Equilibration process of the QGP and its connection to jet physics

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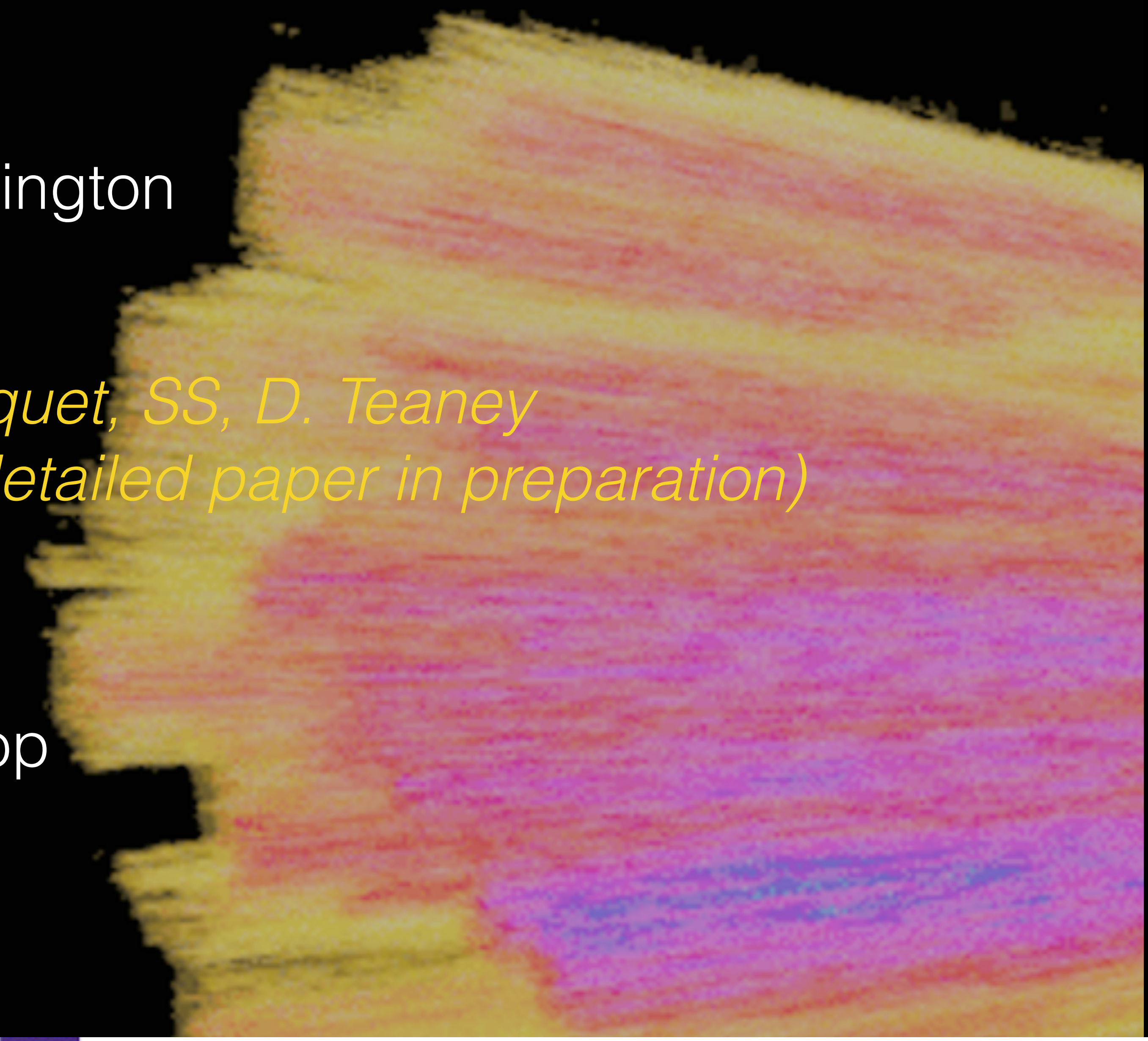
Based on
A. Kurkela, A. Mazeliauskas, J.-F. Paquet, SS, D. Teaney
(QM proceeding arXiv:1704.05242; detailed paper in preparation)



Santa Fe Jets & Heavy Flavor Workshop
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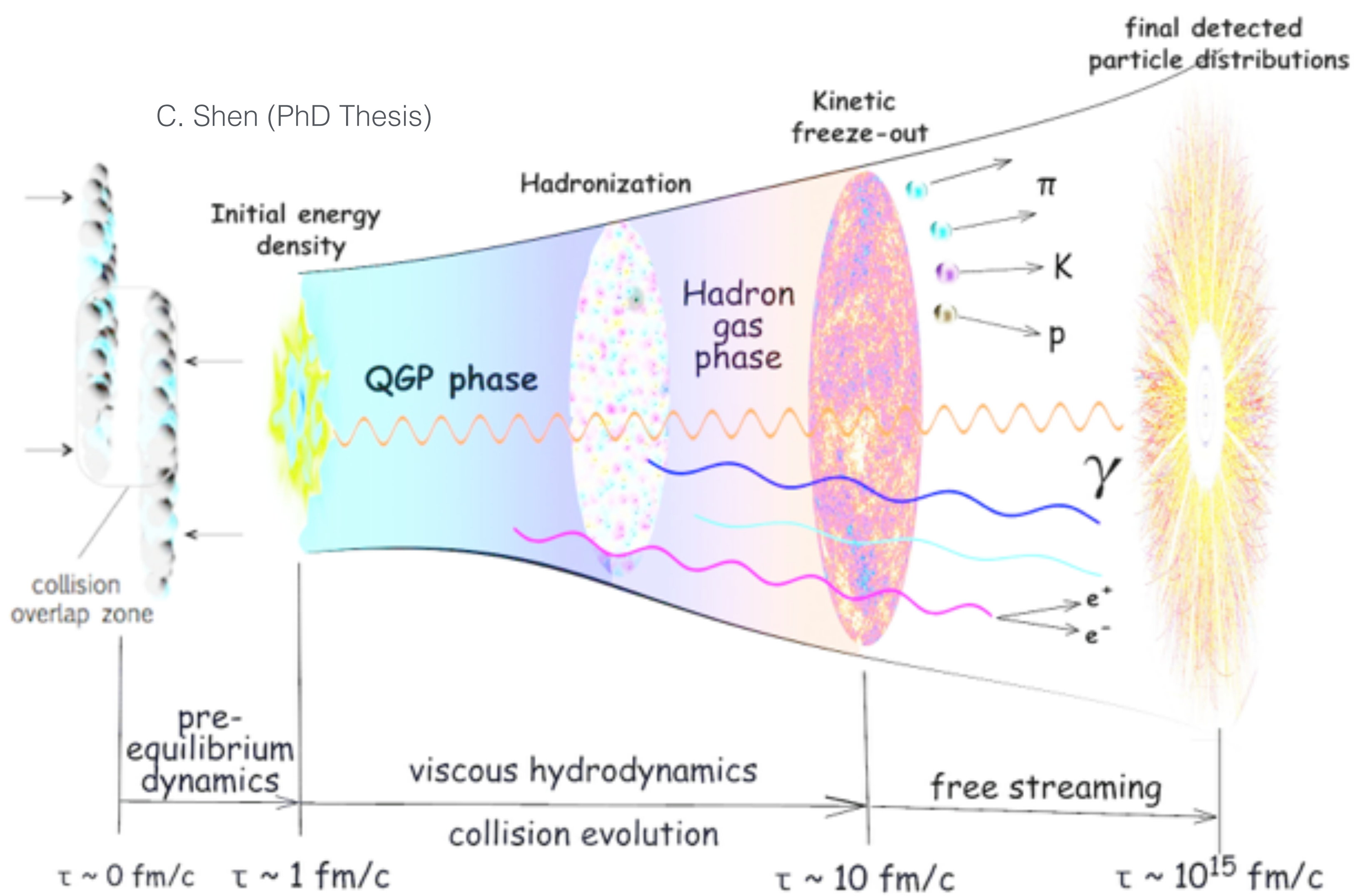
UNIVERSITY *of* WASHINGTON

Space-time picture of HIC

Extremely successful phenomenology based on hydrodynamic models of space-time evolution starting from τ~1fm/c

C. Shen (PhD Thesis)



Goal: Develop theoretical description of pre-equilibrium stage for complete description of space-time dynamics

Outline

Early time dynamics & equilibration process

— Microscopic dynamics & connections to jet physics

Description of early-time dynamics by macroscopic d.o.f.

— Energy momentum tensor & non-eq. response function

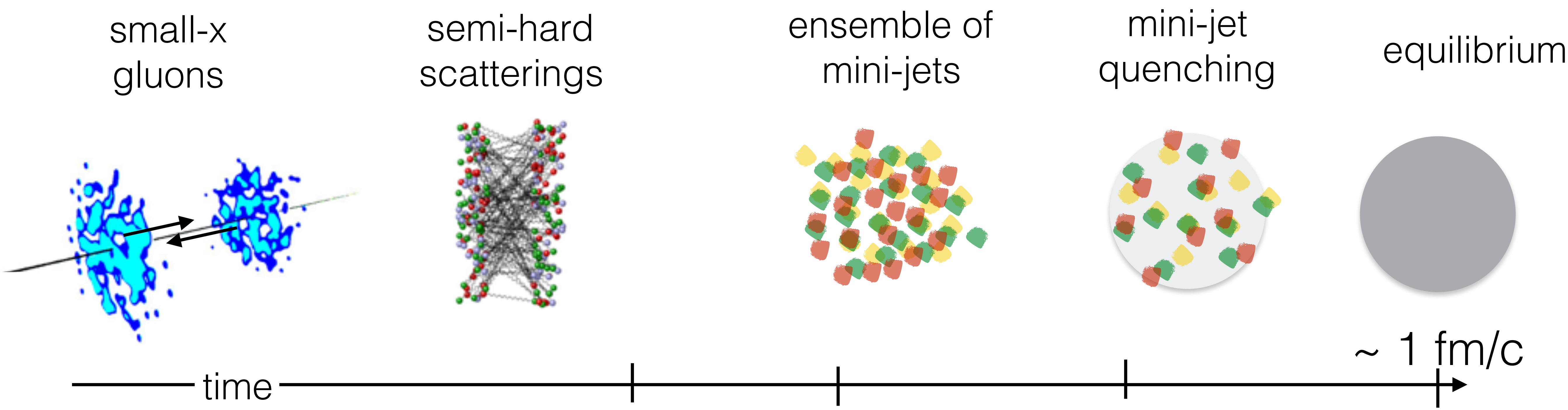
Event-by-event simulation of pre-equilibrium dynamics

— consistent matching to rel. visc. hydrodynamics

Conclusions & Outlook

Early time dynamics & equilibration process

Canonical picture at weak coupling:



Starting with the collision of heavy-ions a sequence of processes eventually leads to the formation of an equilibrated QGP

Key questions:

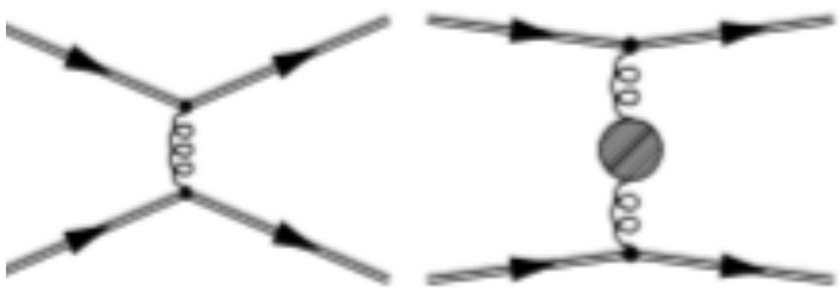
How does ensemble of mini-jets thermalize?

When and to what extent can this process be described macroscopically e.g. in terms of visc. hydrodynamics?

Description at (LO) weak coupling

Based on effective kinetic theory of Arnold, Moore, Yaffe (AMY)
(basis for MARTINI jet-quenching Monte Carlo)

$$\left(\partial_\tau - \frac{p_z}{\tau}\right) f(\tau, |\mathbf{p}_\perp|, p_z) = \mathcal{C}[f] = \mathcal{C}_{2\leftrightarrow 2}[f] + \mathcal{C}_{1\leftrightarrow 2}[f]$$



elast. 2<->2 scattering
screened by Debye mass

collinear 1<->2 Bremsstrahlung
incl. LPM efffect
via eff. vertex re-summation

Differences to parton/jet energy loss calculations

- lower p_T - phase space density of on-shell partons (no structure)
- no “background” medium -> non-linear treatment of interactions between mini-jets
- soft & (semi-)hard degrees of freedom all treated within same framework

see Keegan,Kurkela,Mazeliauskas,Teaney JHEP 1608 (2016) 171 for details on numerics

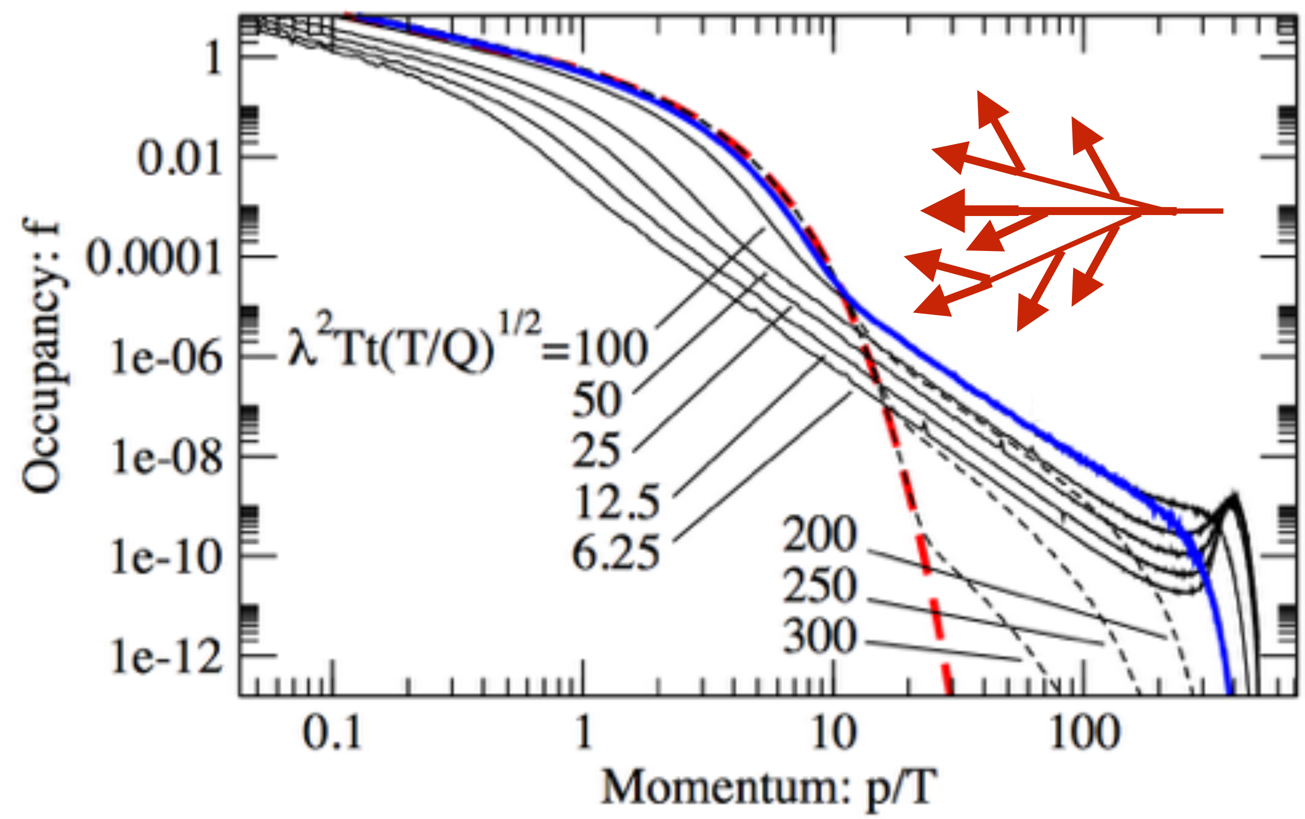
Mini-jet quenching

Interactions between mini-jets (p~Q) induce collinear Bremsstrahlung radiation (p<<Q)

-> Cascades towards low p via multiple (democratic) branchings

Soft fragments p << Q begin to thermalize via elastic/inelastic interactions

-> soft thermal bath T<<Q forms



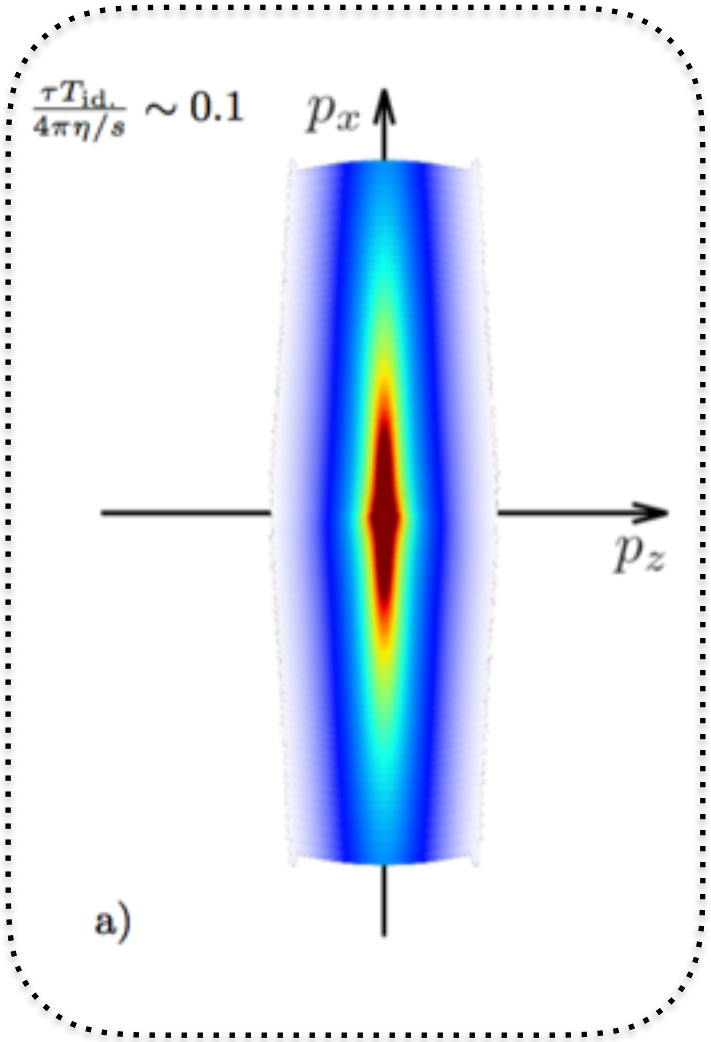
Kurkela, Lu PRL 113 (2014) 182301

Energy continues to flow from p~Q to p~T, increasing the temperature of the bath

-> Soft bath begins to dominate screening & scattering

Subsequently the situation is analogous to parton energy loss; mini-jets loose all their energy to soft bath heating it up to the final temperature.

Equilibration process at weak coupling



Semi-hard gluons produced around mid-rapidity have $p_T >> p_z$
-> initial phase-space distribution is highly anisotropic

Non-equilibrium plasma subject to rapid long. expansion
-> depletion of phase space density

Equilibration of expanding plasma proceeds as three step process described by “bottom-up” scenario

Baier, Mueller, Schiff, Son PLB502 (2001) 51-58

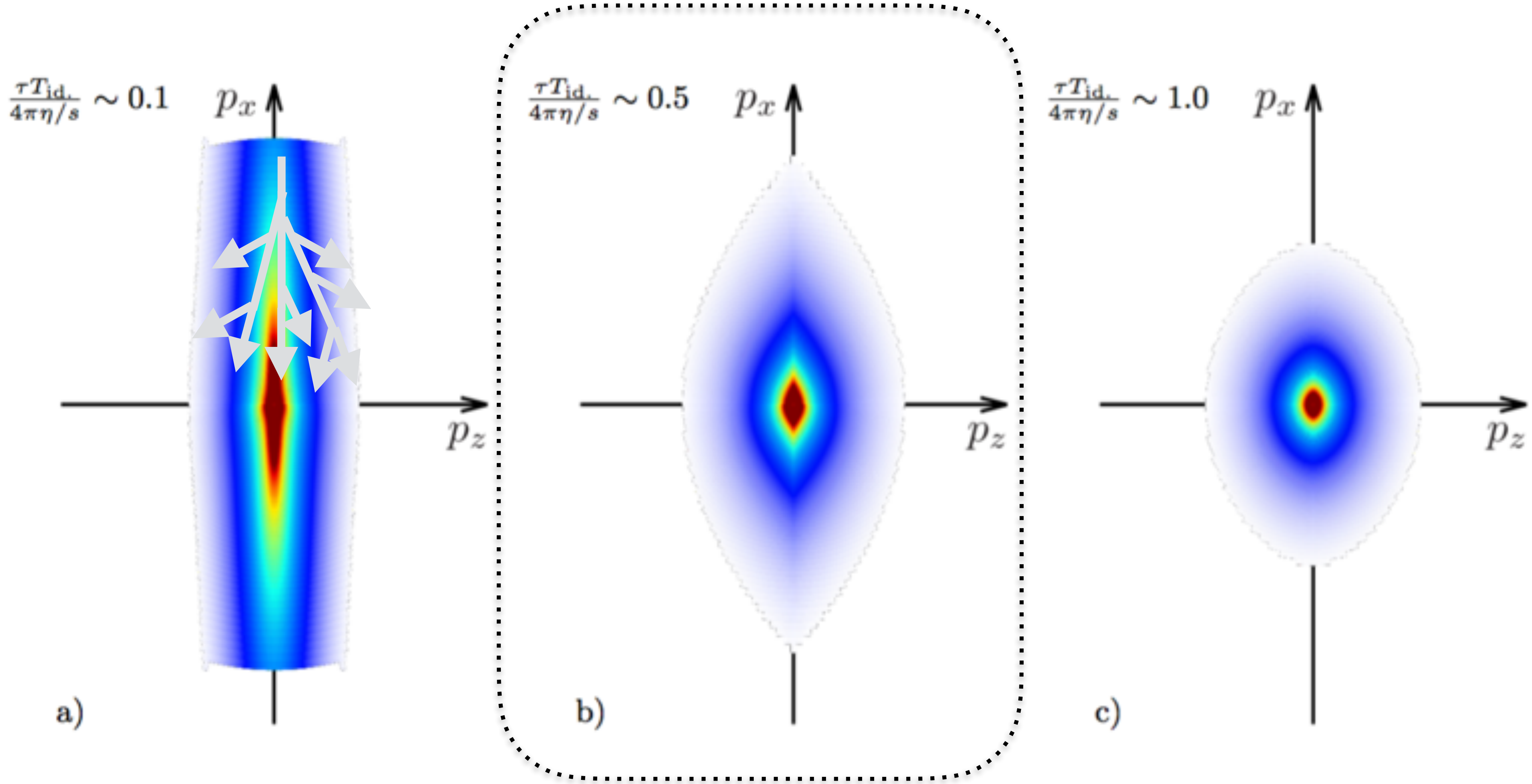
Phase I: Quasi-particle description becomes applicable.
Elastics scattering dominant but insufficient to isotropize system

c.f. Berges,Boguslavski,SS, Venugopalan, PRD 89 (2014) no.7, 074011

Equilibration process at weak coupling

Equilibration proceeds as three step process described by "bottom-up" scenario

Baier, Mueller, Schiff, Son PLB502 (2001) 51-58



Phase II: Mini-jets undergo a radiative break-up cascade eventually leading to formation of soft thermal bath

c.f. Kurkela, Zhu PRL 115 (2015) 182301

Equilibration process at weak coupling

Equilibration proceeds as three step process described by "bottom-up" scenario

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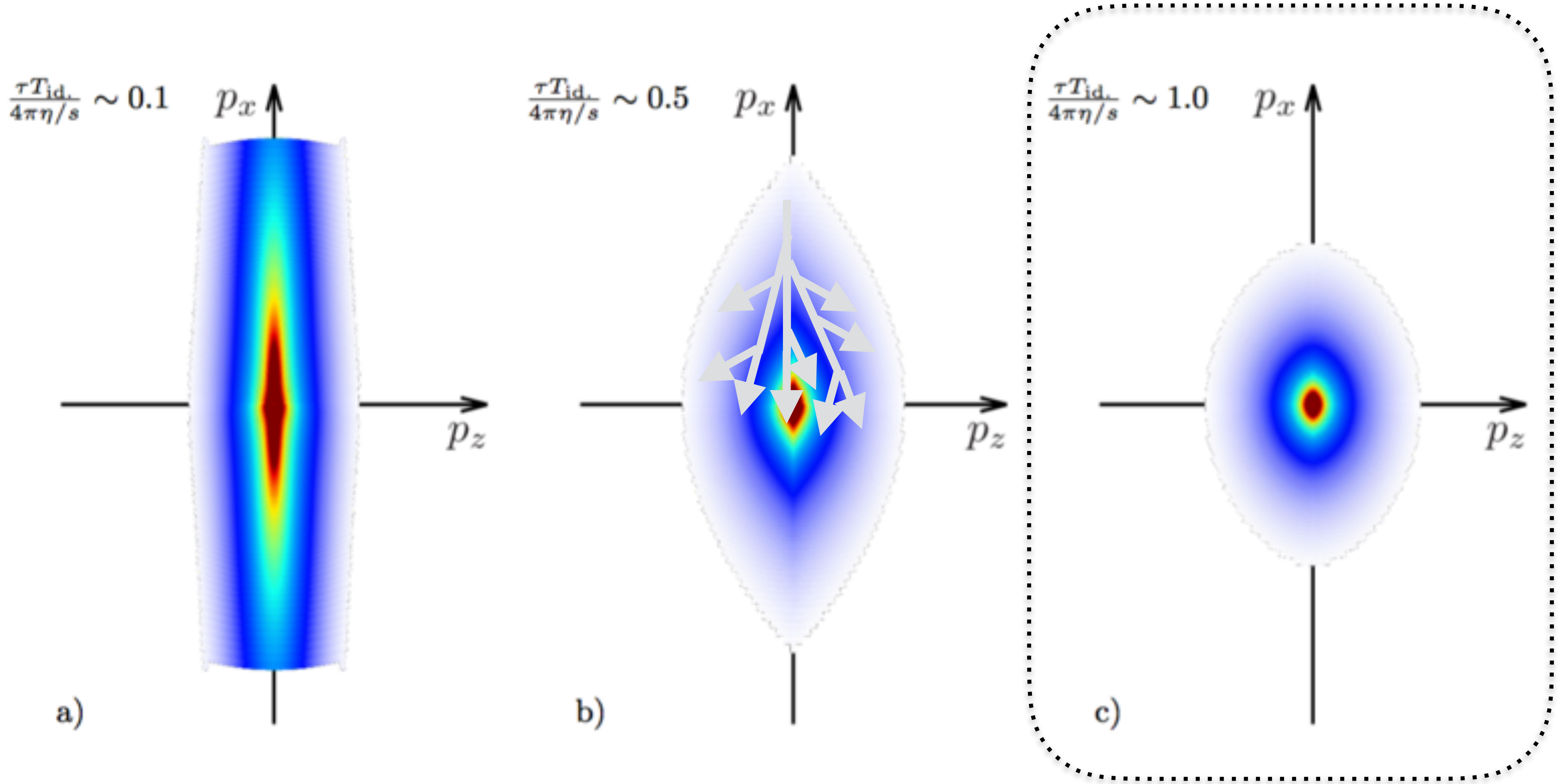

Phase III: Quenching of mini-jets in soft thermal bath transfers energy to soft sector leading to isotropization of plasma

c.f. Kurkela, Zhu PRL 115 (2015) 182301

Equilibration process at weak coupling

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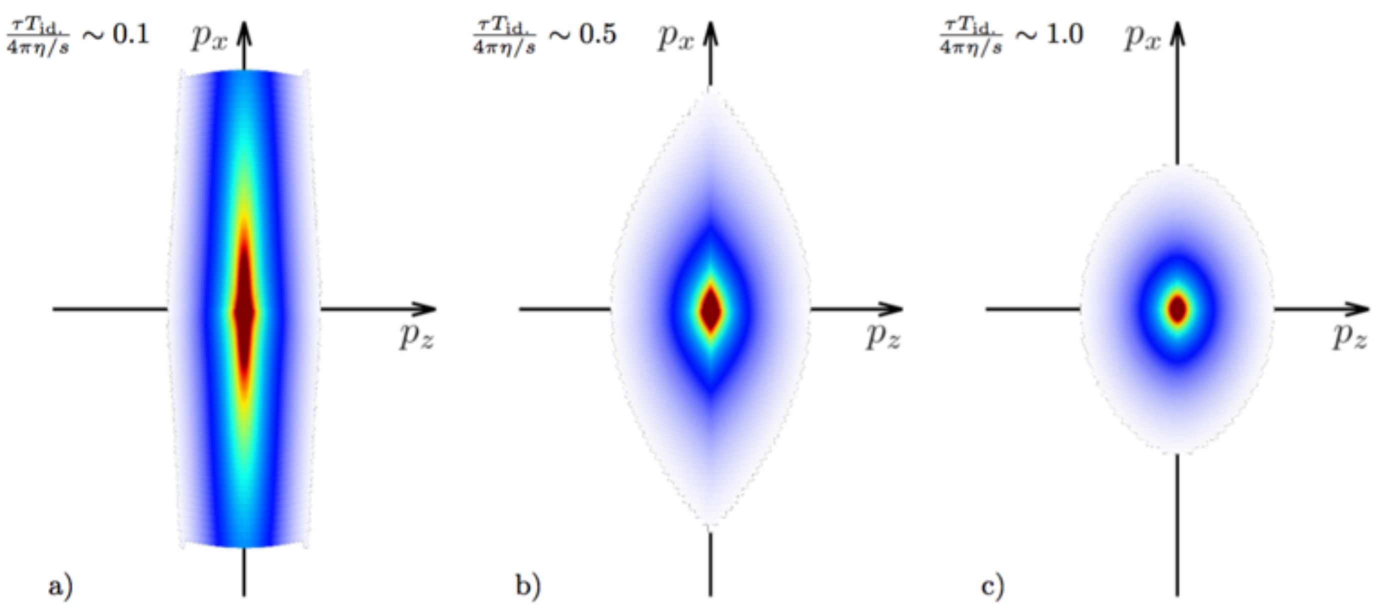
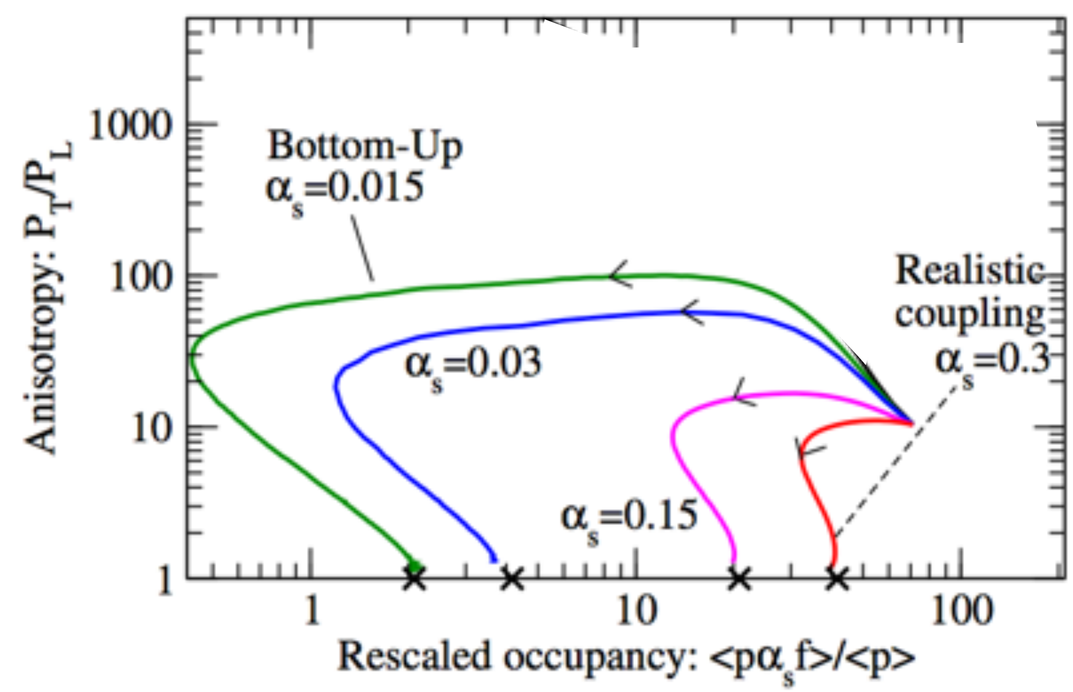
Kurkela, Zhu PRL 115 (2015) 182301





Beyond very early times equilibration process similar to parton-energy loss in thermal medium

Equilibration time determined by the time-scale for a mini-jet ($p \sim Q_s$) to loose all its energy to soft thermal bath

Onset of hydrodynamic behavior

Since the system is highly anisotropic initially $P_L << P_T$, one of the key questions is to understand evolution of anisotropy of $T^{\mu\nu}$

Viscous hydrodynamics begins to describe evolution of energy momentum tensor starting on time scales ~1 fm/c for realistic values of α_s (~0.3) at RHIC & LHC energies

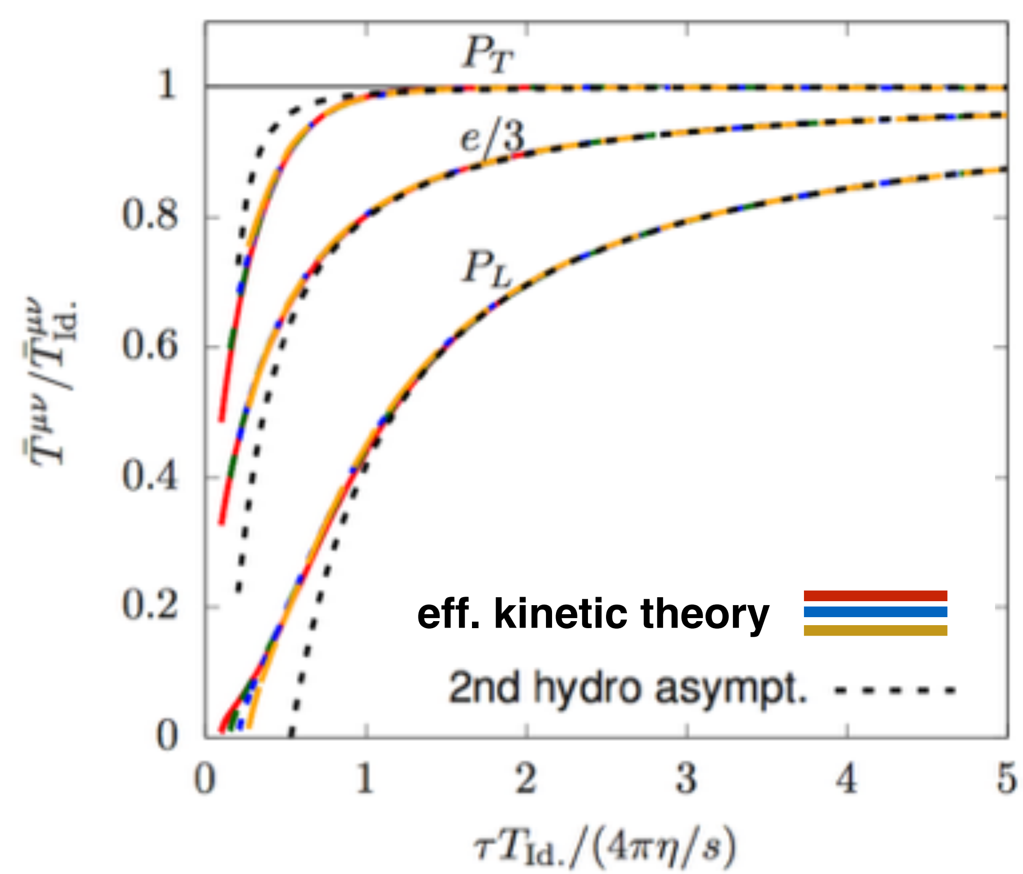
e.g. $T_{Initial}$ ~1 GeV, η/s ~$3/4\pi$, τ_{Hydro}~0.8 fm/c

Kurkela, Zhu PRL 115 (2015) 182301

Kurkela, Mazeliauskas, Paquet, SS, Teaney (in preparation)

-> in-line with heavy-ion phenomenology

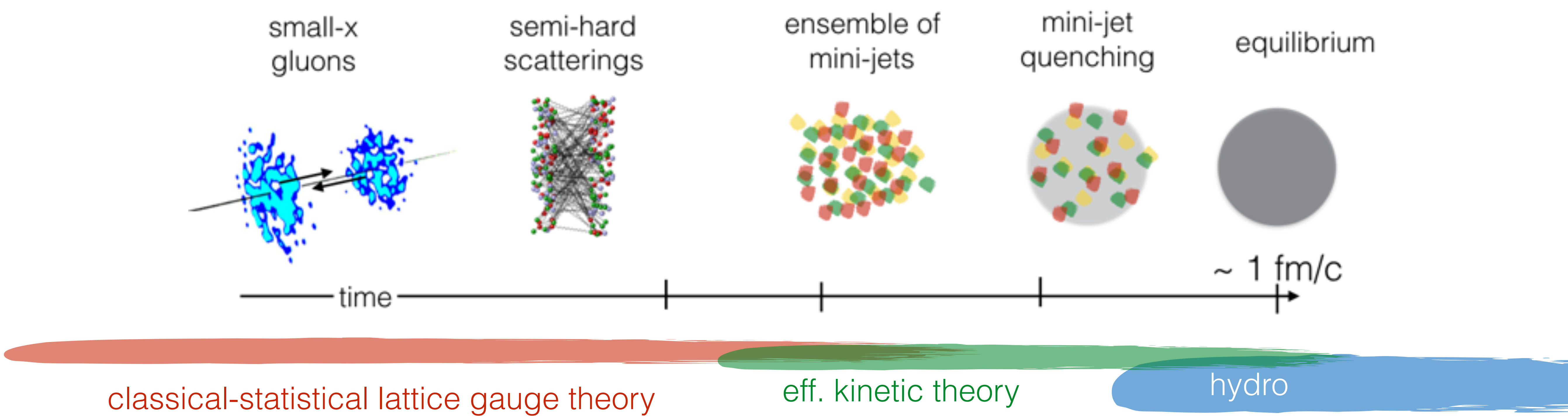
Similar to strong coupling picture viscous hydrodynamics becomes applicable when pressure anisotropies are still O(1) and microscopic physics is still somewhat jet-like



c.f. Kurkela, Zhu PRL 115 (2015) 182301
Kurkela, Mazeliauskas, Paquet, SS, Teaney (in preparation)

Early time dynamics & equilibration process

Based on combination of weak-coupling methods a complete description of early-time dynamics can be achieved



Brute force calculation challenging but possible (e.g. in p+p/A)
(Greif, Greiner, Schenke, SS, Xu, Phys.Rev. D96 (2017) no.9, 091504)

Ultimately for the purpose of describing soft physics of the medium, we are mostly interested in calculation of energy-momentum tensor

-> Exploit memory loss to use macroscopic degrees of freedom for description of pre-equilibrium dynamics

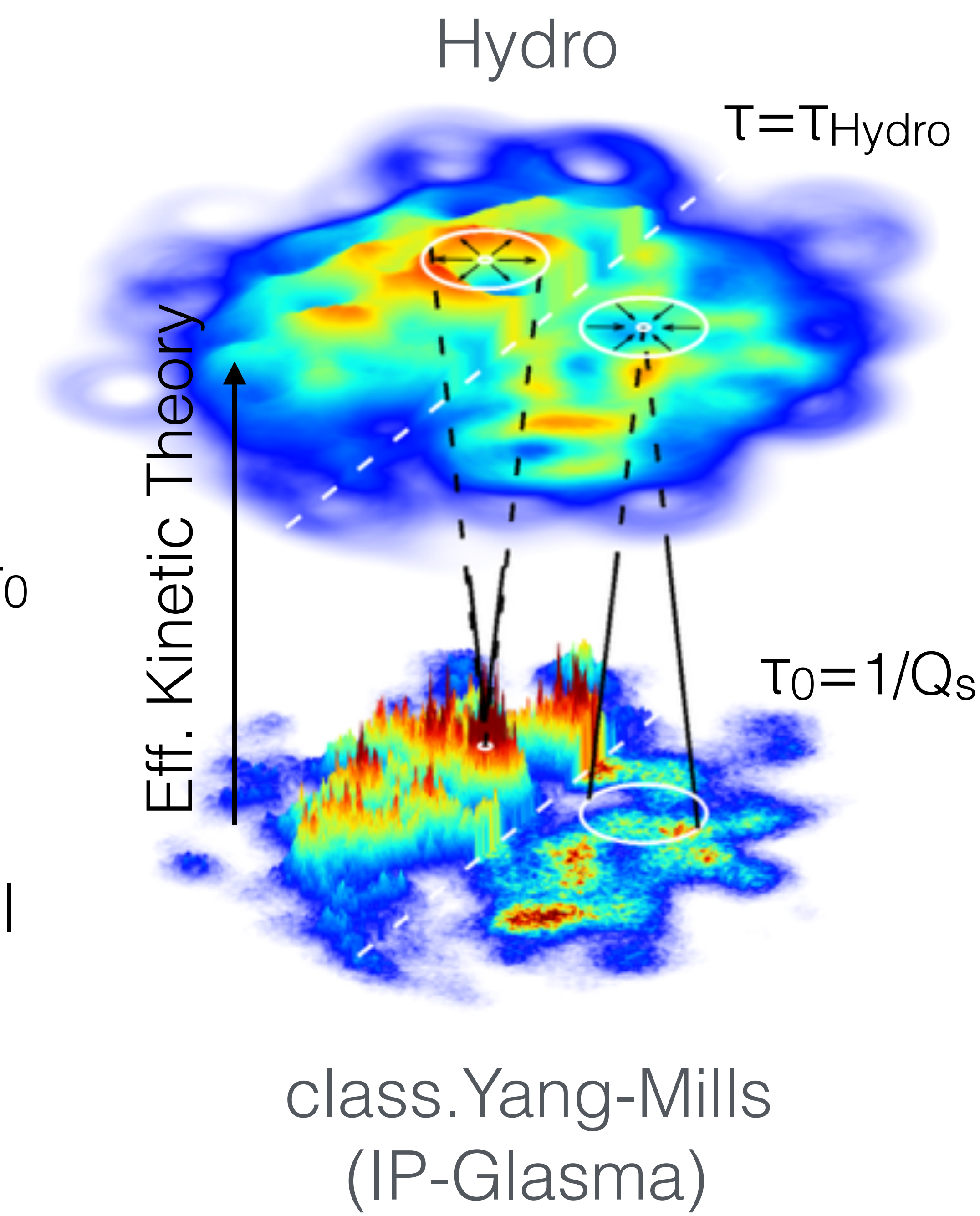
Macroscopic pre-equilibrium evolution

Extract energy-momentum tensor $T^{\mu\nu}(x)$ from initial state model (e.g. IP-Glasma)

Evolve $T^{\mu\nu}$ from initial time $\tau_0 \sim 1/Q_s$ to hydro initialization time τ_{Hydro} using eff. kinetic theory description

Causality restricts contributions to $T^{\mu\nu}(x)$ to be localized from causal disc $|x-x_0| < \tau_{Hydro}-\tau_0$ useful to decompose into a local average $T^{\mu\nu}_{BG}(x)$ and fluctuations $\delta T^{\mu\nu}(x)$

Since in practice size of causal disc is small $\tau_{Hydro}-\tau_0 << R_A$ fluctuations $\delta T^{\mu\nu}(x)$ around local average $T^{\mu\nu}_{BG}(x)$ are small and can be treated in a linearized fashion



Keegan,Kurkela, Mazeliauskas, Teaney JHEP 1608 (2016) 171
Kurkela, Mazeliauskas, Paquet, SS, Teaney (in preparation)

Macroscopic pre-equilibrium evolution

Effective kinetic description needs phase-space distribution $f(\tau,p,x)$

Memory loss: Details of initial phase-space distribution become irrelevant as system approaches local equilibrium

Can describe evolution of $T^{\mu\nu}$ in kinetic theory in terms of a representative phase-space distribution

$$f(\tau,p,x) = f_{BG}(Q_s(x)\tau, p/Q_s(x)) + \delta f(\tau,p,x)$$

where f_{BG} characterizes typical momentum space distribution, and δf can be chosen to represent local fluctuations of initial energy momentum tensor, e.g. energy density $\delta T^{\tau\tau}$ and momentum flow $\delta T^{\tau i}$

Energy perturbations:

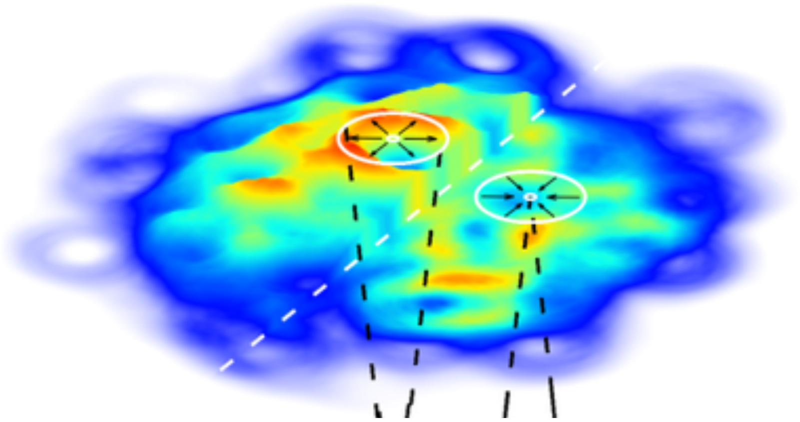
$$\delta f_s(\tau_0,p,x) \propto \frac{\delta T^{\tau\tau}(x)}{T^{\tau\tau}_{BG}(x)} \times \frac{\partial}{\partial Q_s(x)} f_{BG}\Big(\tau_0, p/Q_s(x)\Big)$$

local amplitude

representative form of phase-space distribution

Macroscopic pre-equilibrium evolution

Energy-momentum tensor on the hydro surface can be reconstructed directly from initial conditions according to

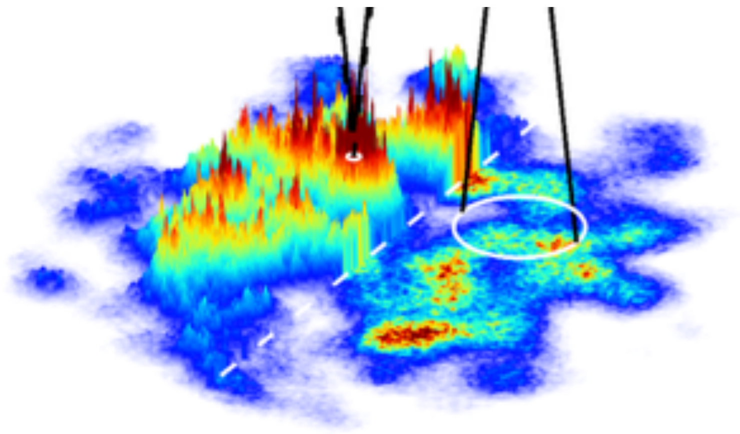


$$T^{\mu\nu}(\tau,x) = T^{\mu\nu}_{BG}\Big(Q_s(x)\tau\Big) + \int_{Disc} G^{\mu\nu}_{\alpha\beta}\Big(\tau,\tau_0,x,x_0,Q_s(x)\Big)\ \delta T^{\alpha\beta}(\tau_0,x_0)$$

non-equilibrium evolution of (local) average background

non-equilibrium Greens function of energy-momentum tensor

Effective kinetic theory simulations only need to be performed once to compute background evolution and Greens functions



Scaling variables

Background evolution and Greens functions still depend on variety of variables e.g. $Q_s(x)$ (local energy scale), α_s, (coupling constant) …

-> Identify appropriate scaling variables to reduce complexity

Since ultimately evolution will match onto visc. hydrodynamics, check wether hydrodynamics admits scaling solution

1st order hydro: $$T^{\tau\tau}(\tau) = T^{\tau\tau}_{Ideal}(\tau)\Big(1 - \frac{8}{3}\frac{\eta/s}{T_{eff}\tau} + ...\Big)$$

where $T^{\tau\tau}_{Ideal}(\tau)$ is the Bjorken energy density and $T_{eff} = \tau^{-1/3} \lim_{\tau\to\infty} T(\tau)\tau^{1/3}$

Natural candidate for scaling variable is $x_s = T_{eff}\tau/(\eta/s)$

(evolution time / equilibrium relaxation time)

Background — Scaling & Equilibration time

Scaling property extends well beyond hydrodynamic regime; non-equilibrium evolution of background $T^{\mu\nu}$ is a unique function of $x_s = T_{eff}\tau/(\eta/s)$

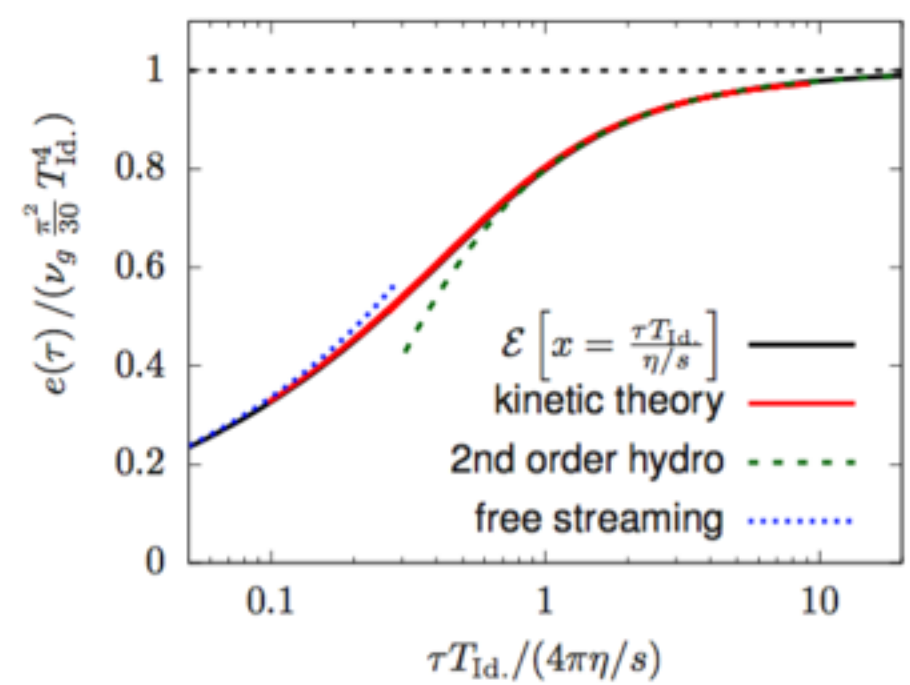
-> near equilibrium physics (η/s) determines time scale for mini-jet quenching

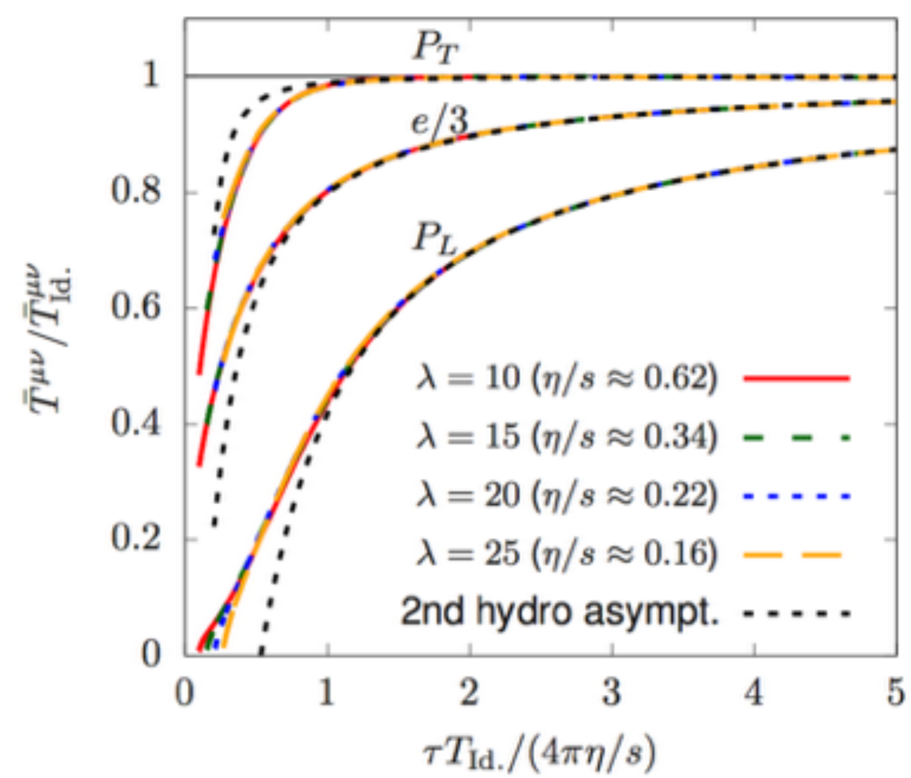
Estimate of minimal time scale for applicability of visc. hydrodynamics

$$\tau_{\text{hydro}} \approx 0.85\,\text{fm}\left(\frac{4\pi(\eta/s)}{2}\right)^{\frac{3}{2}}\left(\frac{1.6\,\text{GeV}}{\langle\tau e^{3/4}\rangle}\right)^{1/2}$$

Kurkela, Zhu PRL 115 (2015) 182301

Kurkela, Mazeliauskas, Paquet, SS, Teaney (in preparation)





Greens functions

Greens functions describe evolution of energy/momentum perturbations on top of a (locally) homogenous boost-invariant background

-> Description of perturbations in Fourier space

Decomposition in a complete basis of tensors leaves a total of 10 independent functions, e.g. for energy perturbations

energy response

$$\tilde{G}^{\tau\tau}_{\tau\tau}(\tau,\tau_0,\mathbf{k}) = \tilde{G}^{s}_{s}(\tau,\tau_0,|\mathbf{k}|) ,$$

momentum response

$$\tilde{G}^{\tau i}_{\tau\tau}(\tau,\tau_0,\mathbf{k}) = \frac{\mathbf{k}^i}{|\mathbf{k}|}\tilde{G}^{v}_{s}(\tau,\tau_0,|\mathbf{k}|) ,$$

shear stress response

$$\tilde{G}^{ij}_{\tau\tau}(\tau,\tau_0,\mathbf{k}) = \tilde{G}^{t,\delta}_{s}(\tau,\tau_0,|\mathbf{k}|)\ \delta^{ij} + \tilde{G}^{t,k}_{s}(\tau,\tau_0,|\mathbf{k}|)\ \frac{\mathbf{k}^i\mathbf{k}^j}{|\mathbf{k}|^2}$$

Numerically computed in eff. kinetic theory by solving linearized Boltzmann equation on top of non-equilibrium background

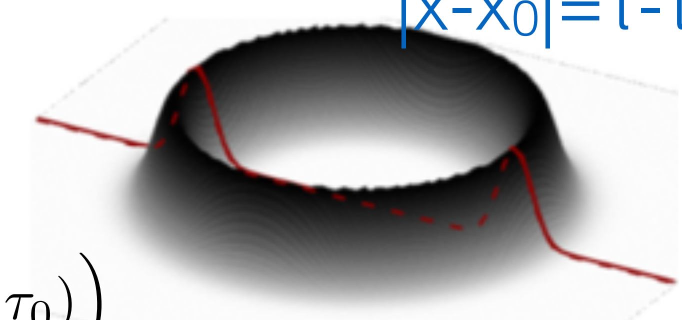
$$\left(\partial_\tau + \frac{i\mathbf{p}_\perp\mathbf{k}_\perp}{p} - \frac{p_z}{\tau}\right)\widetilde{\delta f}(\tau,|\mathbf{p}_\perp|,p_z;\ \mathbf{k}_\perp) = \delta\mathcal{C}[f,\widetilde{\delta f}]$$

same approach as in parton energy loss calculation a la MARTINI/ColBT, except now considering typical d.o.f. and non-eq background

Greens functions

Free-streaming:

coordinate space



Energy-momentum perturbations propagate as a concentric wave traveling at the speed of light

energy/momentum response:

$$G_s^{s/v}(\tau,\tau_0,\mathbf{x}-\mathbf{x}_0)=\frac{1}{2\pi(\tau-\tau_0)}\delta\Big(|\mathbf{x}-\mathbf{x}_0|-(\tau-\tau_0)\Big)$$

Hydrodynamic response functions in the limit of large times $x_s \gg 1$ and small wave-number $k\,(\tau\text{-}\tau_0) \ll x_s^{1/2}$

(c.f. Vredevoogd,Pratt PRC79 (2009) 044915, Keegan,Kurkela, Mazeliauskas, Teaney JHEP 1608 (2016) 171)

energy response: $$\tilde{G}_s^s(\tau,\tau_0,k) = \tilde{G}_s^s(\tau,\tau_0,k=0)\left(1-\frac{1}{2}k^2(\tau-\tau_0)^2\,\tilde{s}_s^{(2)}+...\right),$$

momentum response: $$\tilde{G}_s^v(\tau,\tau_0,k) = \tilde{G}_s^s(\tau,\tau_0,k=0)\left(\quad k(\tau-\tau_0)\,\tilde{s}_v^{(1)}+...\right),$$

shear response: determined by hydrodynamic constitutive relations

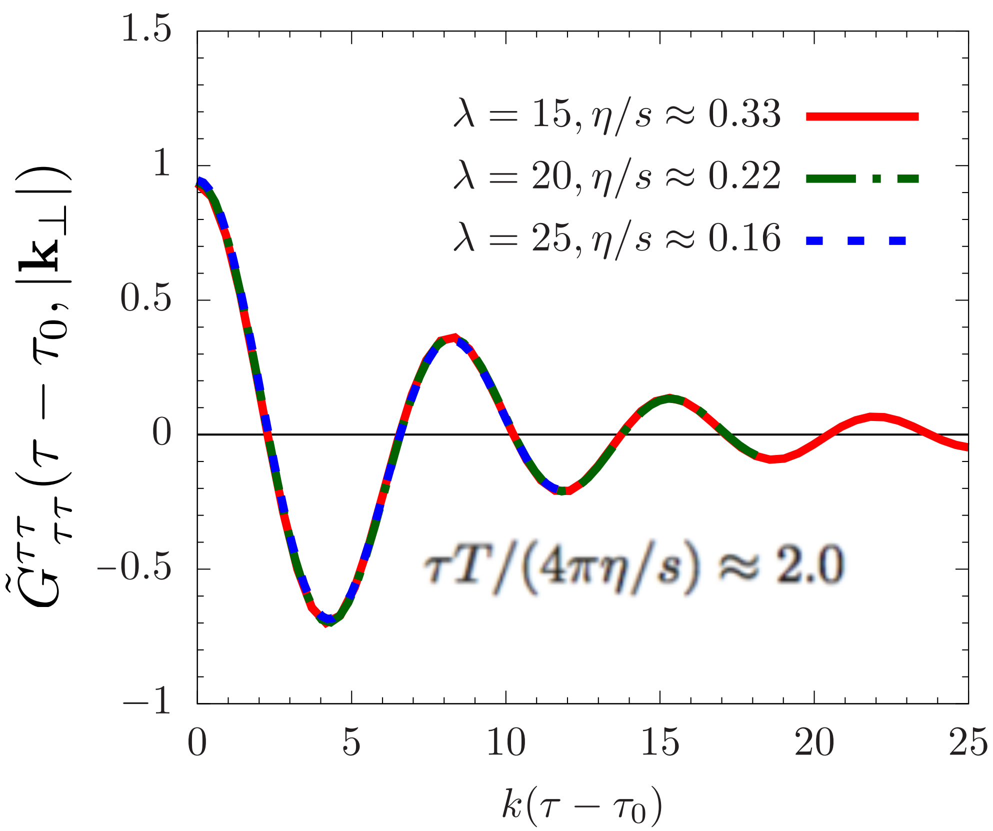
$$\tilde{G}_s^s(\tau,\tau_0,k=0)=\left(\frac{T^{\tau\tau}(\tau_0)}{T^{\tau\tau}(\tau)}\right)\left(\frac{3T^{\tau\tau}(\tau)-T^{\eta}_{\eta}(\tau)}{3T^{\tau\tau}(\tau_0)-T^{\eta}_{\eta}(\tau_0)}\right) \qquad \tilde{s}_s^{(2)}=\frac{1}{2}\,,\quad \tilde{s}_v^{(1)}=\frac{1}{2}+\frac{1}{2}\frac{\eta/s}{\tau T_{\rm id.}}\,,$$

background evolution

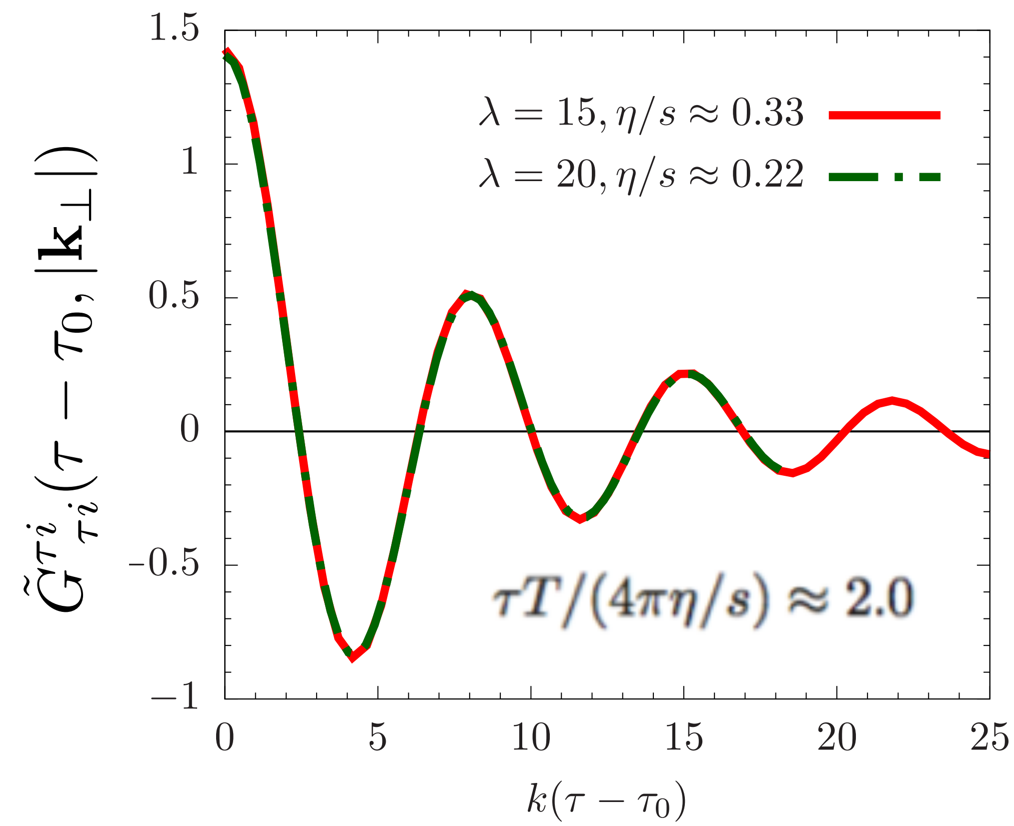
“long wave-length constants”

Greens functions — Scaling variables

Energy response
to energy perturbation



Momentum response
to momentum perturbations



Non-equilibrium Greens functions show universal scaling
in $x_s = T_{eff}\tau/(\eta/s)$ and $k(\tau - \tau_0)$ beyond hydro limit

Satisfy hydrodynamic constitutive relations for sufficiently large
times $x_s >> 1$ and long wave-length $k\,(\tau-\tau_0) << x_s^{1/2}$

KoMPoST

Scaling properties ensure that pre-equilibrium evolution of energy momentum tensor can be expressed in terms of

Background: $T^{\mu\nu}_{BG}(x_s)$ Greens-functions: $G^{\mu\nu}_{\alpha\beta}\left(x_s, \frac{x-x_0}{\tau-\tau_0}\right)$

computed once and for all in numerical kinetic theory simulation

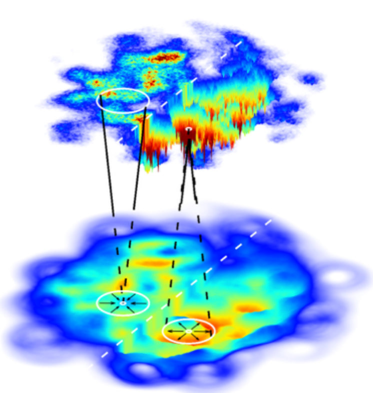
Dependence of coupling constant **α_s** has been re-expressed in terms of physical parameter **η/s**, can now perform event-by-event simulations for variety of macroscopic physical parameters

General framework for event-by-event pre-equilibrium dynamics (KoMPoST):

Input: Out-of-equilibrium energy-momentum tensor; η/s

non-equilibrium evolution in linear response formalism

Output: Energy-momentum tensor at τ_{Hydro} when visc. hydro becomes applicable



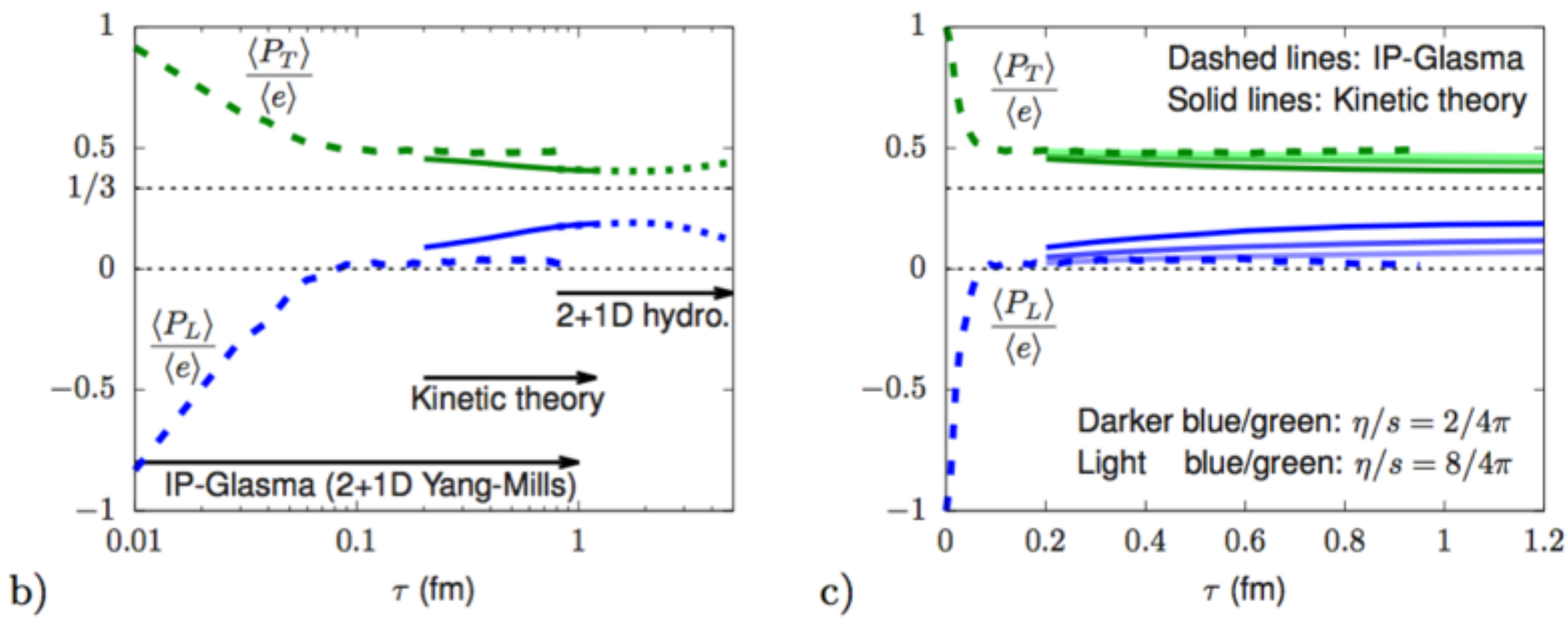
Event-by-event pre-equilibrium evolution

1) Evolve class. Yang-Mills fields to early time τ_0 = 0.2 fm/c (IP-Glasma)

2) Macroscopic pre-equilibrium evolution to hydro initialization time τ_{Hydro}

3) Hydrodynamic evolution from τ_{Hydro} ($\eta/s = 2/(4\pi)$ | conformal EoS)

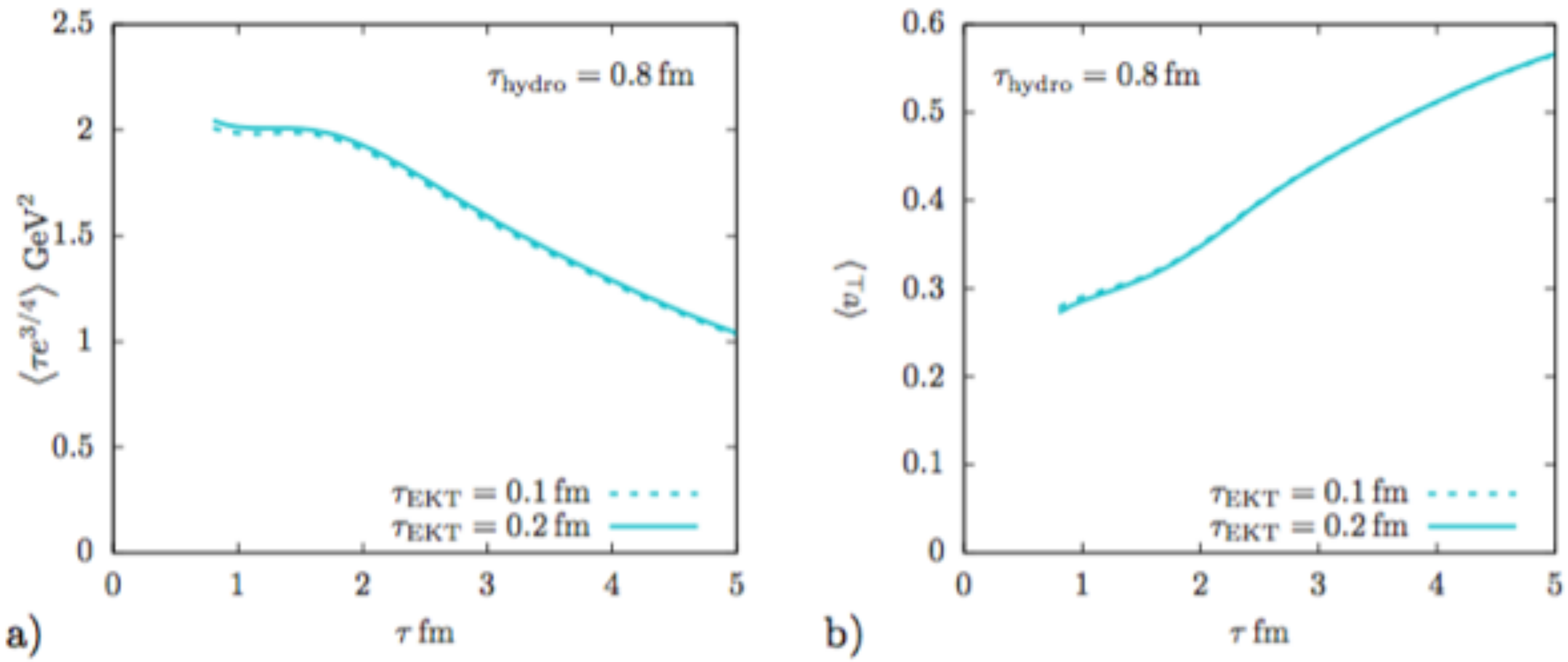
Energy/pressure evolution in central Pb+Pb collision



Based on combination of weak-coupling methods can consistently describe early-time dynamics until onset of hydro

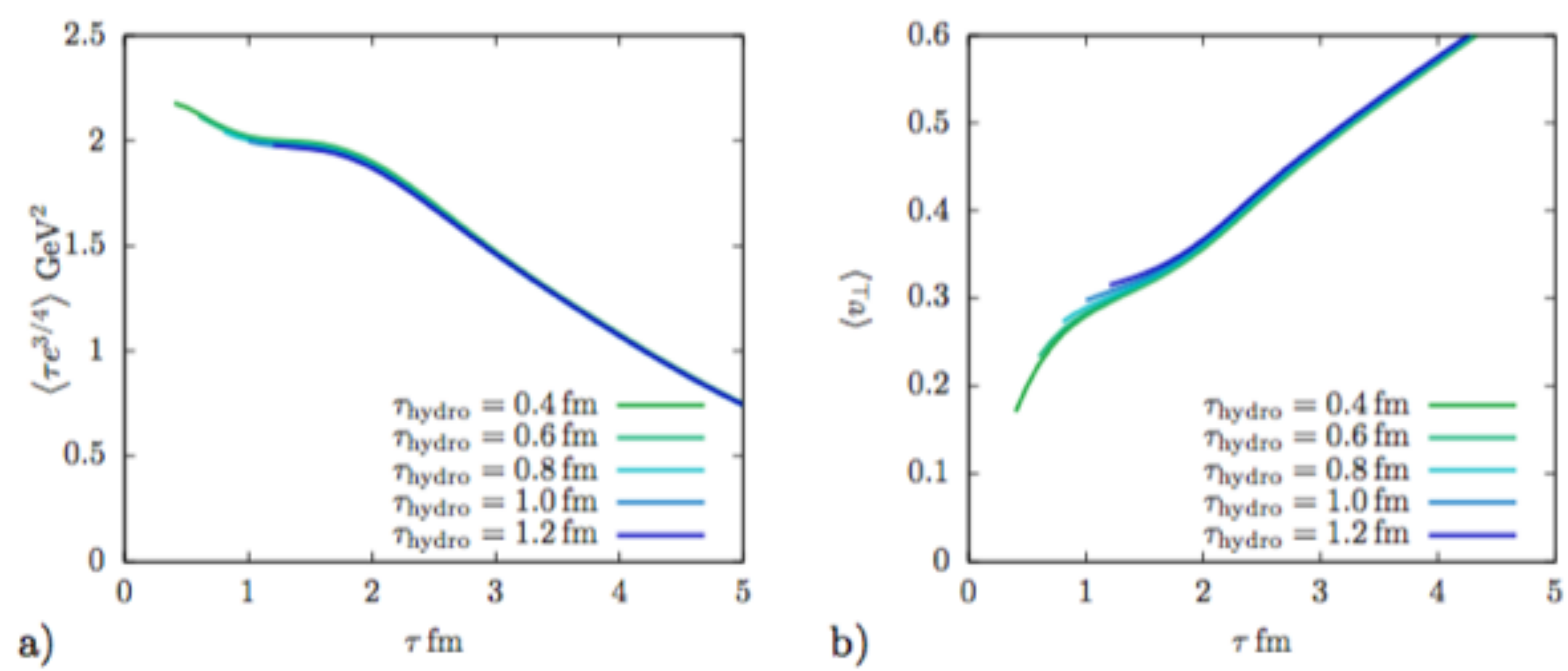
Event-by-event pre-equilibrium evolution

Energy density & radial flow in central Pb+Pb collision



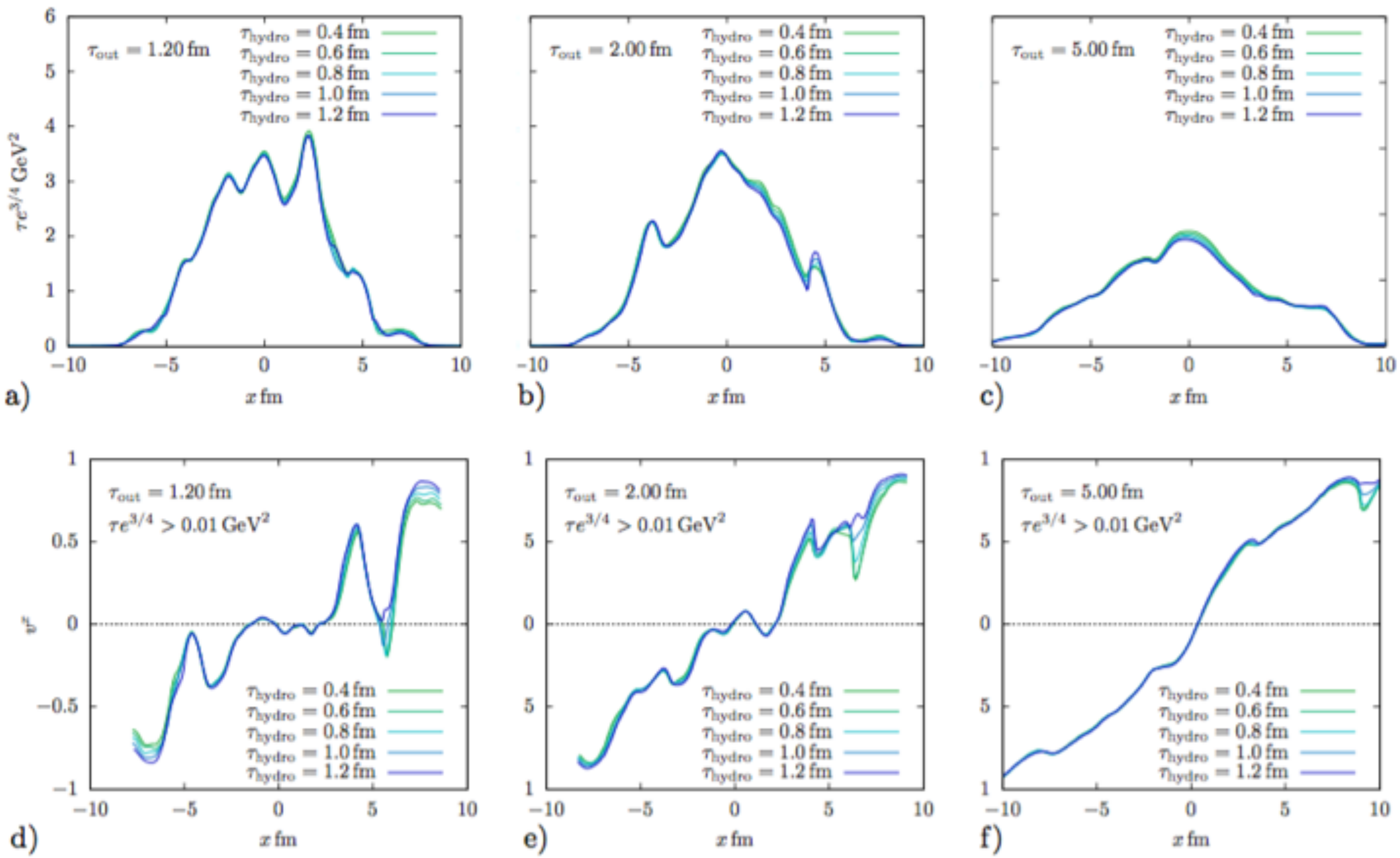
Overlap in the range of validity ensures smooth transition from CYM to EKT to Hydro

No sensitivity to switching times τ_{EKT} , τ_{Hydro} in sensible range



Event-by-event pre-equilibrium evolution

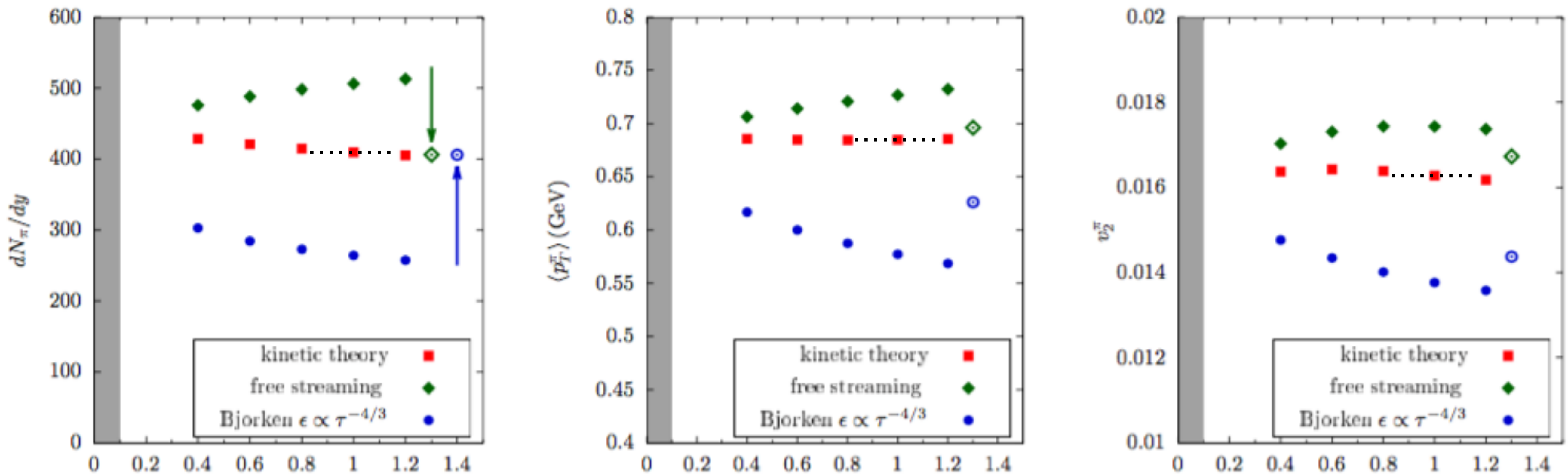
Energy density profile in Pb+Pb collision



Even with QCD EoS sensitivity to switching time τ_{Hydro} from pre-equilibrium to hydro is negligible

Event-by-event pre-equilibrium evolution

Hadronic observables in single (MC-Glauber) Pb+Pb event:



τ_{Hydro} [fm/c]

Very little to no sensitivity to switching time τ_{Hydro} from pre-equilibrium to hydro for dN/dy, <p_T>, <v_2>, …

Conclusions & Outlook

Significant progress in understanding early time dynamics of heavy-ion collisions from weak-coupling perspective

-> similarities between equilibration and parton energy loss

Development of macroscopic description of pre-equilibrium dynamics which enables event-by-event description of heavy-ion collisions from beginning to end

-> could be interesting for jet-energy disposition into medium

So far focus of equilibration studies has been on typical d.o.f. semi-hard gluons; next up

Quark production & chemical equilibration

Electro-magnetic and hard probes

Explore signatures of pre-equilibrium stage in small systems