n$  of a perturbative (small) parameter  $Q/\Lambda_\chi$

$$\mathcal{L}_{\text{eff}} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \dots + \mathcal{L}^{(n)} + \dots$$



- \* The coefficients of the expansion, **Low Energy Constants (LECs)**, are unknown and need to be fixed by comparison with exp data, or take them from LQCD
- \* The systematic expansion in  $Q$  naturally has the feature

$$\langle \mathcal{O} \rangle_{1\text{-body}} > \langle \mathcal{O} \rangle_{2\text{-body}} > \langle \mathcal{O} \rangle_{3\text{-body}}$$

- \* A theoretical error due to the truncation of the expansion can be assigned

## $\pi$ , $N$ and $\Delta$ Strong Vertices



$$\begin{aligned}
 H_{\pi NN} &= \frac{g_A}{F_\pi} \int d\mathbf{x} N^\dagger(\mathbf{x}) [\boldsymbol{\sigma} \cdot \nabla \pi_a(\mathbf{x})] \tau_a N(\mathbf{x}) \quad \longrightarrow \quad V_{\pi NN} = -i \frac{g_A}{F_\pi} \frac{\boldsymbol{\sigma} \cdot \mathbf{k}}{\sqrt{2} \omega_k} \tau_a \sim Q^1 \times Q^{-1/2} \\
 H_{\pi N\Delta} &= \frac{h_A}{F_\pi} \int d\mathbf{x} \Delta^\dagger(\mathbf{x}) [\mathbf{S} \cdot \nabla \pi_a(\mathbf{x})] T_a N(\mathbf{x}) \quad \longrightarrow \quad V_{\pi N\Delta} = -i \frac{h_A}{F_\pi} \frac{\mathbf{S} \cdot \mathbf{k}}{\sqrt{2} \omega_k} T_a \sim Q^1 \times Q^{-1/2}
 \end{aligned}$$

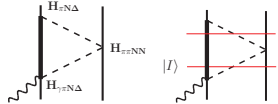
$g_A \simeq 1.27$ ;  $F_\pi \simeq 186 \text{ MeV}$ ;  $h_A \sim 2.77$  (fixed to the width of the  $\Delta$ )  
 are ‘known’ LECs

$$\begin{aligned}
 \pi_a(\mathbf{x}) &= \sum_{\mathbf{k}} \frac{1}{\sqrt{2} \omega_k} [c_{\mathbf{k},a} e^{i\mathbf{k} \cdot \mathbf{x}} + \text{h.c.}] , \\
 N(\mathbf{x}) &= \sum_{\mathbf{p}, \sigma\tau} b_{\mathbf{p}, \sigma\tau} e^{i\mathbf{p} \cdot \mathbf{x}} \chi_{\sigma\tau} ,
 \end{aligned}$$

## (Naïve) Power Counting

Each contribution to the  $T_{fi}$  scales as

$$\underbrace{\left( \prod_{i=1}^N Q^{\alpha_i - \beta_i} \right)}_{H_1 \text{ scaling}} \times \underbrace{Q^{-(N-1)}}_{\text{denominators}} \times \underbrace{Q^{3L}}_{\text{loop integration}}$$



$\alpha_i$  = # of derivatives (momenta) in  $H_1$ ;

$\beta_i$  = # of  $\pi$ 's;

$N$  = # of vertices;  $N - 1$  = # of intermediate states;

$L$  = # of loops

$$H_1 \text{ scaling} \sim \underbrace{Q^1}_{H_{\pi N \Delta}} \times \underbrace{Q^1}_{H_{\pi \pi N N}} \times \underbrace{Q^0}_{H_{\pi \gamma N \Delta}} \times Q^{-2} \sim Q^0$$

$$\text{denominators} \sim \frac{1}{E_i - H_0} |I\rangle \sim \frac{1}{2m_N - (m_\Delta + m_N + \omega_\pi)} |I\rangle = -\frac{1}{m_\Delta - m_N + \omega_\pi} |I\rangle \sim \frac{1}{Q} |I\rangle$$

$$Q^1 = Q^0 \times Q^{-2} \times Q^3$$

\* This power counting also follows from considering Feynman diagrams, where loop integrations are in 4D

## χEFT nucleon-nucleon potential at LO

$$v_{\text{NN}}^{\text{LO}} = \underbrace{\text{Contact Term}}_{v_{\text{CT}}} + \underbrace{\text{One-Pion Exchange}}_{\text{OPE } v^{\pi}} \sim Q^0$$

$$T_{fi}^{\text{LO}} = \langle N'N' | H_{\text{CT},1} | NN \rangle + \sum_{|I\rangle} \langle N'N' | H_{\pi NN} | I \rangle \frac{1}{E_i - E_I} \langle I | H_{\pi NN} | NN \rangle$$

### Leading order nucleon-nucleon potential in χEFT

$$v_{\text{NN}}^{\text{LO}} = v_{\text{CT}} + v_{\pi} = C_S + C_T \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 - \frac{g_A^2}{F_{\pi}^2} \frac{\boldsymbol{\sigma}_1 \cdot \mathbf{k} \boldsymbol{\sigma}_2 \cdot \mathbf{k}}{\omega_k^2} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$$

\* Configuration space \*

$$v_{12} = \sum_p v_{12}^p(r) O_{12}^p; \quad O_{12} = 1, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2, S_{12} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$$

$$S_{12} = 3 \boldsymbol{\sigma}_1 \cdot \hat{\mathbf{r}} \boldsymbol{\sigma}_2 \cdot \hat{\mathbf{r}} - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$$

## One Pion Exchange in Configuration Space

$$v_{NN}^{\text{LO}} = \underbrace{\text{Diagram 1}}_{v_{\text{CT}}} + \underbrace{\text{Diagram 2} + \text{Diagram 3}}_{\text{OPE } v^\pi} \sim Q^0$$

The diagrams represent the leading-order nucleon-nucleon potential. The first diagram is a contact term (CT) represented by a black dot where two lines cross. The second and third diagrams represent one-pion exchange (OPE), showing two vertical lines (nucleons) connected by a dashed line (pion) with momentum  $k$ . The second diagram shows the pion being emitted from the first nucleon and absorbed by the second, while the third shows the opposite. The entire expression is proportional to  $Q^0$ .

### One-Pion-Exchange Potential (OPEP)

$$v_\pi(\mathbf{k}) = -\frac{g_A^2}{F_\pi^2} \frac{\boldsymbol{\sigma}_1 \cdot \mathbf{k} \boldsymbol{\sigma}_2 \cdot \mathbf{k}}{\omega_k^2} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$$

$$v_\pi(\mathbf{r}) = \frac{f_{\pi NN}^2}{4\pi} \frac{m_\pi}{3} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \left[ T_\pi(r) S_{12} + \left[ Y_\pi(r) - \frac{4\pi}{m_\pi^3} \delta(\mathbf{r}) \right] \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \right]$$

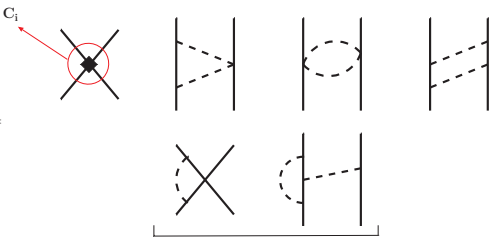
$$Y_\pi(r) = \frac{e^{-m_\pi r}}{m_\pi r}$$

$$T_\pi(r) = \left( 1 + \frac{3}{m_\pi r} + \frac{3}{m_\pi^2 r^2} \right) Y_\pi(r)$$

$$S_{12} = 3 \boldsymbol{\sigma}_1 \cdot \hat{\mathbf{r}} \boldsymbol{\sigma}_2 \cdot \hat{\mathbf{r}} - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$$



## $\chi$ EFT nucleon-nucleon potential at NLO (without $\Delta$ 's)

$$v_{NN}^{\text{NLO}} =$$


renormalize  $C_S$ ,  $C_T$ , and  $\underline{g}_A$

$\sim Q^2$

- \* At NLO there are 7 LEC's,  $C_i$ , fixed so as to reproduce nucleon-nucleon scattering data (order of  $k$  data)
- \*  $C_i$ 's multiply contact terms with 2 derivatives acting on the nucleon fields ( $\nabla N$ )
- \* Loop-integrals contain ultraviolet divergences reabsorbed into  $g_A$ ,  $C_S$ ,  $C_T$ , and  $C_i$ 's (for example, use dimensional regularization)

\* Configuration space \*

$$v_{12} = \sum_p v_{12}^p(r) O_{12}^p; \quad O_{12} = [1, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2, S_{12}, \mathbf{L} \cdot \mathbf{S}] \otimes [1, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2]$$

## Technicalities: The Cutoff

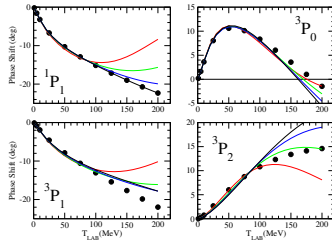
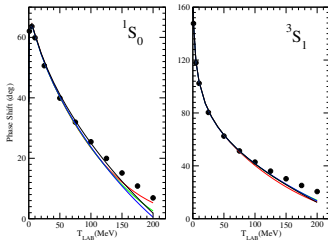
\*  $\chi$ EFT operators have a power law behavior in  $Q$

1. introduce a regulator to kill divergencies at large  $Q$ , e.g.,  $C_\Lambda = e^{-(Q/\Lambda)^n}$
2. pick  $n$  large enough so as to not generate spurious contributions

$$C_\Lambda \sim 1 - \left(\frac{Q}{\Lambda}\right)^n + \dots$$

3. for each cutoff  $\Lambda$  re-fit the LECs
  4. ideally, your results should be cutoff-independent
- \* In  $r_{ij}$ -space this corresponds to cutting off the short-range part of the operators that make the matrix elements diverge at  $r_{ij} = 0$

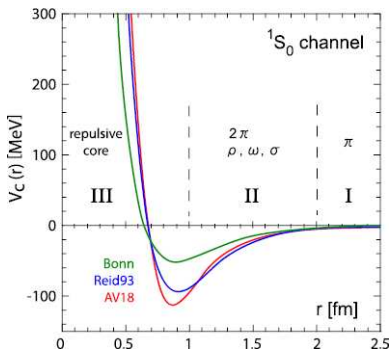
# Determining LEC's: fits to $np$ phases \* up to $T_{\text{LAB}} = 100\text{MeV}$ NLO Chiral Potential



LS-equation regulator  $\sim \exp(-2Q^4/\Lambda^4)$ , (cutting off momenta  $Q \gtrsim 3-4 m_\pi$ ),  
 $\Lambda = 500, 600, \text{ and } 700$  MeV

\* F.Gross and A.Stadler PRC **78**, 104405 (2008)

## Nucleon-nucleon potential



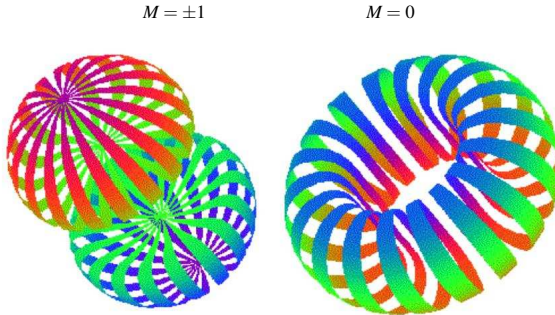
Aoki *et al.* [Comput.Sci.Disc.1\(2008\)015009](#)

CT = Contact Term (short-range);

OPE = One Pion Exchange (range  $\sim \frac{1}{m_\pi}$ );

TPE = Two Pion Exchange (range  $\sim \frac{1}{2m_\pi}$ )

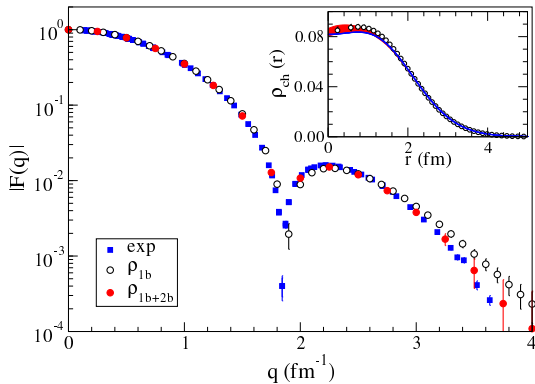
## Nucleon-Nucleon Potential and the Deuteron



Constant density surfaces for a polarized deuteron in the  $M = \pm 1$  (left) and  $M = 0$  (right) states

Carlson and Schiavilla [Rev.Mod.Phys.70\(1998\)743](#)

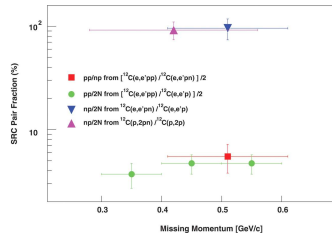
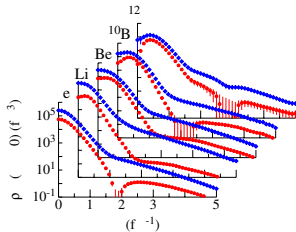
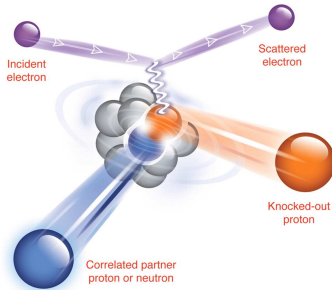
# Shape of Nuclei



Lovato *et al.*

PRL111(2013)092501

# Back-to-back $np$ and $pp$ Momentum Distributions

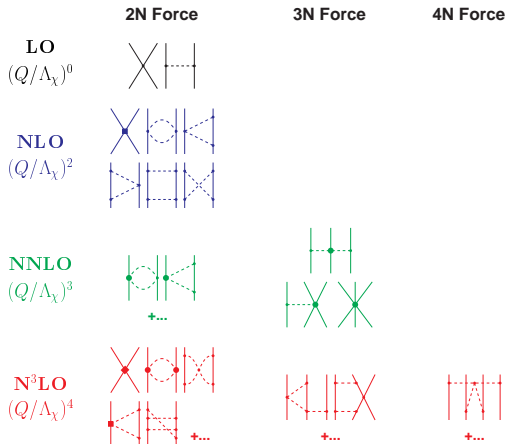


Wiringa *et al.* [PRC89\(2014\)024305](#)

JLab, Subedi *et al.* [Science320\(2008\)1475](#)

Nuclear properties are strongly affected by **two-nucleon** interactions!

# $\chi$ EFT many-body potential: Hierarchy



Machleidt & Sammarruca - PhysicaScripta91(2016)083007

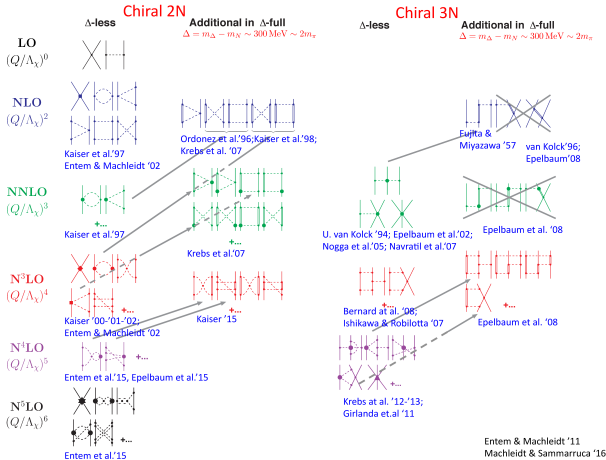
CT = Contact Term (short-range);

OPE = One Pion Exchange (range  $\sim \frac{1}{m_\pi}$ );

TPE = Two Pion Exchange (range  $\sim \frac{1}{2m_\pi}$ )



# Nuclear Interactions and the role of the $\Delta$



Courtesy of Maria Piarulli

\* N<sup>3</sup>LO with  $\Delta$  nucleon-nucleon interaction constructed by Piarulli *et al.* in [PRC91\(2015\)024003-PRC94\(2016\)054007-arXiv:1707.02883](https://arxiv.org/abs/1707.02883) with  $\Delta$ 's fits  $\sim 2000$  ( $\sim 3000$ ) data up 125 (200) MeV with  $\chi^2/\text{datum} \sim 1$ ;

\* N<sup>2</sup>LO with  $\Delta$  3-nucleon force fits  $^3\text{H}$  binding energy and the  $nd$  scattering length

# “Phenomenological” aka “Conventional” aka “Traditional” aka “Realistic” Two- and Three- Nucleon Potentials

L A A M L T A

$$H = \sum_i K_i + \sum_{i < j} v_{ij} + \sum_{i < j < k} V_{ijk}$$

$K_i$  - e ati isti i eti e e g  $m_n - m_p$  effe ts i ded

A g e 18  $v_{ij} = v_{ij}^7 + v_{ij}^{\pi} + v_{ij}^I + v_{ij}^S = \sum_p(r_{ij}) O_{ij}^p$

- 18 s i t e s s i - i t i s s i e t e a t s
- f M a d s t g a d S B t e s i d e d
- e d i a t a e a t s t t e
- t s i e g e P A 3 d a t a i t h  $\chi^2/d f$  l l

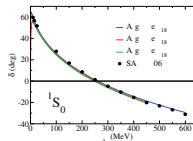
i i g a S t s S h i a i a P 51 (1 5)

a a i i s  $V_{ijk} = V_{ijk}^{2\pi} + V_{ijk}^{3\pi} + V_{ijk}^R$

- a a h a s s t a d d  $2\pi$   $P$ - a e
- s h t - a g e e s i f a t t e s a t a t i
- i i s a d d s  $2\pi$   $S$ - a e  $3\pi$  i g s
- t i d e e t a  $T$  3 2 i t e a t i
- i i s - h a s f a a e t e s t t 23 e e s i  $A \leq 10$  e i

P i e e P a d h a i a d e i i g a a s P 64 014001 (2001)

P i e e A P P 1011 143 (2008)

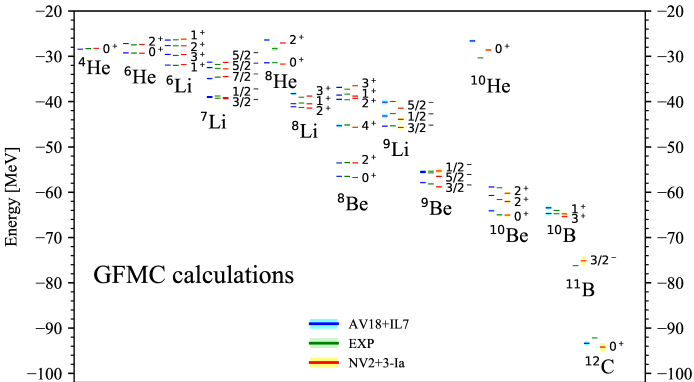


Courtesy of Bob Wiringa

\* AV18 fitted up to 350 MeV, reproduces phase shifts up to  $\sim 1$  GeV \*

\* IL7 fitted to 23 energy levels, predicts hundreds of levels \*

## Spectra of Light Nuclei

M. Piarulli *et al.* - arXiv:1707.02883

- \* one-pion-exchange physics dominates \*
- \* it is included in **both** chiral and “conventional” potentials \*

## Three-body forces

$$H = T + V = \sum_{i=1}^A t_i + \sum_{i < j} v_{ij} + \sum_{i < j < k} V_{ijk} + \dots$$

$$V_{ijk} \sim (0.2 - 0.9) v_{ij} \sim (0.15 - 0.6) H$$

$$v_{\pi} \sim 0.83 v_{ij}$$

<sup>10</sup>B VMC code output

$$T_i + V_{ij} = -38.2131 \ (0.1433) \quad + \ V_{ijk} = -46.7975 \ (0.1150)$$

$$T_i = 290.3220 \ (1.2932) \quad V_{ij} = -328.5351 \ (1.1983) \quad V_{ijk} = -8.5844 \ (0.0892)$$

## Chiral Potentials (Incomplete List of Credits)

- \* van Kolck *et al.*; [PRL72\(1994\)1982-PRC53\(1996\)2086](#)
- \* Kaiser, Weise *et al.*; [NPA625\(1997\)758-NPA637\(1998\)395](#)
- \* Epelbaum, Glöckle, Meissner<sup>\*</sup>; [RevModPhys81\(2009\)1773](#) and references therein
- \* Entem and Machleidt<sup>\*</sup>; [PhysRept503\(2011\)1](#) and references therein

\* Chiral Potentials suited for Quantum Monte Carlo calculations \*

- \* Gezerlis *et al.* [PRL111\(2013\)032501-PRC90\(2014\)054323](#);  
Lynn *et al.* [PRL113\(2014\)192501](#)
- \* Piarulli *et al.*<sup>\*</sup> [PRC91\(2015\)024003-PRC94\(2016\)054007-arXiv:1707.02883](#) (with  $\Delta$ 's)

\* Potentials fitted and used in many-body calculations

# Nuclear Many-body Interaction

## \* Observations \*

- \* Nuclear two-body forces contain a number of parameters (up to  $\sim 40$ ) fitted to a large  $\sim 4k$  ( $\sim 3k$ ) data base up to 350 ( $\sim 200$ ) MeV in the case of AV18 (Chiral) model
- \* Intermediate and long components are described in terms of one- and two-pion exchange potentials
- \* Short-range repulsion core described by Contact Terms in Chiral Formulations and special functions in AV18
- \* Chiral Formulation requires a study of variation with respect to cut off
- \* The AV18 has fixed cutoffs
- \* Due to a cancellation between kinetic and two-body contribution, three-body potentials are necessary to reach agreement with the data
- \* Three-body potentials Illinois fitted to 23 energy levels predicts hundreds levels
- \* Three-body potentials Chiral fitted to triton binding energy and  $nd$ -scattering length only! Strong validation of the Microscopic picture of the nucleus

## Observations continuation

### \* Chiral Effective Field Theory \*

- \* Chiral Formulation of Nuclear Physics is extremely successful
- \* But limited to low-energies ( $\sim 200$  MeV)
- \* Inclusion of the  $\Delta$  possibly allows for applications to higher energies

### \* “Conventional” Formulation \*

- \* “Conventional” Formulation of Nuclear Physics is extremely successful
- \* But hard to be systematically improved
- \* “Conventional” AV18 Interaction has a range of applicability as  $\sim 1$  GeV

Fundamental Physics with Electroweak Probes of Light Nuclei

June 12 - July 13, 2018

S. Bacca, R. J. Hill, S. Pastore, D. Phillips

Contacts

<http://www.int.washington.edu/>

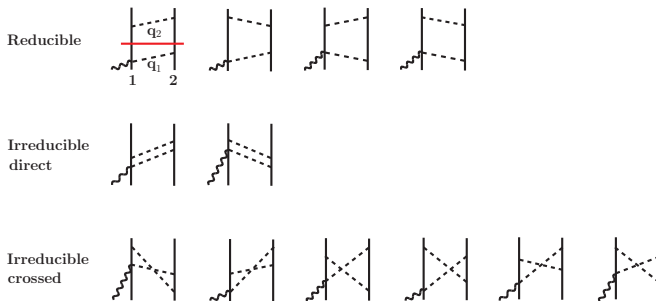
[saori.pastore@gmail.com](mailto:saori.pastore@gmail.com)

[saori@lanl.gov](mailto:saori@lanl.gov)



## Technicalities I: Reducible Contributions

4 interaction Hamiltonians  $\longrightarrow$  4! time ordered diagrams



$$|\Psi\rangle \simeq |\phi\rangle + \frac{1}{E_i - H_0} v^\pi |\phi\rangle + \dots$$

$$\langle \Psi_f | \mathbf{j} | \Psi_i \rangle \simeq \langle \phi_f | \mathbf{j} | \phi_i \rangle + \langle \phi_f | v^\pi \frac{1}{E_i - H_0} \mathbf{j} + \text{h.c.} | \phi_i \rangle + \dots$$

\* Need to carefully subtract contributions generated by the iterated solution of the Schrödinger equation