Approximate Methods for Nuclei II

natalie.jachowicz@ugent.be

Outline

6. Approximate methods for nuclei (II) (2 hours).

The polarization propagator; RPA approach; RPA equations; many-body diagrams; meson exchange currents and 2p2h terms. Continuum RPA.

Mean field description of the nucleus and beyond : long range correlations, collective excitations and random phase approximation ; interactions at low energy ; short range correlations, meson exchange currents and their influence on genuine quasi-elastic and 2-nucleon knockout neutrino-induced processes.

- Basic nuclear models
- Long range correlations : propagators, RPA and low energy processes
- Short range correlations
- Meson exchange corrections

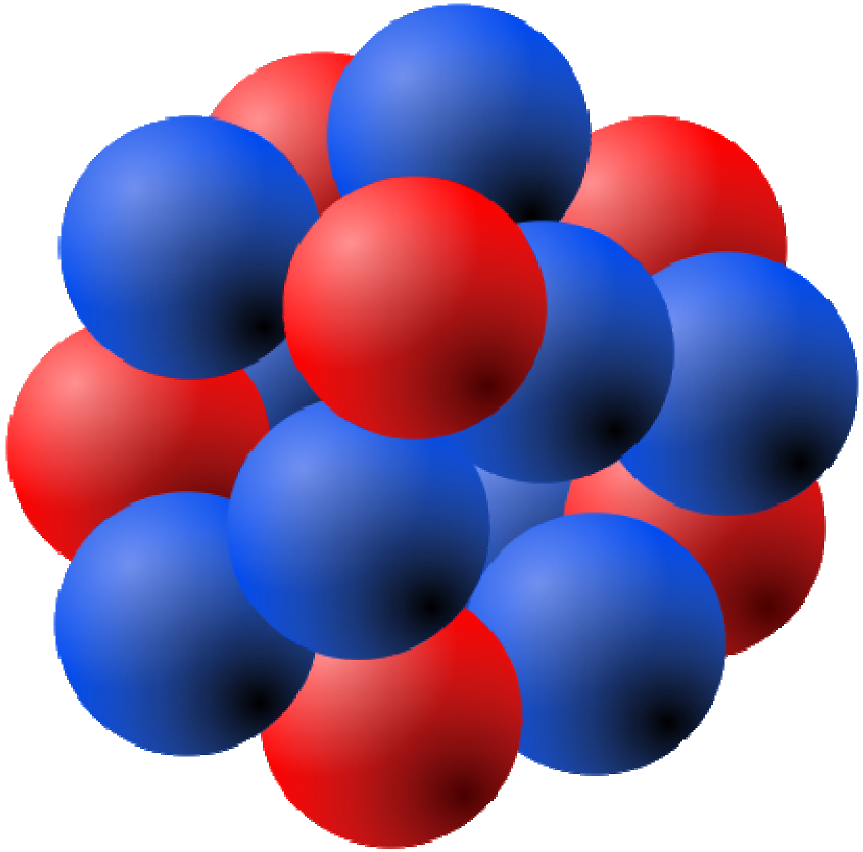
Modeling the nucleus

The nucleus is a mesoscopic system :

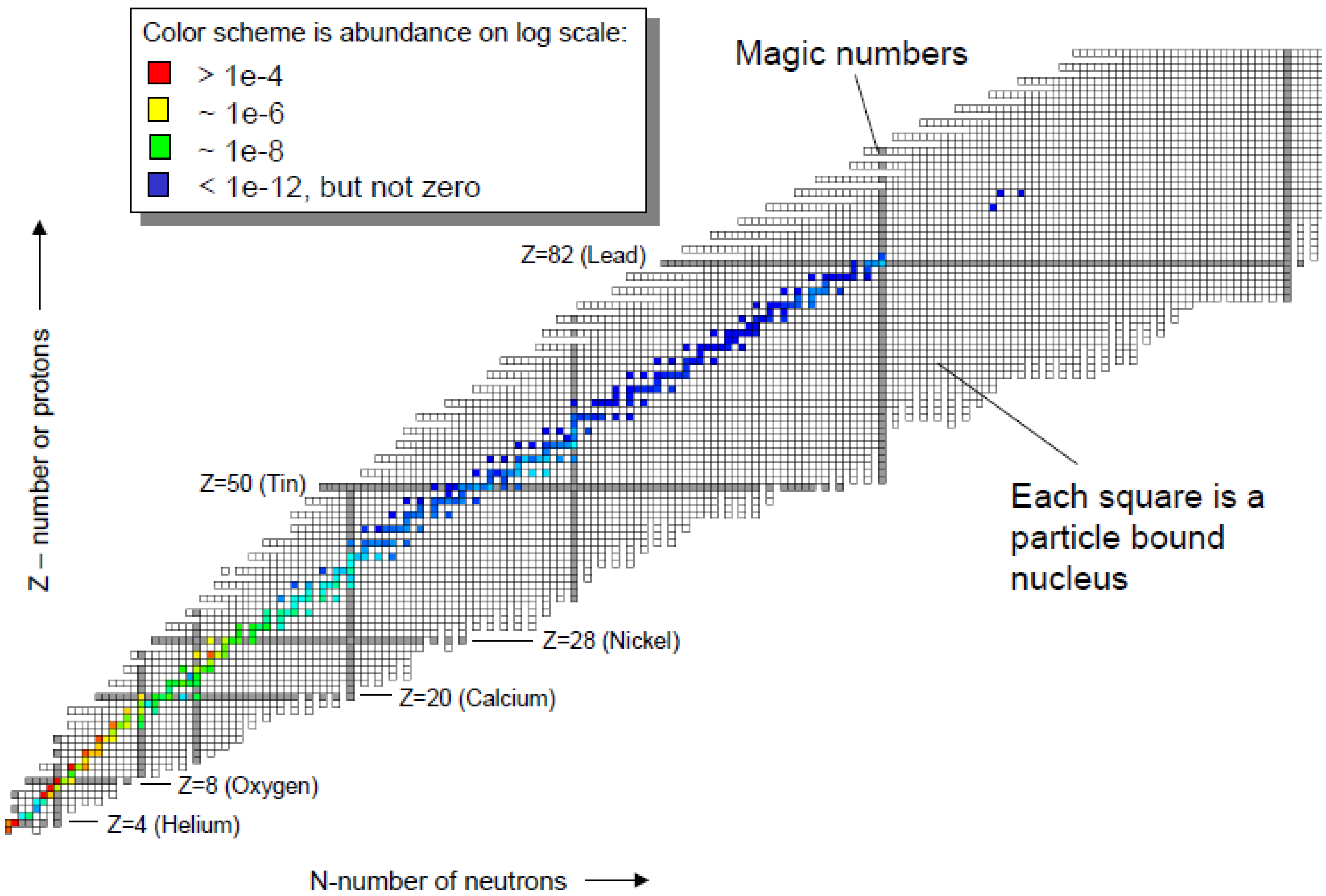
-usually too big for few-body techniqes

-usually too small for statistical methods

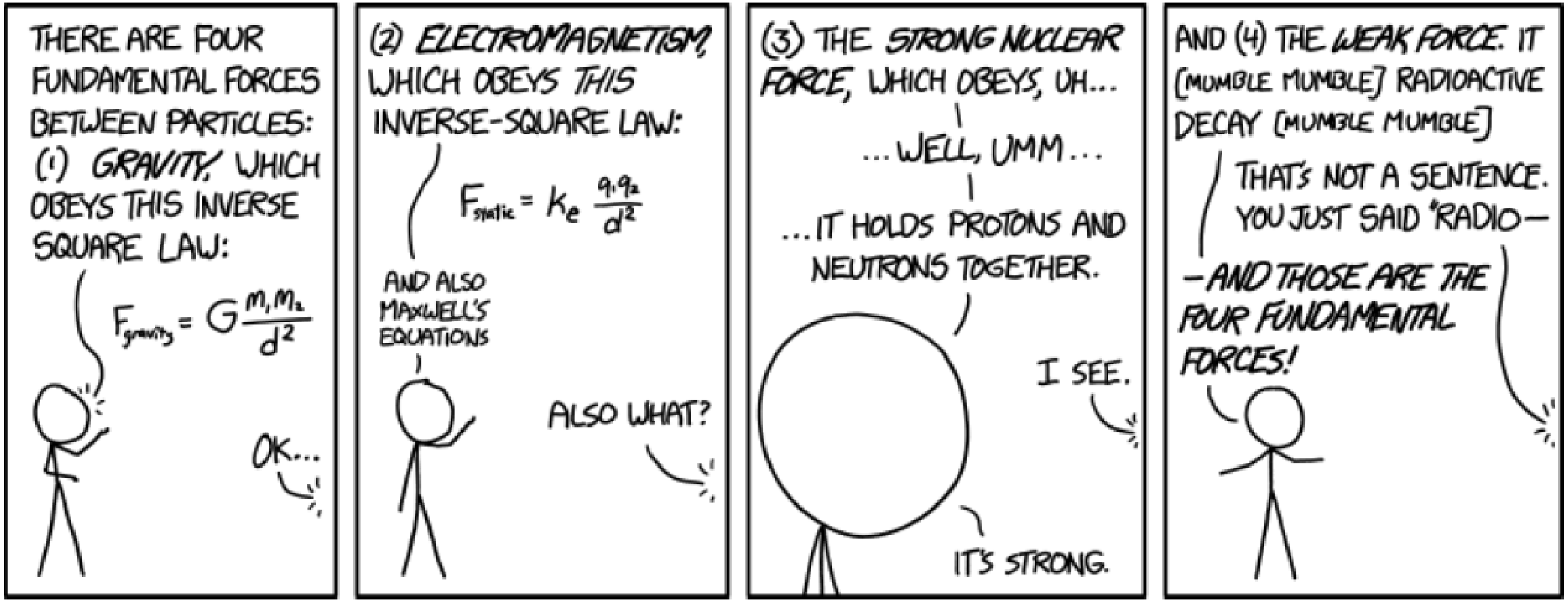
Nuclear physics is hard work !



Abundances of nuclei on the chart of nuclides:

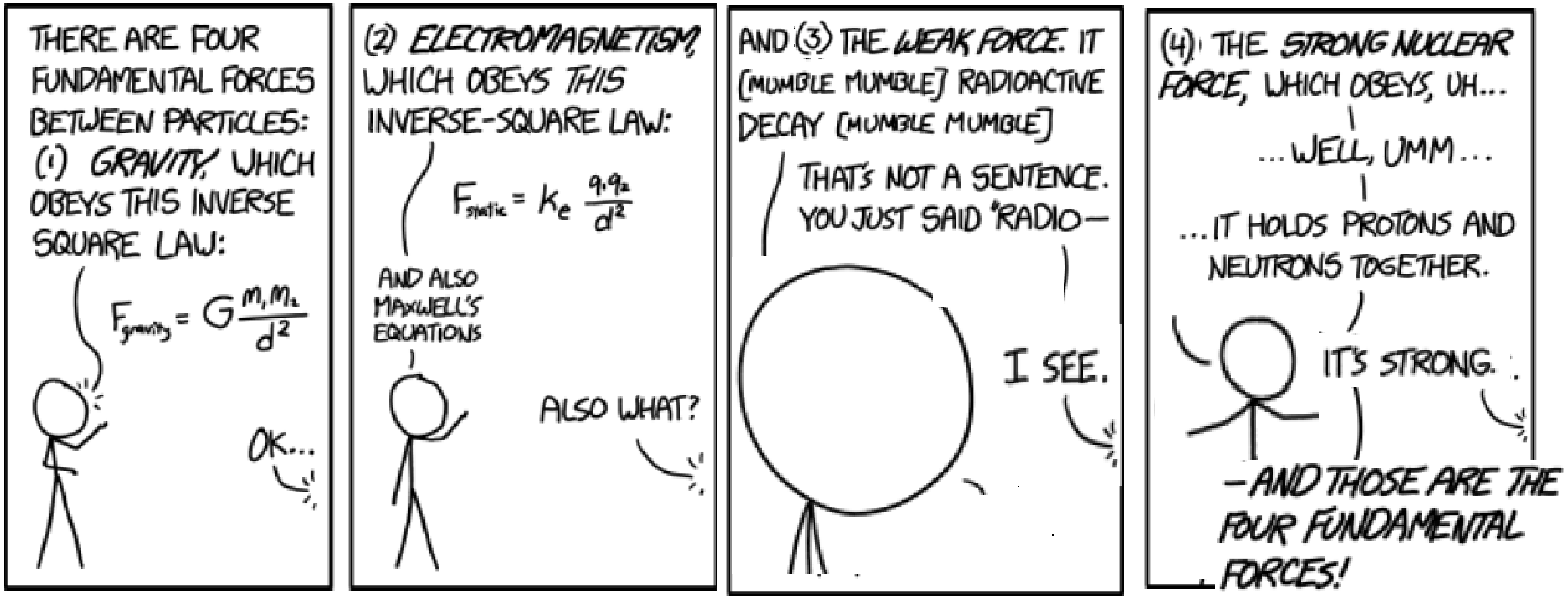


Nuclear dynamics is governed by the nuclear force = the residue of the strong interaction

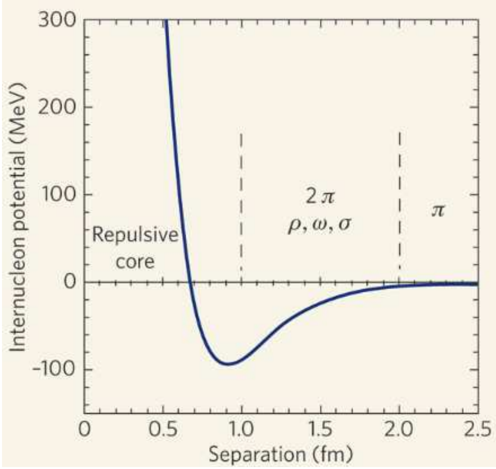


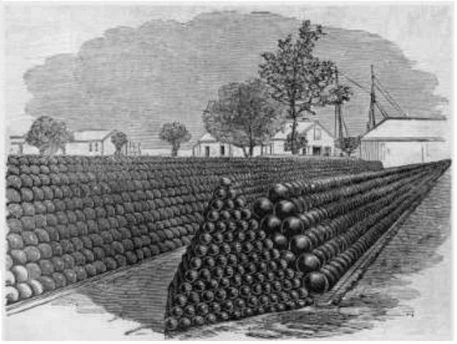
And (quite) a bit by the Coulomb interaction

- The nucleons inside the nucleus are bound via the nuclear force. This force is understood as a residual effect of the strong force, which is the force that binding quarks together to form the nucleons.
- To a large extent, the nuclear force can be understood in terms of the exchange of virtual light mesons such as the π, ρ and ω meson

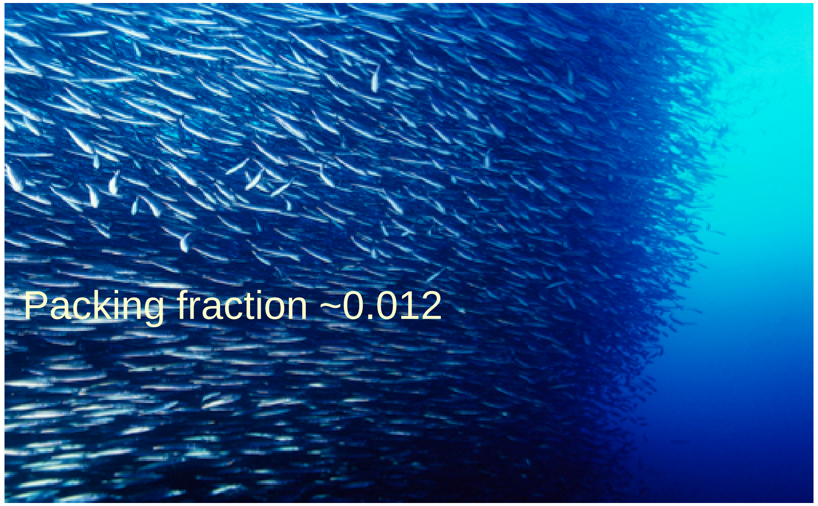


- Nuclear radius $\approx 1.2 A^{\frac{1}{3}}$ fm
- Nucleon is a diffuse system
  - Hard core (repulsion) ≈ 0.5 fm
  - RMS charge radius from (e,e') $= 0.897(18)$ fm
- $0.07 \lesssim \text{NPF} \lesssim 0.42$
  - closest packing fraction of spheres ≈ 0.74
  - packing fraction of Argon liquid ≈ 0.032
  - packing fraction of Argon gas $\approx 3.75 \cdot 10^{-5}$
- The nuclear medium is a rather **dense quantum liquid**

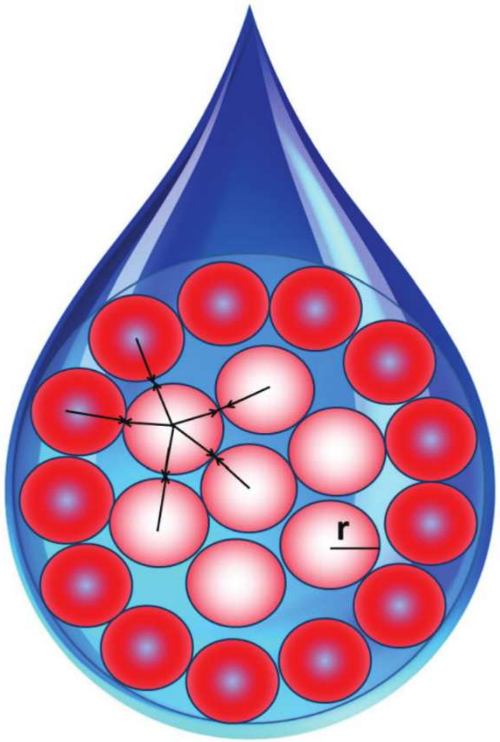




C. Colle, PhD, UGent 2017



Very approximate : (almost) no correlations : the liquid drop model

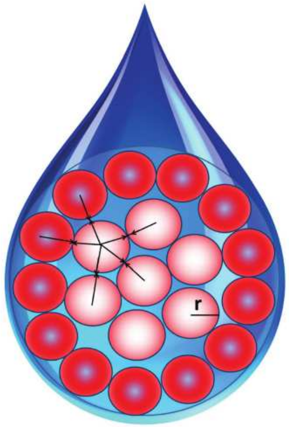


$$M(Z,N) = Z\,M_H + N\,M_n - BE$$

Binding energy

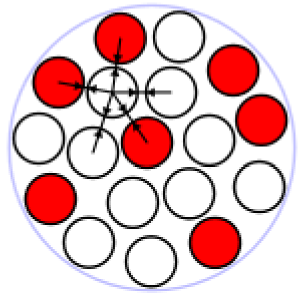
Bulk description of the nucleus
Liquid drop of uniform density

Very approximate : (almost) no correlations : the liquid drop model



$$M(Z,N) \;=\; Z\,M_H + N\,M_n - BE$$

$$BE \;=\; C_1 A$$



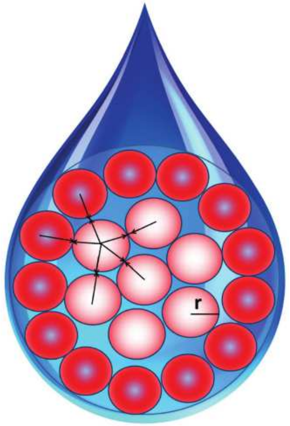
Volume

Volume term

- Binding energy per nucleon is ~constant for stable nuclei over the whole mass table
- The nuclear force is short-range, attractive and saturated
- $V \;\propto\; A$

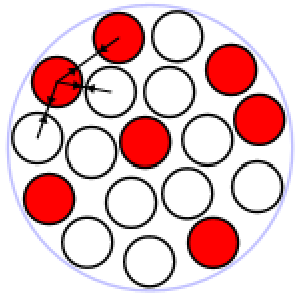
  $R \;=\; r_0 A^{1/3} = 1.28 fm A^{1/3}$

Very approximate : (almost) no correlations : the liquid drop model



$$M(Z,N) \quad = \quad Z\,M_H + N\,M_n - BE$$

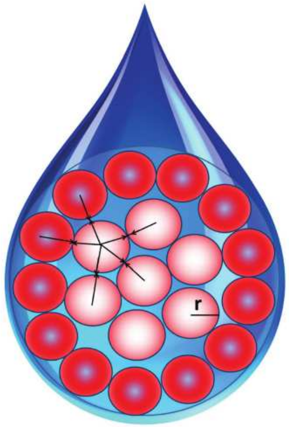
$$BE \quad = \quad C_1 A - C_2 A^{2/3}$$



Surface term

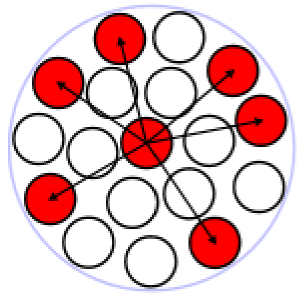
- Nucleons at the surface are not surrounded by other nucleons to attract
- Surface nucleons have reduced binding compared to those in the nuclear interior

Very approximate : (almost) no correlations : the liquid drop model



$$M(Z,N) \quad = \quad Z\,M_H + N\,M_n - BE$$

$$BE \quad = \quad C_1 A - C_2 A^{2/3} - C_3 \frac{Z^2}{A^{1/3}}$$

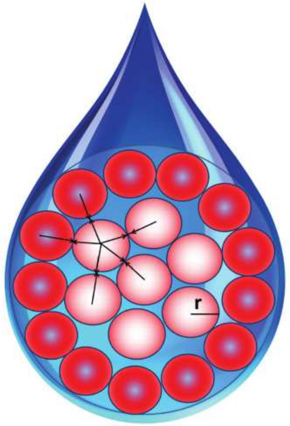


Coulomb

Coulomb term

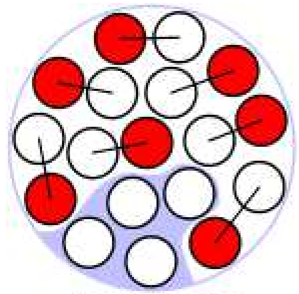
- Coulomb force is repulsive and long range
- This term reduces the number of protons in large nuclei

Very approximate : (almost) no correlations : the liquid drop model



$$M(Z,N) \;=\; Z\,M_H + N\,M_n - BE$$

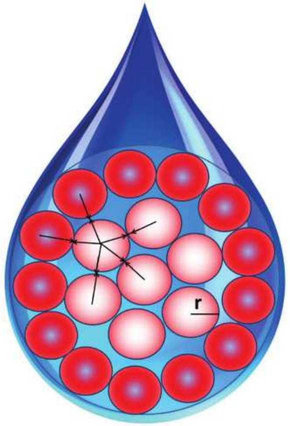
$$BE \;=\; C_1 A - C_2 A^{2/3} - C_3 \frac{Z^2}{A^{1/3}} - C_4 \left(\frac{A}{2} - Z\right)^2$$



Symmetry term

- Pauli principle
- This term tends to keep the number of protons and neutrons equal

Very approximate : (almost) no correlations : the liquid drop model



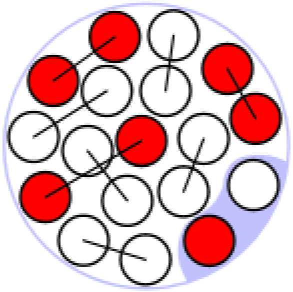
$$M(Z,N) \quad = \quad Z\,M_H + N\,M_n - BE$$

$$BE \quad = \quad C_1 A - C_2 A^{2/3} - C_3 \frac{Z^2}{A^{1/3}} - C_4 \left(\frac{A}{2} - Z\right)^2 + \delta_{N,Z}$$

Pairing term

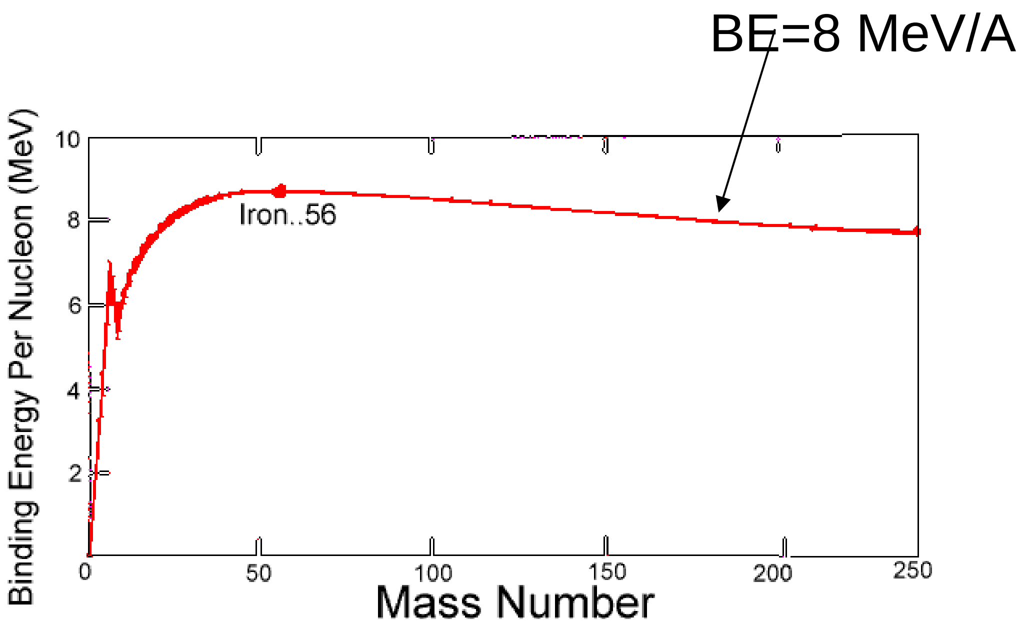
- Pairing force
- Protons and neutrons that are paired tend to have enhanced binding energy

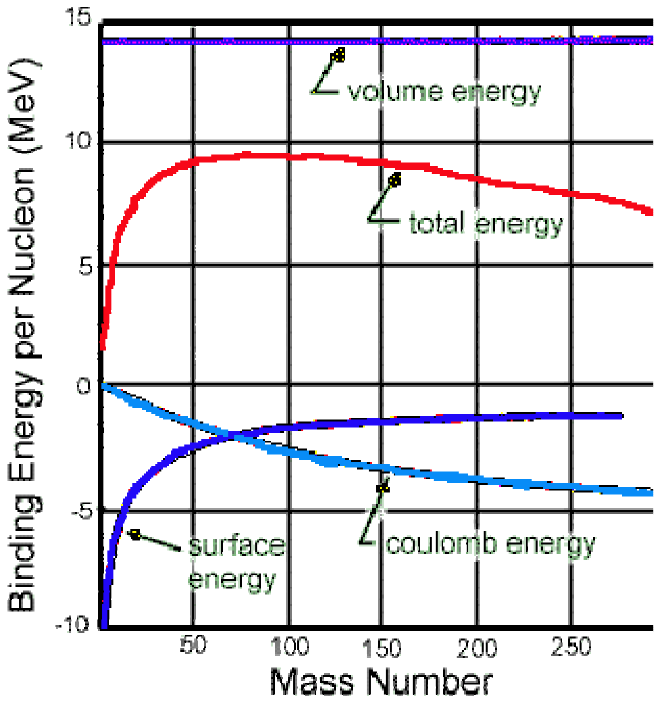
$$\begin{array}{lll} + & \delta & \text{(even-even)} \\ + & 0 & \text{(odd-even /even-odd)} \\ - & \delta & \text{(odd-odd)} \end{array}$$



Pairing

Very approximate : (almost) no correlations : the liquid drop model





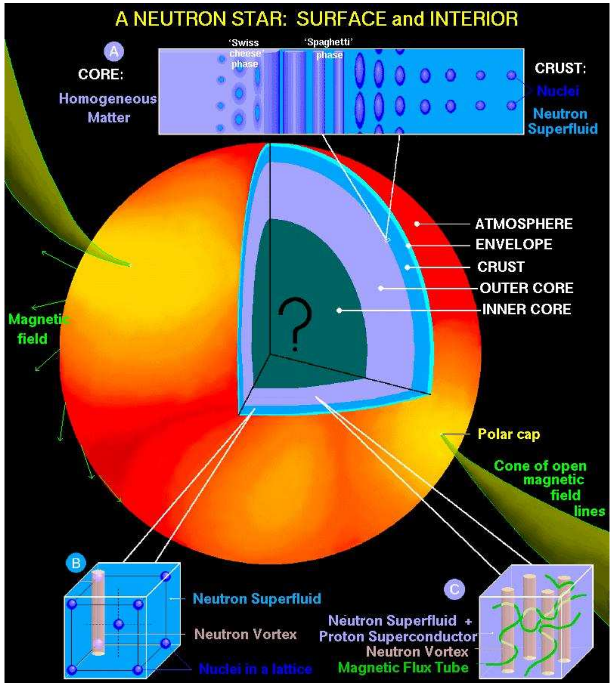
$$BE = C_1 A - C_2 A^{2/3} - C_3 \frac{Z^2}{A^{1/3}} - C_4 \left(\frac{A}{2} - Z \right)^2 + \delta_{N,Z}$$

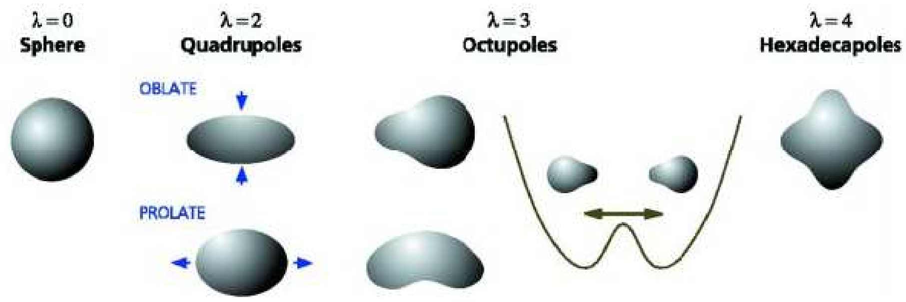
To boldly go … Neutron Star Stability

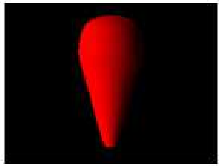
$$BE = C_1 A - C_2 \cancel{A^{2/3}} - C_3 \cancel{\frac{Z^2}{A^{1/3}}} - C_4 \left(\frac{A}{2} - Z\right)^2 + \delta_{\cancel{N},Z} + \frac{3}{5} G \frac{M^2}{R}$$

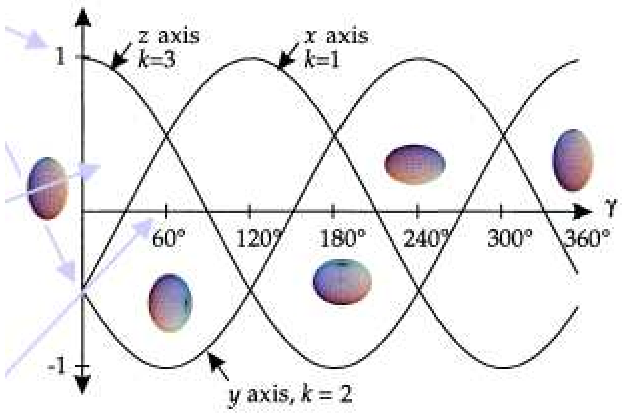
$$+ \frac{3}{5} G \frac{M^2 A^{-1/3}}{r_0}$$

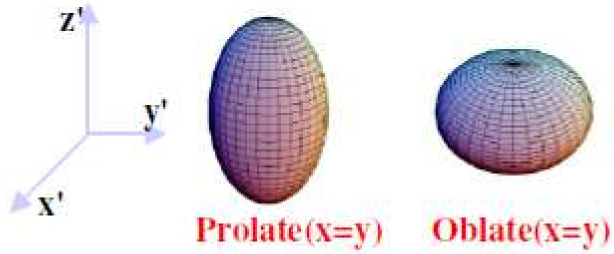
$$\begin{aligned} A &\simeq 5 \times 10^{55} \\ R &\simeq 4.3\,\mathrm{km} \\ M &\simeq 0.045 M_\odot \end{aligned}$$

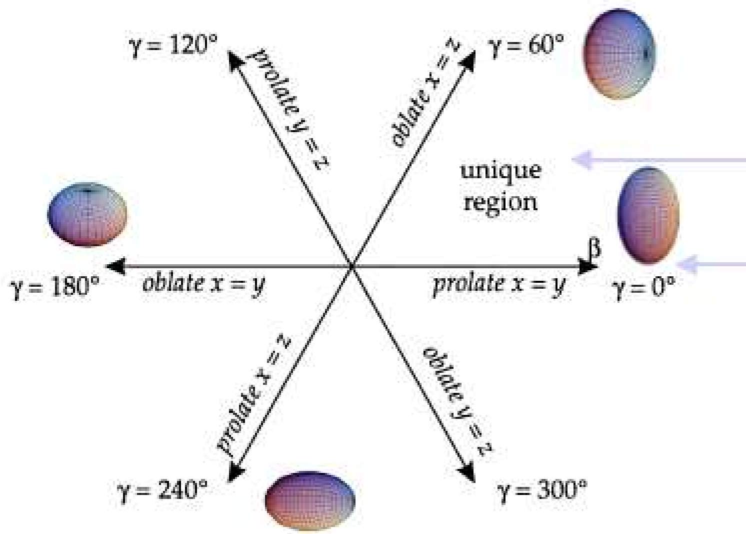










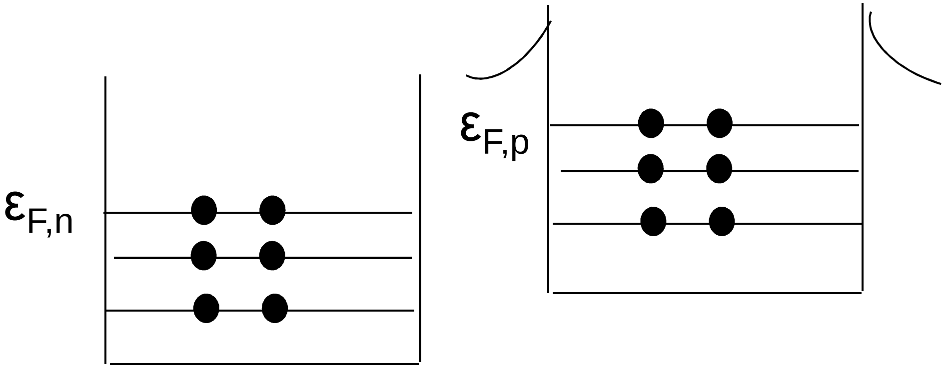


[Octupolevibration]

[vibrations]

Slightly less approximate : (still almost) no correlations : the Fermi gas model

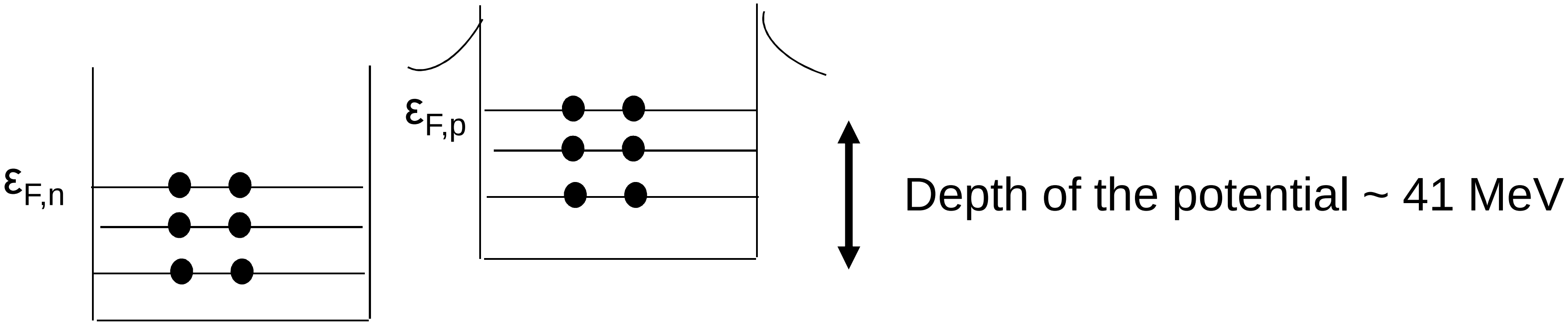
Easiest microscopic independent particle model



$$
\begin{aligned}
n &= 2\frac{V}{(2\pi\hbar)^3}\int_0^{p_F} d^3p = 2\frac{V}{(2\pi\hbar)^3}\left(\frac{4}{3}\pi p_F^3\right) \\
\Rightarrow &\quad p_F = \hbar\left(\frac{3n\pi}{V}\right)^{1/3} \sim 247\,\mathrm{MeV/c} \\
&\quad E_F = \frac{p_F^2}{2m} \sim 33\,\mathrm{MeV}
\end{aligned}
$$

Slightly less approximate : (still almost) no correlations : the Fermi gas model

Easiest microscopic independent particle model



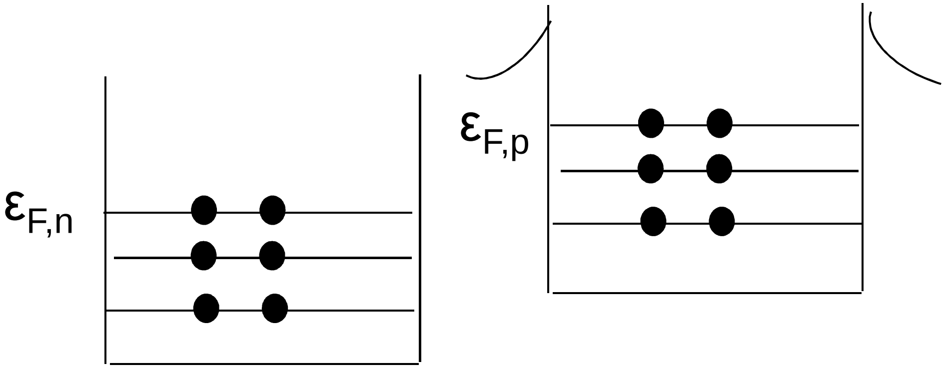
$$n = 2\frac{V}{(2\pi\hbar)^3}\int_0^{p_F} d^3p = 2\frac{V}{(2\pi\hbar)^3}\left(\frac{4}{3}\pi p_F^3\right)$$

$$\Rightarrow \quad p_F = \hbar\left(\frac{3n\pi}{V}\right)^{1/3} \sim 247\,\mathrm{MeV/c}$$

$$E_F = \frac{p_F^2}{2m} \sim 33\,\mathrm{MeV}$$

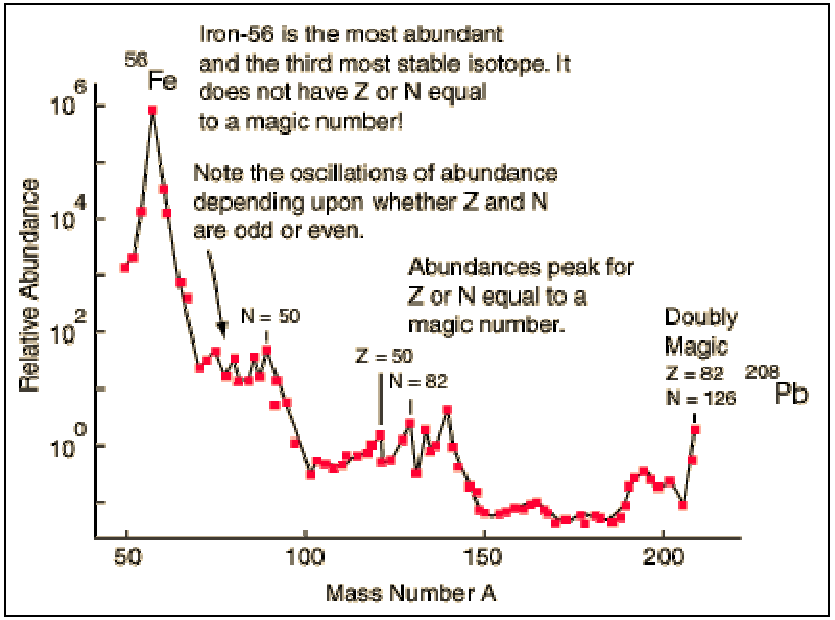
Slightly less approximate : (still almost) no correlations : the Fermi gas model

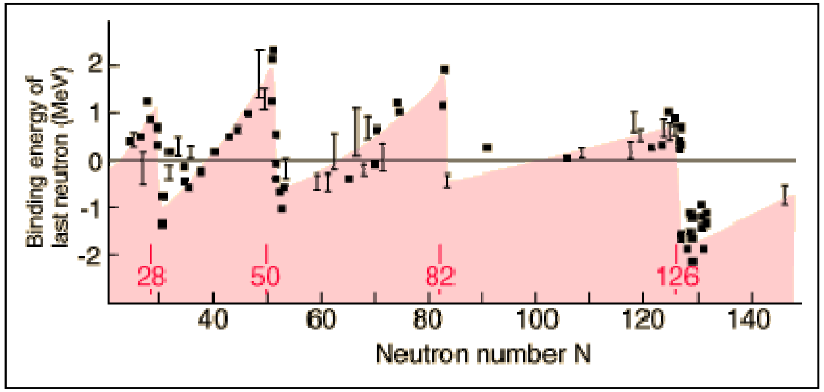
Easiest microscopic independent particle model

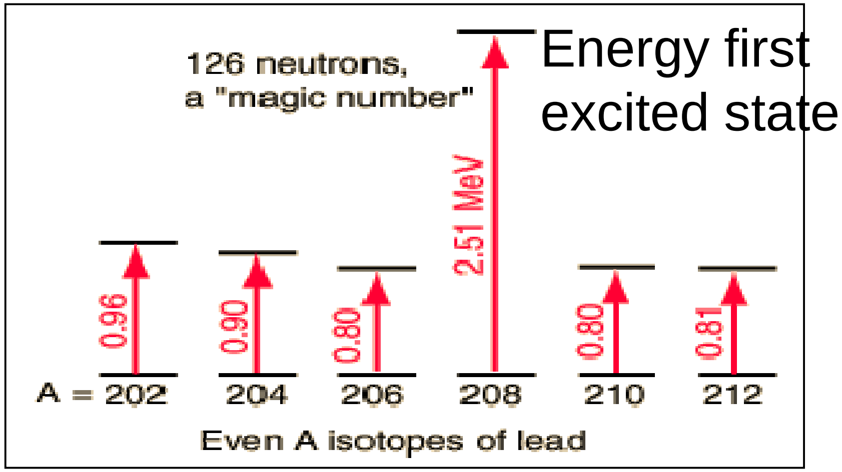


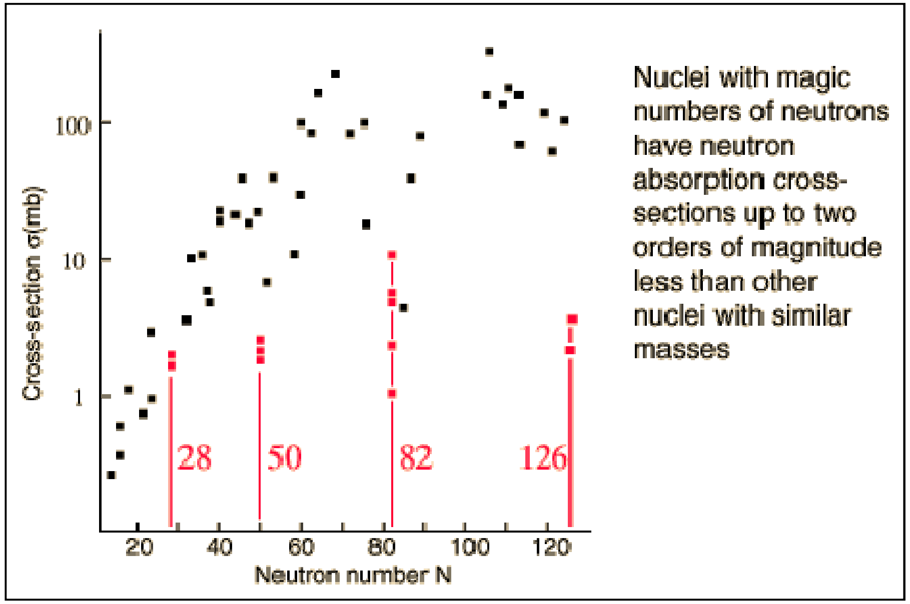
$$\langle E \rangle = \frac{\int_0^{E_F} E_{kin} d^3p}{\int_0^{p_F} d^3p} = \frac{1}{2m} \frac{\int_0^{E_F} p^2 d^3p}{\int_0^{p_F} d^3p} = \frac{3}{5} E_F \sim 20\text{MeV}$$

Not too approximate : already quite some correlations : the mean field model (or shell-model)









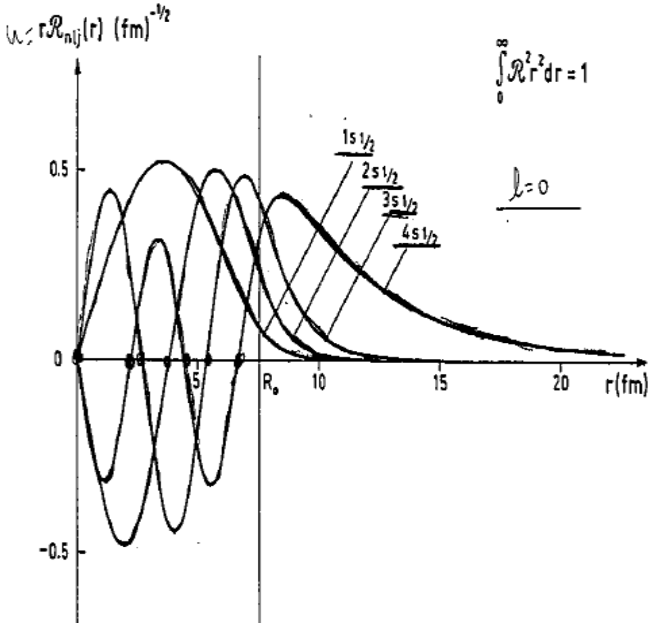
Not too approximate : already quite some correlations : the mean field model (or shell-model)

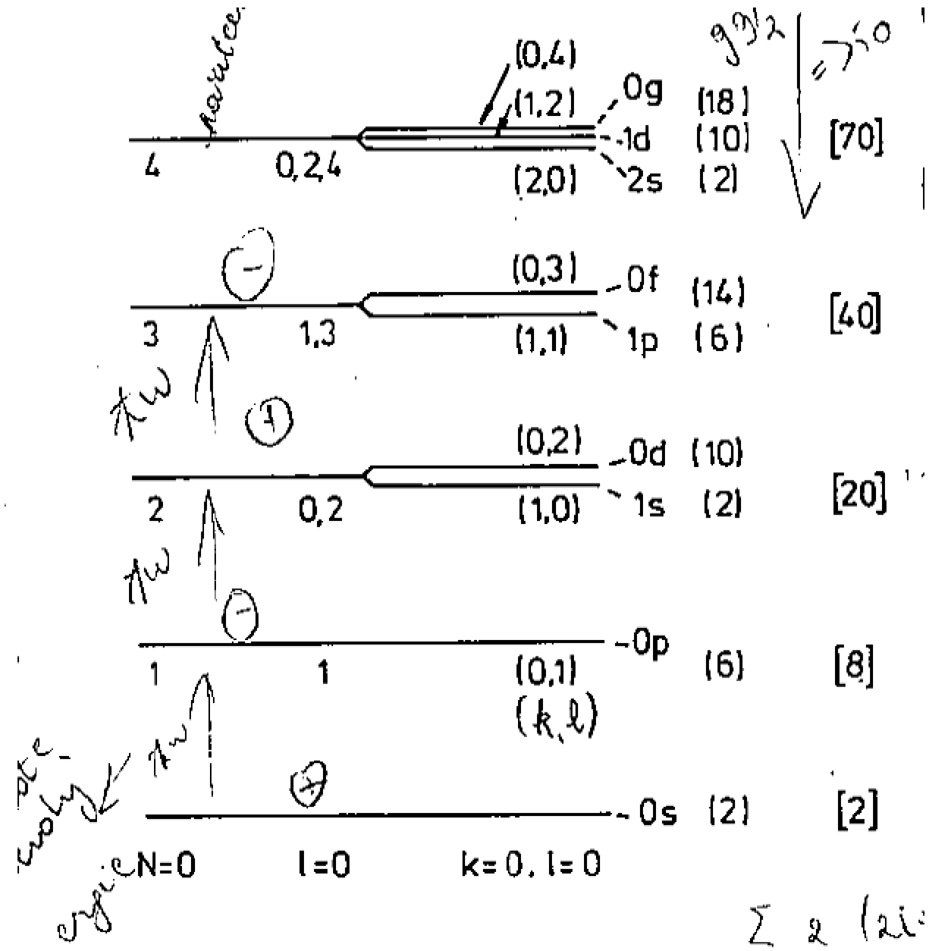
→solve Schrödinger equation for nucleon in nuclear potential

$$\varphi(\vec{r}) = u(r) Y_m^l(\Omega)$$

spherical

Harmonic oscillator potential

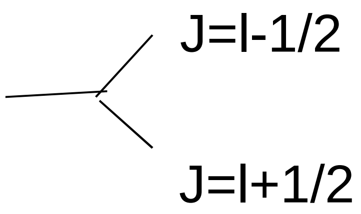


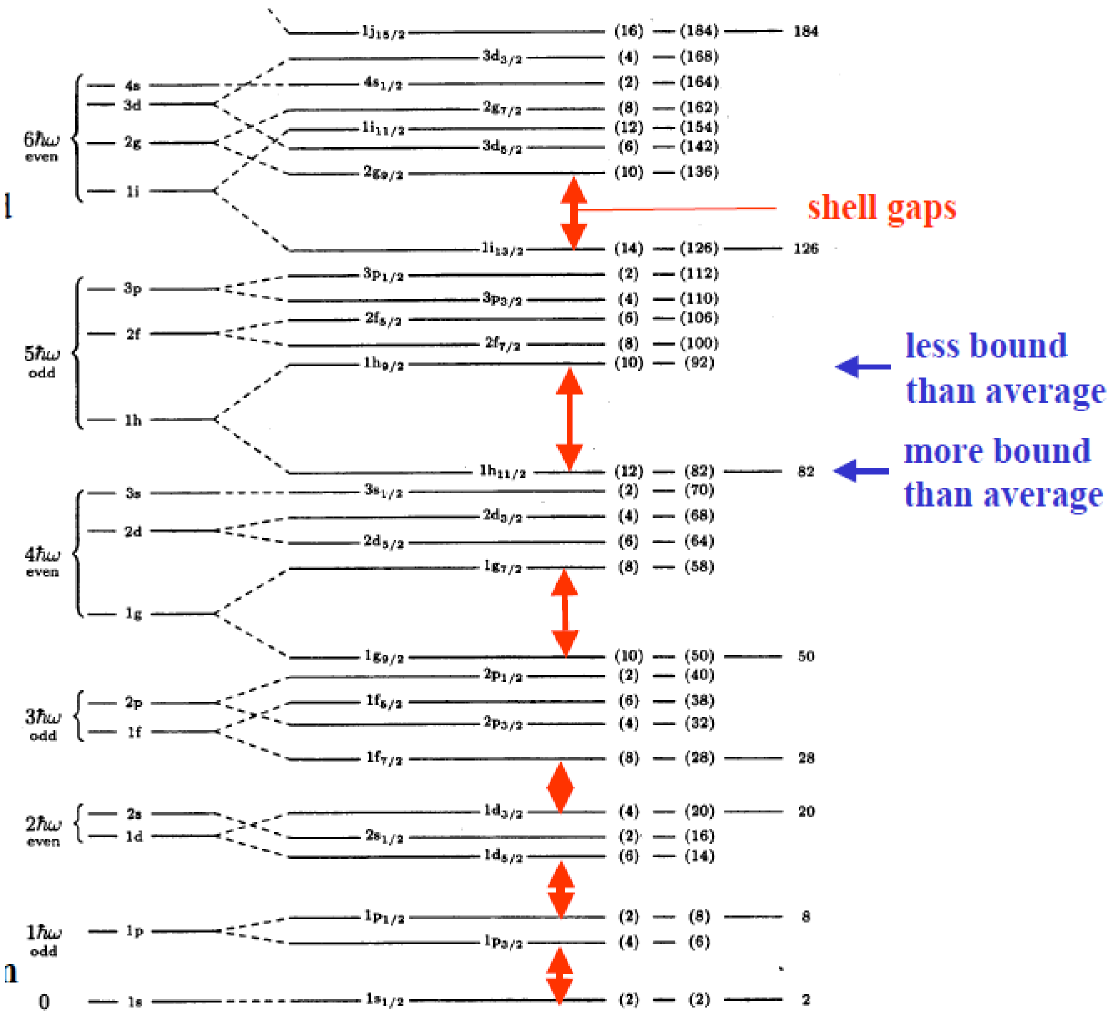


Not too approximate : already quite some correlations : the mean field model (or shell-model)

→Work harder : add spin-orbit term

$$\widehat{H} \rightarrow \widehat{H} + \xi(r)\bar{l} \cdot \bar{s}$$





Magic numbers

Independent particle picture ???

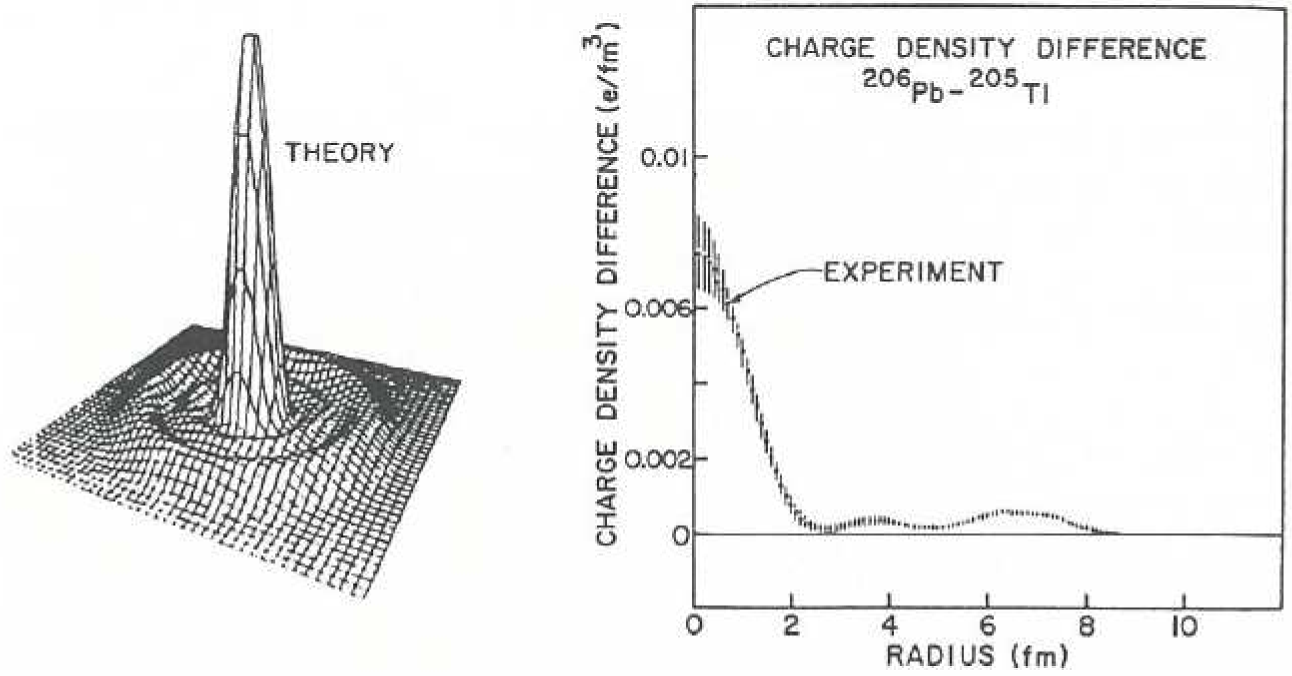


Fig. 3.18. The nuclear density distribution for the least bound proton in ^{206}Pb. The shell-model predicts the last ($3s_{1/2}$) proton in ^{206}Pb to have a sharp maximum at the centre, as shown at the left-hand side. On the right-hand side the nuclear charge density difference $\varrho_c\,(^{206}\mathrm{Pb}) - \varrho_c\,(^{205}\mathrm{Tl}) = \varphi^2_{3s_{1/2}}(r)$ is given [taken from (Frois 1983) and Doe 1983)]

$$\rho(r) = \sum_{b<F} \varphi_b^*(r)\varphi_b(r)$$

Not too approximate : already quite some correlations : the mean field model (or shell-model)

Woods-Saxon potential :

$$V(r) = V_0 \frac{1}{1 + e^{\left(\frac{r-R}{a}\right)}}$$

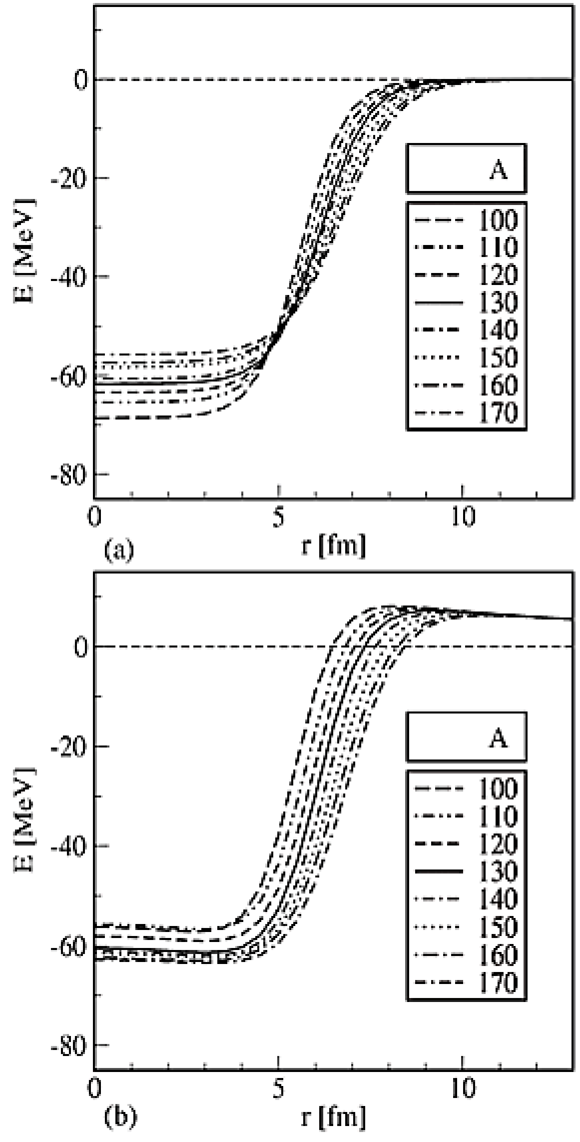


Figure 1. Woods-Saxon potential for neutrons (a) and protons (b) along the Sn isotopic chain.

The Hartree-Fock mean field

- Nucleons are moving independent from each other in a mean field
- How do we obtain a reliable and consistent mean field ?

$$H = \sum_i T_i + \frac{1}{2}\sum_{i,j} V_{i,j}$$

$$H = \sum_i \left(T_i + U(r_i)\right) + \left(\frac{1}{2}\sum_{i,j} V_{i,j} - \sum_i U(r_i)\right)$$

$$H = \sum_i h_0(i) + H_{res}(i,j)$$

Residual interaction

The Hartree-Fock recipe :

Nucleons fill up a number of orbitals and form a density that can be written in terms of the occupied states as :

$$\rho(r) = \sum_{b<F} \varphi_b^*(r)\varphi_b(r)$$

The potential at a position r', generated by the nucleon-nucleon two-body interaction V(r,r') is given by

$$U_H(r') = \sum_{b<F} \int \varphi_b^*(r)V(r,r')\varphi_b(r)dr$$

$$- \quad \frac{\hbar^2}{2m}\Delta\varphi_i(r) + \sum_{b<f} \int \varphi_b^*(r')V(r,r')\varphi_b(r')dr' \cdot \varphi_i(r)$$

$$- \quad \sum_{b<f} \int \varphi_b^*(r')V(r,r')\varphi_b(r)\varphi_i(r')dr' = \varepsilon_i\varphi_i(r)$$

$$-\frac{\hbar^2}{2m}\Delta\varphi_i(r) + U_H(r)\varphi_i(r) - \int U_F(r,r')\varphi_i(r')dr' = \varepsilon_i\varphi_i(r)$$

$$U_H(r') = \sum_{b<F}\int \varphi_b^*(r)V(r,r')\varphi_b(r)dr$$

$$U_F(r,r') = \sum_{b<F}\varphi_b^*(r')V(r,r')\varphi_b(r)$$

Hartree-Fock recipe :Start with an initial guess for either the average field or the wave functions and use V(r,r') to solve the coupled equations to obtain better values

e.g.

$$\begin{tikzcd}
U^{(0)}_{H(F)}(\boldsymbol{r}) \arrow[d] & & U^{(1)}_{H(F)}(\boldsymbol{r}) \arrow[d] & & U^{(2)}_{H(F)}(\boldsymbol{r}) \arrow[d] \\
\varphi_i^{(0)}(\boldsymbol{r}) \arrow[urr] & & \varphi_i^{(1)}(\boldsymbol{r}) \arrow[urr] & & \varphi_i^{(2)}(\boldsymbol{r}) \\
\varepsilon_i^{(0)} & & \varepsilon_i^{(1)} & & \varepsilon_i^{(2)}
\end{tikzcd}$$

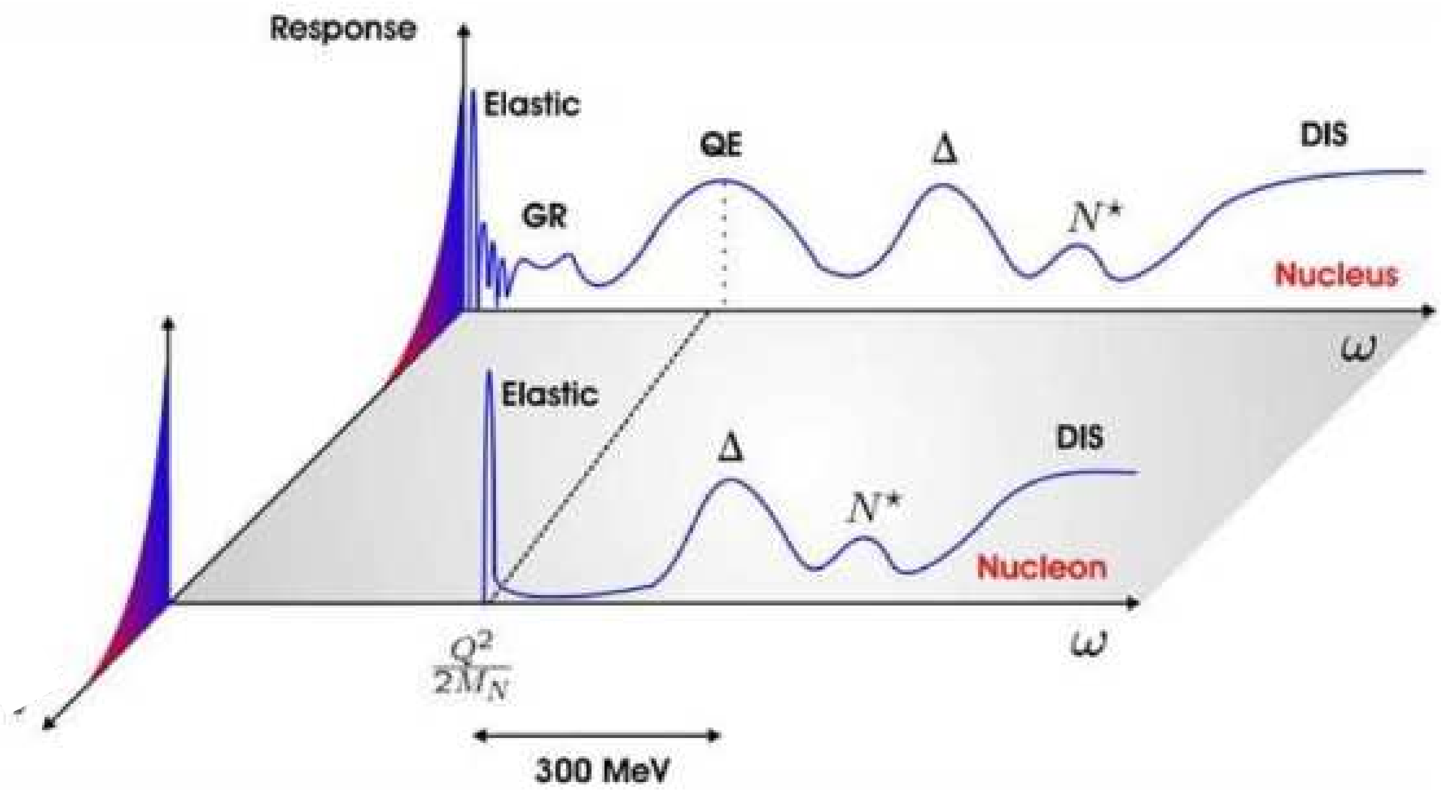
Wave function for the nucleus = Slater determinant

$$\Psi_{1,2,\ldots,A}(r_1, r_2, \ldots, r_A) = \frac{1}{\sqrt{A!}} \begin{vmatrix} \varphi_1(r_1) & \varphi_2(r_2) & \cdots & \varphi_A(r_A) \\ \varphi_2(r_1) & \varphi_3(r_2) & \cdots & \varphi_1(r_A) \\ \vdots & & & \end{vmatrix}$$

$$E_0 = \sum_{i=1}^{A} \varepsilon_i$$

Antisymmetrization takes into account the Pauli principle

Interactions …

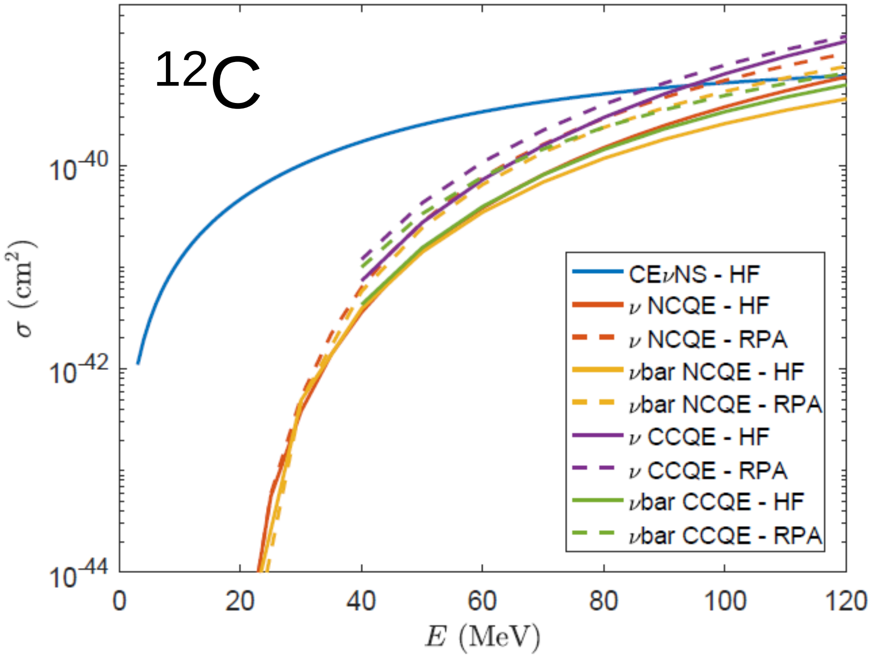


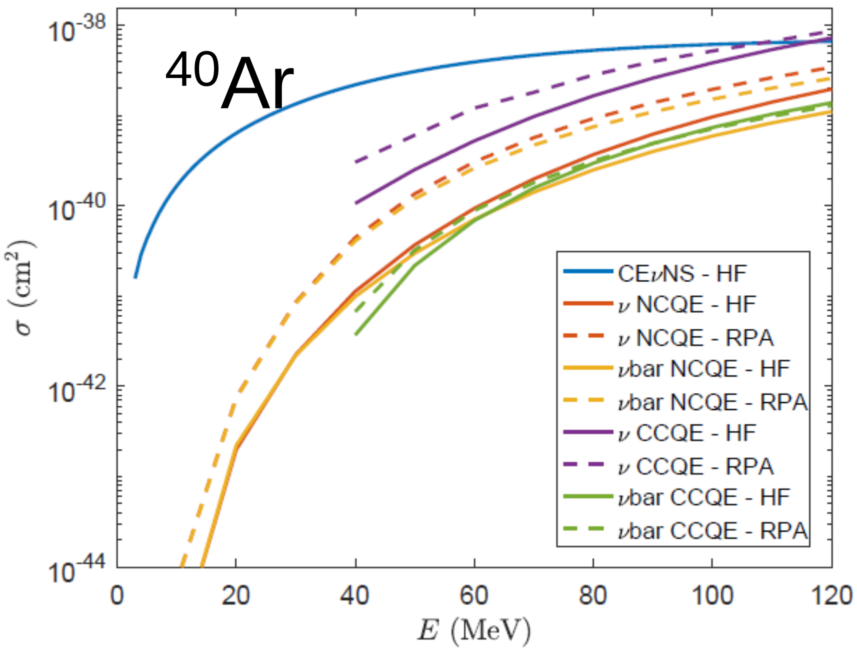
Coherent (elastic) scattering : CEνNS

$$\frac{\mathrm{d}\sigma}{\mathrm{d}T} = \frac{G_F^2}{4\pi} Q_W^2 |F(-2MT)|^2 M \left(1 - \frac{T}{E} - \frac{MT}{2E^2}\right)$$

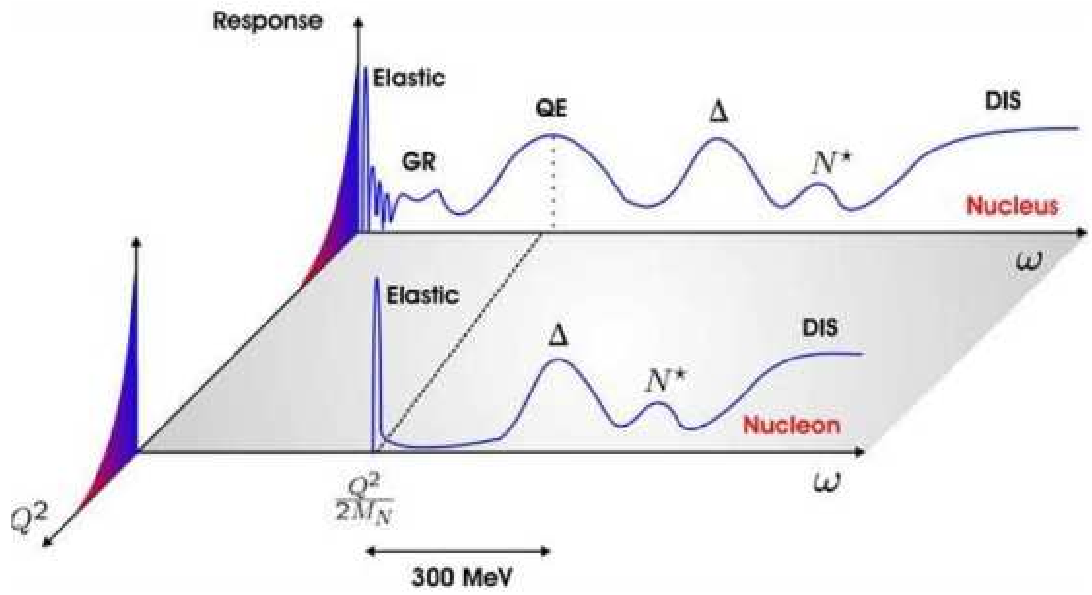
$$F(q^2) = \frac{4\pi}{Q_W} \int \left((1 - 4\sin^2\theta_W)\rho_p(|\mathbf{r}|) - \rho_n(|\mathbf{r}|)\right) \frac{\sin(|\mathbf{q}||\mathbf{r}|)}{|\mathbf{q}||\mathbf{r}|} \mathbf{r}^2 \mathrm{d}|\mathbf{r}|$$

$$\rho_\tau(|\mathbf{r}|) = \frac{1}{4\pi|\mathbf{r}|^2} \sum_\alpha (2j_\alpha + 1) R_\alpha^2(|\mathbf{r}|)$$





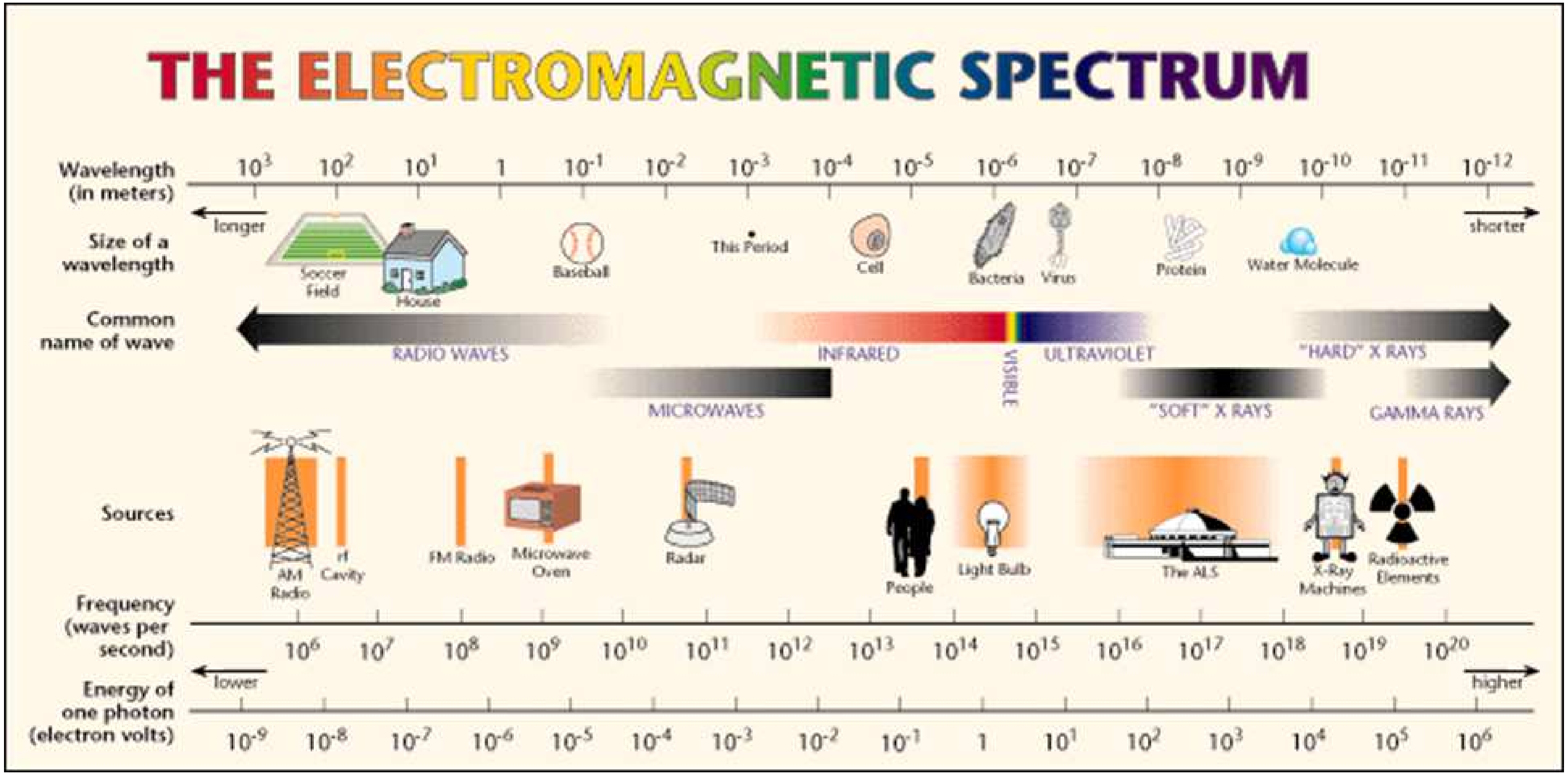
The Random Phase approximation



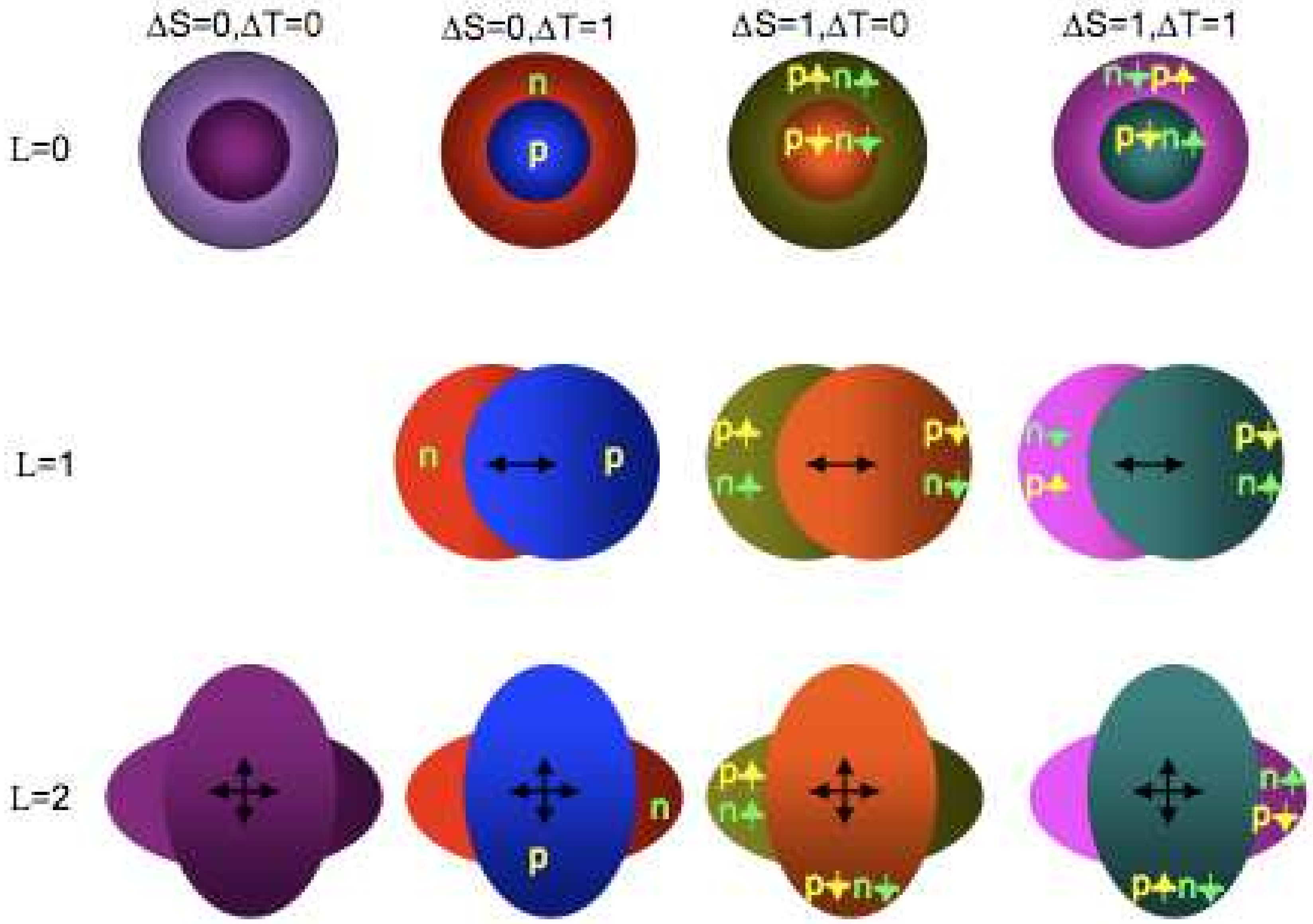
- Long-range correlations are correlations over the whole size of the nucleus
- They can redistribute the incoming energy transfer to the nucleus over all the nuclear constituents.
- They manifest themselves in collective excitations such as giant resonances

The Random Phase approximation

Long-range correlations = low energy phenomena



Long-range correlations = probing collective effects



https://cyclotron.tamu.edu/research/nuclear-structure/

RPA … what's in a name

David Bohm &David Pines '52 – Condensed matter physics

Quantum mechanical interactions interaction between electrons

The Hamiltonian corresponding to a system of individual electrons is re-expressed such that the long-range part of the Coulomb interactions between electrons is described in terms of collective fields

This leads to the description of organized behavior of electrons brought along by long-range Coulomb interactions that couple together the motion of many electrons simultaneously = plasma oscillations

Neglecting the coupling between plasma vibrations of different momenta

Propagators (in a nutshell)

Time evolution of a single-particle system characterized by quantum numbers α at time t_0 :

$$|\alpha, t_0; t\rangle = e^{-\frac{i}{\hbar}H(t-t_0)}\,|\alpha; t_0\rangle$$

$$i\hbar\frac{\partial}{\partial t}\,|\alpha, t_0; t\rangle = H\,|\alpha, t_0; t\rangle$$

Time-dependent Schrödinger equation

… or written in terms of wave functions :

$$\int d\vec{r}'\,|\vec{r}'\rangle\,\langle\vec{r}'| = \mathbb{1}$$

$$\begin{aligned}
\Psi(\vec{r}, t) &= \langle\vec{r}|\alpha; t_0, t\rangle \\
&= \langle\vec{r}|\,e^{-\frac{i}{\hbar}H(t-t_0)}\,|\alpha, t_0\rangle \\
&= \int d\vec{r}'\,\langle\vec{r}|\,e^{-\frac{i}{\hbar}H(t-t_0)}\,|\vec{r}'\rangle\,\langle\vec{r}'|\,\alpha, t_0\rangle
\end{aligned}$$

Ref. : Many-body Theory exposed ! WH Dickhoff, D. Van Neck

$$\begin{aligned}\Psi(\vec{r},t) &= \int d\vec{r}'\ \langle\vec{r}|\, e^{-\frac{i}{\hbar}H(t-t_0)}\, |\vec{r}'\rangle\, \langle\vec{r}'|\ \alpha, t_0\rangle \\ &= i\hbar \int d\vec{r}' G(\vec{r},\vec{r}';t,t_0)\Psi(\vec{r}',t_0)\end{aligned}$$

Propagator or Green's function : $G(\vec{r},\vec{r}';t,t_0) = -\frac{i}{\hbar}\langle\vec{r}|\, e^{-\frac{i}{\hbar}H(t-t_0)}\, |\vec{r}'\rangle$

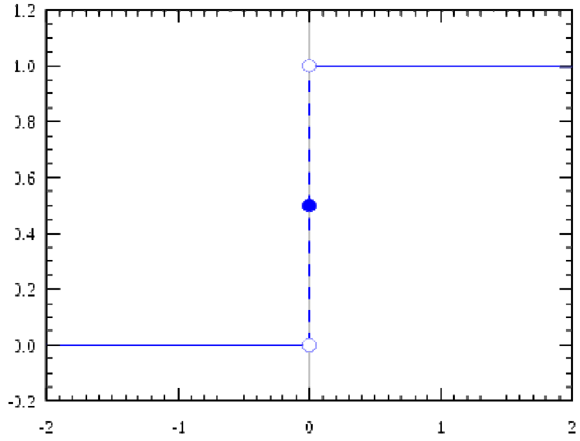
$$\boxed{\mathbb{1} = \sum_n |n\rangle\langle n|}$$

Plug in unity operator

$$\begin{aligned}G(\vec{r},\vec{r}';t,t_0) = &= -\frac{i}{\hbar}\langle 0|\, \hat{a}_r e^{-\frac{i}{\hbar}H(t-t_0)}\hat{a}^\dagger_{r'}\, |0\rangle \\ &= -\frac{i}{\hbar}\sum_n \langle 0|\, \hat{a}_r\, |n\rangle\langle n|\, \hat{a}^\dagger_{r'}\, |0\rangle\, e^{-\frac{i}{\hbar}\varepsilon_n(t-t_0)} \\ &= \frac{-i}{\hbar}\sum_n u(\vec{r})u_n^*(\vec{r}')e^{-\frac{i}{\hbar}\varepsilon_n(t-t_0)}\end{aligned}$$

Fourier transform :

Impose causality



$$\theta(t-t_0) = -\frac{1}{2\pi i}\int dE' \frac{e^{-\frac{i}{\hbar}E'(t-t_0)}}{E'+i\eta}$$

$$\begin{aligned} G(\vec{r},\vec{r}';E) &= -\frac{i}{\hbar}\int_{-\infty}^{+\infty} d(t-t_0) e^{\frac{i}{\hbar}E(t-t_0)}\theta(t-t_0)\sum_n u(\vec{r})u_n^*(\vec{r}')e^{-\frac{i}{\hbar}\varepsilon_n(t-t_0)} \\ &= \sum_n \int dE'\delta(E-E'-\varepsilon_n)\frac{1}{E'+i\eta}u_n(\vec{r})u_n^*(\vec{r}') \\ &= \sum_n \frac{u_n(\vec{r})u_n^*(\vec{r}')}{E-\varepsilon_n+i\eta} \\ &= \langle\vec{r}|\frac{1}{E-H+i\eta}|\vec{r}'\rangle \end{aligned}$$

$$\boxed{\delta(x-a) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{i(x-a)t}dt}$$

Or more general :

$$\boxed{G(\alpha,\beta;E) = \langle 0|\, a_\alpha \frac{1}{E-H+i\eta} a_\beta^\dagger \,|0\rangle}$$

Any set of single-particle states

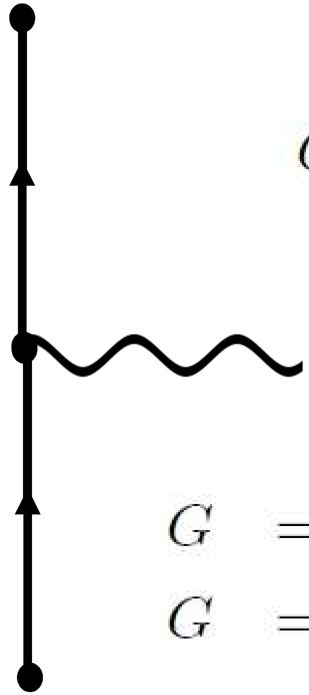
Any Hamiltonian

$$\widehat{H} = \widehat{H}_0 + \widehat{V}$$

$$\widehat{H}_0 = \sum_\alpha \varepsilon_\alpha a_\alpha^\dagger a_\alpha$$

$$G(\alpha,\beta;E) = \langle 0| a_\alpha \frac{1}{E - H + i\eta} a_\beta^\dagger |0\rangle$$

$$\Rightarrow \quad G^{(0)}(\alpha,\beta;E) = \langle 0| a_\alpha \frac{1}{E - H_0 + i\eta} a_\beta^\dagger |0\rangle = \frac{\delta_{\alpha,\beta}}{E - \varepsilon_\alpha + i\eta}$$



$$G = \frac{1}{E - H + i\eta}$$

Plug in : $\boxed{\frac{1}{A-B} = \frac{1}{A} + \frac{1}{A} B \frac{1}{A-B}}$ Use B=V

$$\begin{aligned} G &= G^{(0)} + G^{(0)} V G \\ G &= G^{(0)} + G^{(0)} V G^{(0)} + G^{(0)} V G^{(0)} V G^{(0)} + \dots \end{aligned}$$

$$\begin{aligned} \langle\alpha| \frac{1}{E - H + i\eta} |\beta\rangle &= \langle\alpha| \frac{1}{E - H_0 + i\eta} |\beta\rangle \\ &+ \sum_{\gamma,\delta} \langle\alpha| \frac{1}{E - H_0 + i\eta} |\gamma\rangle \langle\gamma| V |\delta\rangle \langle\delta| \frac{1}{E - H + i\eta} |\beta\rangle \end{aligned}$$

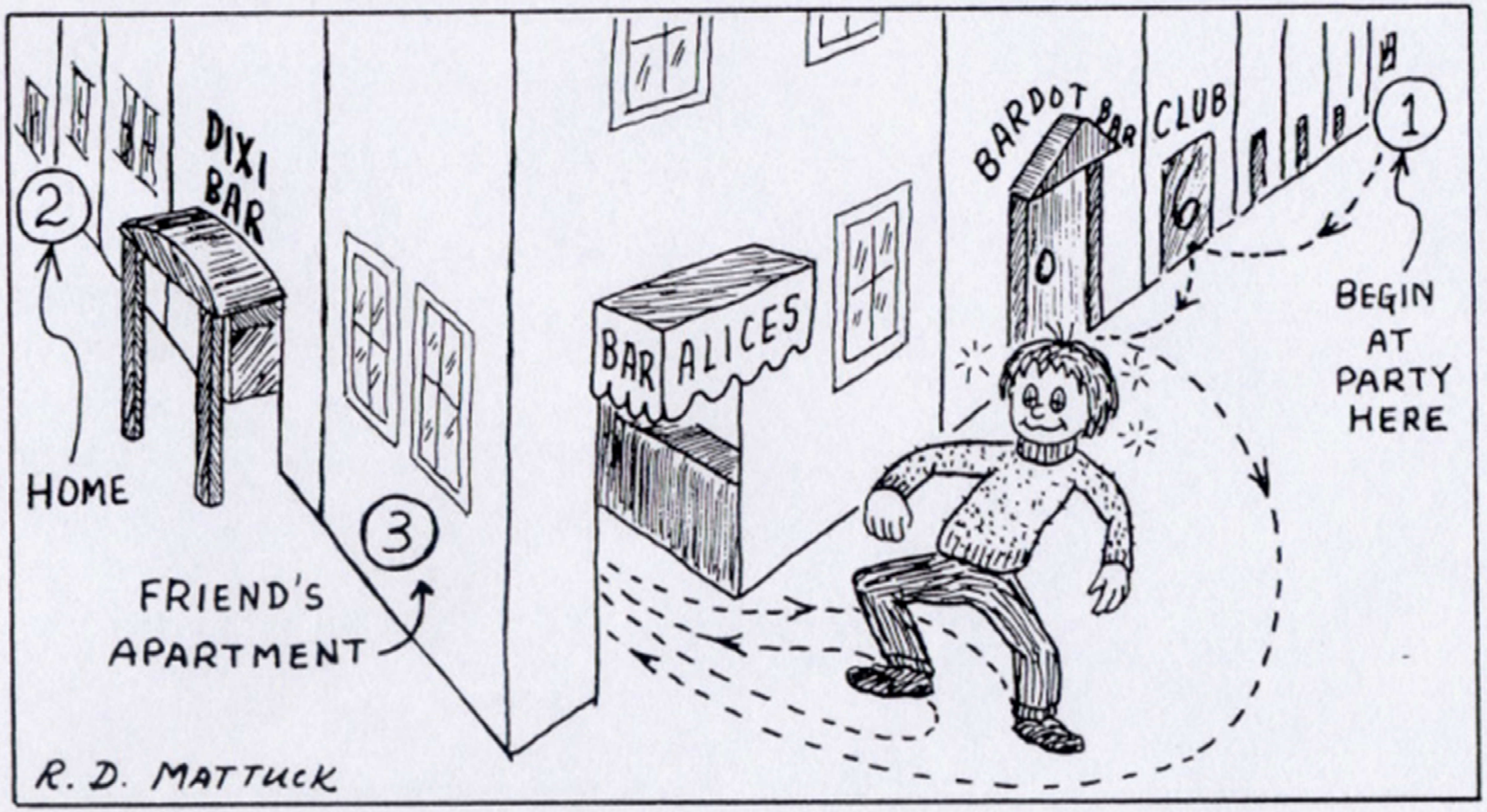
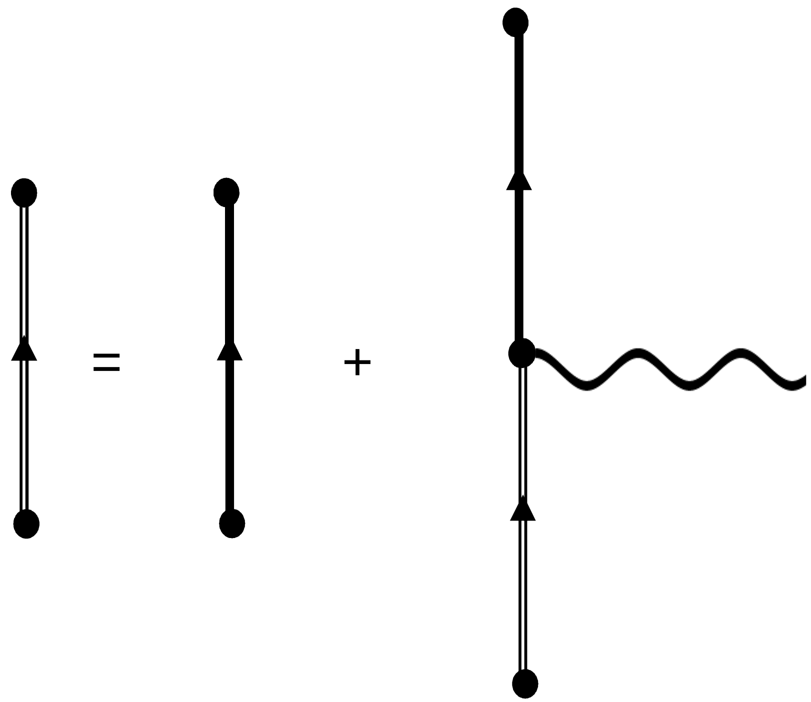


Fig. 1.1 *Propagation of Drunken Man*

(Reproduced with the kind permission of *The Encyclopedia of Physics*)

$$
\begin{aligned}
G &= G^{(0)} + G^{(0)} V G \\
G &= G^{(0)} + G^{(0)} V G^{(0)} + G^{(0)} V G^{(0)} V G^{(0)} + \dots
\end{aligned}
$$



N-body systems

N-body state can be written as

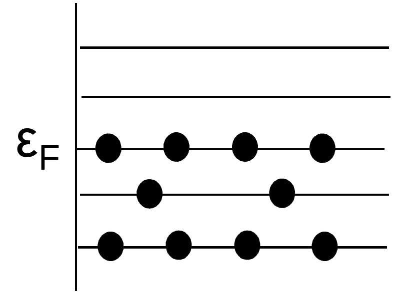
$$|\Phi_n\rangle = |\lambda_1\lambda_2....\lambda_N\rangle = a^\dagger_{\lambda_1} a^\dagger_{\lambda_2}....a^\dagger_{\lambda_N} |0\rangle$$

With energy

$$E_n = \sum_{i=1}^{N} \varepsilon_{\lambda_i}$$

The Fermi sea is written as

$$|\Phi_0\rangle = \prod_{\lambda_i \leq F} a^\dagger_{\lambda_i} |0\rangle$$



$$\begin{aligned} \widehat{H}_0 a^\dagger_\alpha |\Phi_0\rangle &= (E_0 + \varepsilon_\alpha)\, a^\dagger_\alpha |\Phi_0\rangle \quad \alpha > F \\ \widehat{H}_0 a_\alpha |\Phi_0\rangle &= (E_0 - \varepsilon_\alpha)\, a^\dagger_\alpha |\Phi_0\rangle \quad \alpha < F \end{aligned}$$

So now we also need hole propagation !

N-body systems

In general, an N-body state can be written as

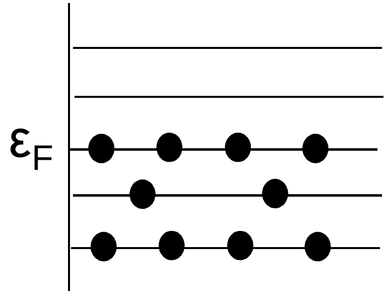
$$|\Phi_n\rangle = |\lambda_1 \lambda_2 .... \lambda_N\rangle = a^\dagger_{\lambda_1} a^\dagger_{\lambda_2} .... a^\dagger_{\lambda_N} |0\rangle$$

With energy

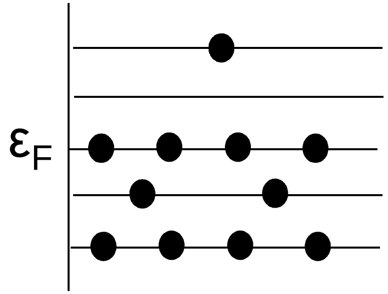
$$E_n = \sum_{i=1}^{N} \varepsilon_{\lambda_i}$$

The Fermi sea is written as

$$|\Phi_0\rangle = \prod_{\lambda_i \le F} a^\dagger_{\lambda_i} |0\rangle$$



$$\begin{aligned} \widehat{H}_0 a^\dagger_\alpha |\Phi_0\rangle &= (E_0 + \varepsilon_\alpha)\, a^\dagger_\alpha |\Phi_0\rangle \quad \alpha > F \\ \widehat{H}_0 a_\alpha |\Phi_0\rangle &= (E_0 - \varepsilon_\alpha)\, a^\dagger_\alpha |\Phi_0\rangle \quad \alpha < F \end{aligned}$$



So now we also need hole propagation !

N-body systems

In general, an N-body state can be written as

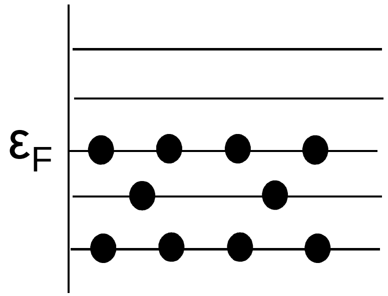
$$|\Phi_n\rangle = |\lambda_1\lambda_2....\lambda_N\rangle = a^\dagger_{\lambda_1} a^\dagger_{\lambda_2}....a^\dagger_{\lambda_N}\,|0\rangle$$

With energy

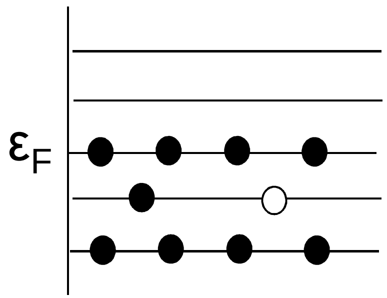
$$E_n = \sum_{i=1}^{N} \varepsilon_{\lambda_i}$$

The Fermi sea is written as

$$|\Phi_0\rangle = \prod_{\lambda_i \leq F} a^\dagger_{\lambda_i}\,|0\rangle$$



$$\begin{aligned} \widehat{H}_0 a^\dagger_\alpha\,|\Phi_0\rangle &= (E_0 + \varepsilon_\alpha)\,a^\dagger_\alpha\,|\Phi_0\rangle \qquad \alpha > F \\ \widehat{H}_0 a_\alpha\,|\Phi_0\rangle &= (E_0 - \varepsilon_\alpha)\,a^\dagger_\alpha\,|\Phi_0\rangle \qquad \alpha < F \end{aligned}$$



So now we also need hole propagation !

Interacting system : $H_0 \rightarrow H$

$$G(\alpha, \beta; E) = \langle 0 | a_\alpha \frac{1}{E - H + i\eta} a^\dagger_\beta | 0 \rangle$$

Single-particle

Particle states above Fermi leve

$$G(\alpha, \beta; E) = \sum_m \frac{\langle \Phi_0^N | a_\alpha | \Phi_m^{N+1} \rangle \langle \Phi_m^{N+1} | a^\dagger_\beta | \Phi_0^N \rangle}{E - (E_m^{N+1} - E_0^N) + i\eta} + \sum_n \frac{\langle \Phi_0^N | a^\dagger_\beta | \Phi_n^{N-1} \rangle \langle \Phi_n^{N-1} | a_\alpha | \Phi_0^N \rangle}{E - (E_0^N - E_n^{N-1}) - i\eta}$$

Hole states below Fermi level

One particle (hole) in an N-body system

Lehmann representation of the single-particle progagator

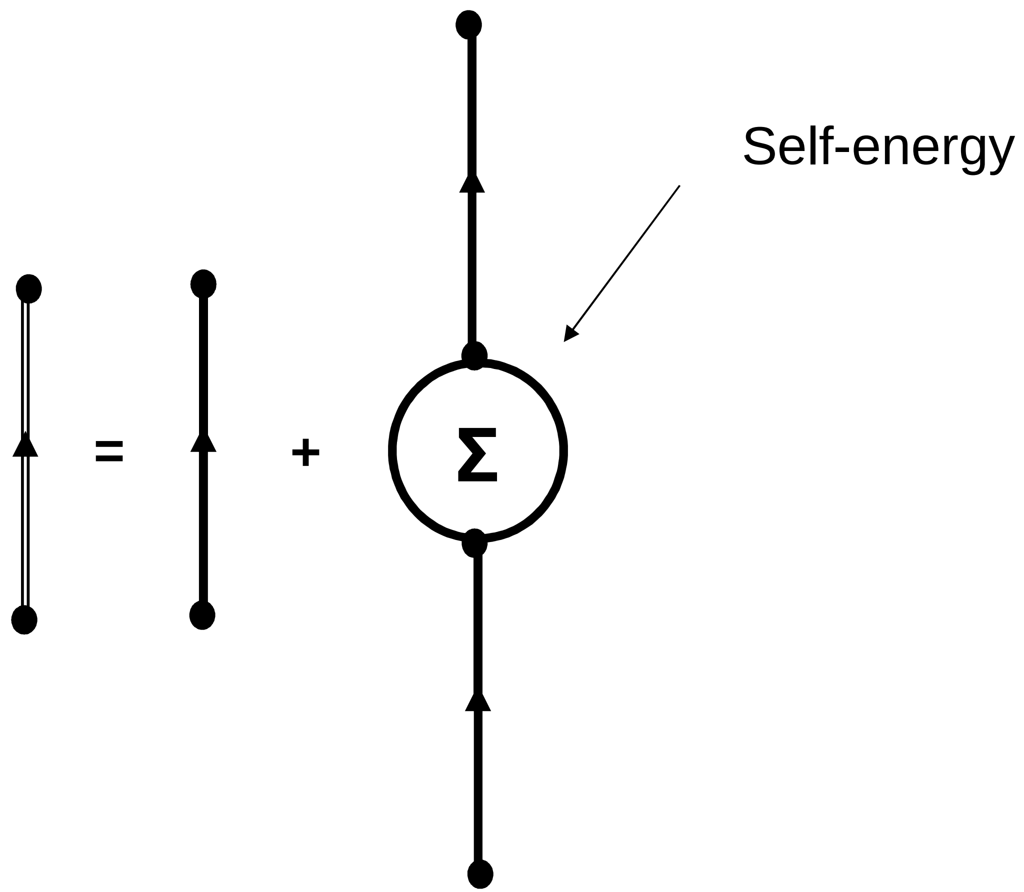
$H = H_0 + U + V$

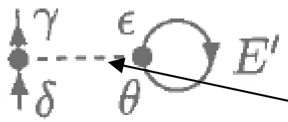
Auxiliary potential

2-body interaction

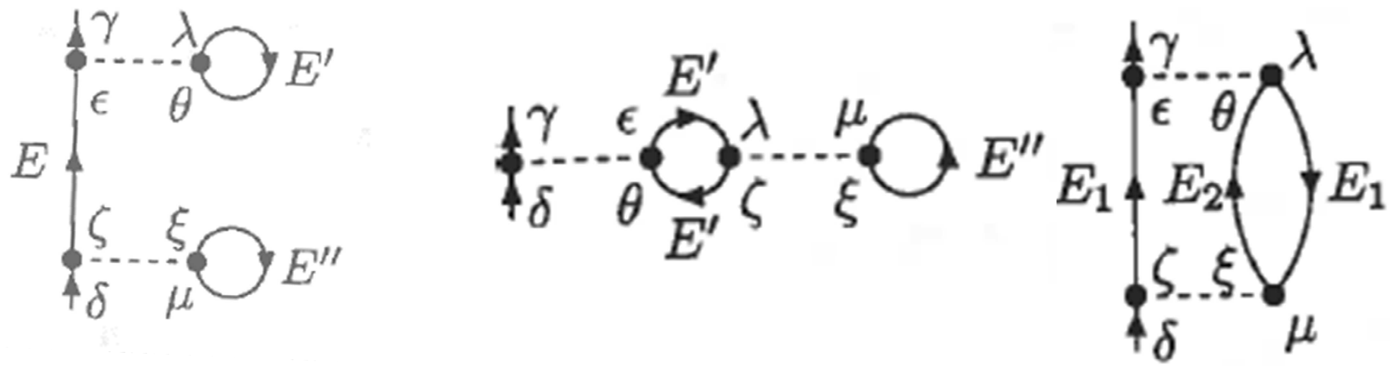
$$
\begin{aligned}
G &= G^{(0)} + G^{(0)} V G \\
G &= G^{(0)} + G^{(0)} V G^{(0)} + G^{(0)} V G^{(0)} V G^{(0)} + \ldots
\end{aligned}
$$

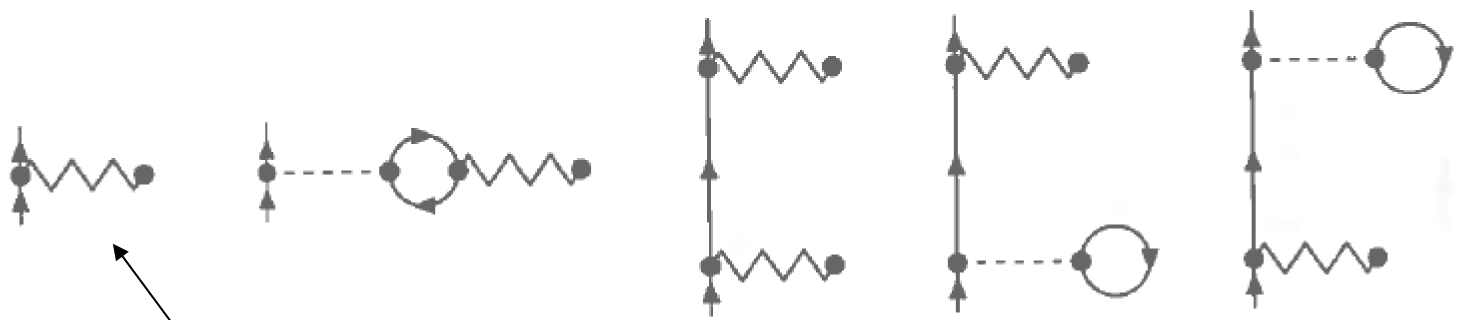
Dyson Equation :



Σ = e.g. 

2-body interaction V



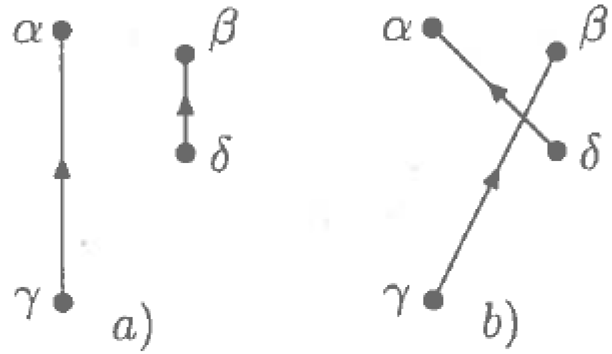


Axiliary field U

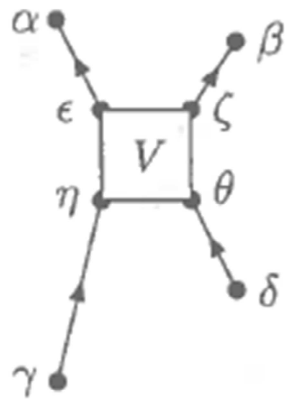
...

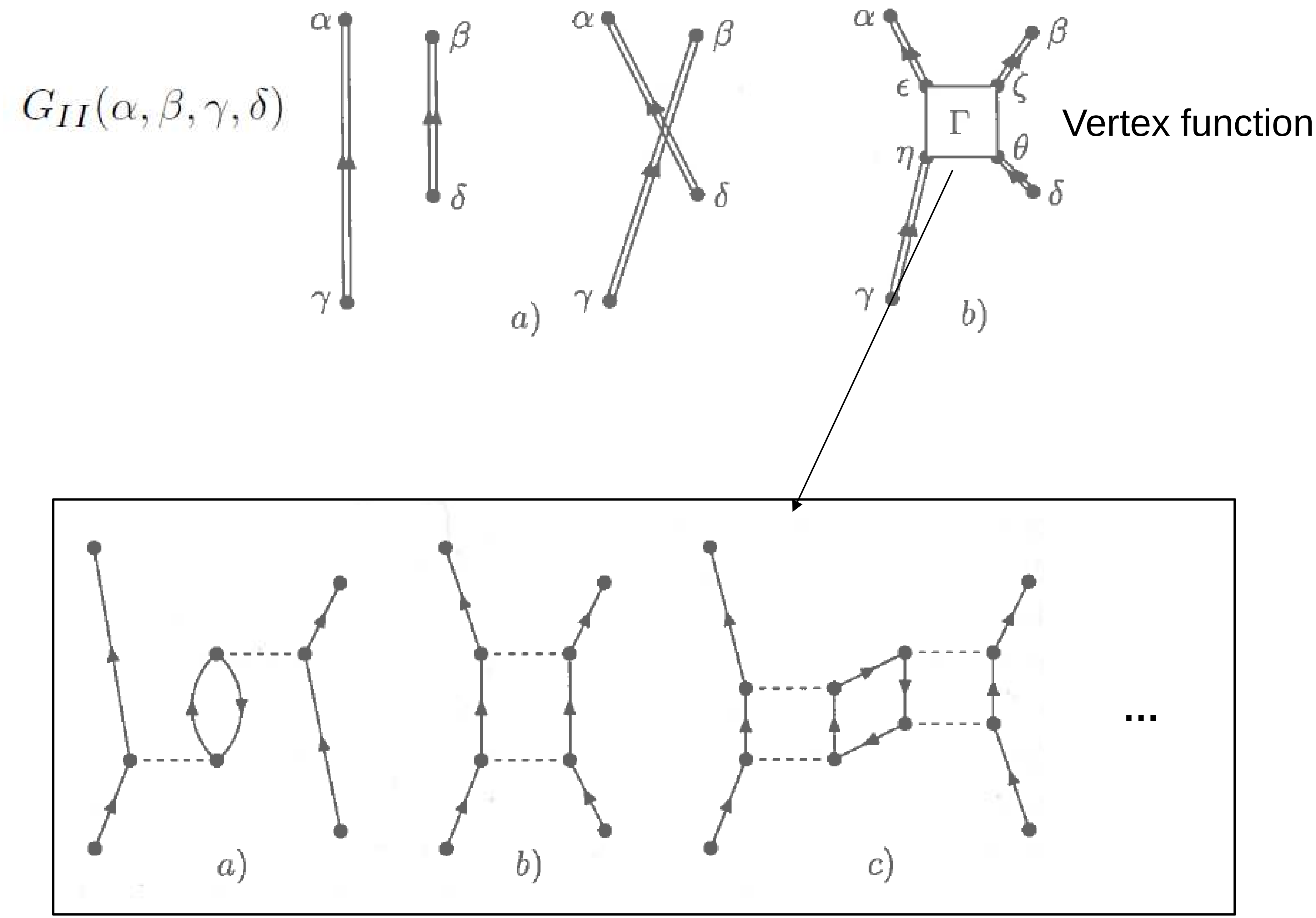
Two-particle propagator

Non-interacting $G_{II}^{(0)}(\alpha, \beta, \gamma, \delta)$

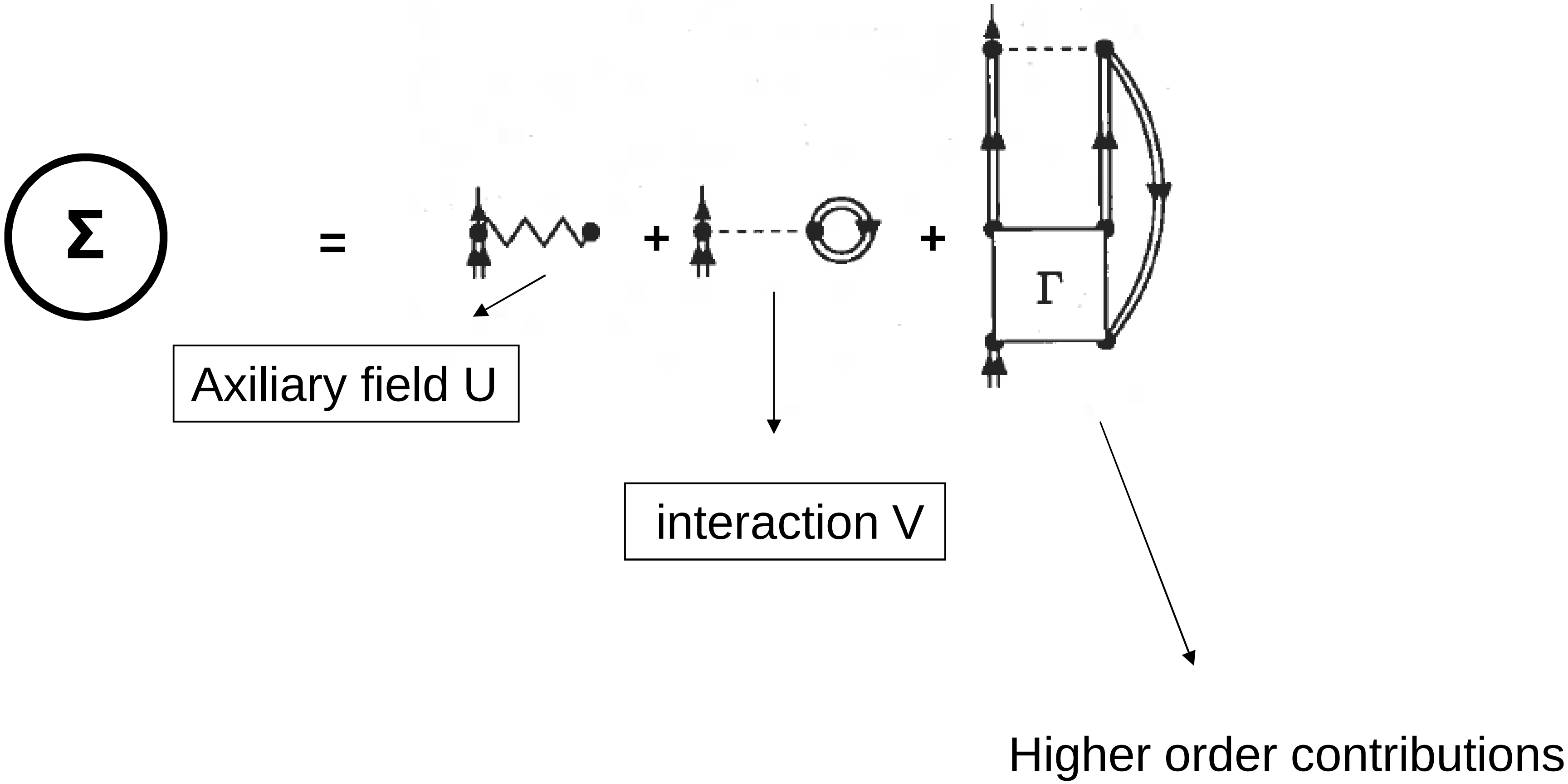


$G_{II}^{(1)}(\alpha, \beta, \gamma, \delta)$





$G_{II}(\alpha, \beta, \gamma, \delta)$
α β γ δ
a)
α β γ δ ϵ ζ η θ Γ
b)
Vertex function
a)
b)
c)
...



Reformulation of the self-energy of a dressed particle

Hartree-Fock propagator

$$\widehat{H} = \widehat{T} + \widehat{V} = \left(\widehat{T} + \widehat{U}\right) + \left(\widehat{V} - \widehat{U}\right)$$

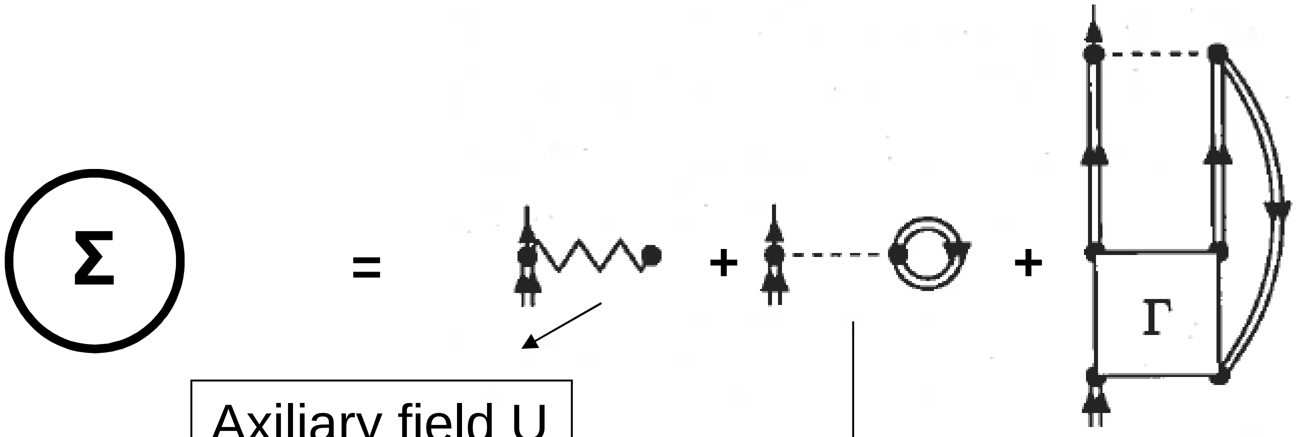
$$\widehat{H}_0 = \widehat{T} + \widehat{U} = \sum_{\alpha} \varepsilon_\alpha a_\alpha^\dagger a_\alpha$$

$$G^{(0)}(\alpha, \beta; E) = \delta_{\alpha,\beta} \left[\frac{\theta(\alpha - F)}{E - \varepsilon_\alpha + i\eta} + \frac{\theta(F - \alpha)}{E - \varepsilon_\alpha - i\eta} \right]$$

$$\downarrow$$

$$G^{HF}(\alpha, \beta; E) = G^{(0)}(\alpha, \beta; E) + \sum_{\gamma,\delta} G^{(0)}(\alpha, \gamma; E) \Sigma^{HF}(\gamma, \delta) G^{HF}(\delta, \beta; E)$$

$$\Sigma^{HF}(\gamma, \delta; E) = -\langle \gamma | U | \delta \rangle - i \int \frac{dE'}{2\pi} \sum_{\mu\nu} \langle \gamma\mu | V | \delta\nu \rangle G^{HF}(\nu\mu; E')$$



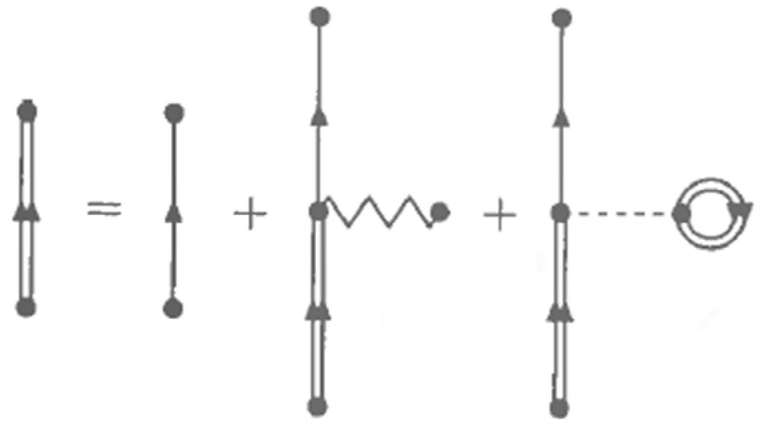
Axiliary field U

interaction V

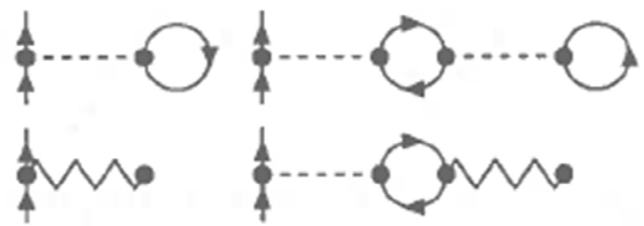
Higher order contributions

Reformulation of the self-energy of a dressed particle

$$G^{HF}(\alpha,\beta;E) = G^{(0)}(\alpha,\beta;E) + \sum_{\gamma,\delta} G^{(0)}(\alpha,\gamma;E)\Sigma^{HF}(\gamma,\delta)G^{HF}(\delta,\beta;E)$$



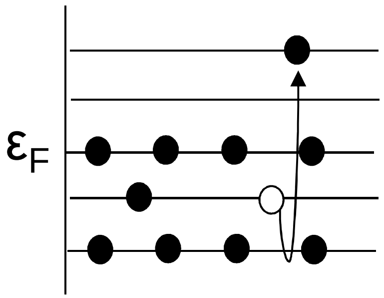
$$\Sigma^{HF}(\gamma,\delta;E) = -\langle\gamma|\,U\,|\delta\rangle - i\int \frac{dE'}{2\pi}\sum_{\mu\nu}\langle\gamma\mu|\,V\,|\delta\nu\rangle\, G^{HF}(\nu\mu;E')$$



Mean field already contains correlations !

Excited states : Particle-hole or polarization propagator

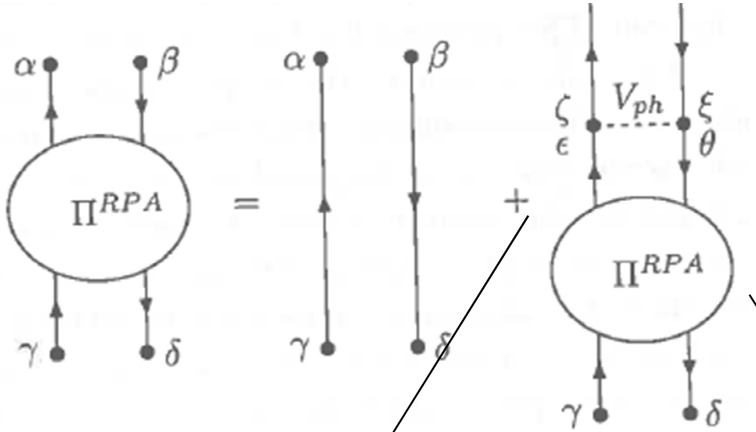
$$\Pi(\alpha,\beta^{-1};\gamma,\delta^{-1};E) = \sum_{n\neq 0} \frac{\langle \Phi_0^N | \, a_\beta^\dagger a_\alpha \, | \Phi_n^N \rangle \langle \Phi_n^N | \, a_\gamma^\dagger a_\delta \, | \Phi_0^N \rangle}{E - (E_n^N - E_0^N) + i\eta} - \sum_{n\neq 0} \frac{\langle \Phi_0^N | \, a_\gamma^\dagger a_\delta \, | \Phi_n^N \rangle \langle \Phi_n^N | \, a_\beta^\dagger a_\alpha \, | \Phi_0^N \rangle}{E + (E_n^N - E_0^N) - i\eta}$$



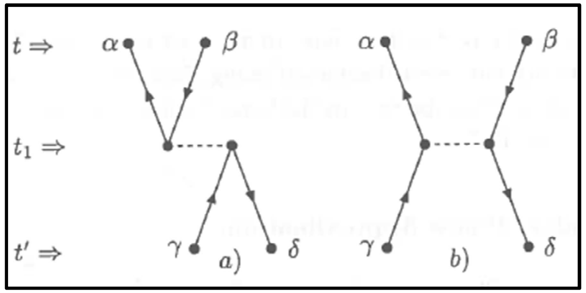
$$\Pi^{(0)}(\alpha,\beta^{-1};\gamma,\delta^{-1};E) = \int \frac{dE'}{2\pi} G^{(0)}(\alpha,\gamma;E+E') G^{(0)}(\delta^{-1},\beta^{-1};E')$$

=

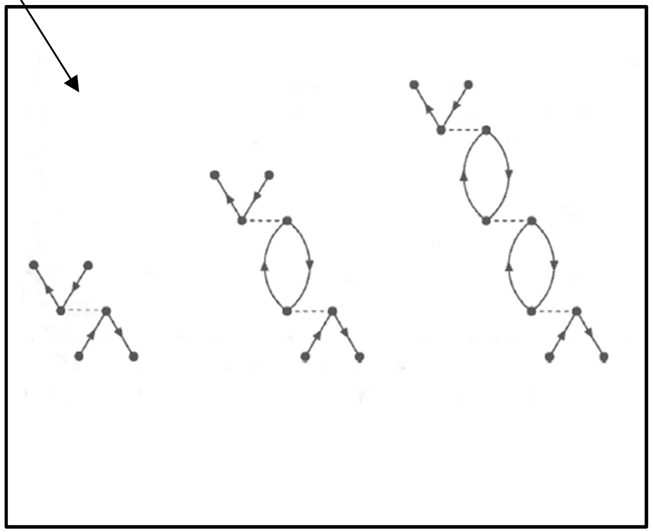
$$\Pi^{RPA}(\alpha,\beta^{-1};\gamma,\delta^{-1};E)=\Pi^{(0)}(\alpha,\beta^{-1};\gamma,\delta^{-1};E)$$
$$+\sum_{\varepsilon\theta\zeta\xi}\Pi^{(0)}(\alpha,\beta^{-1};\zeta,\xi^{-1};E)\left\langle\zeta\xi^{-1}\right|V_{ph}\left|\varepsilon\theta^{-1}\right\rangle\Pi^{RPA}(\varepsilon,\theta^{-1};\gamma,\delta^{-1};E)$$



First order



Higher order RPA diagrams



2-body interaction V ?

Landau-Migdal

$$V = c_0\{f_0(\rho) + f_0'(\rho)\vec{\tau}_1\vec{\tau}_2 + g_0(\rho)\vec{\sigma}_1\vec{\sigma}_2 + g_0'(\rho)\vec{\sigma}_1\vec{\sigma}_2\vec{\tau}_1\vec{\tau}_2\}$$

$$f(\rho(r)) = (1 - \rho(r))\ f^{ext}\ +\ \rho(r)\ f^{int}$$

$$\rho(r)\ =\ \frac{1}{1 + e^{\frac{r-R}{a}}}$$

2-body interaction V ?

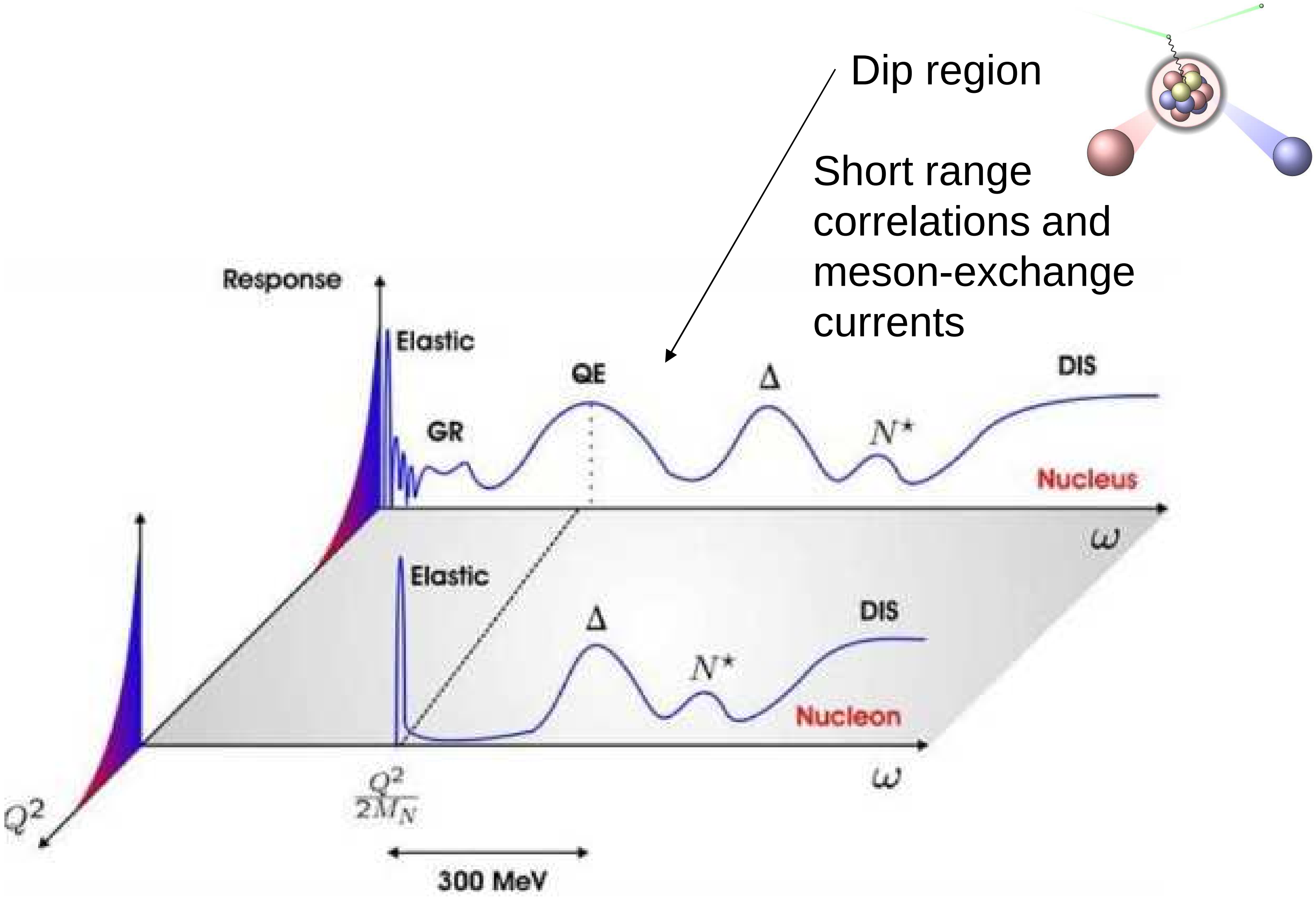
Skyrme

$$
\begin{aligned}
V(\vec{r_1},\vec{r_2}) \quad = \quad & t_0\,(1+x_0\hat{P}_\sigma)\,\delta(\vec{r}_1-\vec{r}_2) \\
& -\frac{1}{8}t_1\left[(\overleftarrow{\nabla}_1-\overleftarrow{\nabla}_2)^2\,\delta(\vec{r}_1-\vec{r}_2)+\delta(\vec{r}_1-\vec{r}_2)\,(\overrightarrow{\nabla}_1-\overrightarrow{\nabla}_2)^2\right] \\
& +\frac{1}{4}t_2(\overleftarrow{\nabla}_1-\overleftarrow{\nabla}_2)\,\delta(\vec{r}_1-\vec{r}_2)\,(\overrightarrow{\nabla}_1-\overrightarrow{\nabla}_2) \\
& +iW_0\,(\vec{\sigma}_1+\vec{\sigma}_2)\cdot(\overleftarrow{\nabla}_1-\overleftarrow{\nabla}_2)\times\delta(\vec{r}_1-\vec{r}_2)\,(\overrightarrow{\nabla}_1-\overrightarrow{\nabla}_2) \\
& +\frac{1}{6}t_3\,(1-x_3)\,(1+\hat{P}_\sigma)\,\rho\,\frac{(\vec{r}_1+\vec{r}_2)}{2}\,\delta(\vec{r}_1-\vec{r}_2) \\
& +\frac{e^2}{|\vec{r}_1-\vec{r}_2|}\;+\;x_3t_3\,\delta(\vec{r}_1-\vec{r}_2)\,\delta(\vec{r}_1-\vec{r}_3) \\
& -\frac{1}{24}t_4\left\{\left[(\overleftarrow{\nabla}_1-\overleftarrow{\nabla}_2)^2+(\overleftarrow{\nabla}_2-\overleftarrow{\nabla}_3)^2+(\overleftarrow{\nabla}_3-\overleftarrow{\nabla}_1)^2\right]\right\} \\
& \qquad\qquad \delta(\vec{r}_1-\vec{r}_2)\,\delta(\vec{r}_1-\vec{r}_3)+\;\delta(\vec{r}_1-\vec{r}_2)\,\delta(\vec{r}_1-\vec{r}_3) \\
& \qquad\quad \left\{\left[(\overrightarrow{\nabla}_1-\overrightarrow{\nabla}_2)^2+(\overrightarrow{\nabla}_2-\overrightarrow{\nabla}_3)^2+(\overrightarrow{\nabla}_3-\overleftarrow{\nabla}_1)^2\right]\right\}.
\end{aligned}
$$

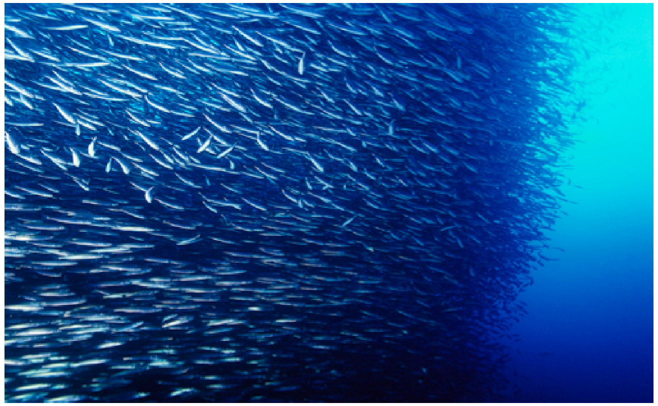
Other correlations

Dip region

Short range correlations and meson-exchange currents

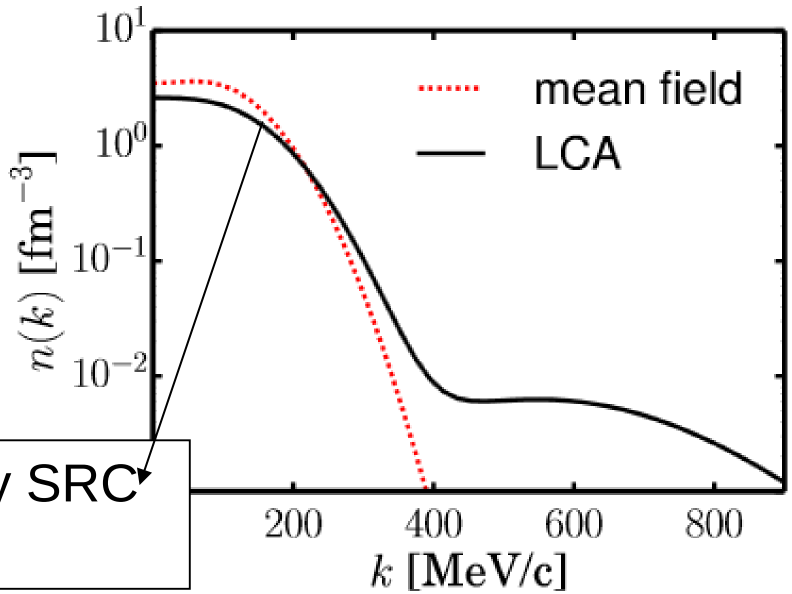


Short-range correlations



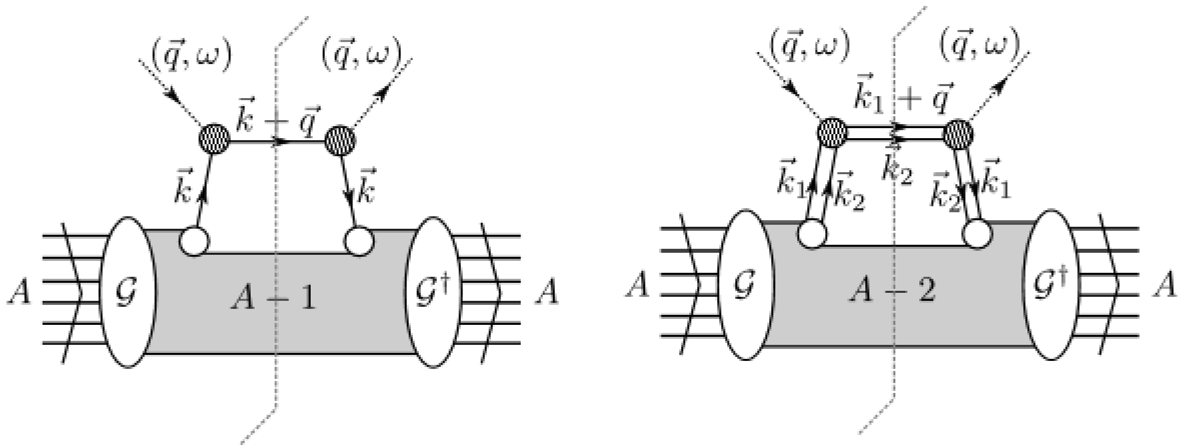
- The short-range repulsive character of this force, which correlates with the Pauli exclusion principle, results in a large mean free path of the nucleons with respect to the size of the nucleus
- In an independent particle model nucleons move independently from each other in a mean field
- This approach fails to capture short-range features of nucleon-nucleon correlations

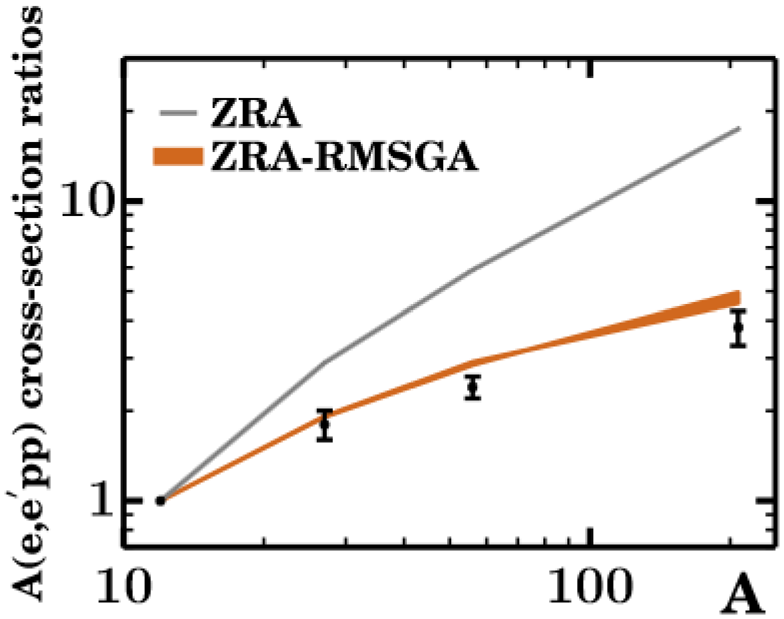
- SRC : short-range repulsive, tensor component of the nuclear force
- Individual nucleons receive large momenta compared to the Fermi momentum



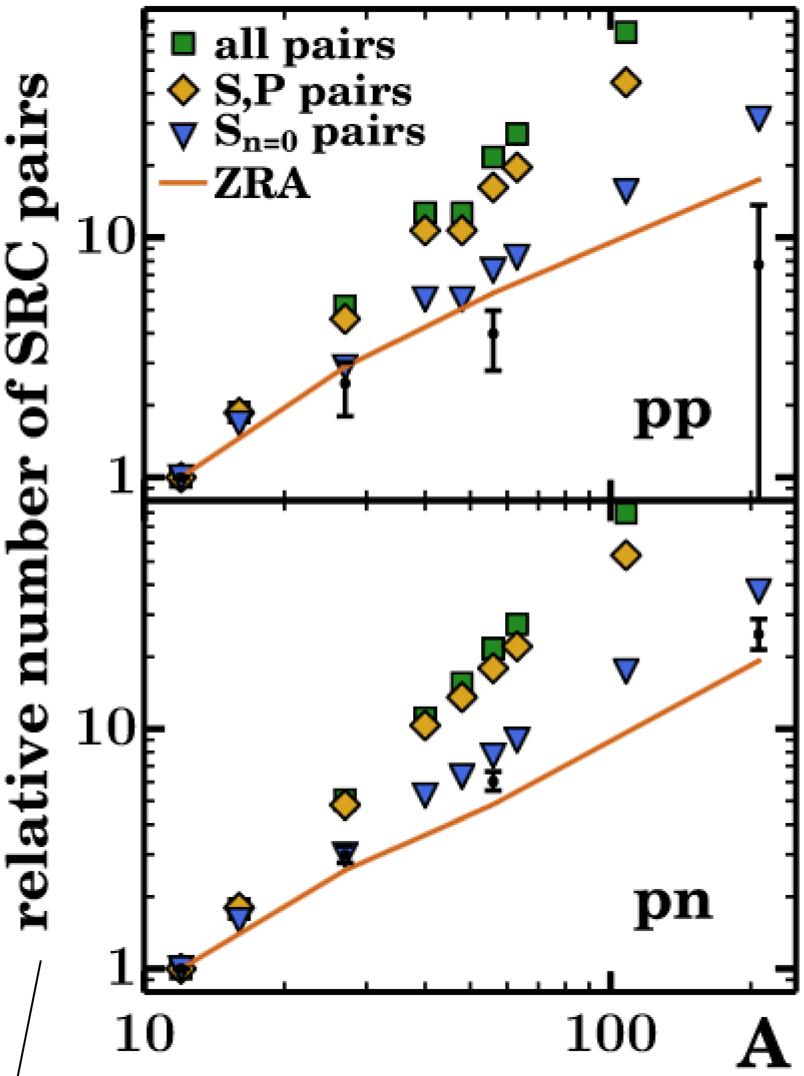
IPM single-particle orbitals are depleted by SRC and higher energy levels are populated

- Short-range correlations affect 1nucleon and 2-nucleon knockout processes
- The SRC-prone nucleon pairs are predominantly in a back-to-back configuration with a small center-of-mass and high relative momentum
- Final-state interactions for the outgoing nucleons affect the experimental observations





C. Colle, W. Cosyn, and J. Ryckebusch, Phys. Rev. C **93**, 034608 (2016)



To ^{12}C

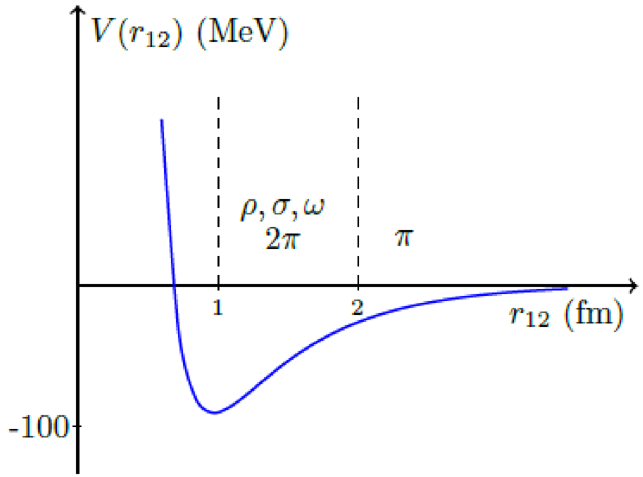
- mass dependence ~ $A^{1.12}$:soft !
- Predominantly pn, s-pairs
- Universal over mass tabel
- Tensor force dominates at short distances

C. Colle, W. Cosyn, and J. Ryckebusch, Phys. Rev. C **93**, 034608 (2016)
C. Colle, W. Cosyn, J. Ryckebusch, and M. Vanhalst, Phys. Rev. C **89**, 024603 (2014)
C. Colle, O. Hen, W. Cosyn, I. Korover, E. Piasetzky, J. Ryckebusch, and L. B. Weinstein, Phys. Rev. C **92**, 024604 (2015).

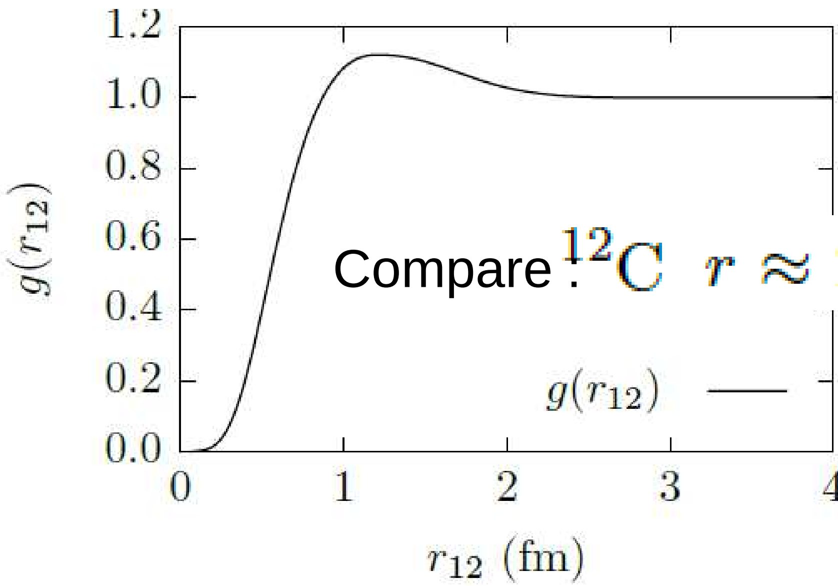
Data : CLAS A(e,e',pN) data

Two-body density :

$$\rho^{[2]}(\boldsymbol{r}_1,\boldsymbol{r}_2)=\underbrace{\rho^{[1]}(\boldsymbol{r}_1)\rho^{[1]}(\boldsymbol{r}_2)}_{\text{Independent particle model}}\,g(r_{12})$$

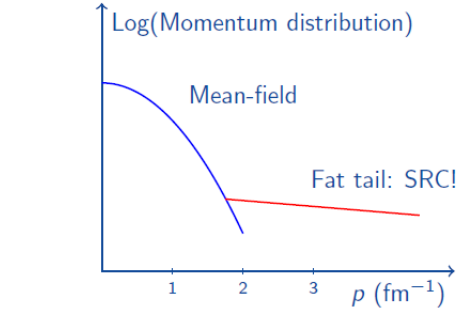


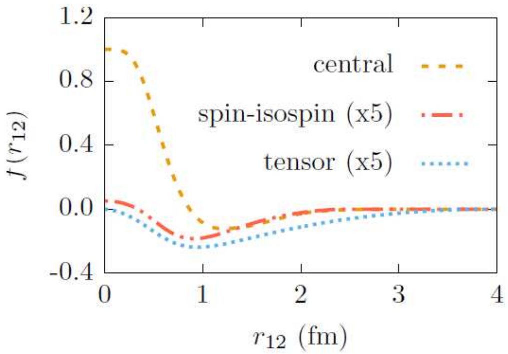
Correlation function

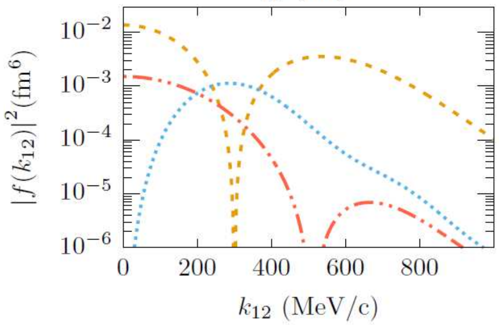


Compare : $^{12}\mathrm{C}$ $r \approx 2.86$ fm

Short-range correlations







Gearheart (1994) Pieper (1992)

$$|\Psi\rangle = \frac{1}{\sqrt{\mathcal{N}}}\widehat{\mathcal{G}}|\Phi\rangle \qquad \text{with} \qquad \widehat{\mathcal{G}} \approx \widehat{\mathcal{S}}\left(\prod_{i<j}^{A}\left[1+\widehat{l}(i,j)\right]\right)$$

$$\widehat{l}(i,j) = -\,g_c(r_{ij}) + f_{\sigma\tau}(r_{ij})\,(\vec{\sigma}_i\cdot\vec{\sigma}_j)\,(\vec{\tau}_i\cdot\vec{\tau}_j) + f_{t\tau}(r_{ij})\widehat{S}_{ij}\,(\vec{\tau}_i\cdot\vec{\tau}_j)\,,$$

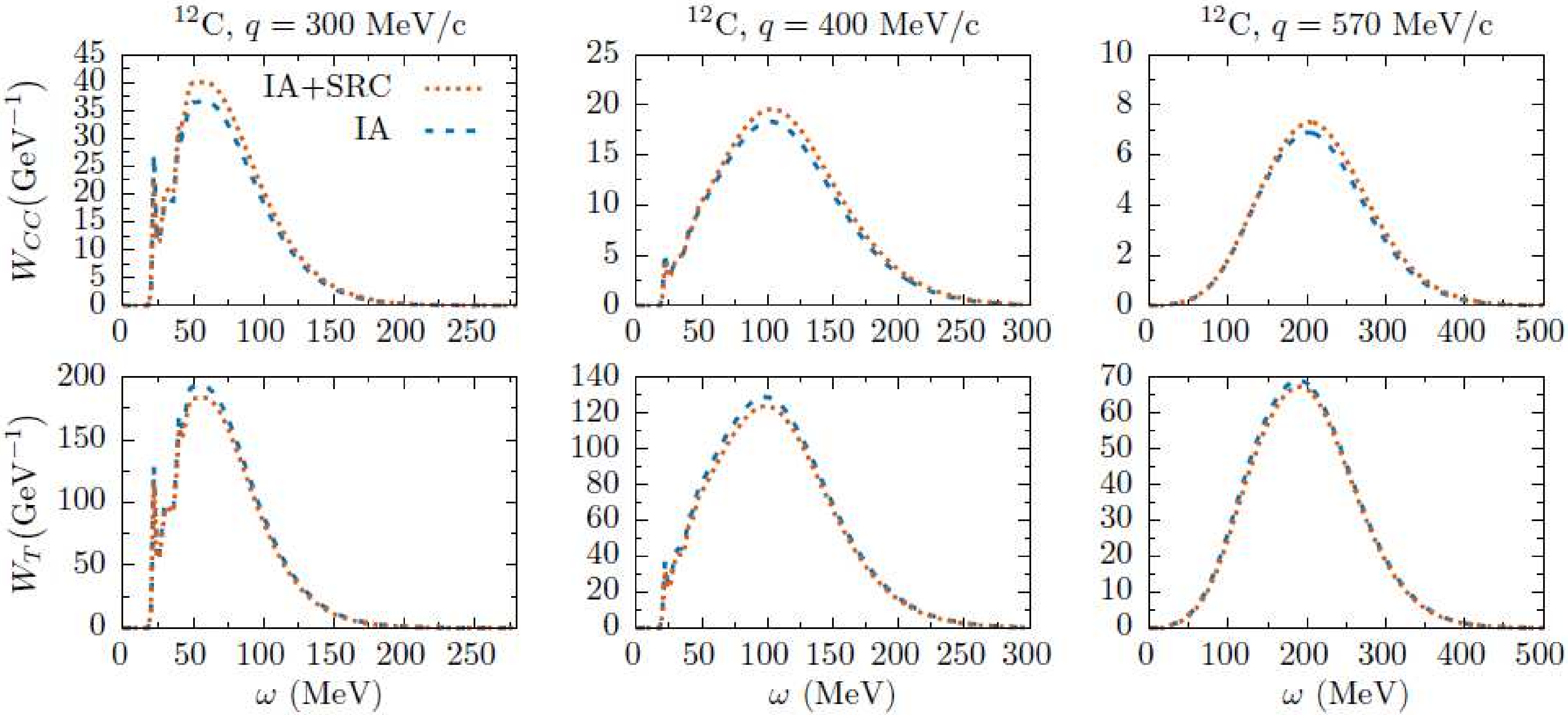
Shifting the complexity induced by correlations from the wave functions to the operators

$$\langle\Psi_{\mathrm{f}}|\widehat{J}_{\mu}^{\mathrm{nucl}}|\Psi_{\mathrm{i}}\rangle = \frac{1}{\sqrt{\mathcal{N}_{\mathrm{i}}\mathcal{N}_{\mathrm{f}}}}\langle\Phi_{\mathrm{f}}|\widehat{J}_{\mu}^{\mathrm{eff}}|\Phi_{\mathrm{i}}\rangle$$

$$\widehat{J}_{\mu}^{\mathrm{eff}} \approx \sum_{i=1}^{A}\widehat{J}_{\mu}^{[1]}(i) + \sum_{i<j}^{A}\widehat{J}_{\mu}^{[1],\mathrm{in}}(i,j) + \left[\sum_{i<j}^{A}\widehat{J}_{\mu}^{[1],\mathrm{in}}(i,j)\right]^{\dagger}$$

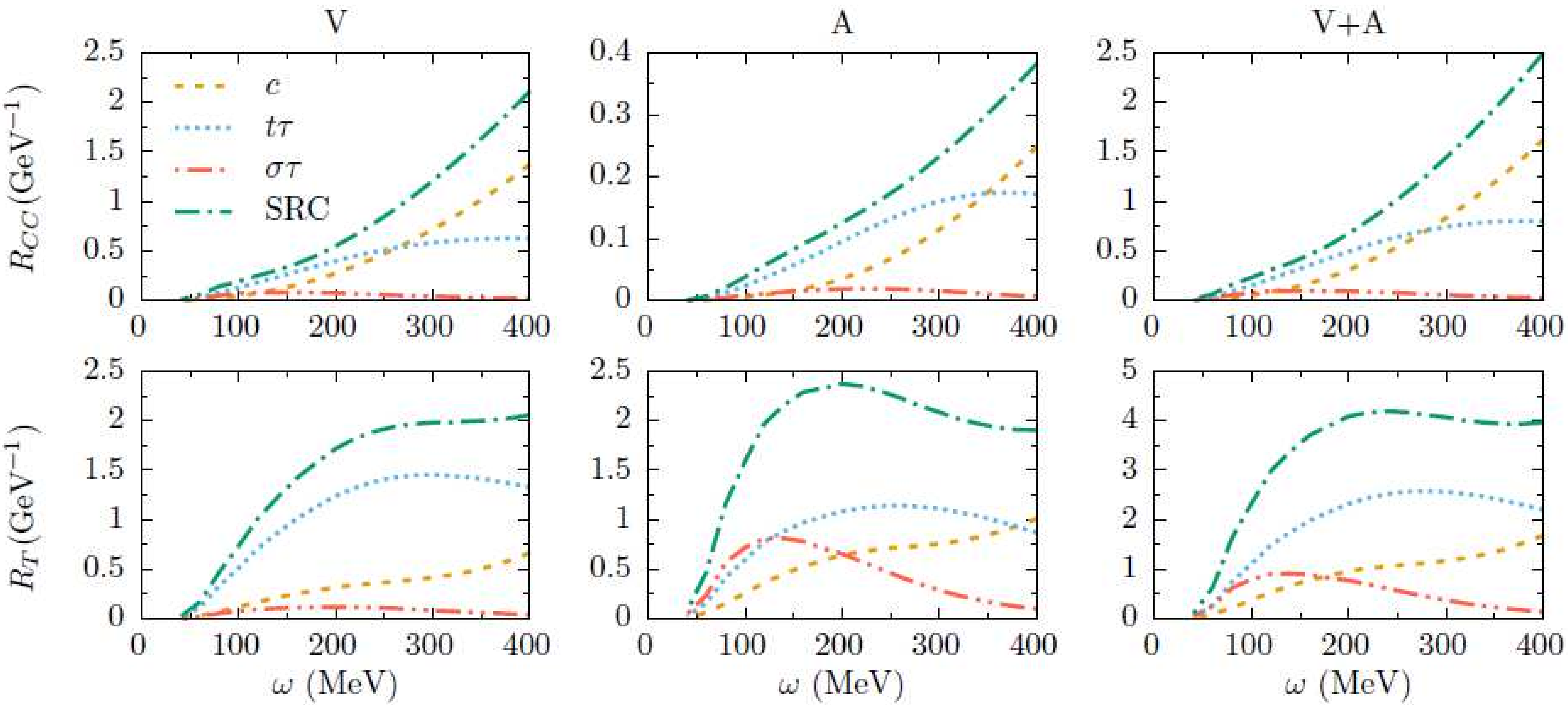
$$\widehat{J}_{\mu}^{[1],\mathrm{in}}(i,j) = \left[\widehat{J}_{\mu}^{[1]}(i) + \widehat{J}_{\mu}^{[1]}(j)\right]\widehat{l}(i,j)$$

SRC neutrinos 1p1h



- Reduction of transverse response
- Enhancement of Coulomb-longitudinal

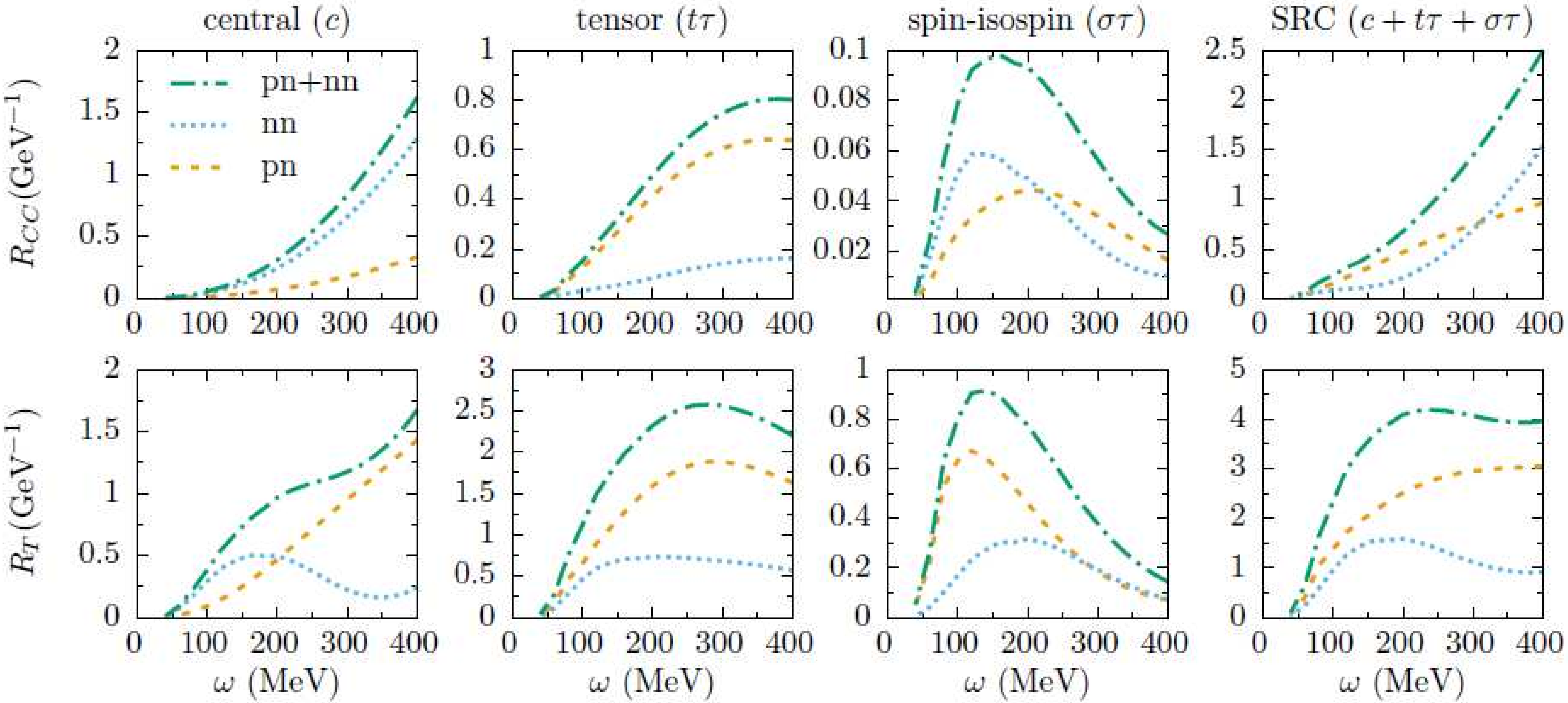
$q = 400$ MeV/c



- Vector and axial contributions have comparable strength
- Tensor often dominates, but not for all kinematics

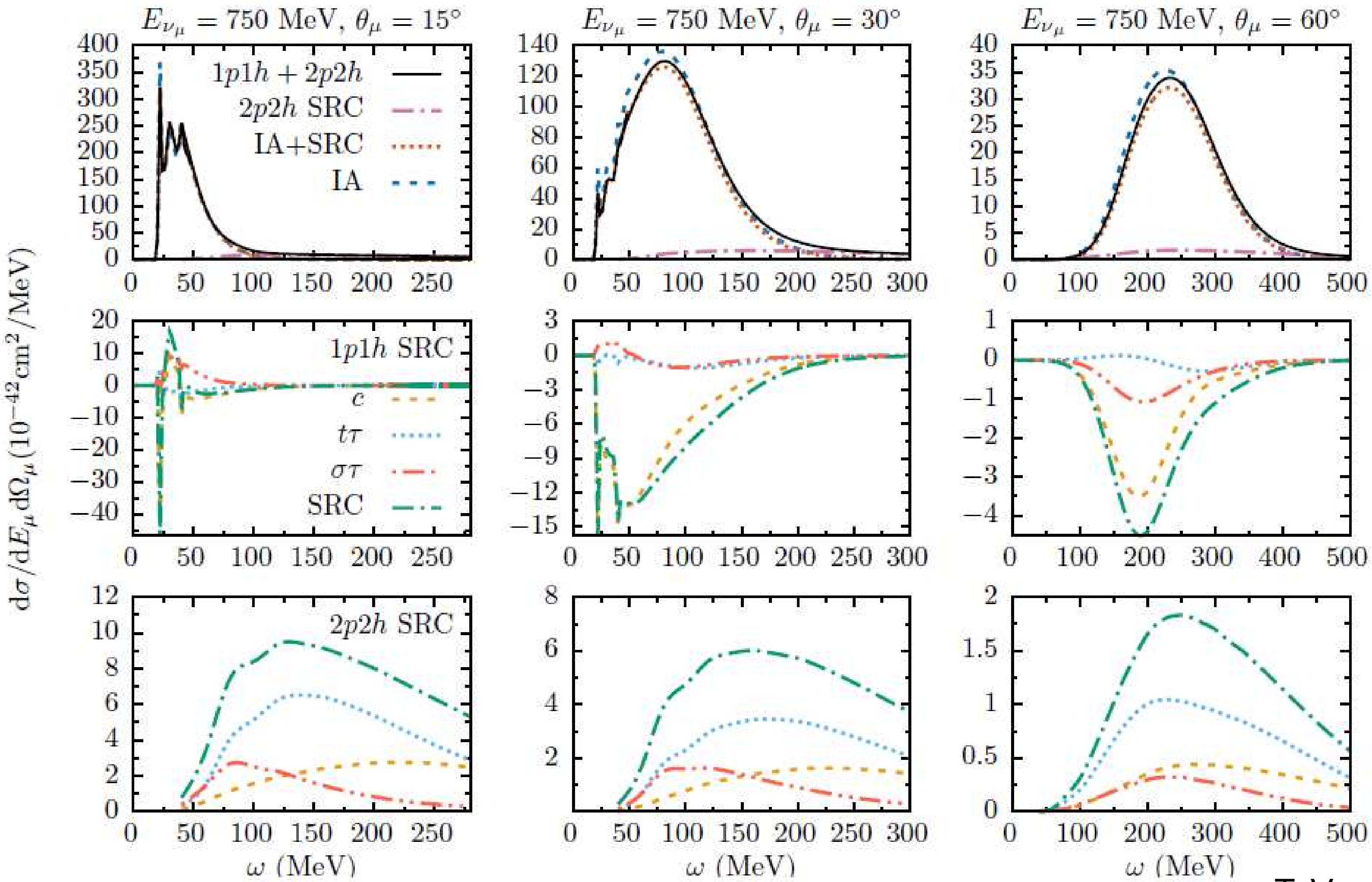
T. Van Cuyck, N. Jachowicz, R. González–Jiménez, *et al.* Phys. Rev. **C94**, 024611 (2016).

$q = 400$ MeV/c



- Vector and axial contributions have comparable strength
- Tensor often dominates, but not for all kinematics
- pn pairs dominate

SRC neutrinos 1p1h+2p2h



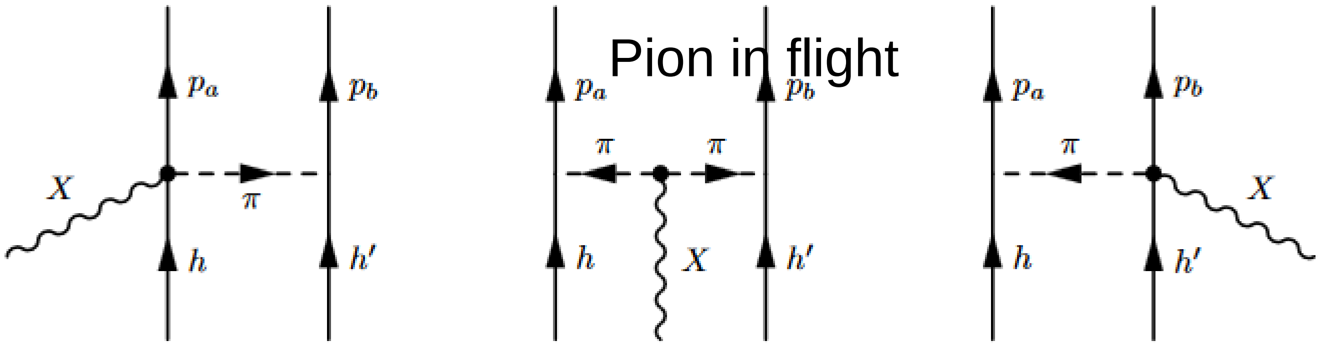
T. Van Cuyck et al
Phys. Rev. C 94, 024611 (2016)

Meson-exchange currents

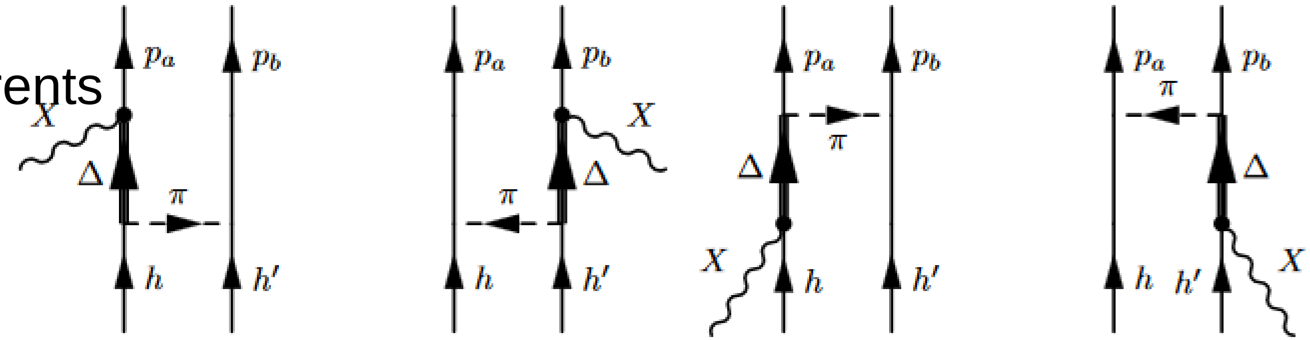
When an electroweak boson interacts with a pair of nucleons which are correlated through the exchange of a meson, this will cause the knockout of one or both of the particles from the nucleus. The boson was interacting with a current consisting of two nucleons, a two-body current, called a MEC

Seagull

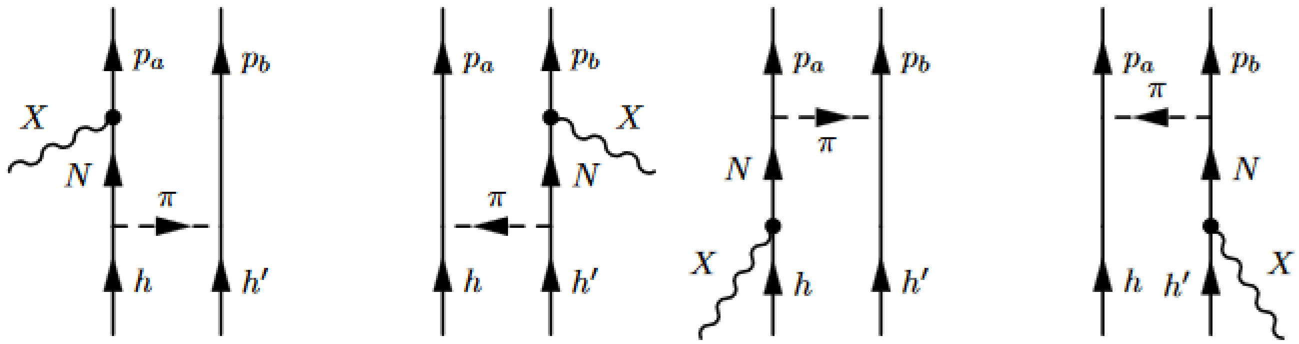
Pion in flight



Delta currents



Correlation currents

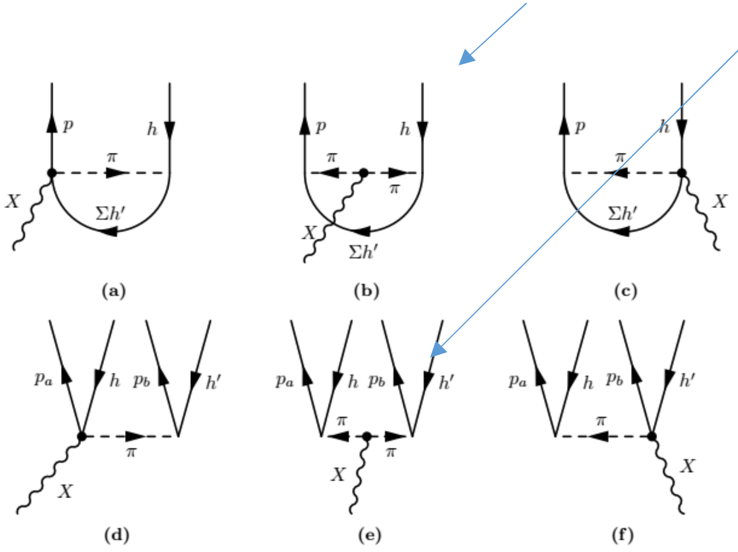


Already included in mean field models !

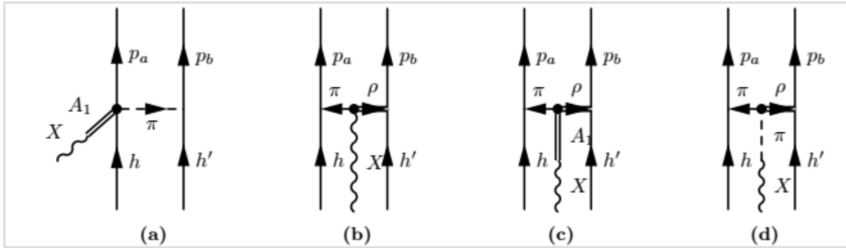
Contributions of heavier mesons :

$m_\pi \approx 135\text{MeV}$, $m_\rho \approx 775\text{MeV}$, $m_\omega \approx 782\text{MeV}$

MEC in 1p1h and 2p2h



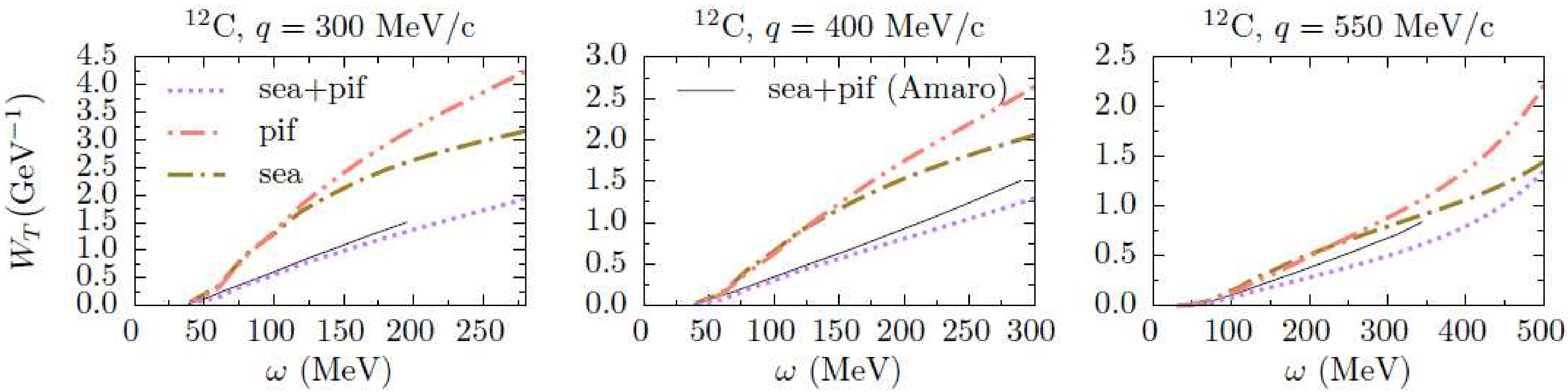
Axial contributions :



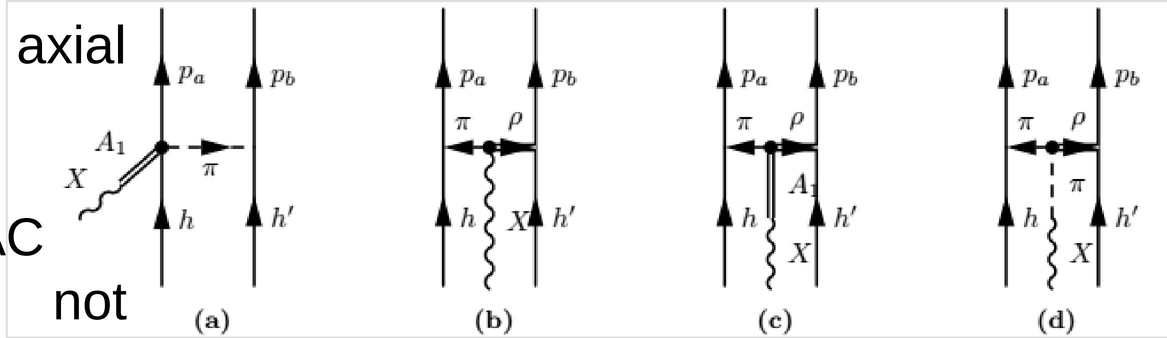
$$\widehat{\rho}_A^{[2],\mathrm{axi}}(\boldsymbol{q}) = \frac{i}{g_A}\left(\frac{f_{\pi NN}}{m_\pi}\right)^2 (\boldsymbol{I}_V)\left(F_\pi(\boldsymbol{q}_2^2)\,\Gamma_\pi^2(\boldsymbol{q}_2^2)\frac{\boldsymbol{\sigma}_2\cdot\boldsymbol{q}_2}{\boldsymbol{q}_2^2+m_\pi^2} - F_\pi(\boldsymbol{q}_1^2)\,\Gamma_\pi^2(\boldsymbol{q}_1^2)\frac{\boldsymbol{\sigma}_1\cdot\boldsymbol{q}_1}{\boldsymbol{q}_1^2+m_\pi^2}\right)$$

I. Towner, Nucl. Phys.A542, 631 (1992)

^{12}C(e, e')

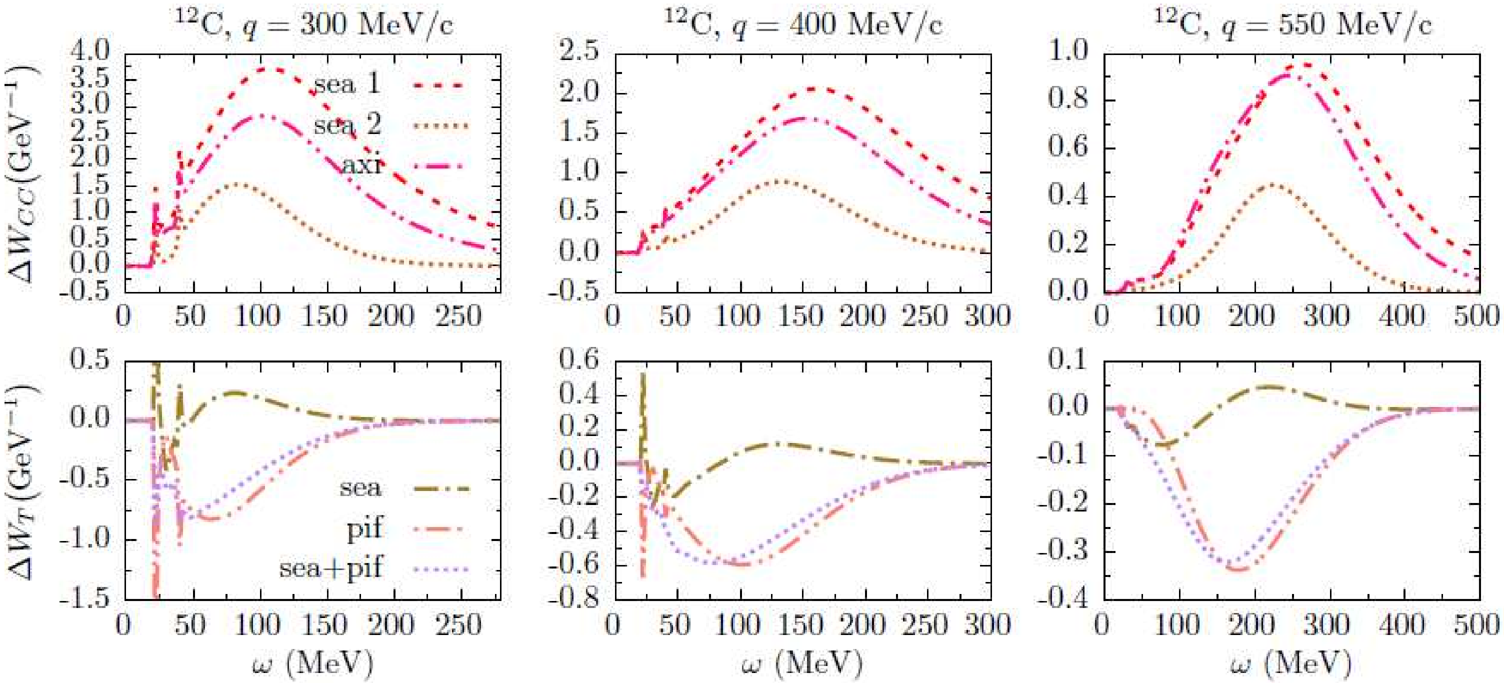


- Only seagull have axial counterpart
- timelike
- Partially constrained by PCAC
- Non-relativistic reduction not unambiguous

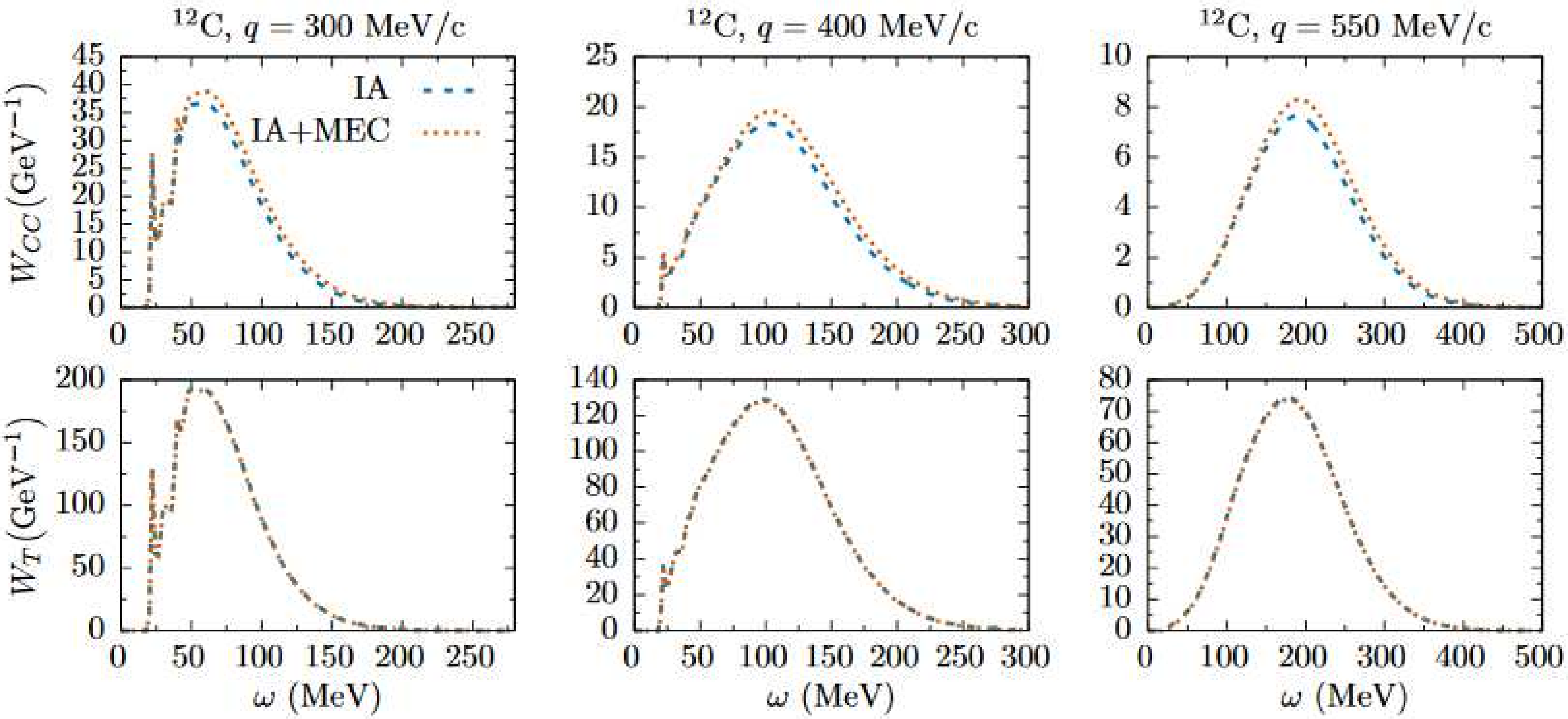


I. Towner, Nucl. Phys.A542, 631 (1992)

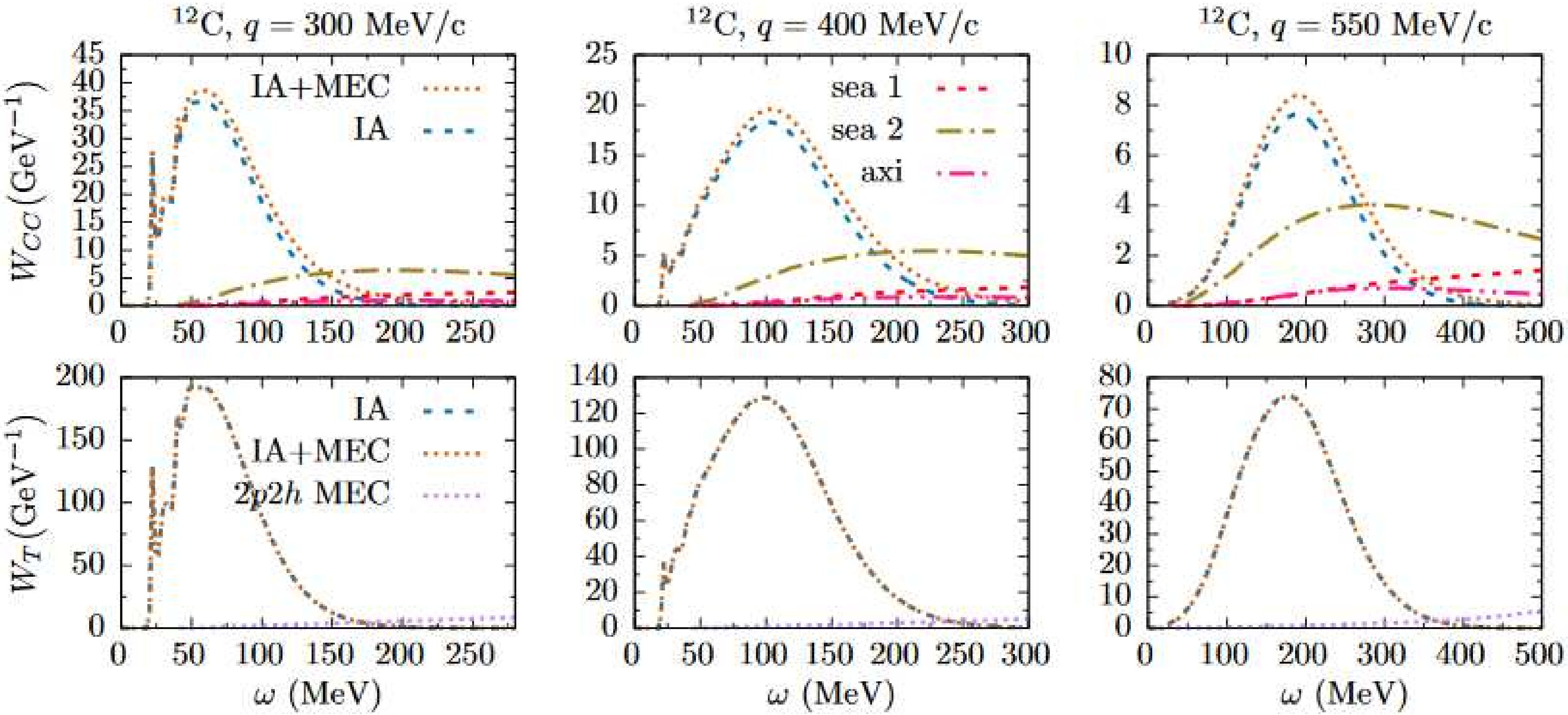
$$\hat{\rho}_A^{[2],\mathrm{axi}}(\boldsymbol{q}) = \frac{i}{g_A}\left(\frac{f_{\pi NN}}{m_\pi}\right)^2 (\boldsymbol{I}_V)\left(F_\pi(\boldsymbol{q}_2^2)\,\Gamma_\pi^2(\boldsymbol{q}_2^2)\frac{\boldsymbol{\sigma}_2\cdot\boldsymbol{q}_2}{\boldsymbol{q}_2^2+m_\pi^2} - F_\pi(\boldsymbol{q}_1^2)\,\Gamma_\pi^2(\boldsymbol{q}_1^2)\frac{\boldsymbol{\sigma}_1\cdot\boldsymbol{q}_1}{\boldsymbol{q}_1^2+m_\pi^2}\right)$$



Seagull and pif in1p1h



Seagull and pif in 2p2h



T. Van Cuyck, N. Jachowicz, R. González–Jiménez, *et al.* PRC95, 054611 (2017)

2-nucleon knockout :

$$\frac{\mathrm{d}\sigma^X}{\mathrm{d}E_f \mathrm{d}\Omega_f \mathrm{d}T_b \mathrm{d}\Omega_b \mathrm{d}\Omega_a} = \frac{p_a p_b E_a E_b}{(2\pi)^6} g_{rec}^{-1} \sigma^X \zeta$$
$$\times \big[v_{CC} W_{CC} + v_{CL} W_{CL} + v_{LL} W_{LL} + v_T W_T + v_{TT} W_{TT} + v_{TC} W_{TC}$$
$$+ v_{TL} W_{TL} + h(v_{T'} W_{T'} + v_{TC'} W_{TC'} + v_{TL'} W_{TL'}) \big], \quad (4.$$

with : $$\mathcal{J}_\lambda = \langle\, \Phi_f^{(A-2)}(E^{exc}, J_R M_R);\, \boldsymbol{p}_a m_{s_a};\, \boldsymbol{p}_b m_{s_b} \,|\, \widehat{J}_\lambda(\boldsymbol{q}) \,|\, \Phi_{gs} \,\rangle$$

$$|\, \Phi^{2p2h} \,\rangle = |\, \Phi_f^{(A-2)}(E^{exc}, J_R M_R);\, \boldsymbol{p}_a m_{s_a};\, \boldsymbol{p}_b m_{s_b} \,\rangle_{as}$$

Some issues :

- 7-dimensional integrals
- Need to fold over incoming neutrino flux !
- Huge numbers of diagrams
- Ambiguities in non-relativistic reduction of contributions
- Little constraints for axial parameters

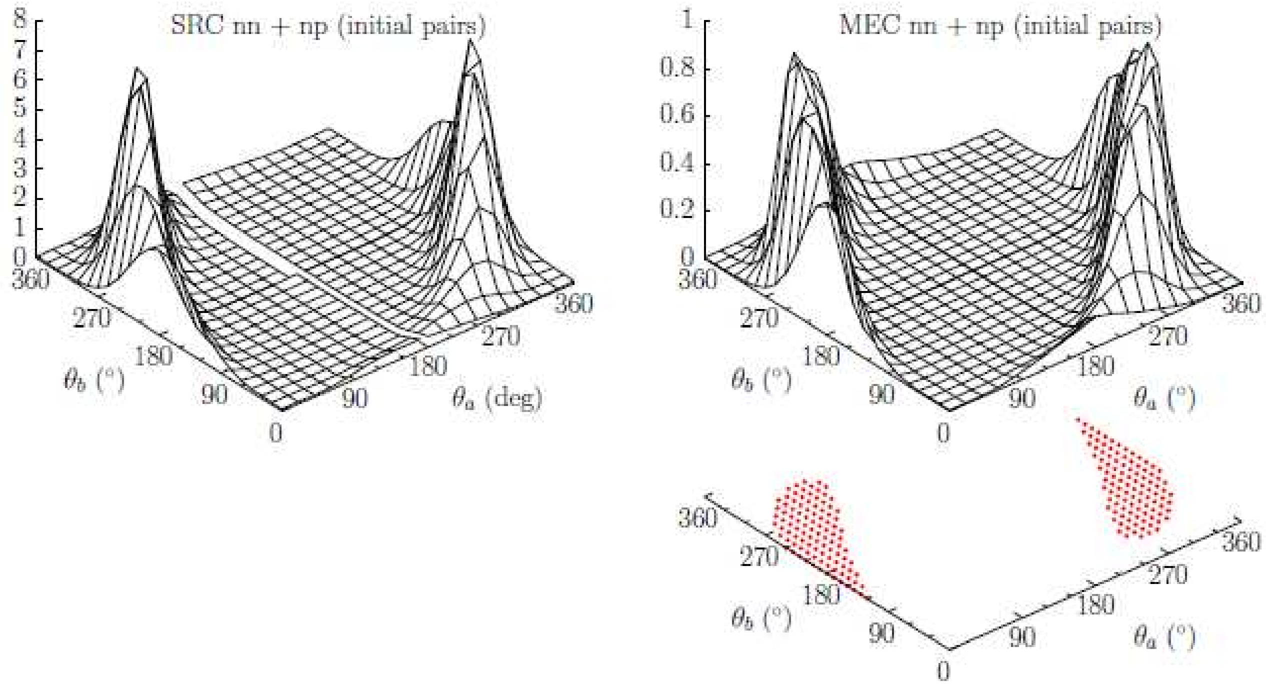


Figure 4.5: The $^{12}C(\nu_\mu, \mu^- N_a N_b)$ cross section (N_a = p, N_b = p′, n) at $\epsilon_{\nu_\mu} = 750$ MeV, $\epsilon_\mu = 550$ MeV, $\theta_\mu = 15°$ and $T_p = 50$ MeV for in-plane kinematics. Left with SRCs, right with MECs, the bottom plot shows the (θ_a, θ_b) regions with $P_{12} < 300$ MeV/c.

- Strength residing in restricted part of phase space
- $\boldsymbol{p}_b \approx \boldsymbol{p}_b^{ave}$
- Quasi-deuteron kinematics

Some issues :

- 7-dimensional integrals
- Need to fold over incoming neutrino flux !
- Huge numbers of diagrams
- divergences
- Ambiguities in non-relativistic reduction of contributions
- Little constraints for axial parameters

Approximation schemes :

- Reduce number or range of integrations :
  - Frozen approximation (Ruiz-Simo, Amaro et.a l ArXiv1703.1018)
  - Modified convolution approximation (Ruiz-Simo, Amaro et.al, ArXiv1706.06377)
- Choice of subset of diagrams and terms

What about correlations on the neutrino market ?

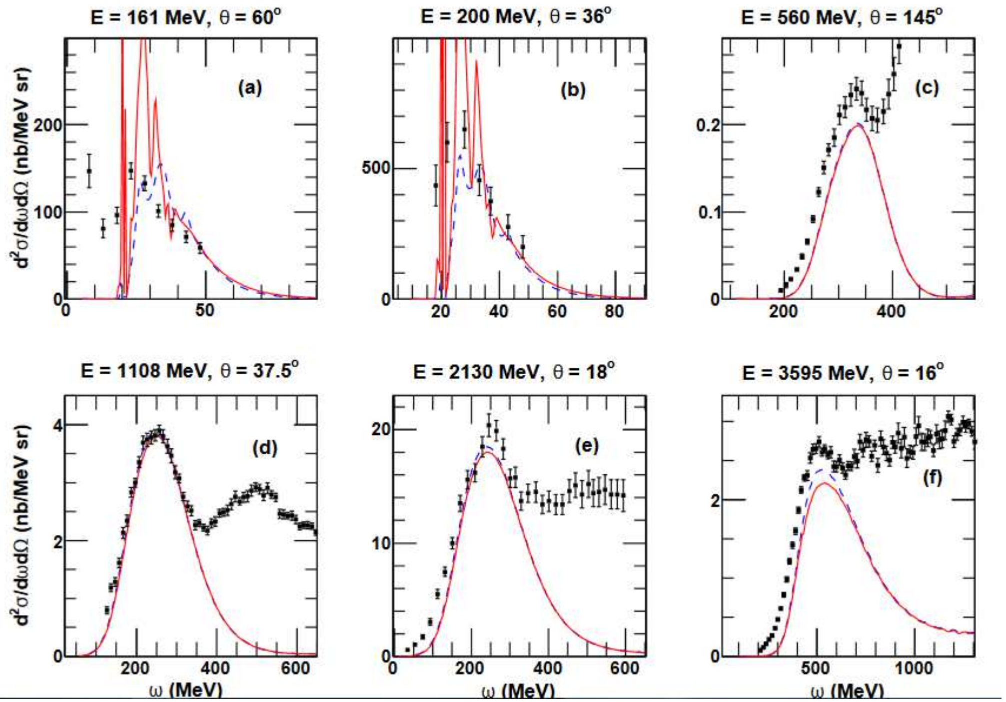
- Neutrino-Nucleus Cross Sections for Oscillation
Experiments
Teppei Katori and Marco Martini
arXiv:1611.07770
- NuSTEC White Paper: Status and Challenges of Neutrino-Nucleus Scattering
arXiv:1706.03621

,

What about correlations on the neutrino market ?

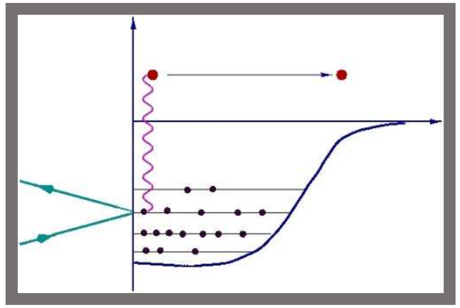
Ghent model

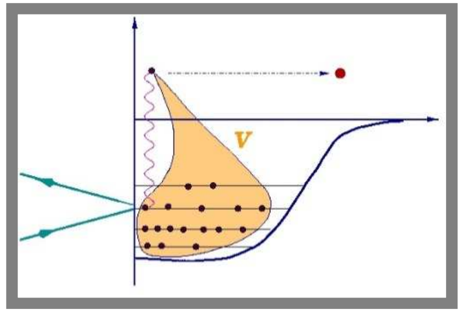
- Hartree-Fock mean field
- Continuum RPA
- Residual interaction : Skyrme

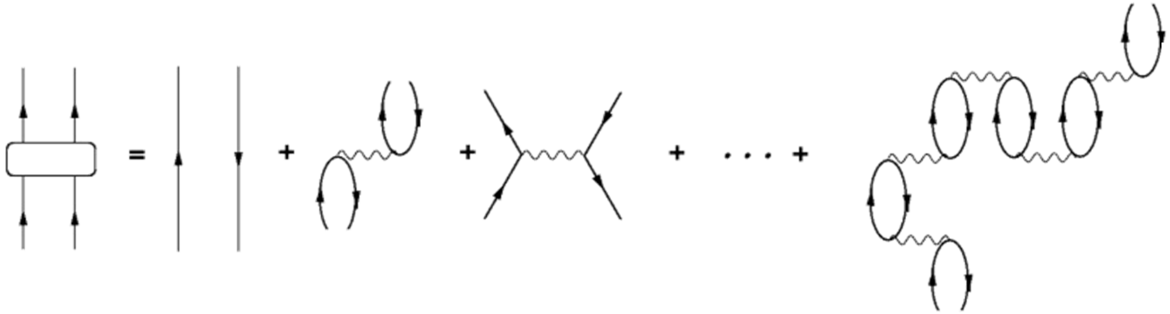


CRPA

- Green's function approach
- Skyrme SkE2 residual interaction
- self-consistent calculations







+ . . .

$$|\Psi_{RPA}\rangle = \sum_{c} \left\{ X_{(\Psi,C)} \left|ph^{-1}\right\rangle - Y_{(\Psi,C)} \left|hp^{-1}\right\rangle \right\}$$

$$\Pi^{(RPA)}(x_1, x_2; \omega) = \Pi^{(0)}(x_1, x_2; \omega) + \frac{1}{\hbar} \int dx \int dx' \; \Pi^{(0)}(x_1, x; \omega) \, \tilde{V}(x, x') \, \Pi^{(RPA)}(x', x_2; \omega)$$

Solving the RPA equations in coordinate space

$$
\begin{aligned}
|\Psi_C(E)\rangle \; = \; & \left|ph^{-1}(E)\right\rangle + \int dx_1 \int dx_2 \;\; \widetilde{V}(x_1, x_2) \\
& \sum_{c'} \mathcal{P} \int d\varepsilon_{p'} \left[\frac{\psi_{h'}(x_1)\psi^{\dagger}_{p'}(x_1, \varepsilon_{p'})}{E - \varepsilon_{p'h'}} \left|p'h'^{-1}(\varepsilon_{p'h'})\right\rangle \right. \\
& \left. - \; \frac{\psi^{\dagger}_{h'}(x_1)\psi_{p'}(x_1, \varepsilon_{p'})}{E + \varepsilon_{p'h'}} \left|h'p'^{-1}(-\varepsilon_{p'h'})\right\rangle \right] \; \left\langle \Psi_0 \left| \hat{\psi}^{\dagger}(x_2)\hat{\psi}(x_2) \right| \Psi_C(E) \right\rangle
\end{aligned}
$$

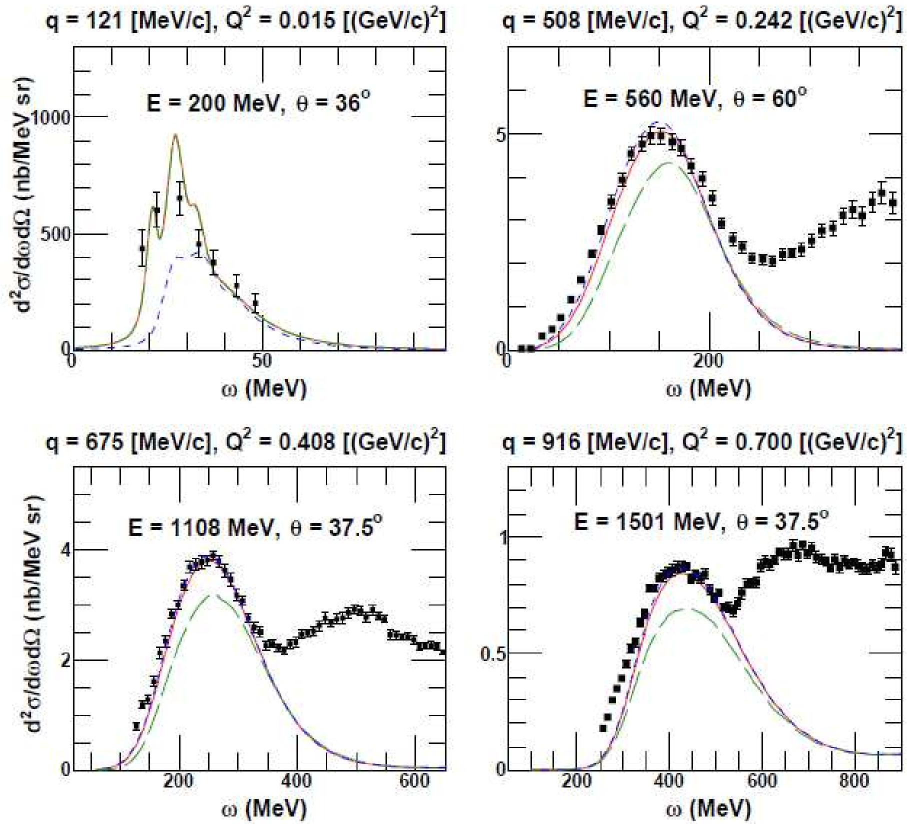
What we really need is transition densities :

$$
\begin{aligned}
\langle \Psi_0 || X_{\eta J} || \Psi_C(J;E) \rangle_r \quad = \quad & - \; \langle h || X_{\eta J} || p(\varepsilon_{ph}) \rangle_r \\
+ \quad & \sum_{\mu,\nu} \int dr_1 \int dr_2 \;\; U^J_{\mu\nu}(r_1, r_2) \;\; \mathcal{R} \left(R^{(0)}_{\eta\mu;J}(r, r_1; E) \right) \;\; \langle \Psi_0 || X_{\nu J} || \Psi_C(J;E) \rangle_{r_2}
\end{aligned}
$$

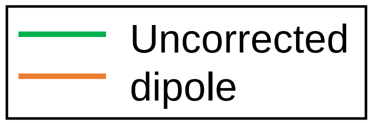
So in the end we have to solve a set of coupled equations, that after discretizing on a mesh in coordinate space, translates into a matrix inversion :

$$
\rho_C^{RPA} \;\; = \; - \, \frac{1}{1 - R\,U} \;\; \rho_C^{HF}
$$

•Regularization of the residual interaction :



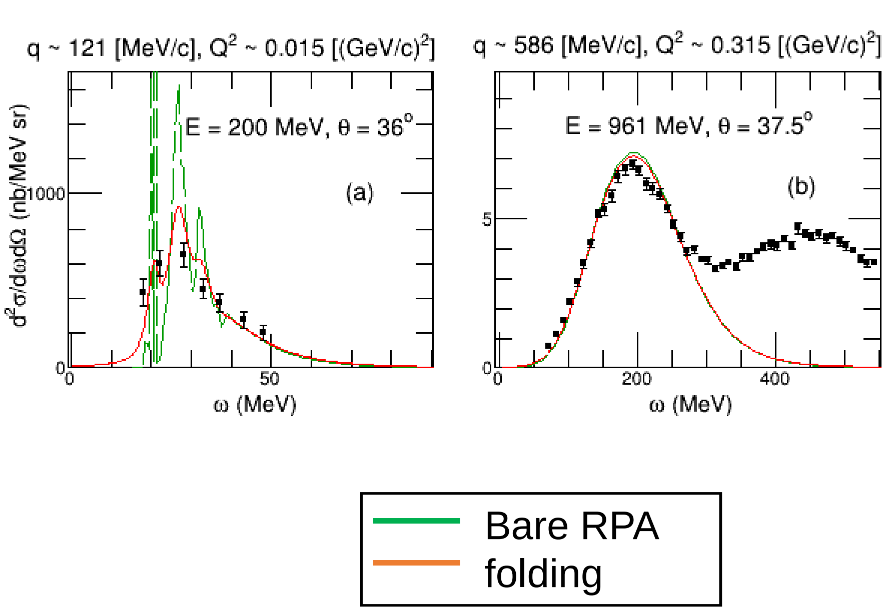
$$V(Q^2) \rightarrow V(Q^2 = 0)\ \frac{1}{(1+\frac{Q^2}{\Lambda^2})^2}$$



- Final state interactions :

-taken into account through the calculations of the wave function of the outgoing nucleon in the (real) nuclear potential generated using the Skyrme force

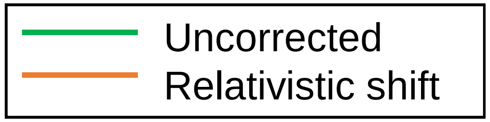
-influence of the spreading width of the particle states is implemented through a folding procedure

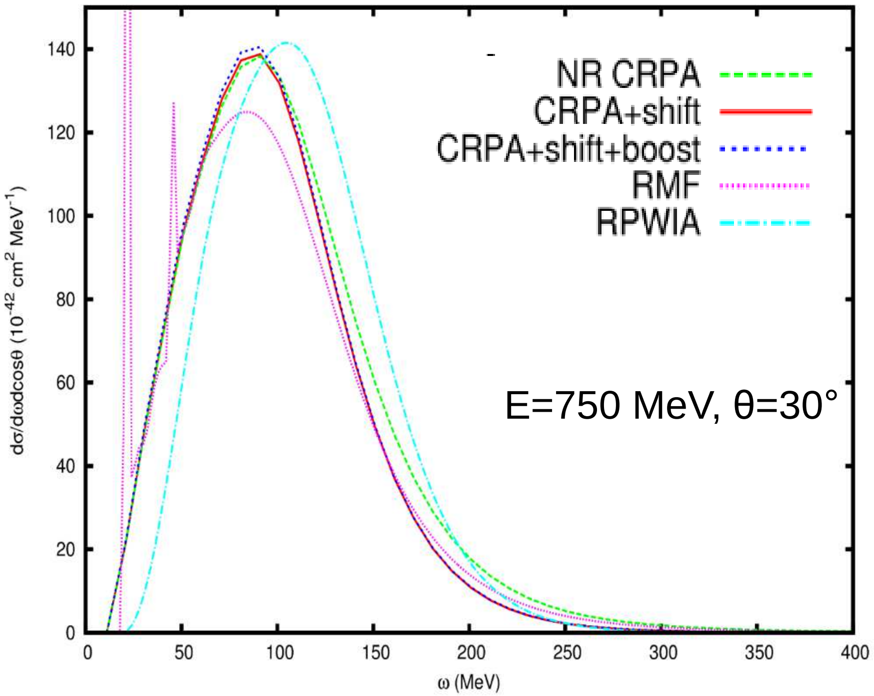


$$R'(q,\omega') = \int_{-\infty}^{\infty} d\omega \; R(q,\omega) \; L(\omega,\omega'),$$

$$L(\omega,\omega') = \frac{1}{2\pi}\left[\frac{\Gamma}{(\omega-\omega')^2 + (\Gamma/2)^2}\right].$$

•Relativistic corrections at higher energies (S. Jeschonnek and T. Donnelly, PRC57, 2438 (1998)):





Shift :

$$\lambda \to \lambda(\lambda + 1) \qquad \lambda = \omega/2M_N$$

- The outgoing nucleon obtains the correct relativistic momentum

$$p = \sqrt{T^2 + 2MT}$$

- Shifts the QE peak to the correct relativistic position

Boost :

$$R^{\mathrm{V}}_{\mathrm{CC}}(q,\omega) \;\to\; \frac{q^2}{q^2-\omega^2}\, R^{\mathrm{V}}_{\mathrm{CC}}(q,\omega)\,,$$

$$R^{\mathrm{A}}_{\mathrm{LL}}(q,\omega) \;\to\; \left(1+\frac{q^2-\omega^2}{4m^2}\right) R^{\mathrm{A}}_{\mathrm{LL}}(q,\omega)\,,$$

$$R^{\mathrm{V}}_{\mathrm{T}}(q,\omega) \;\to\; \frac{q^2-\omega^2}{q^2}\, R^{\mathrm{V}}_{\mathrm{T}}(q,\omega)\,,$$

$$R^{\mathrm{A}}_{\mathrm{T}}(q,\omega) \;\to\; \left(1+\frac{q^2-\omega^2}{4m^2}\right) R^{\mathrm{A}}_{\mathrm{T}}(q,\omega)\,,$$

$$R^{\mathrm{VA}}_{\mathrm{T'}}(q,\omega) \;\to\; \sqrt{\frac{q^2-\omega^2}{q^2}}\,\sqrt{1+\frac{q^2-\omega^2}{4m^2}}\, R^{\mathrm{VA}}_{\mathrm{T'}}(q,\omega)\,.$$

•Coulomb correction for the outgoing lepton in charged-current interactions :

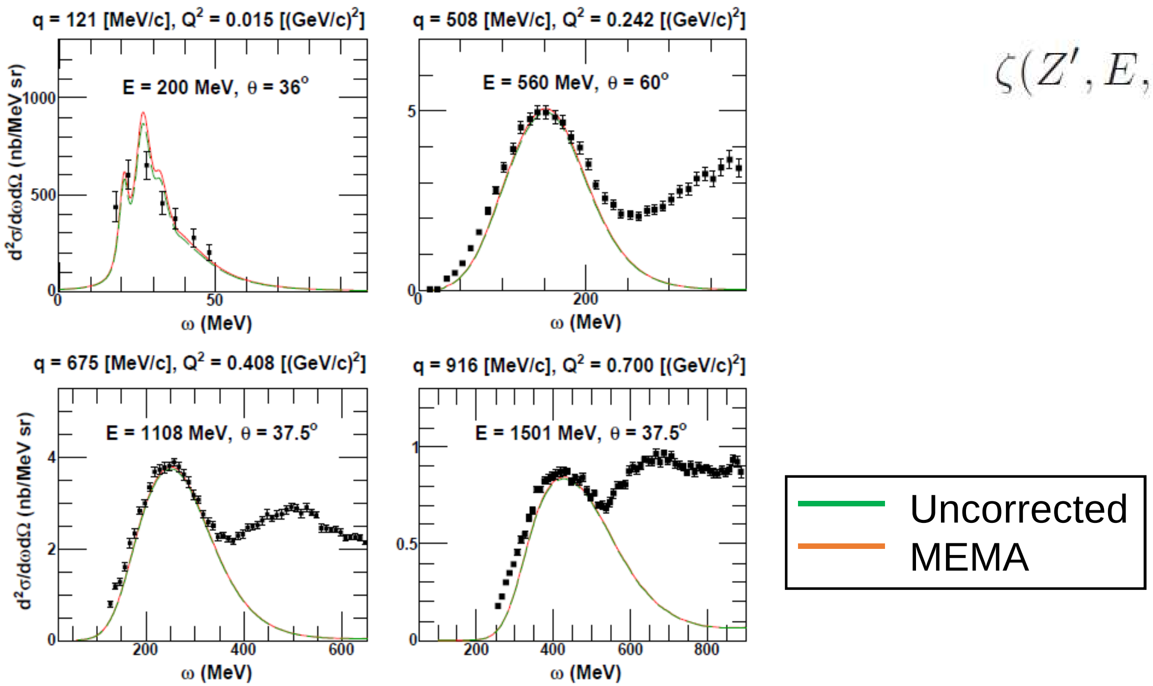
- ✓ Low energies : Fermi function

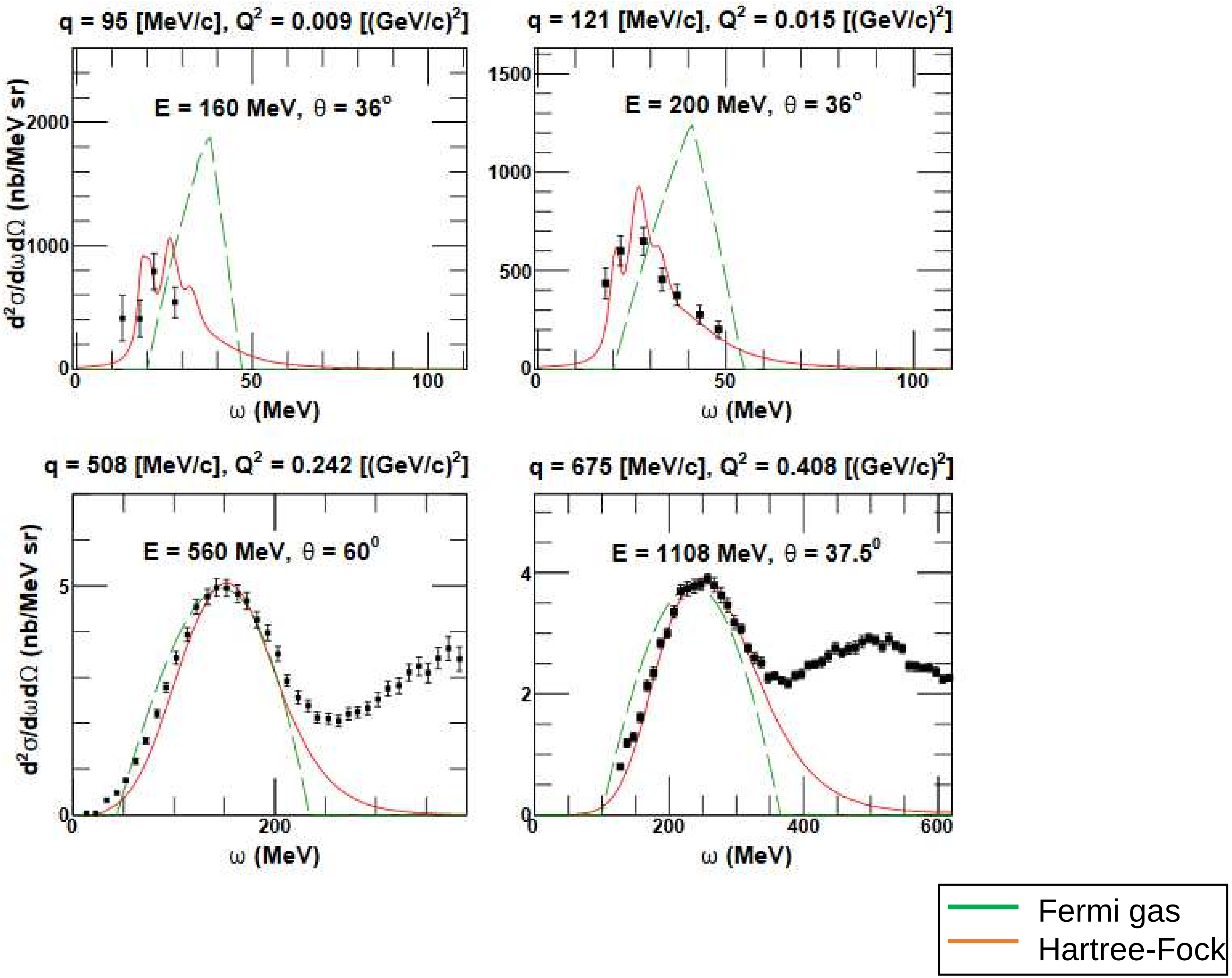
$$F(Z',E) = \frac{2\pi\eta}{1-e^{-2\pi\eta}} \qquad \eta \sim \mp Z'\alpha$$

- ✓ High energies : modified effective momentum approximation (J. Engel, PRC57,2004 (1998))

$$q_{eff} = q + 1.5\left(\frac{Z'\alpha\hbar c}{R}\right), \qquad \Psi_l^{eff} = \zeta(Z',E,q)\,\Psi_l\,,$$

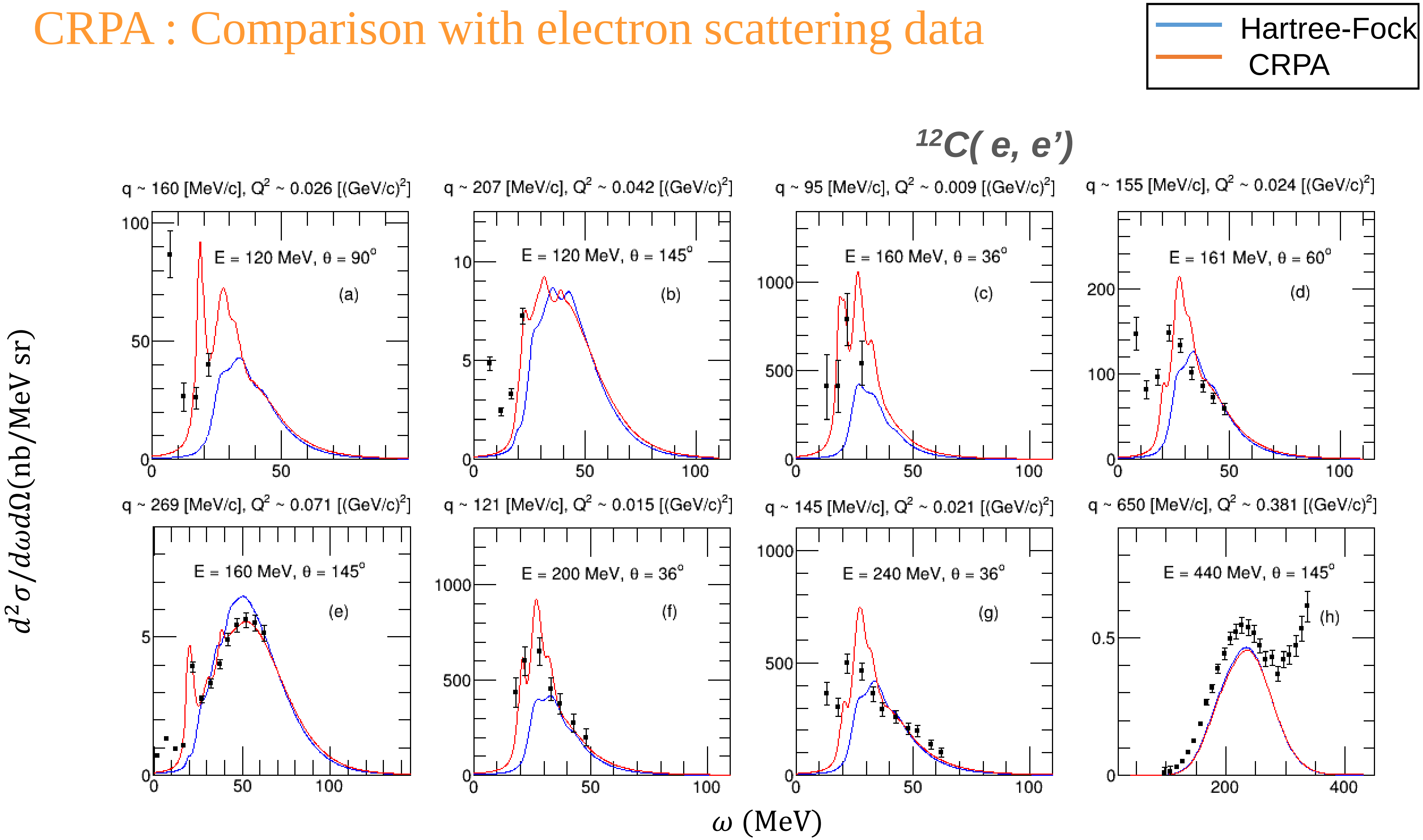
$$\zeta(Z',E,q) = \sqrt{\frac{q_{eff}E_{eff}}{qE}}$$



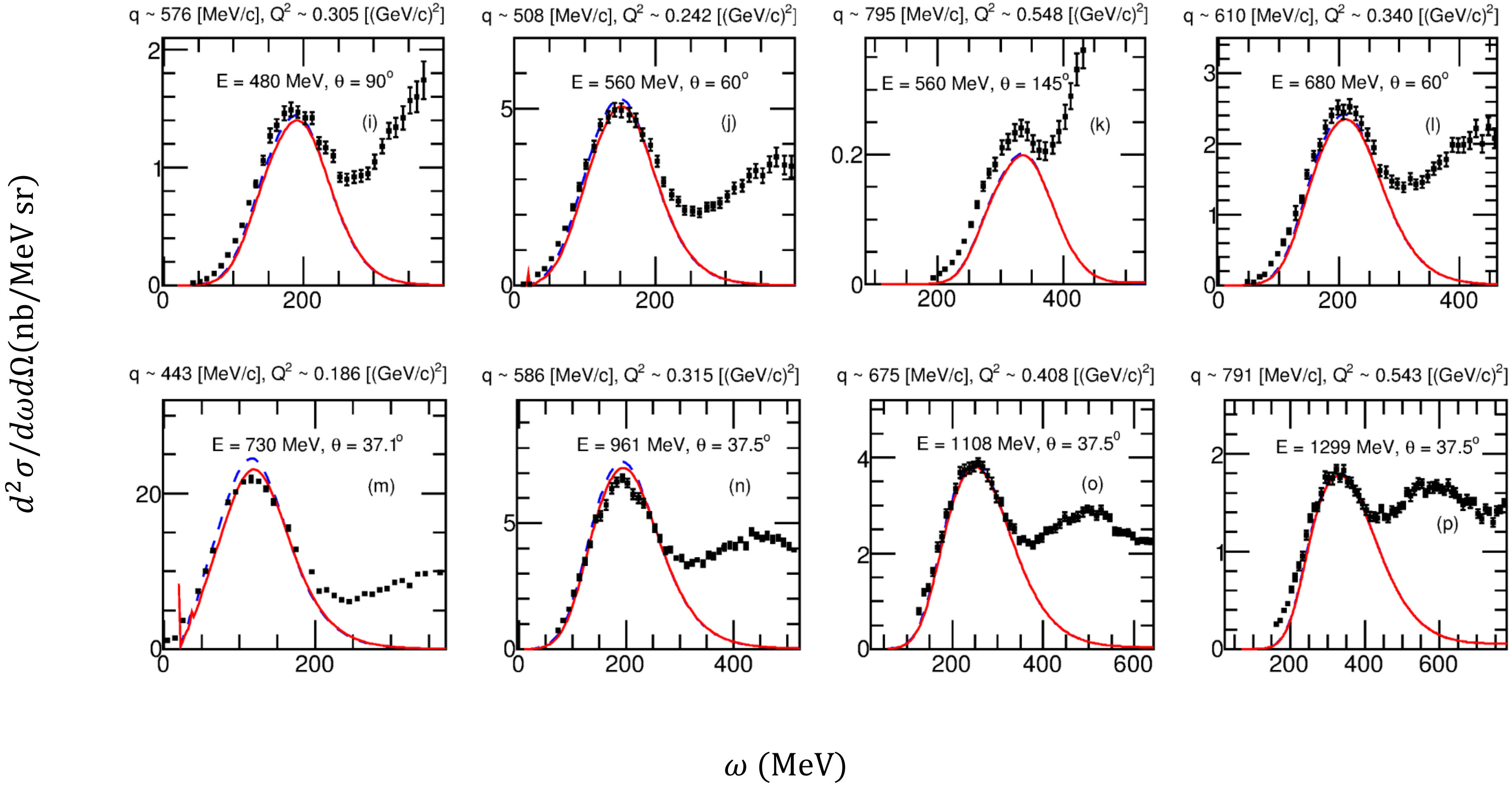


q = 95 [MeV/c], Q² = 0.009 [(GeV/c)²]
E = 160 MeV, θ = 36°
q = 121 [MeV/c], Q² = 0.015 [(GeV/c)²]
E = 200 MeV, θ = 36°
q = 508 [MeV/c], Q² = 0.242 [(GeV/c)²]
E = 560 MeV, θ = 60⁰
q = 675 [MeV/c], Q² = 0.408 [(GeV/c)²]
E = 1108 MeV, θ = 37.5⁰
d²σ/dωdΩ (nb/MeV sr)
ω (MeV)
Fermi gas
Hartree-Fock

CRPA : Comparison with electron scattering data



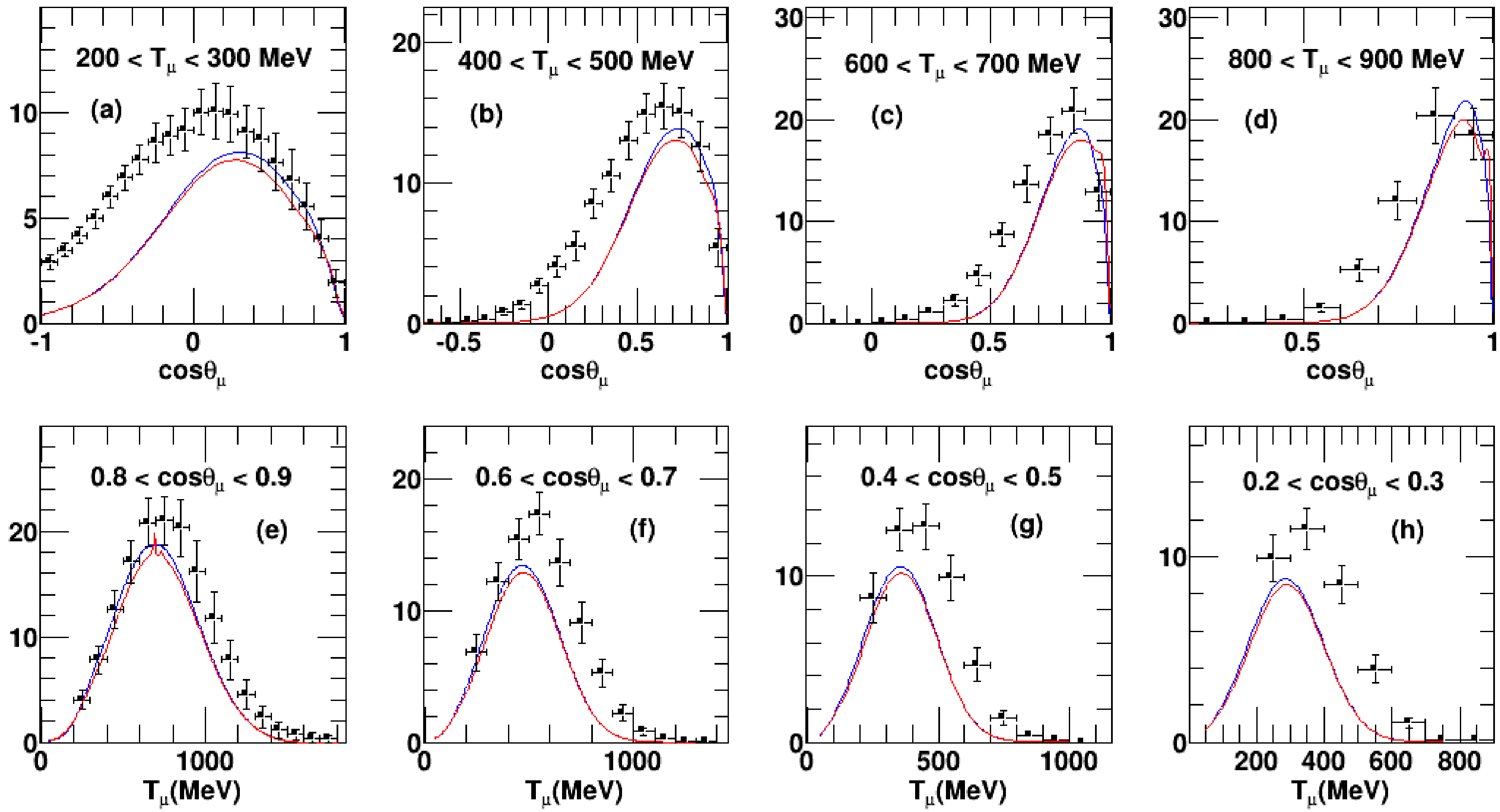
P. Barreau et al., Nucl. Phys. A402, 515 (1983), J. S. O'Connell et al., Phys. Rev. C35, 1063 (1987), R. M. Sealock et al., Phys. Rev. Lett.62, 1350 (1989).,D. S. Bagdasaryan et al., YERPHI-1077-40-88 (1988),D. B. Day et al., Phys. Rev. C 48, 1849 (1993).,D. Zeller, DESY-F23-73-2 (1973).



V. Pandey, N. Jachowicz, M. Martini, *et al.* Phys. Rev. **C94**, 054609 (2016)

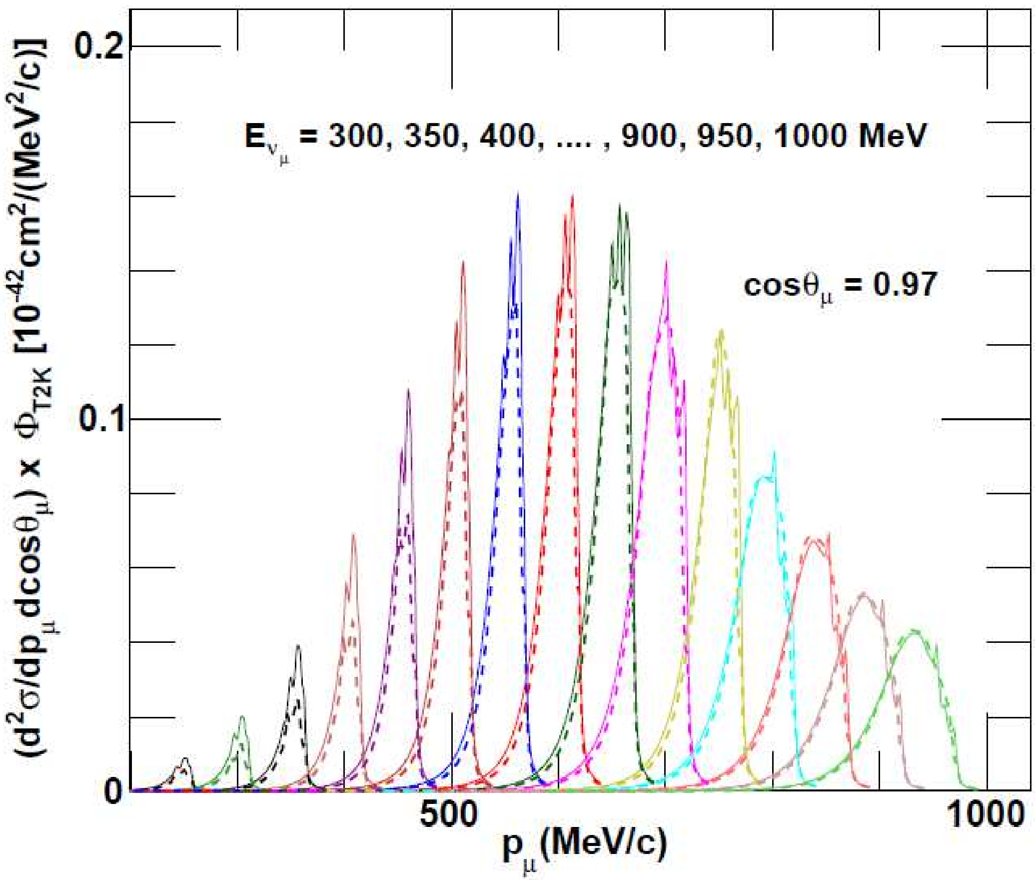
MiniBooNe ν_μ

- Satisfactory general agreement
- Good agreement for forward scattering
- Missing strength for low T_μ , backward scattering

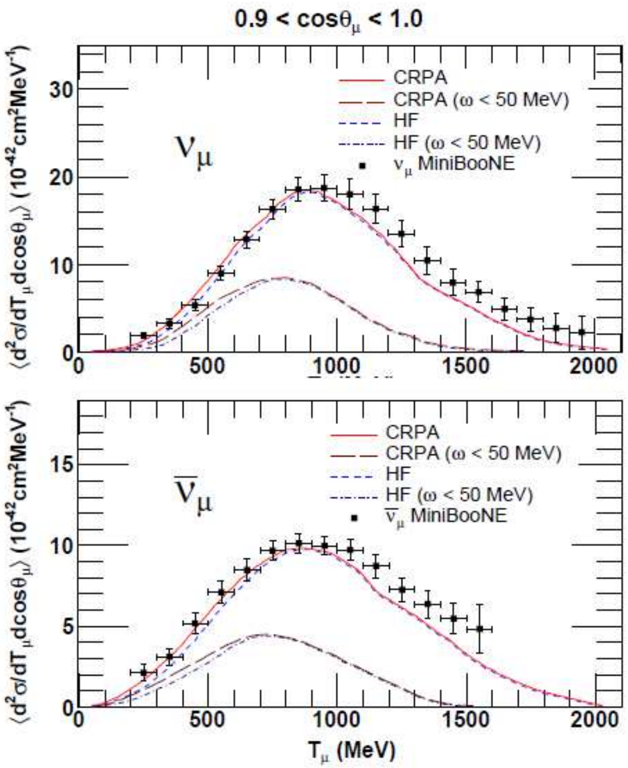


V. Pandey et al, Phys. Rev. C 92, 024606 (2015)

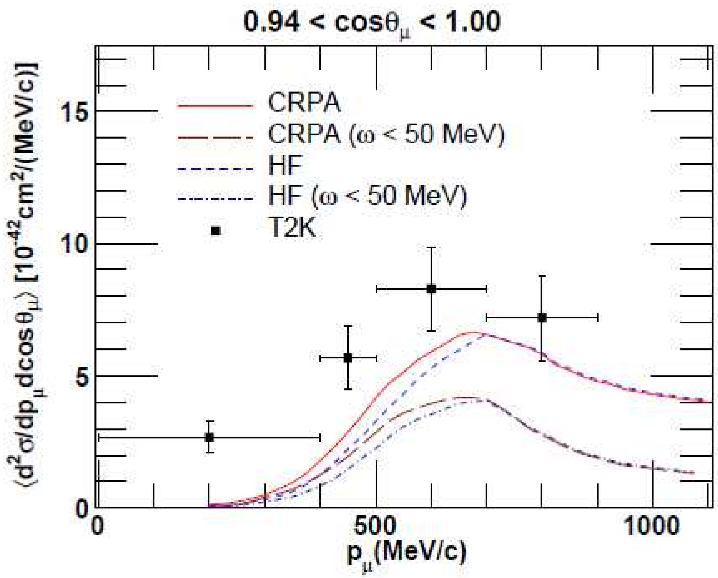
Forward scattering

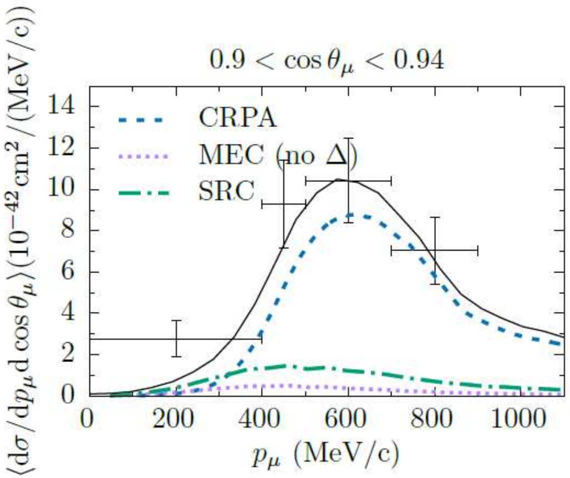


MiniBooNe



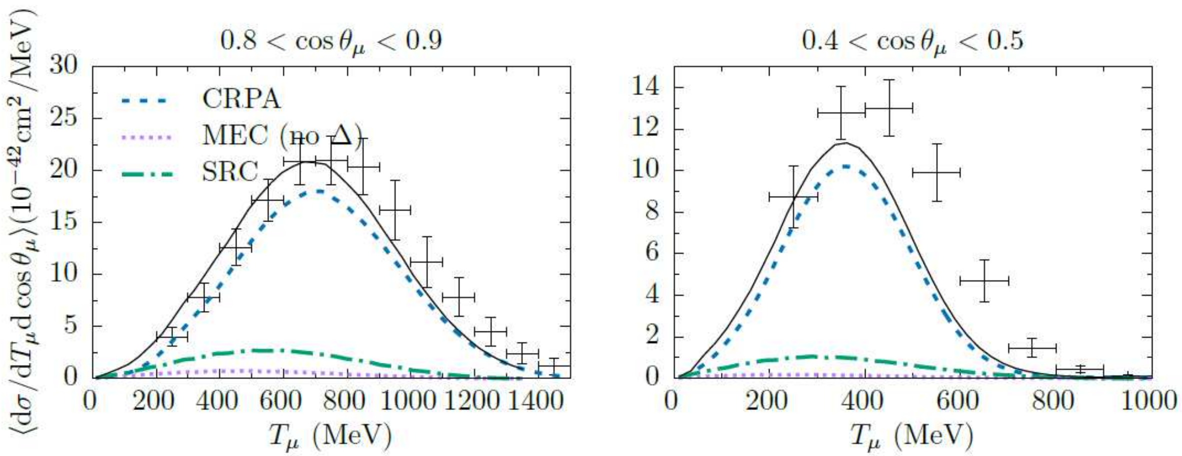
T2K

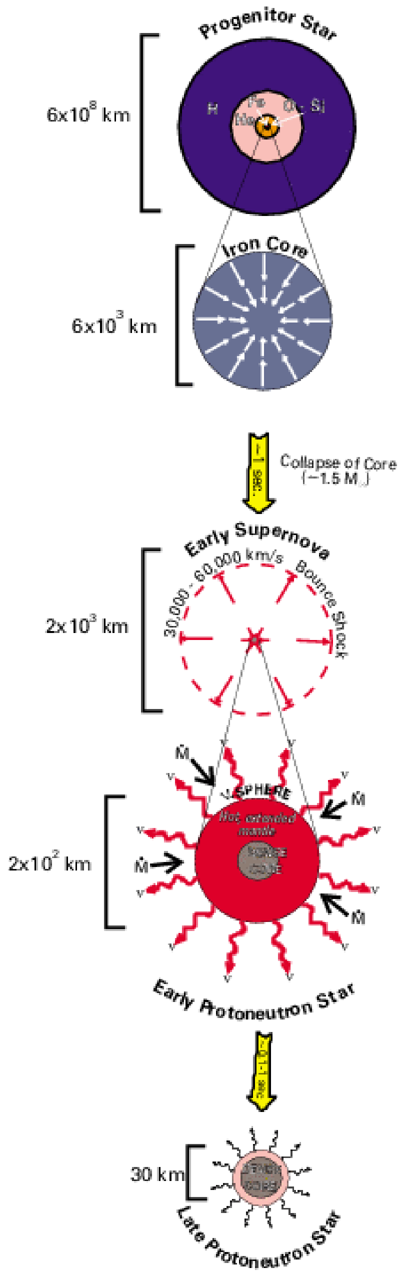




T2K

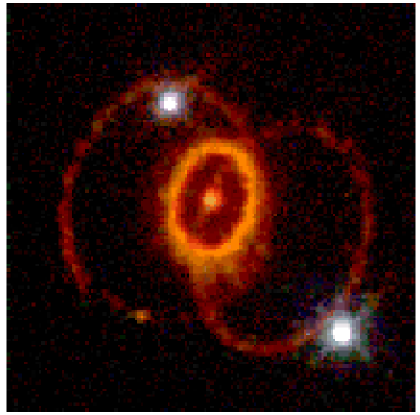
MiniBooNe

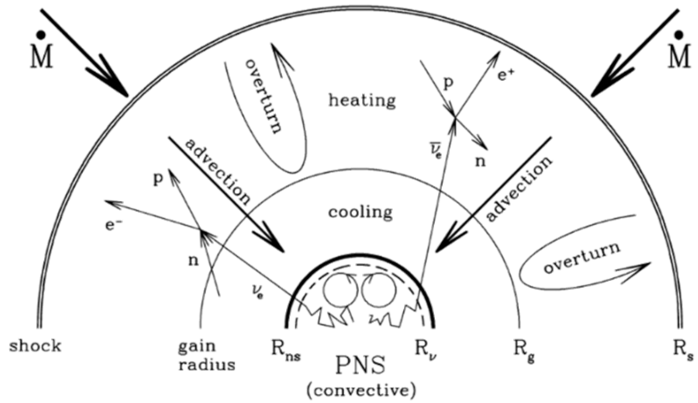




Neutrinos in a core-collapse supernova

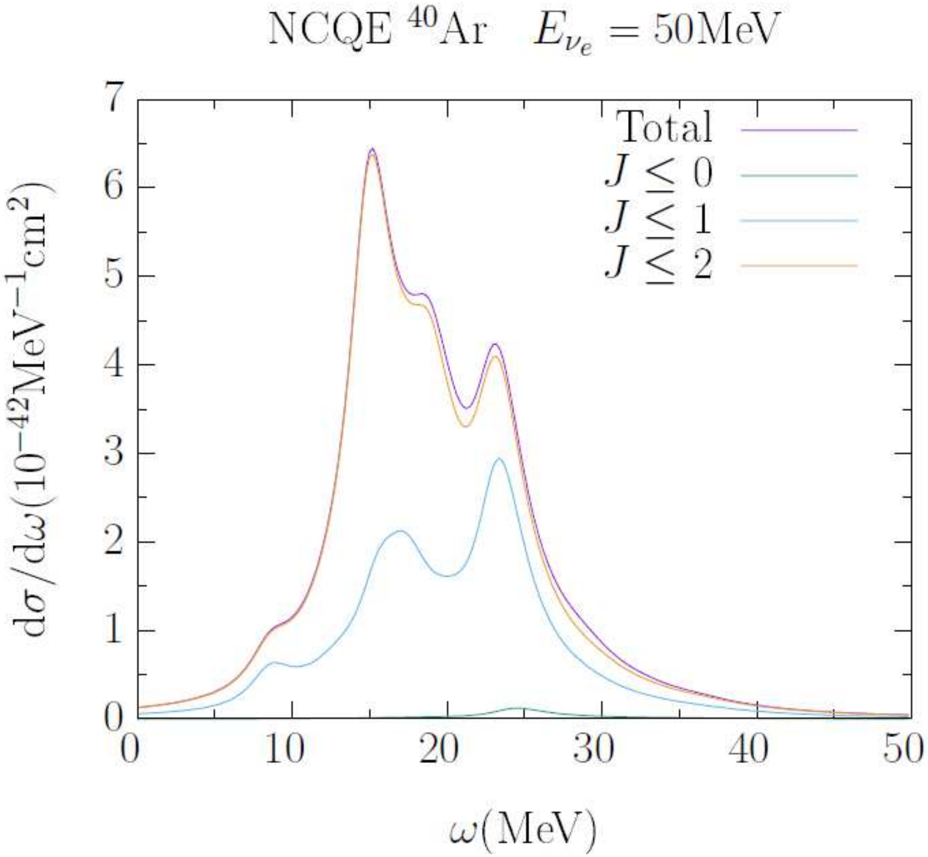
- weak interactions are important neutrinos are produced in the neutronization processes characterizing the gravitational collapse
- neutrinos are responsible for the cooling of the proto-neutron star
- neutrinonucleosynthesis
- energy deposition by neutrinos might reheat the stalled shock wave and cause a delayed explosion
- terrestrial detection of supernova neutrinos

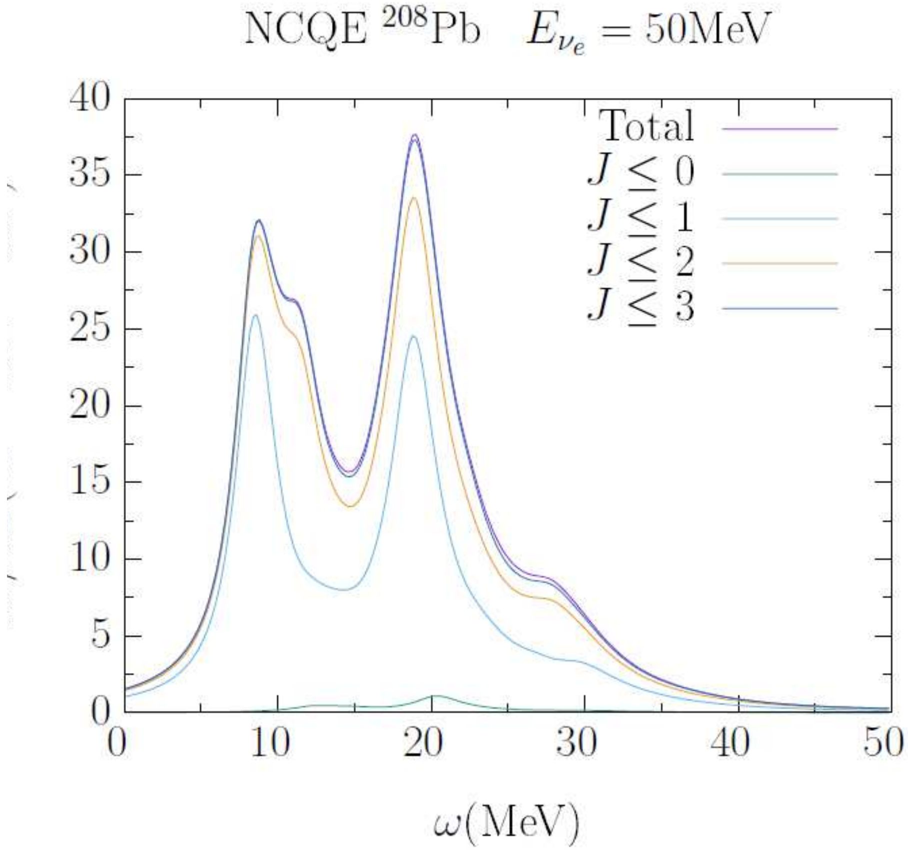




H.-T. Janka astro-ph/0008432

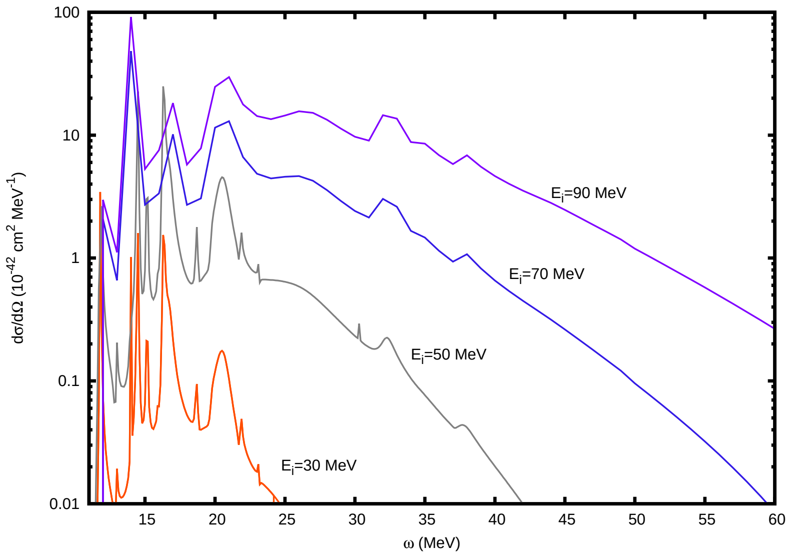
Neutrino scattering results at low energies :

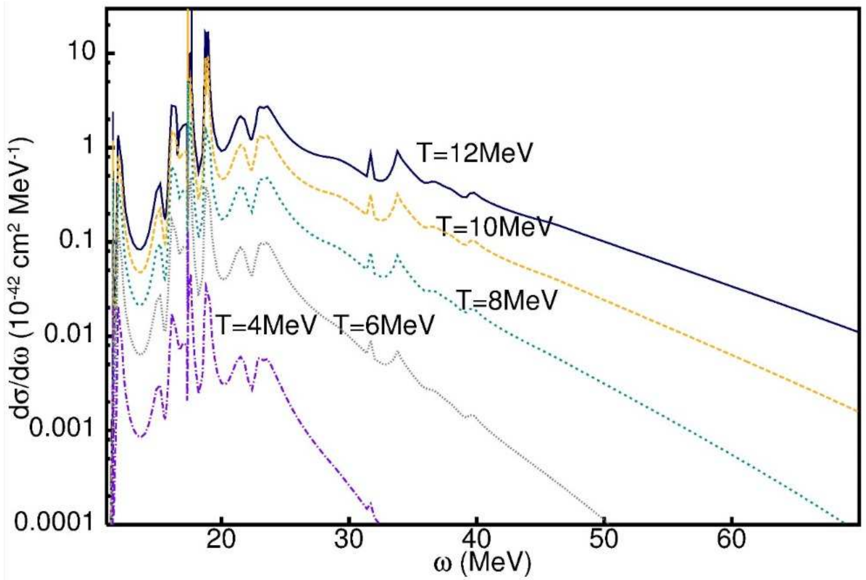




Folded cross sections supernova neutrino spectra :

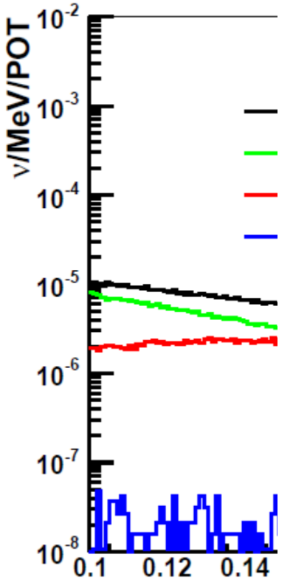
$^{16}O(\nu, \nu') \, ^{16}O^*$

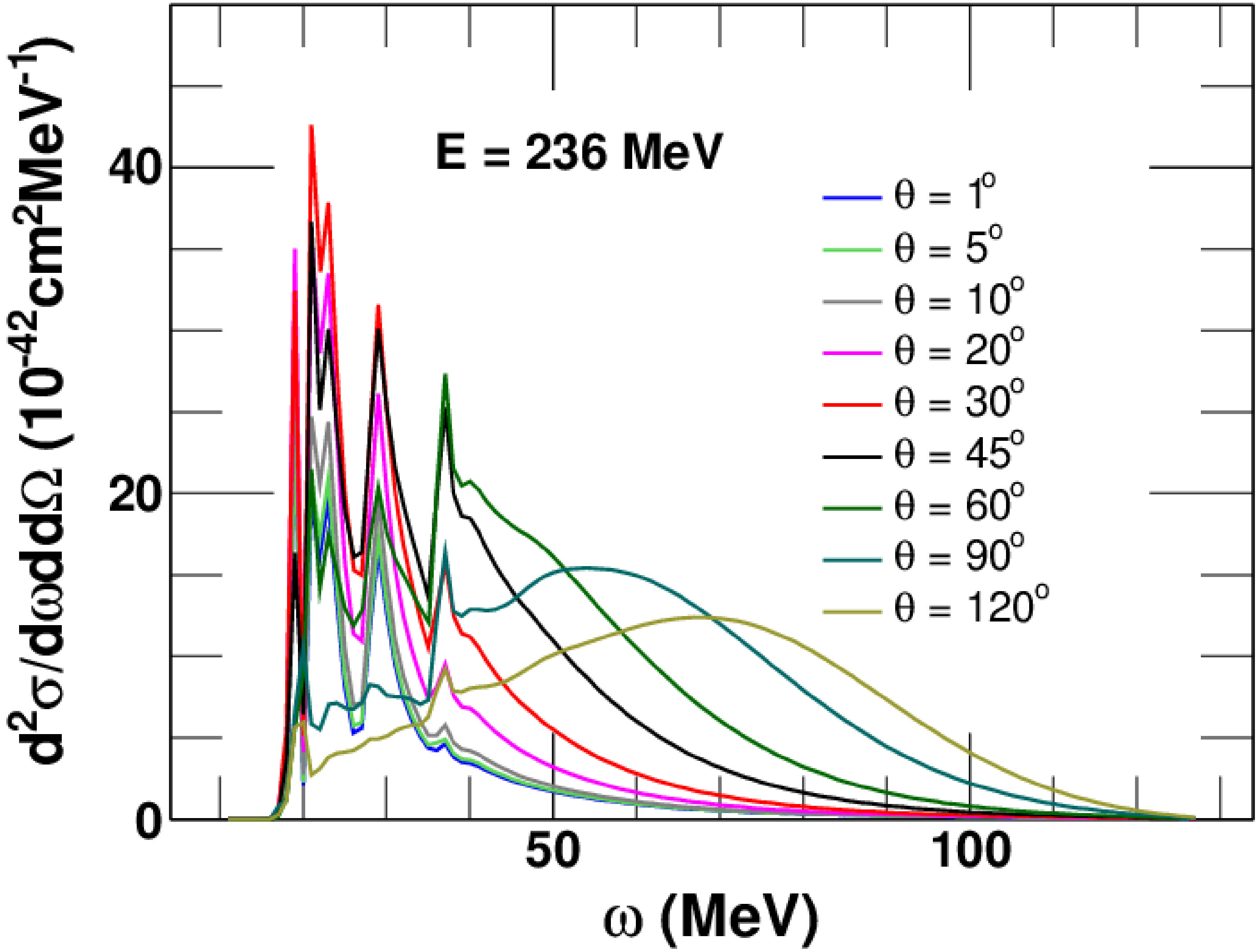




236 MeV neutrinos

- Protons on Carbon generate Kaons
- Kaons-at-rest- decay … primarily in ν_μ
- with an en



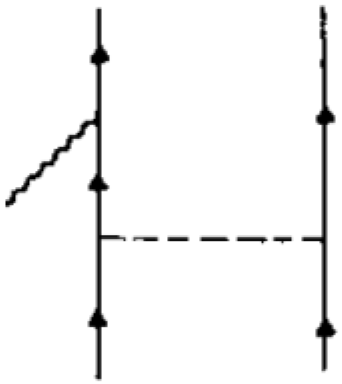


Different diagrammatic languages are used …

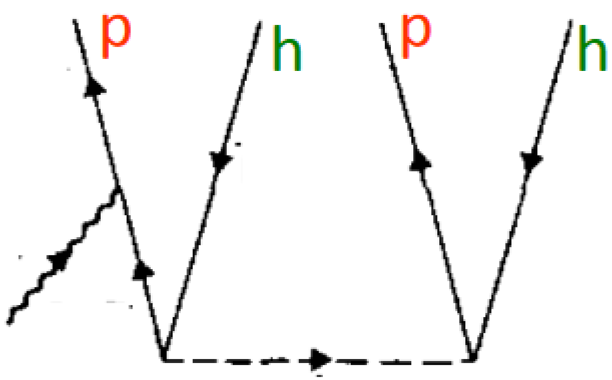
Two particle-two hole sector (2p-2h)

Three equivalent representations of the same process

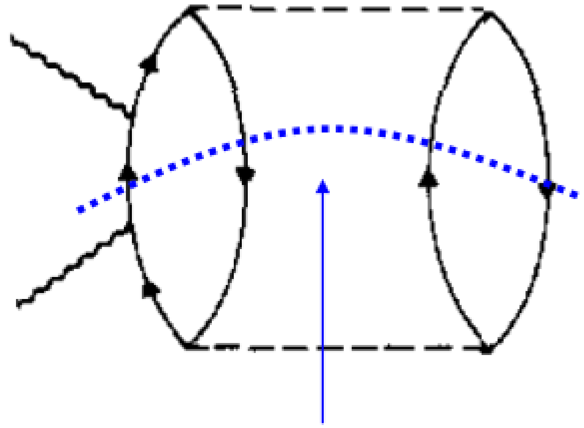
body current



2p-2h matrix element



2p-2h response

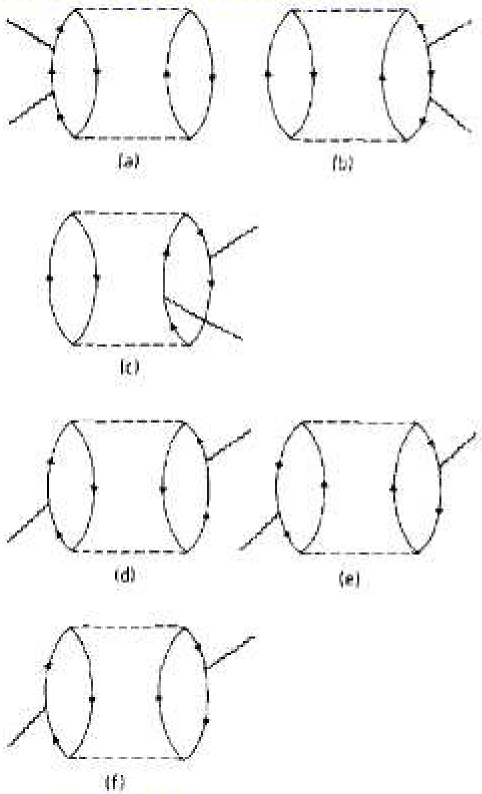


Cut
(optical theorem)

Final state: two particles-two holes

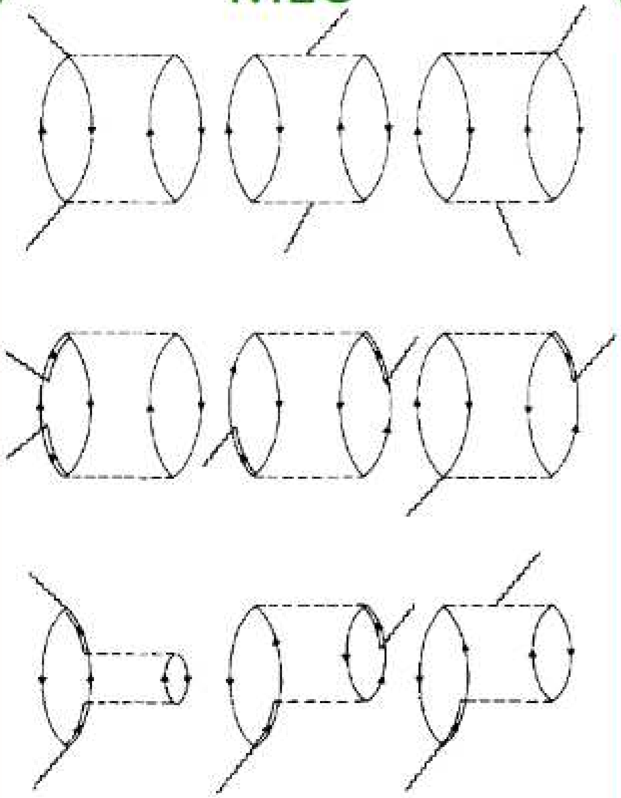
Some diagrams for 2p-2h responses

NN correlations



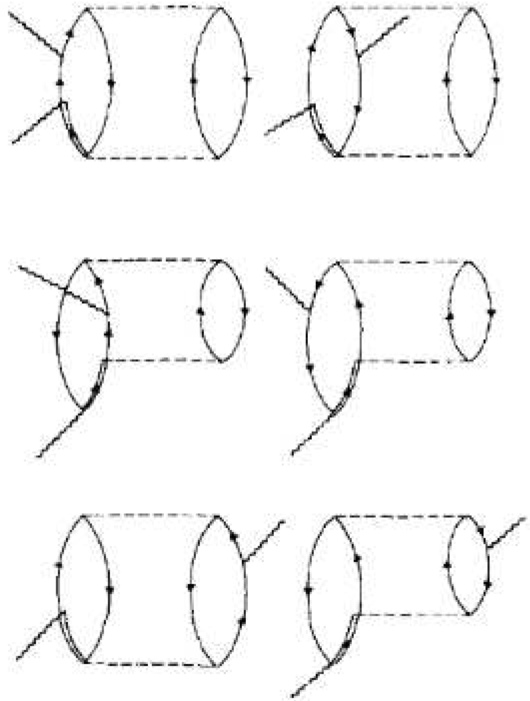
16 diagrams

MEC



49 diagrams

NN correlation-MEC interference



56 diagrams

(Some already included in a mean field)

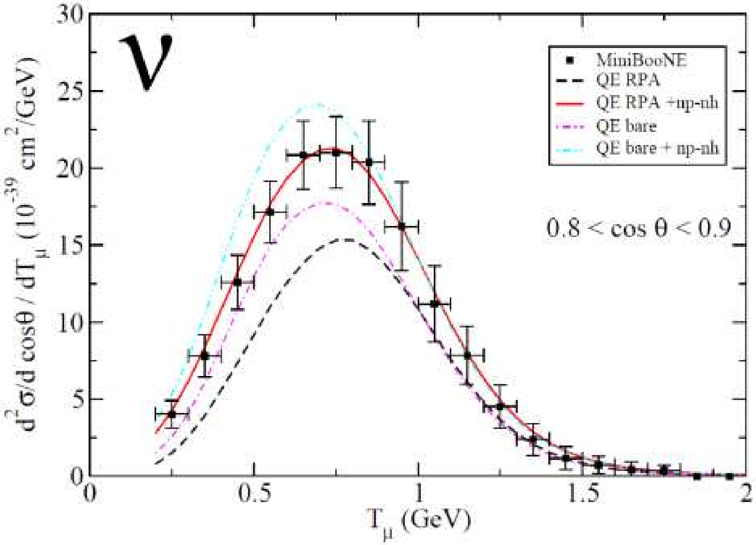
What about correlations in models on the neutrino market ?

Martini, Ericson model

- Starting point :Local Fermi gas, relativized
- RPA equations are solved in momentum space : Lindhard equation
- Correlations are introduced with a Landau-Migdal interaction : effective parametrization of pion- and exchange, contact force, non-relativistic reduction for MEC
- Includes delta degrees of freedom in RPA (Ericson-Ericson Lorentz-Lorentz effect)

References (non-exhaustive list): PRC 80, 065501 (2009); PRC 81, 045502 (2010); PRC 84, 055502 (2011); PRD 85, 093012 (2012); PRD 87, 013009 (2013); PRC 87, 065501 (2013); PRC 90, 025501 (2014); PRC 91, 035501 (2015); PRC 94, 015501 (2016), etc.

Martini, Ericson model

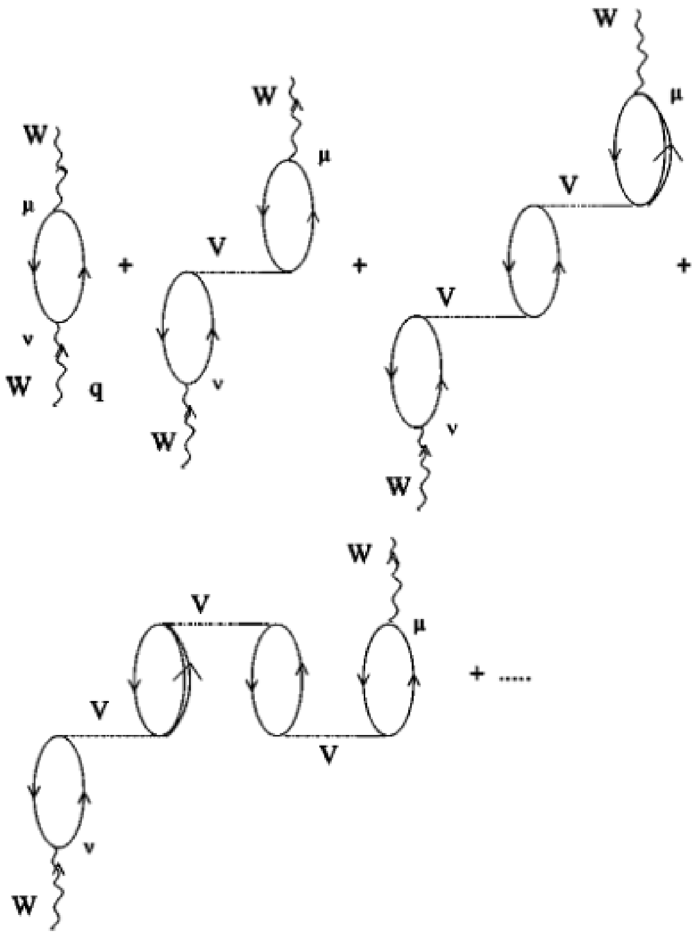


First to explain MiniBooNE QE data including np-nh effects, M_A~ 1

What about RPAs on the neutrino market ?

Valencia model (Nieves et al.)

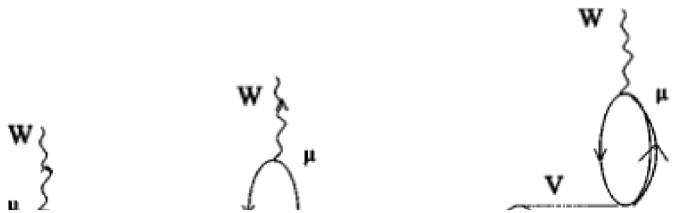
- Starting point :Local Fermi gas
- MEC, correlation with an effective Landau-Migdal type nucleon-nucleon force of pion-, and rho meson correlation currents
- relativistic
- Includes delta degrees of freedom
- Dressed nucleon propagators in the nuclear medium~ spectral function
- Includes SRC effects and 2p-2h degrees of freedom

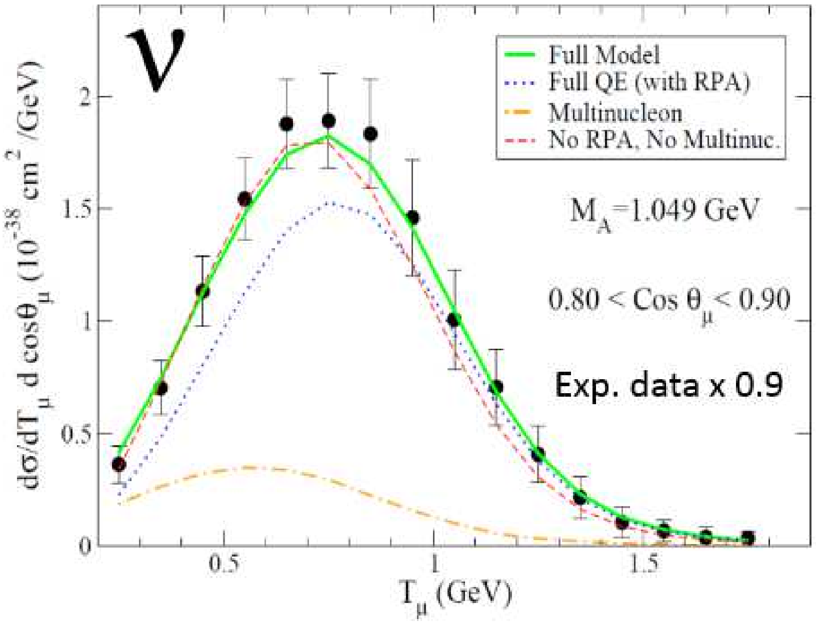


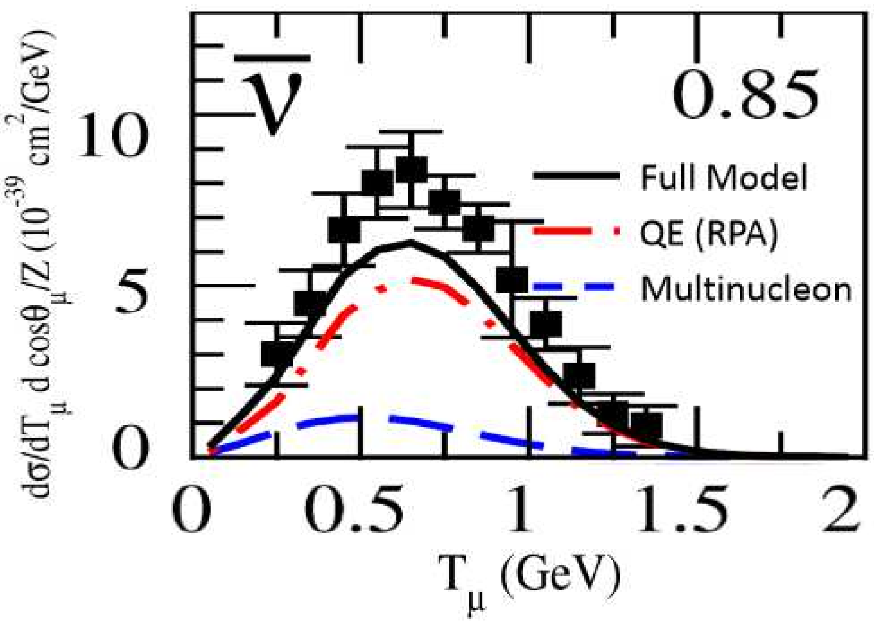
What about RPAs on the neutrino market ?

Valencia model (Nieves et al.)

- Starting point :Local Fermi gas, relativized
- Correlations with an effective Landau-Migdal type nucleon-nucleon force of





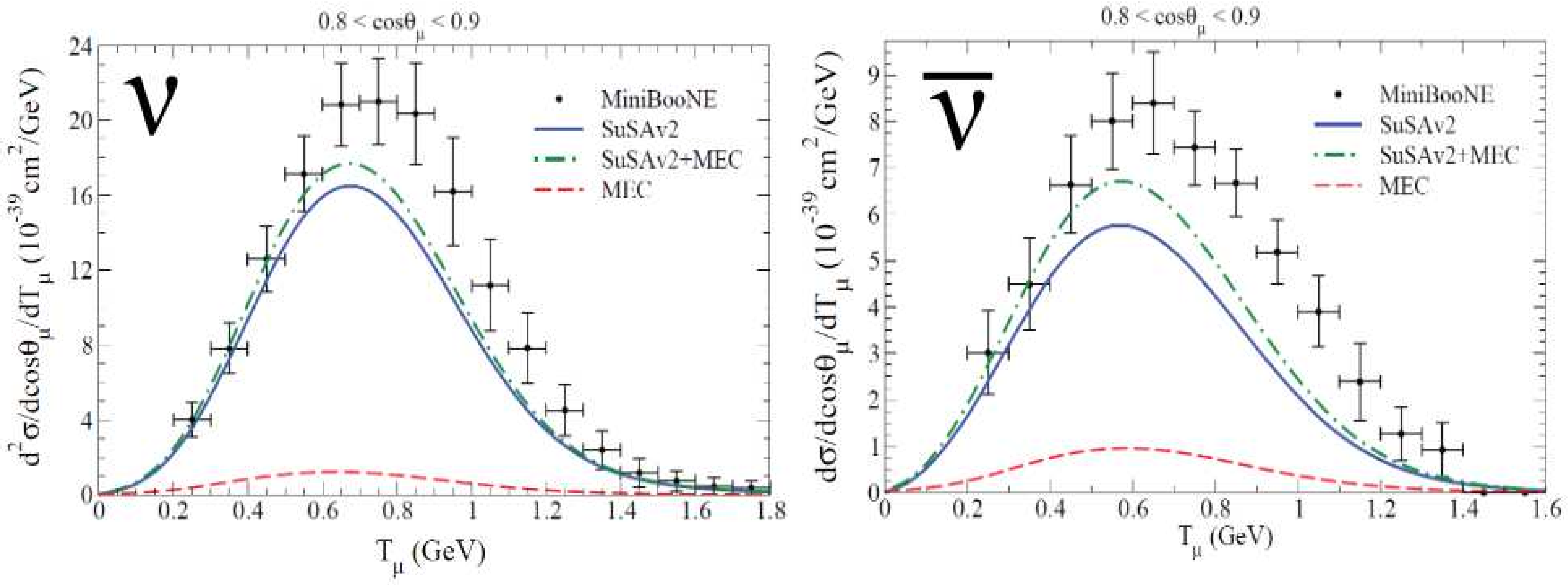


References (non-exhaustive): PRC 83, 045501 (2011); PLB 707, 72 (2012); PRD 85, 113008 (2012); PLB 721, 90 (2013); 88, 113007 (2013), etc.

What about correlations in models on the neutrino market ?

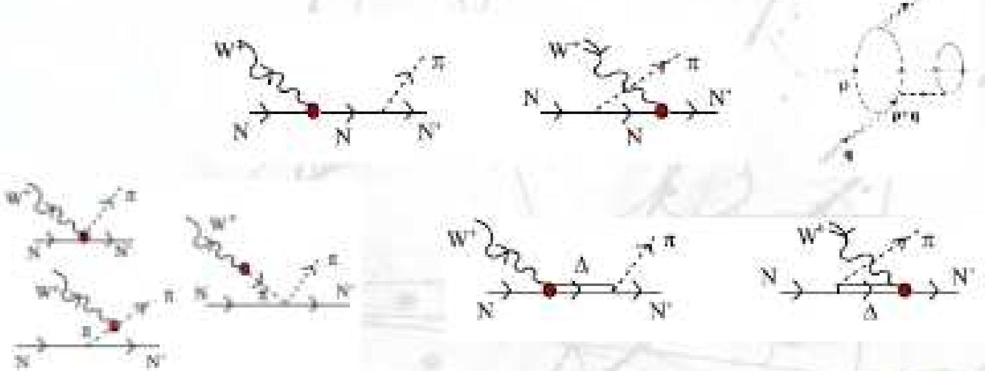
Susa model

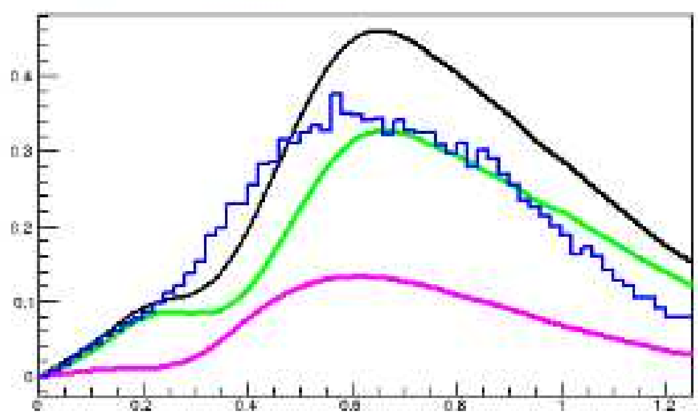
'Effective' scaling approach for genuine QE + MEC correlations

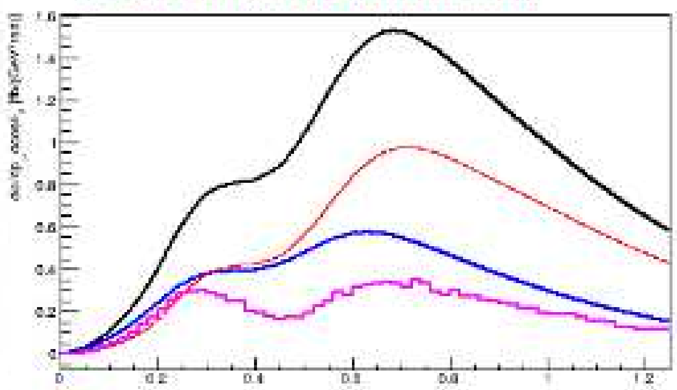


References (non-exhaustive): Phys.Lett.B696,151 (2011) ; Phys.Rev.Lett. 108, 152501 (2012) ; .Phys. Rev.C90, 035501 (2014) ;Phys. Rev. D91, 073004 (2015) etc.

- "A priori" very similar models (microscopic) give very different results.







Models have parameters!

Prediction : we will see more models with parameters to adjust to the data

Models have limits!

Prediction : collaborations th-exp will help to understand model validity.

F.Sanchez, Outlook to 2020. Nuint 2017. Toronto.

Selected Open issues (NuSTEC white paper) http://lanl.arxiv.org/pdf/1706.03621

After the acceptance of two-body currents as relevant contribution to the CCQE cross section, several issues still remain. The most urgent one is that of **agreement between different models**, and **between models and experiments**. Theoretical results need to be compared in a systematic way to all available data, and validated against electron-scattering data. The various **assumptions and differences in models that lead to discrepancies need to be understood**. This would be of great help in **assessing the range of validity of each approach and facilitate the incorporation of more detailed models in generators**

- From a purely theoretical view, the **modeling of outgoing hadrons** and hadronic final-state interactions is an issue that needs increased efforts
- **Interferences between various nuclear effects** and a meticulous study of double counting hazards. It is important to identify model-dependences and basis-dependent separations between different approaches.
- The **development of consistent models able to cover all experimental needs from 200 MeV to 10s of GeVs** is an open issue. None of the theories currently in use cover this vast energy region, models to match and fill the gaps between different predictions need to be developed