

Jet substructure measurements in semi-inclusive jet production

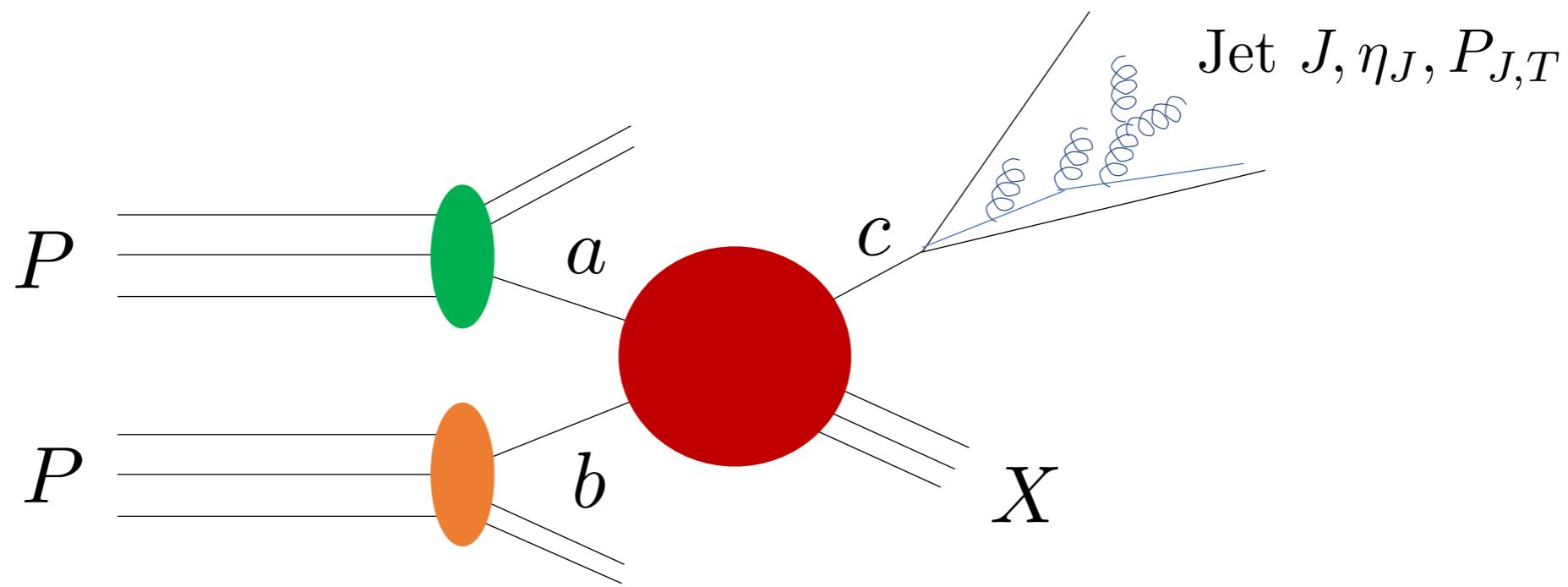
In collaboration with Zhong-Bo Kang and Felix Ringer

Kyle Lee
Stony Brook University

Santa Fe Jets and Heavy Flavor Workshop
01/29/18 - 01/31/18



Process of Interest

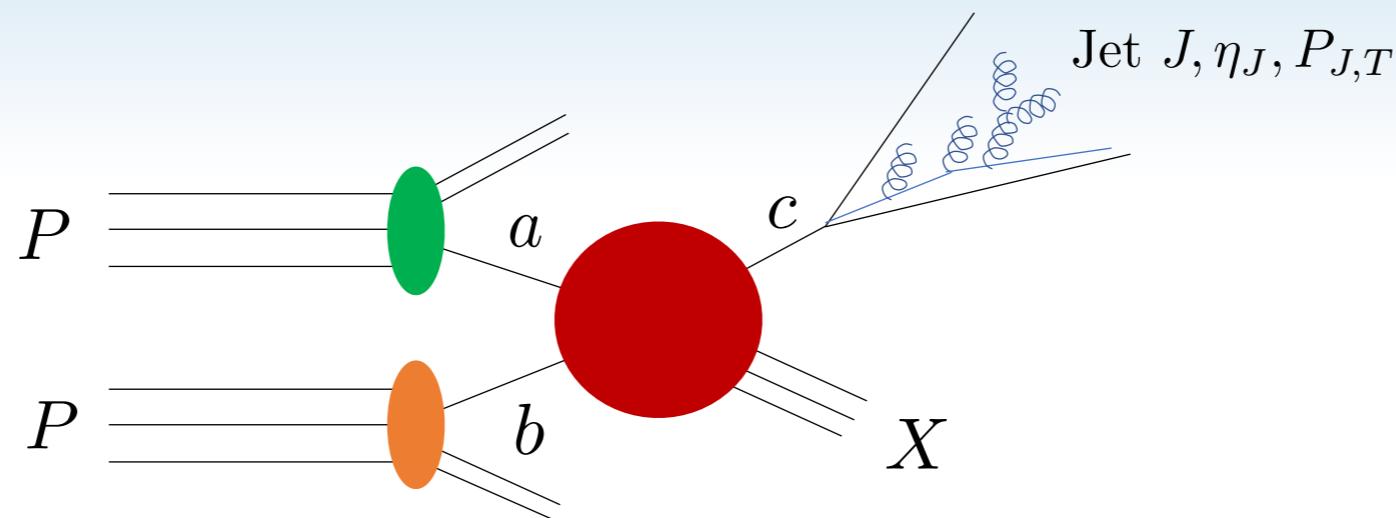


- ▶ We want to study semi-inclusive jet production event:
 $p + p \rightarrow \text{Jet}((\text{with/without}) \text{ substructure}) + X$
- ▶ More statistics. No veto on additional jets.

Plans of this talk

- Inclusive jet production (no substructure)
- Substructure measurements
 - hadron-in-jet (brief)
- Angularities (1801.00790)
 - quark and gluon discrimination
 - Relation to inclusive jet
 - Phenomenology
- Conclusions

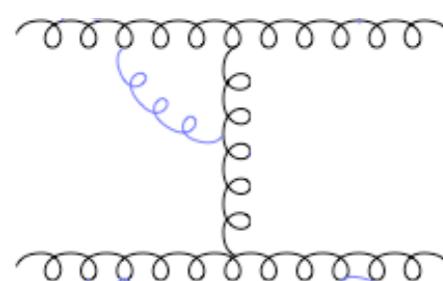
Inclusive Jet Production



- A completely perturbative process (IR safe), perturbative computation can be made.
- Fixed order calculation gives :

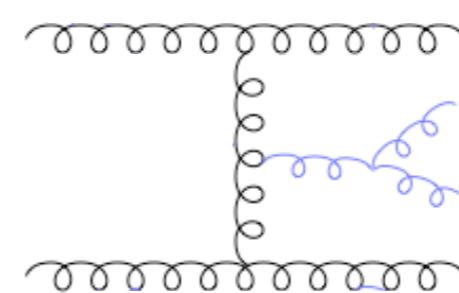
$$E \frac{d\sigma^{pp \rightarrow \text{jet}X}}{dp_T d\eta} \propto \sum_{a,b} \int \frac{dx_a}{x_a} f_a^p(x_a) \int \frac{dx_b}{x_b} f_b^p(x_b) H_{ab}$$

Inclusive Jet Production



NLO 1990

Ellis, Kunszt, Soper '90



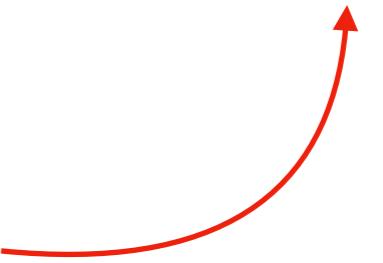
NNLO 2016

Currie, Glover, Pires '16

- A completely perturbative process (IR safe), perturbative computation can be made.
- Fixed order calculation gives :

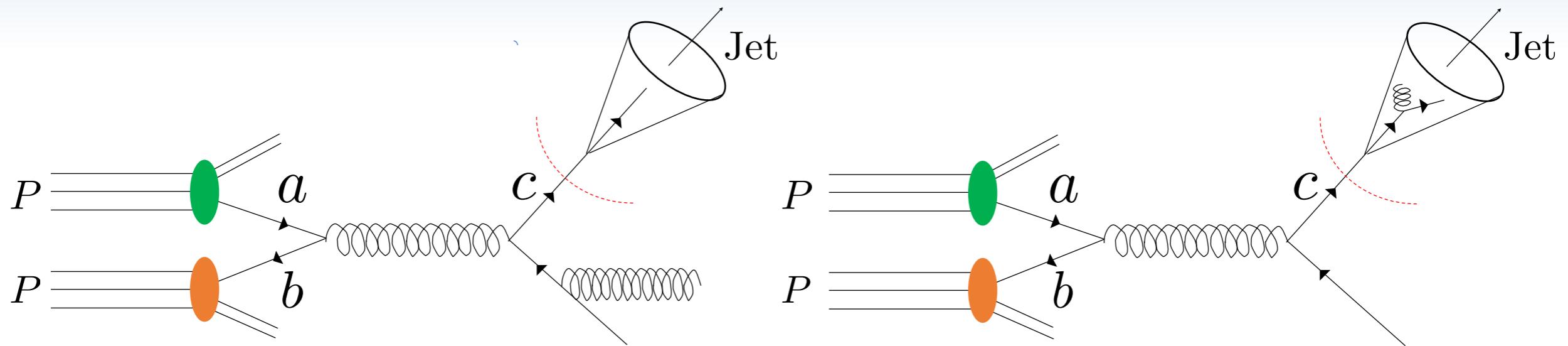
$$E \frac{d\sigma^{pp \rightarrow \text{jet}X}}{dp_T d\eta} \propto \sum_{a,b} \int \frac{dx_a}{x_a} f_a^p(x_a) \int \frac{dx_b}{x_b} f_b^p(x_b) \boxed{H_{ab}}$$

$$H_{ab} = \alpha_s^2 \left(H_{ab}^{(0)} + \alpha_s H_{ab}^{(1)} + \alpha_s^2 H_{ab}^{(2)} + \dots \right)$$



- $\boxed{H_{ab}}$ has $(\alpha_s \ln R)^n$ which must be resummed.

Factorization



Example of NLO diagrams

- Relevant scales :

1. Hard scale: $\mu_H \sim p_T$

2. Jet scale: $\mu_J \sim p_T R$

- For small-R jet, we have hierarchy between the two different scales and jet cross-section is factorized, $H_{ab} \rightarrow \sum_c \int \frac{dz_c}{z_c^2} H_{ab}^c J_c(z_c)$, giving

$$E \frac{d\sigma^{pp \rightarrow \text{jet} X}}{dp_T d\eta} \propto \sum_{a,b,c} \int \frac{dx_a}{x_a} f_a^p(x_a) \int \frac{dx_b}{x_b} f_b^p(x_b) \int \frac{dz_c}{z_c^2} H_{ab}^c J_c(z_c)$$

Semi-inclusive jet function $J_c(z_c, p_T R, \mu)$

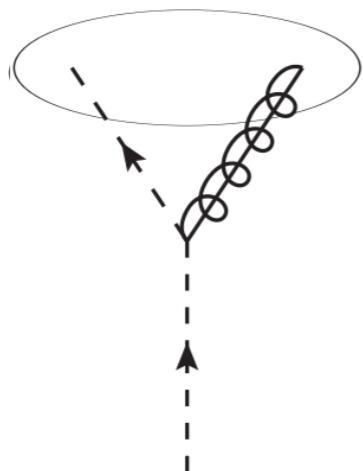
- Using Effective-field theory, we can write operator definition of J_c :

$$J_q(z, p_T R, \mu) = \frac{z}{2N_c} \text{Tr} \left[\frac{\gamma \cdot \bar{n}}{2} \langle 0 | \delta(\omega - \bar{n} \cdot \mathcal{P}) \chi_n(0) | JX \rangle \langle JX | \bar{\chi}_n(0) | 0 \rangle \right]$$

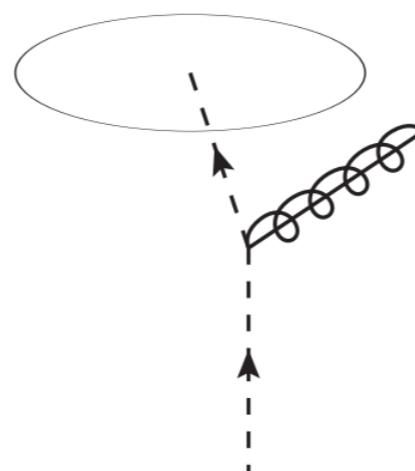
$$J_g(z, p_T R, \mu) = -\frac{z \omega}{2(N_c^2 - 1)} \langle 0 | \delta(\omega - \bar{n} \cdot \mathcal{P}) \mathcal{B}_{n \perp \mu}(0) | JX \rangle \langle JX | \mathcal{B}_{n \perp}^\mu(0) | 0 \rangle,$$

Kang, Ringer, Vitev '16

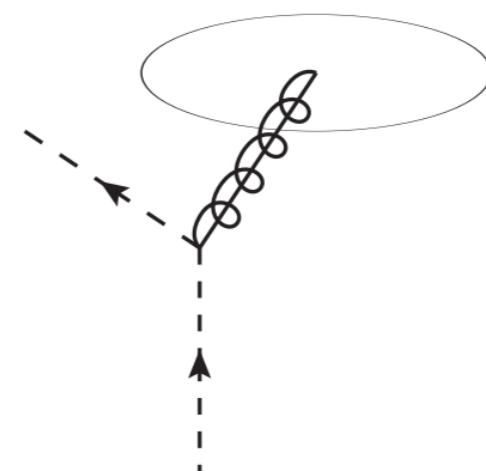
- At NLO, (quark initiated) we have the following diagrams :



(A)



(B)



(C)

Semi-inclusive jet function $J_c(z_c, p_T R, \mu)$

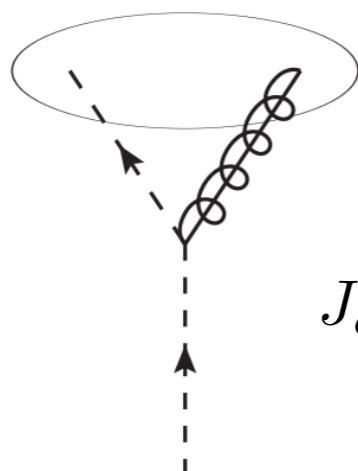
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Kang, Ringer, Vitev '16

- At NLO, (quark initiated) we have the following diagrams : ($L = \ln \frac{\mu^2}{p_T^2 R^2}$)



(A)

$$J_{q \rightarrow qg}(z, p_T R) = \delta(1-z) \frac{\alpha_s}{2\pi} \left[C_F \left(\frac{1}{\epsilon^2} + \frac{3}{2\epsilon} + \frac{1}{\epsilon} L + \left(\frac{1}{2} L^2 + \frac{3}{2} L \right) \right) + d_q^{\text{alg}} \right]$$

Semi-inclusive jet function $J_c(z_c, p_T R, \mu)$

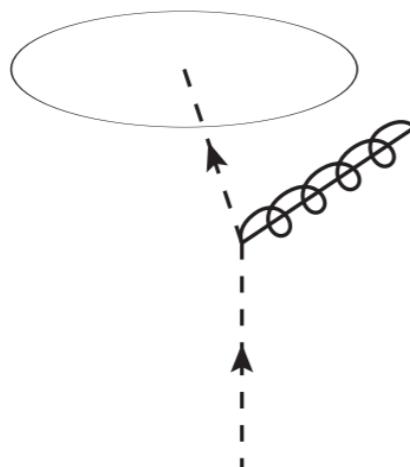
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Kang, Ringer, Vitev '16

- At NLO, (quark initiated) we have the following diagrams : ($L = \ln \frac{\mu^2}{p_T^2 R^2}$)



(B)

$$\begin{aligned} J_{q \rightarrow q(g)}(z, p_T R) = & \frac{\alpha_s C_F}{2\pi} \left[\delta(1-z) \left(-\frac{1}{\epsilon^2} - \frac{1}{\epsilon} L - \frac{1}{2} L^2 + \frac{\pi^2}{12} \right) \right. \\ & \left. + \left(\frac{1}{\epsilon} + L \right) \frac{1+z^2}{(1-z)_+} - 2(1+z^2) \left(\frac{\ln(1-z)}{1-z} \right)_+ - (1-z) \right] \end{aligned}$$

Semi-inclusive jet function $J_c(z_c, p_T R, \mu)$

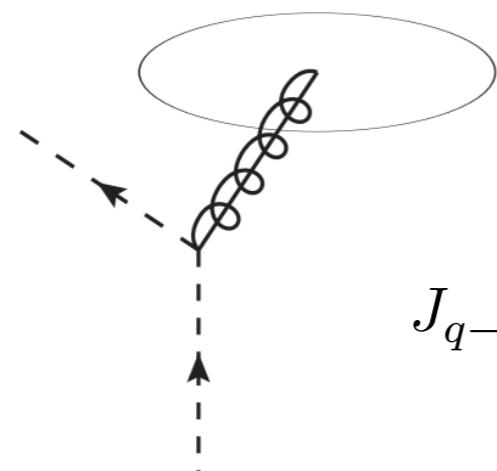
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Kang, Ringer, Vitev '16

- At NLO, (quark initiated) we have the following diagrams : ($L = \ln \frac{\mu^2}{p_T^2 R^2}$)



$$J_{q \rightarrow (q)g}(z, p_T R) = \frac{\alpha_s}{2\pi} \left(\frac{1}{\epsilon} + L \right) P_{gq}(z) - \frac{\alpha_s}{2\pi} [P_{gq}(z) 2 \ln(1-z) + C_F z]$$

(C)

- Adding the contributions, we have cancellation of $\frac{1}{\epsilon^2}$ and L^2 :

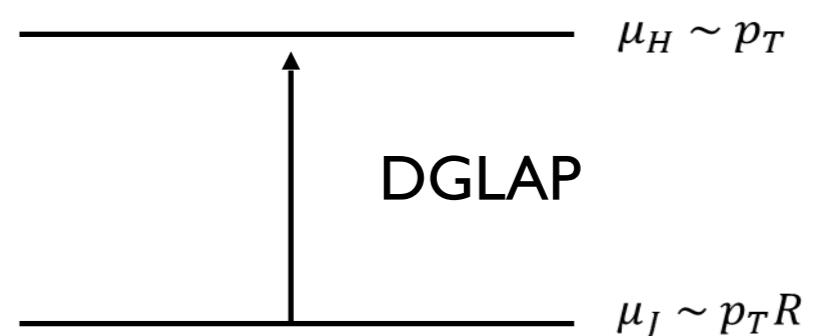
$$\begin{aligned} J_q(z, p_T R) &= J_{q \rightarrow qg}(z, p_T R) + J_{q \rightarrow q(g)}(z, p_T R) + J_{q \rightarrow (q)g}(z, p_T R) \\ &\supset \frac{\alpha_s C_F}{2\pi} \frac{1}{\epsilon} (P_{qq}(z) + P_{gq}(z)) \\ J_g(z, p_T R) &\supset \frac{\alpha_s C_F}{2\pi} \frac{1}{\epsilon} (P_{gg}(z) + 2n_f P_{qg}(z)) \end{aligned}$$

- The UV poles give DGLAP RG equation for J_c :

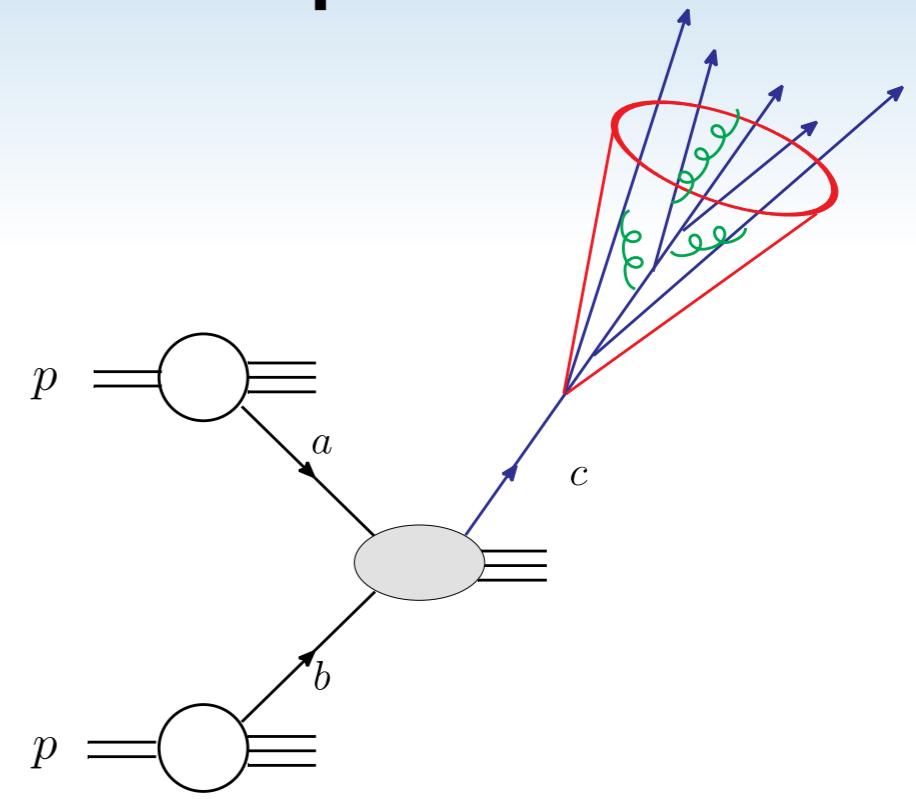
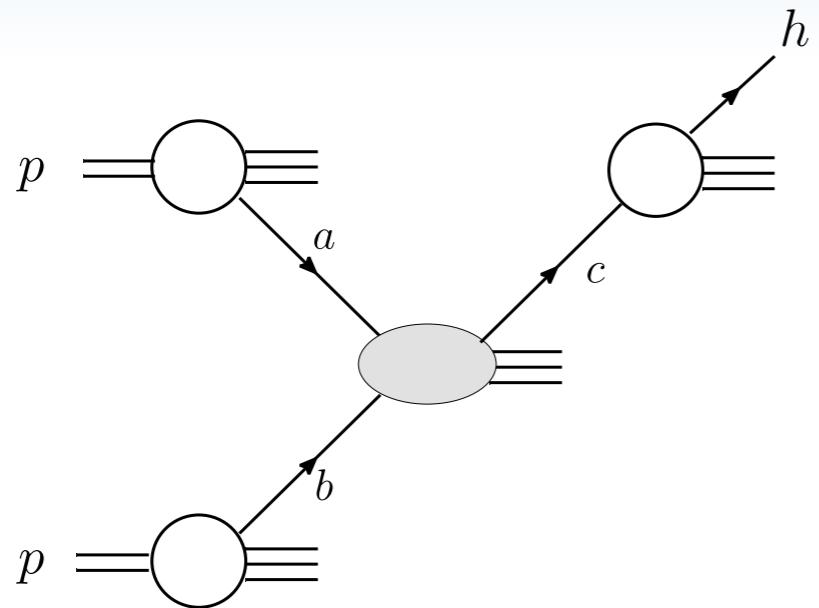
$$\mu \frac{d}{d\mu} J_i(z, p_T R, \mu) = \frac{\alpha_s(\mu)}{\pi} \sum_j \int_z^1 \frac{dz'}{z'} P_{ji}\left(\frac{z}{z'}, \mu\right) J_j(z', p_T R, \mu)$$

where P_{ij} are the standard splitting functions.

- Evolution from μ_J to μ_H gives $(\alpha_s \ln R)^n$ resummations.
- See Liu, Moch, Ringer '18 for joint resummation of threshold log and jet radius logs, and phenomenology.



Comparison with the inclusive hadron production case



Factorization

Jet

$$\frac{d\sigma^{pp \rightarrow \text{jet}X}}{dp_T d\eta} = \sum_{a,b,c} f_a \otimes f_b \otimes H_{ab}^c \otimes J_c + \mathcal{O}(R^2)$$

Hadron

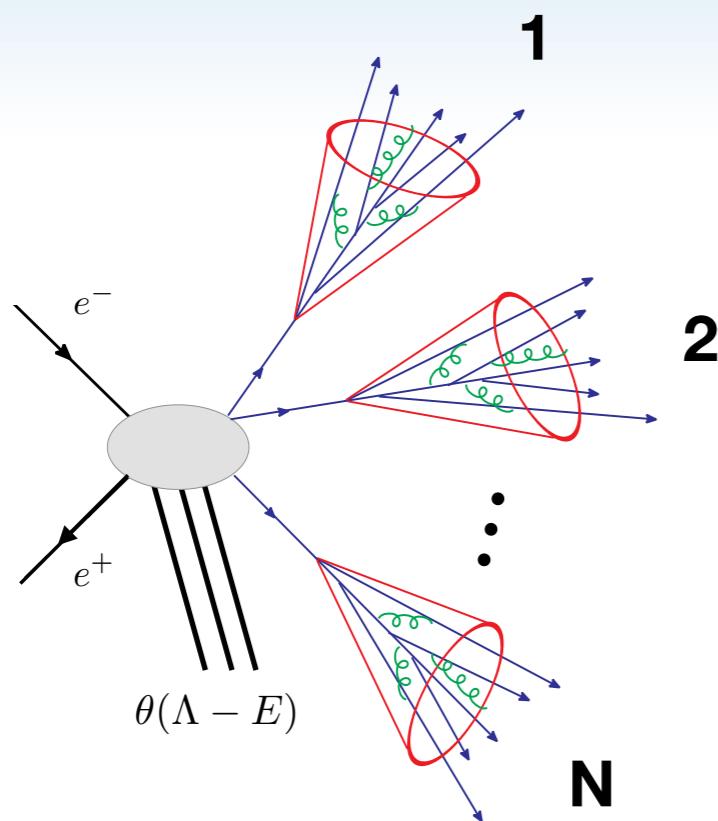
$$\frac{d\sigma^{pp \rightarrow hX}}{dp_T d\eta} = \sum_{a,b,c} f_a \otimes f_b \otimes H_{ab}^c \otimes D_c^h$$

Evolution

$$\mu \frac{d}{d\mu} J_i = \sum_j P_{ji} \otimes J_j$$

$$\mu \frac{d}{d\mu} D_i^h = \sum_j P_{ji} \otimes D_j^h$$

Comparison with the exclusive jet production case

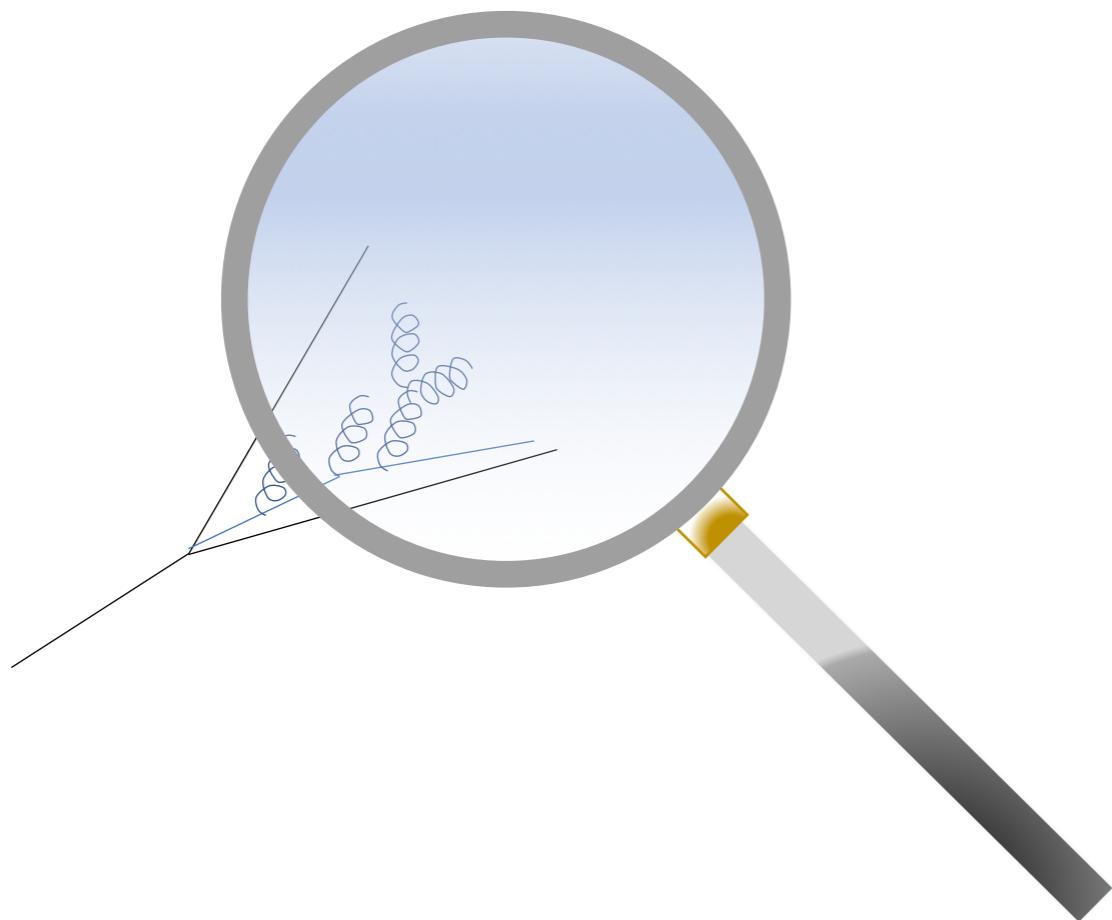


$$d\sigma^{e^+e^- \rightarrow \text{jet}_1 \dots \text{jet}_N} \propto H_{1,2,\dots,N} \otimes J_1^{\text{excl}} \dots J_N^{\text{excl}} S$$

Ellis, Vermilion, Walsh, Hornig, Lee '10

- The operator definition of the J_c^{excl} is similar to the semi-inclusive case, except it has an additional restriction $\delta_{(N(X_n)-1)}$, which restricts amount of energy outside the jet.

Jet Substructure Measurements

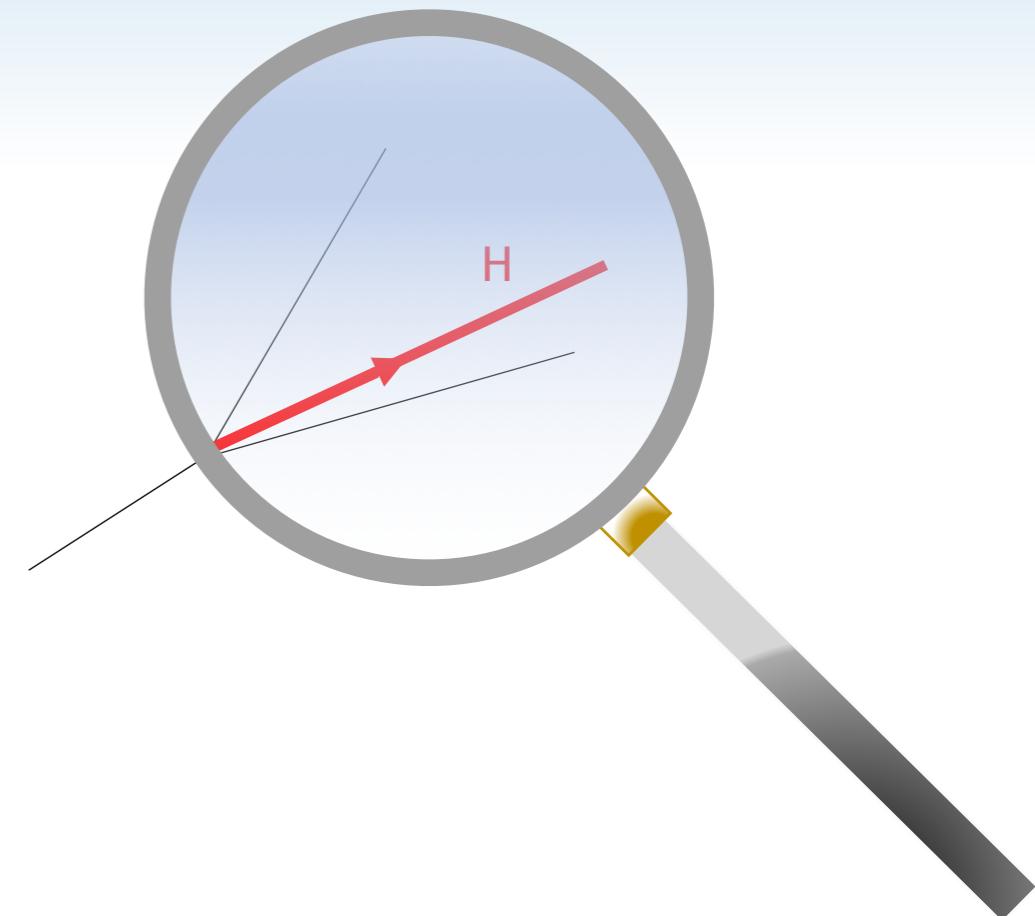
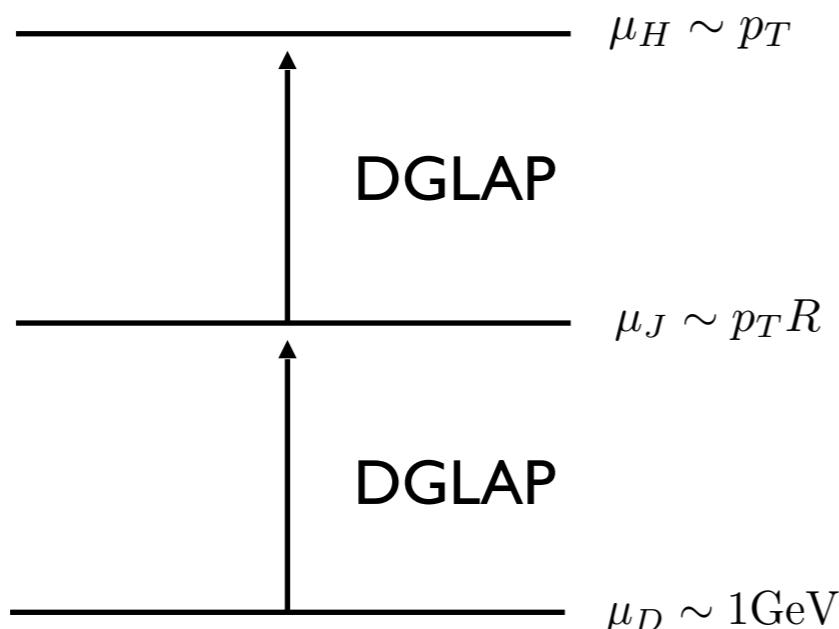


Hadron in jet

- We may try to observe a hadron H inside the jet
- $J_c(z, p_T R, \mu) \rightarrow \mathcal{G}_c^H(z, z_h, p_T R, \mu)$
- \mathcal{G}_c^H no more completely perturbative:

$$\mathcal{G}_i^h(z, z_h, \mu) = \sum_j \int_{z_h}^1 \frac{dz'_h}{z'_h} \mathcal{J}_{ij}(z, z'_h, \mu) D_j^h\left(\frac{z_h}{z'_h}, \mu\right)$$

where $z_h = \frac{\omega_h}{\omega_J}$
- Two DGLAPS



Hadron in jet

Light Charged Hadrons

- We may try to observe a hadron H inside the jet

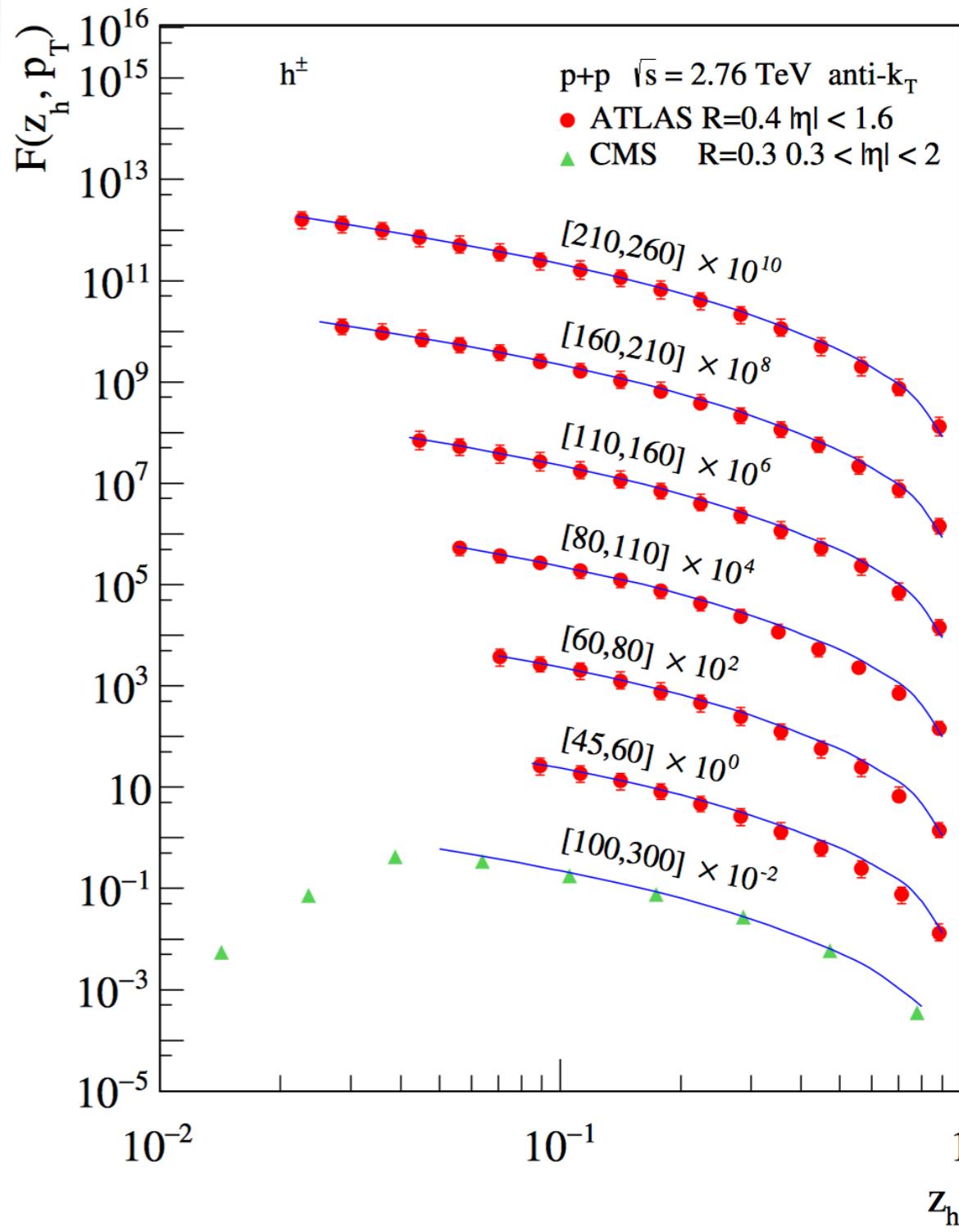
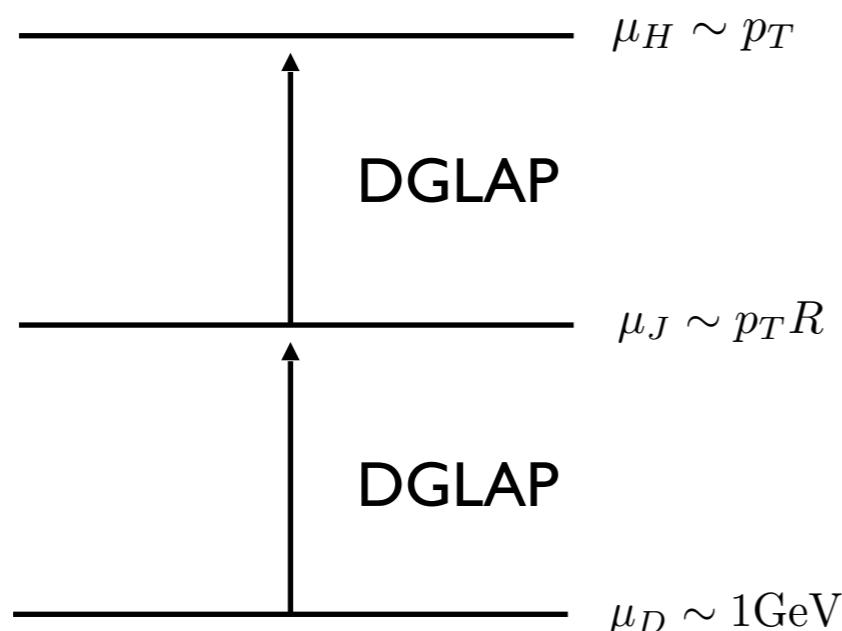
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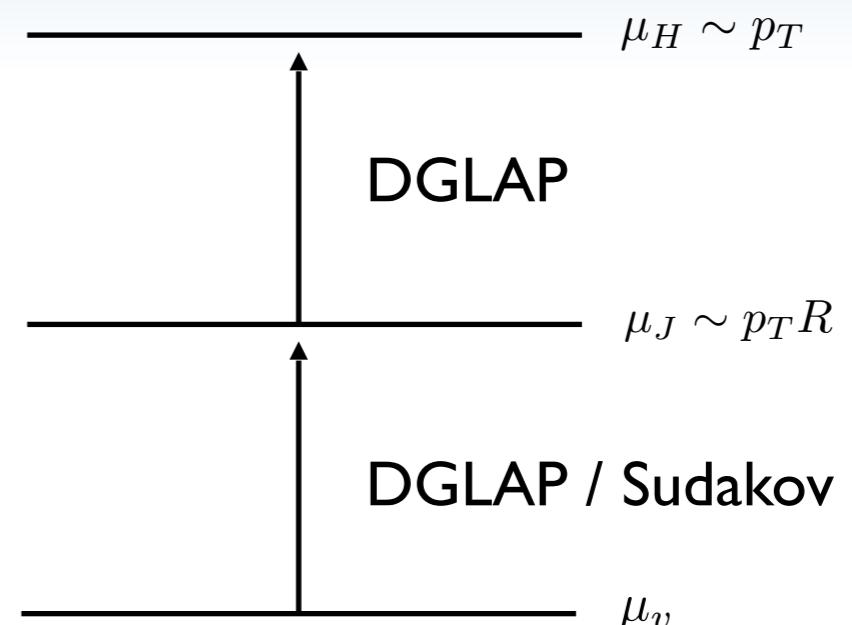
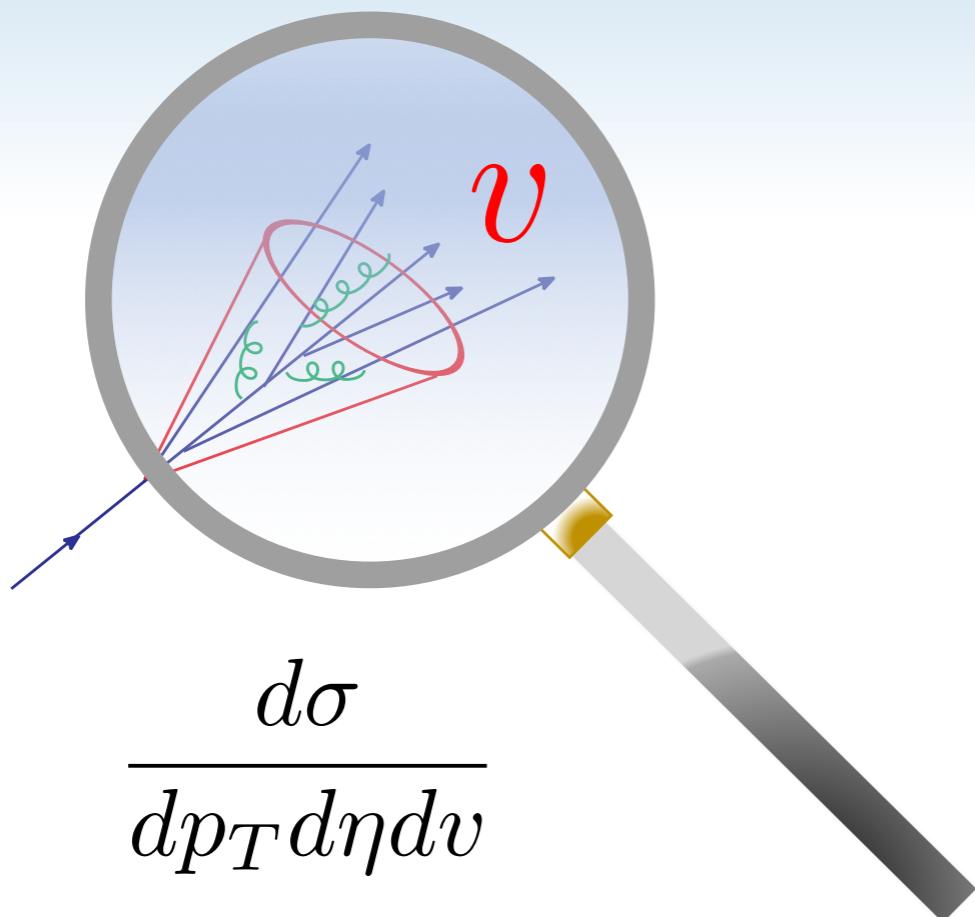
$$\mathcal{G}_i^h(z, z_h, \mu) = \sum_j \int_{z_h}^1 \frac{dz'_h}{z'_h} \mathcal{J}_{ij}(z, z'_h, \mu) D_j^h\left(\frac{z_h}{z'_h}, \mu\right)$$

where $z_h = \frac{\omega_h}{\omega_J}$

- Two DGLAPs



Patterns emerging



- When we measure substructure v from the jet, once we evolve to μ_J the remaining evolution to μ_H is given by DGLAP evolution!
- Two step factorization:
 - a) production of a jet
 - b) probing the internal structure of the jet produced.

Jet angularity

- Thrust was defined as an event shape parameter to understand radiation pattern

$$T = \frac{1}{Q} \max_{\mathbf{t}} \sum_{i \in X} |\mathbf{t} \cdot \mathbf{p}_i| = 1 - \tau_0$$

- $\tau_0 \rightarrow 0$ is equivalent to dijet limit
- A generalized class of IR safe observables, angularity (applied to jet):

$$\tau_a^{e^+ e^-} = \frac{1}{E_J} \sum_{i \in J} E_i \theta_{iJ}^{2-a}$$

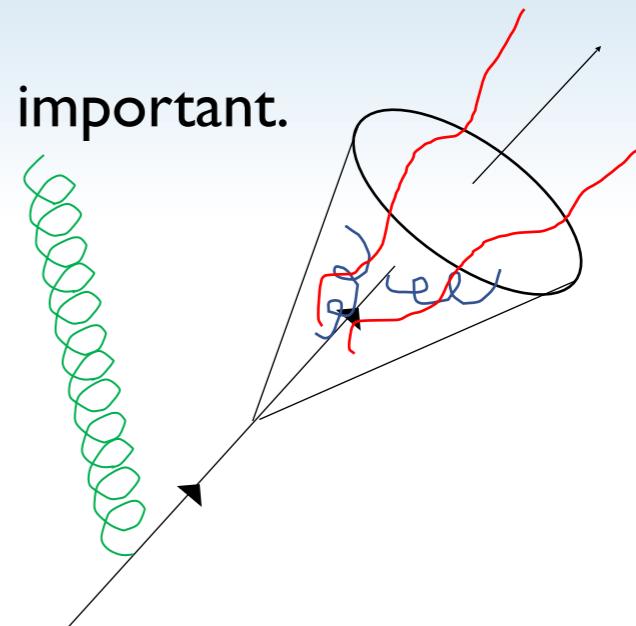
$$\tau_a^{pp} = \frac{1}{p_T} \sum_{i \in J} p_{T,i} (\Delta R_{iJ})^{2-a}$$

- a=0 related to thrust (jet mass)
- a=1 related to jet broadening (sensitive to rapidity divergence)
- Many studies done for exclusive case : *Sterman et al. '03, '08,
Hornig, C. Lee, Ovanesyan '09, Ellis, Vermilion, Walsh, Hornig, C. Lee '10,
Chien, Hornig, C. Lee '15, Hornig, Makris, Mehen '16*

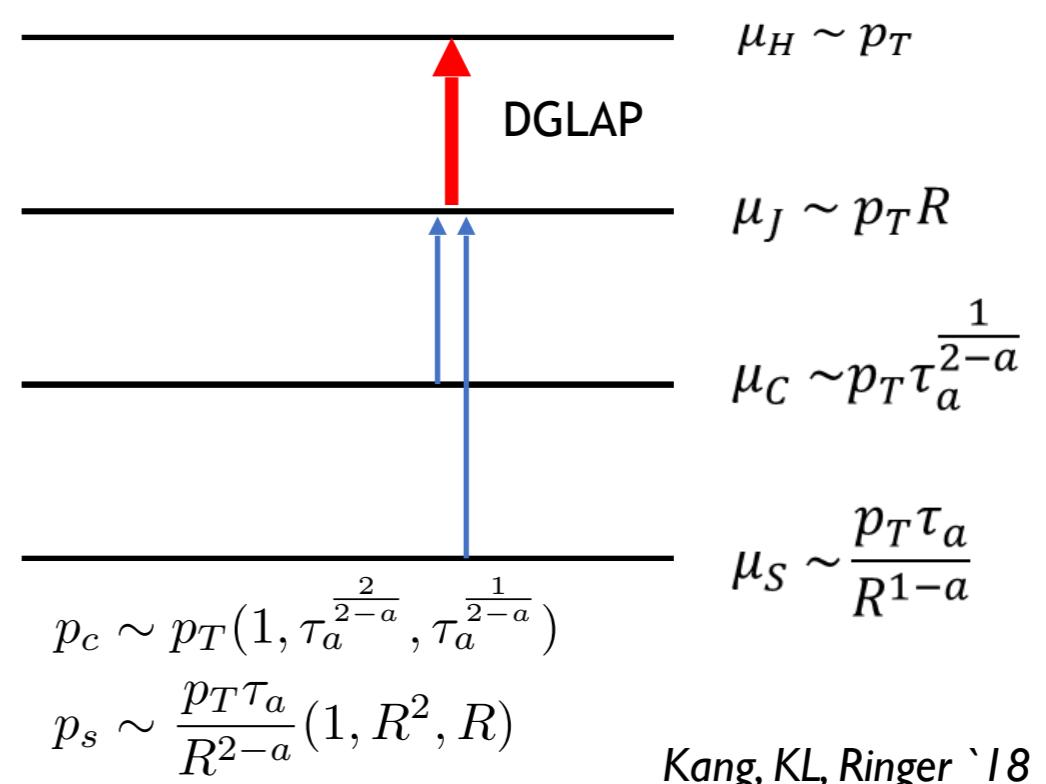
Jet angularity

- When $\tau_a^{\frac{1}{2-a}} \ll R$, $\left(\alpha_s \ln^2 \left(\tau_a^{\frac{1}{2-a}} / R\right)\right)^n$ resummation becomes important.
- Replace $J_c(z, p_T R, \mu) \rightarrow \mathcal{G}_c(z, p_T R, \tau_a, \mu)$
- Refactorize \mathcal{G}_c as

$$\begin{aligned} \mathcal{G}_c(z, p_T R, \tau_a, \mu) &= \sum_i \mathcal{H}_{c \rightarrow i}(z, p_T R, \mu) \\ &\times \int d\tau_a^{C_i} d\tau_a^{S_i} \delta(\tau_a - \tau_a^{C_i} - \tau_a^{S_i}) \mathbf{C}_i(\tau_a^{C_i}, p_T \tau_a^{\frac{1}{2-a}}, \mu) \mathbf{S}_i(\tau_a^{S_i}, \frac{p_T \tau_a}{R^{1-a}}, \mu) \end{aligned}$$



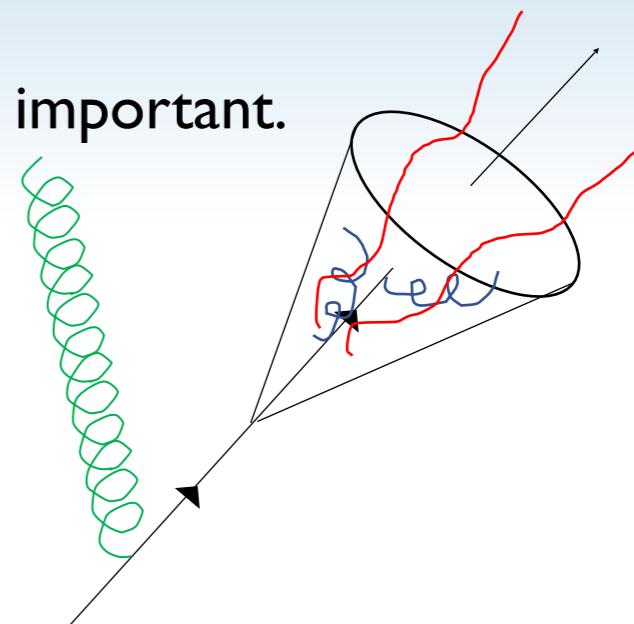
- Each pieces describe physics at different scales.
- $\mu_J \rightarrow \mu_H$ evolution follows DGLAP evolution equation again
- Jointly resums $(\alpha_s \ln R)^n$ and $(\alpha_s \ln^2 \frac{R}{\tau_a^{1/(2-a)}})^n$



Jet angularity

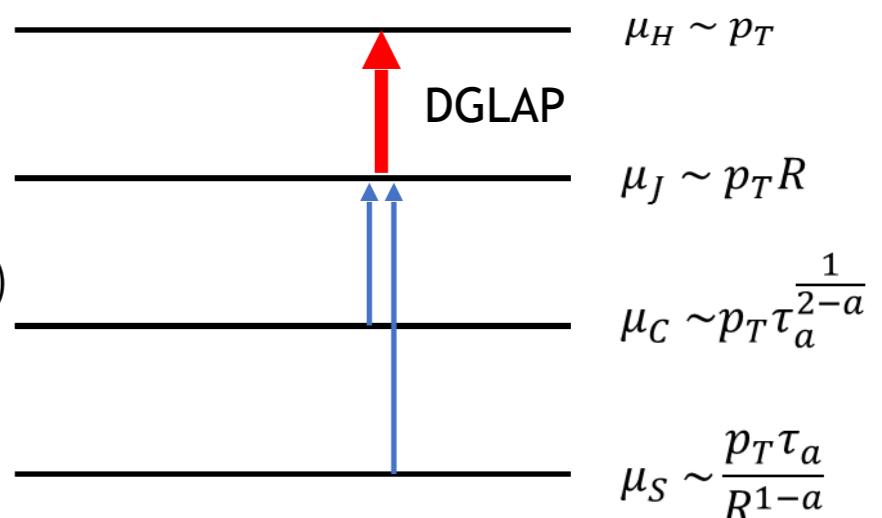
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- $H_{c \rightarrow i}$, C_i and S_i have double poles, which cancel once evolved to μ_J .
- $\mathcal{G}_c(z, p_T R, \tau_a, \mu)$ follows DGLAP from μ_J to μ_H :

$$\mu \frac{d}{d\mu} \mathcal{G}_i(z, p_T R, \tau_a, \mu) = \frac{\alpha_s(\mu)}{\pi} \sum_j \int_z^1 \frac{dz'}{z'} P_{ji}\left(\frac{z}{z'}, \mu\right) \mathcal{G}_j(z', p_T R, \tau_a, \mu)$$



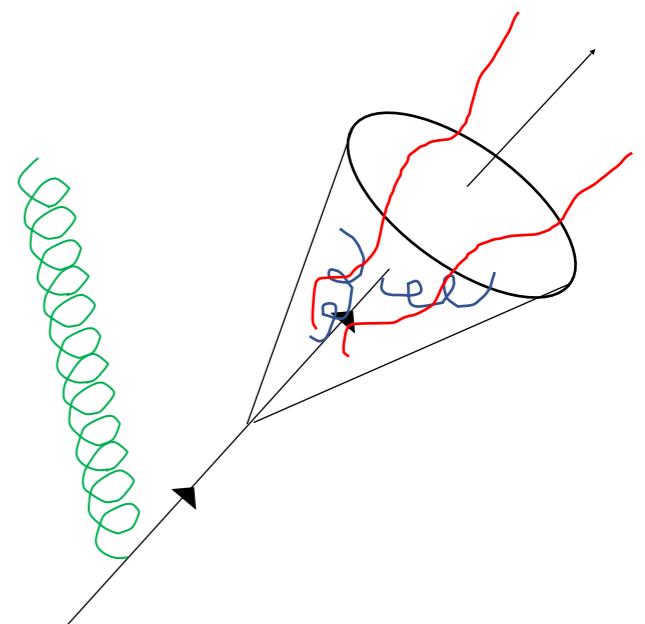
Relation to inclusive jet function

$$\int \frac{d\sigma}{dp_T d\eta d\tau_a} d\tau_a = \frac{d\sigma}{dp_T d\eta} \Leftrightarrow \int_0^\infty d\tau_a \mathcal{G}_i(z, p_T, R, \tau_a, \mu) = J_i(z, p_T, R, \mu)$$

See also Chien, Hornig, C. Lee '15

- Correct integration over substructure should give inclusive case.

$$\mathcal{G}_c(z, p_T R, \tau_a, \mu) = \sum_i \mathcal{H}_{c \rightarrow i}(z, p_T R, \mu) \int d\tau_a^{C_i} d\tau_a^{S_i} \delta(\tau_a - \tau_a^{C_i} - \tau_a^{S_i}) \mathcal{C}_i(\tau_a^{C_i}, p_T \tau_a^{\frac{1}{2-a}}, \mu) \mathcal{S}_i(\tau_a^{S_i}, \frac{p_T \tau_a}{R^{1-a}}, \mu)$$



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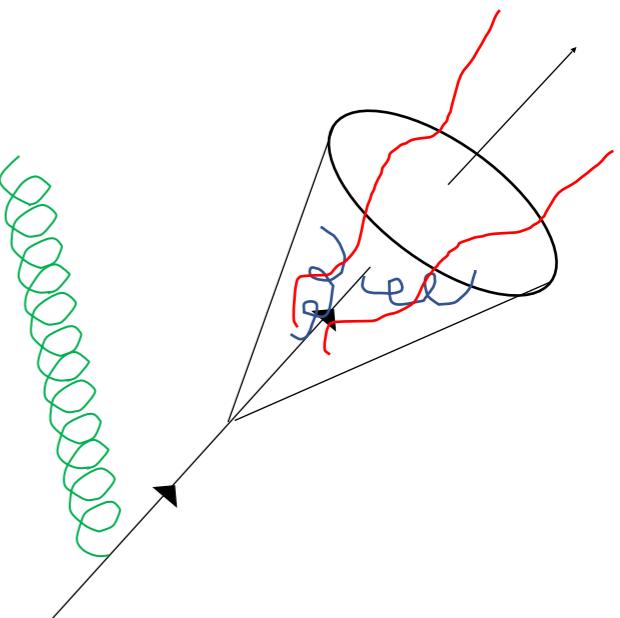
- Correct integration over substructure should give inclusive case.

$$\begin{aligned} \mathcal{G}_c(z, p_T R, \tau_a, \mu) &= \sum_i \mathcal{H}_{c \rightarrow i}(z, p_T R, \mu) \int d\tau_a^{C_i} d\tau_a^{S_i} \delta(\tau_a - \tau_a^{C_i} - \tau_a^{S_i}) \mathcal{C}_i(\tau_a^{C_i}, p_T \tau_a^{\frac{1}{2-a}}, \mu) \mathcal{S}_i(\tau_a^{S_i}, \frac{p_T \tau_a}{R^{1-a}}, \mu) \\ \mathcal{H}_{q \rightarrow q}(z, p_T R, \mu) &= \delta(1-z) - \frac{\alpha_s C_F}{2\pi} \left\{ \delta(1-z) \left[\frac{1}{\epsilon^2} + \frac{3}{2} \frac{1}{\epsilon} + \frac{\ln \left(\frac{\mu^2}{p_T^2 R^2} \right)}{\epsilon} \right] \right. \\ &\quad \left. + \frac{1}{2} \ln^2 \left(\frac{\mu^2}{p_T^2 R^2} \right) + \frac{3}{2} \ln \left(\frac{\mu^2}{p_T^2 R^2} \right) - \frac{\pi^2}{12} \right] \\ &\quad + 2 \left((1+z^2) \frac{\ln(1-z)}{1-z} \right)_+ + (1-z) - P_{qq}(z) \left(\frac{1}{\epsilon} + \ln \left(\frac{\mu^2}{p_T^2 R^2} \right) \right) \right\} \\ \mathcal{H}_{g \rightarrow q}(z, p_T R, \mu) &= \frac{\alpha_s}{2\pi} P_{gq}(z) \left(\frac{1}{\epsilon} + \ln \left(\frac{\mu^2}{p_T^2 R^2} \right) - \ln(1-z)^2 \right) - \frac{\alpha_s C_F}{2\pi} z \\ \mathcal{C}_q(\tau_0, p_T, R, \mu) &= \mathcal{C}_q^{\text{l.p.}}(\tau_0, p_T, \mu) + \Delta C_q^{\text{alg}}(\tau_0, R) \end{aligned}$$

$$\begin{aligned} \int_0^{\tau_0^{\max}} d\tau_0 \mathcal{C}_q^{\text{l.p.}}(\tau_0, p_T, \mu) &= 1 + \frac{\alpha_s C_F}{2\pi} \left\{ \frac{2}{\epsilon^2} + \frac{3}{2\epsilon} - \frac{2}{\epsilon} \ln \left(\frac{\tau_0^{\max} p_T^2}{\mu^2} \right) + \ln^2 \left(\frac{\tau_0^{\max} p_T^2}{\mu^2} \right) \right. \\ &\quad \left. - \frac{3}{2} \ln \left(\frac{\tau_0^{\max} p_T^2}{\mu^2} \right) + \frac{7}{2} - \frac{\pi^2}{2} \right\} \end{aligned}$$

$$\int_0^{\tau_0^{\max}} d\tau_0 \Delta C_q^{k_T}(\tau_0, R) = \frac{\alpha_s C_F}{2\pi} \left(3 - \frac{\pi^2}{3} - 3 \ln 2 + 4 \ln^2 2 \right)$$

$$\int_0^{\tau_0^{\max}} d\tau_0 \mathcal{S}_q(\tau_0, p_T, R, \mu) = 1 + \frac{\alpha_s C_F}{2\pi} \left\{ -\frac{1}{\epsilon^2} + \frac{2}{\epsilon} \ln \left(\frac{\tau_0^{\max} p_T}{\mu R} \right) + \frac{\pi^2}{12} - 2 \ln^2 \left(\frac{\tau_0^{\max} p_T}{\mu R} \right) \right\}$$



Relation to inclusive jet function

$$\int \frac{d\sigma}{dp_T d\eta d\tau_a} d\tau_a = \frac{d\sigma}{dp_T d\eta} \Leftrightarrow \int_0^\infty d\tau_a \mathcal{G}_i(z, p_T, R, \tau_a, \mu) = J_i(z, p_T, R, \mu)$$

See also Chien, Hornig, C. Lee '15

- Correct integration over substructure should give inclusive case.

$$\mathcal{G}_c(z, p_T R, \tau_a, \mu) = \sum_i \mathcal{H}_{c \rightarrow i}(z, p_T R, \mu) \int d\tau_a^{C_i} d\tau_a^{S_i} \delta(\tau_a - \tau_a^{C_i} - \tau_a^{S_i}) \mathcal{C}_i(\tau_a^{C_i}, p_T \tau_a^{\frac{1}{2-a}}, \mu) \mathcal{S}_i(\tau_a^{S_i}, \frac{p_T \tau_a}{R^{1-a}}, \mu)$$

$$\begin{aligned} \mathcal{H}_{q \rightarrow q}(z, p_T R, \mu) = & \delta(1-z) - \frac{\alpha_s C_F}{2\pi} \left\{ \delta(1-z) \left[\frac{1}{\epsilon^2} + \frac{3}{2} \frac{1}{\epsilon} + \frac{\ln \left(\frac{\mu^2}{p_T^2 R^2} \right)}{\epsilon} \right. \right. \\ & \left. \left. + \frac{1}{2} \ln^2 \left(\frac{\mu^2}{p_T^2 R^2} \right) + \frac{3}{2} \ln \left(\frac{\mu^2}{p_T^2 R^2} \right) - \frac{\pi^2}{12} \right] \right. \\ & \left. + 2 \left((1+z^2) \frac{\ln(1-z)}{1-z} \right)_+ + (1-z) - P_{qq}(z) \left(\frac{1}{\epsilon} + \ln \left(\frac{\mu^2}{p_T^2 R^2} \right) \right) \right\} \end{aligned}$$

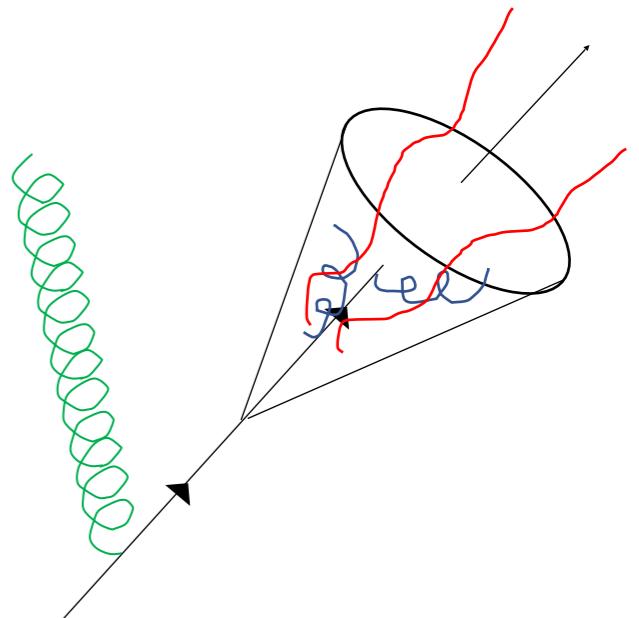
$$\mathcal{H}_{g \rightarrow q}(z, p_T R, \mu) = \frac{\alpha_s}{2\pi} P_{gq}(z) \left(\frac{1}{\epsilon} + \ln \left(\frac{\mu^2}{p_T^2 R^2} \right) - \ln(1-z)^2 \right) - \frac{\alpha_s C_F}{2\pi} z$$

$$\mathcal{C}_q(\tau_0, p_T, R, \mu) = \mathcal{C}_q^{\text{l.p.}}(\tau_0, p_T, \mu) + \Delta \mathcal{C}_q^{\text{alg}}(\tau_0, R)$$

$$\begin{aligned} \int_0^{\tau_0^{\max}} d\tau_0 \mathcal{C}_q^{\text{l.p.}}(\tau_0, p_T, \mu) = & 1 + \frac{\alpha_s C_F}{2\pi} \left\{ \frac{2}{\epsilon^2} + \frac{3}{2\epsilon} - \frac{2}{\epsilon} \ln \left(\frac{\tau_0^{\max} p_T^2}{\mu^2} \right) + \ln^2 \left(\frac{\tau_0^{\max} p_T^2}{\mu^2} \right) \right. \\ & \left. - \frac{3}{2} \ln \left(\frac{\tau_0^{\max} p_T^2}{\mu^2} \right) + \frac{7}{2} - \frac{\pi^2}{2} \right\} \end{aligned}$$

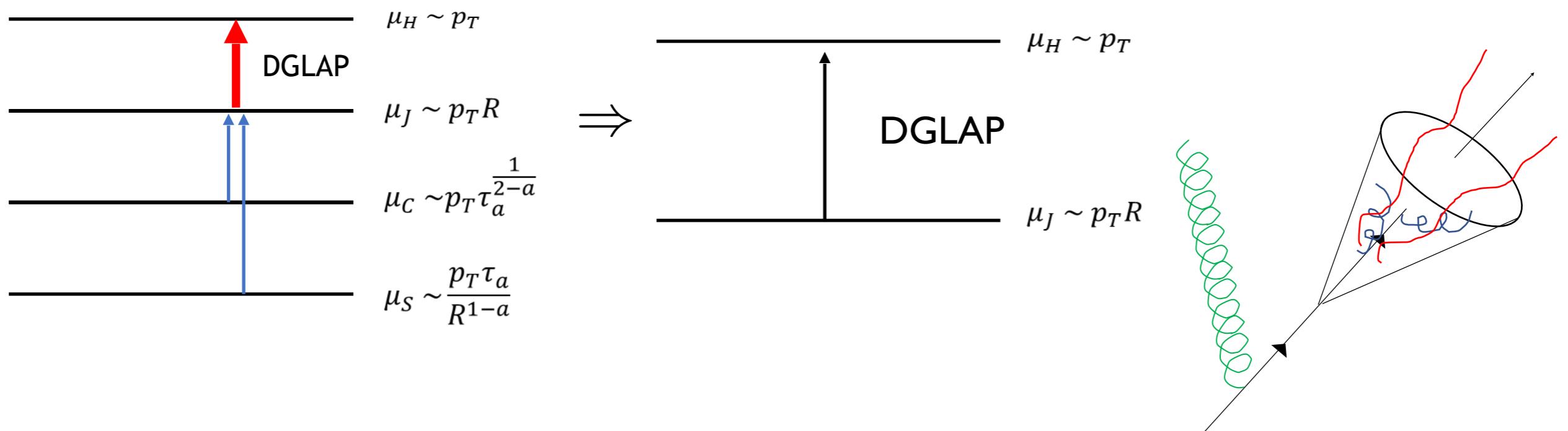
$$\int_0^{\tau_0^{\max}} d\tau_0 \Delta \mathcal{C}_q^{k_T}(\tau_0, R) = \frac{\alpha_s C_F}{2\pi} \left(3 - \frac{\pi^2}{3} - [3 \ln 2 + 4 \ln^2 2] \right)$$

$$\int_0^{\tau_0^{\max}} d\tau_0 \mathcal{S}_q(\tau_0, p_T, R, \mu) = 1 + \frac{\alpha_s C_F}{2\pi} \left\{ -\frac{1}{\epsilon^2} + \frac{2}{\epsilon} \ln \left(\frac{\tau_0^{\max} p_T}{\mu R} \right) + \frac{\pi^2}{12} - 2 \ln^2 \left(\frac{\tau_0^{\max} p_T}{\mu R} \right) \right\}$$

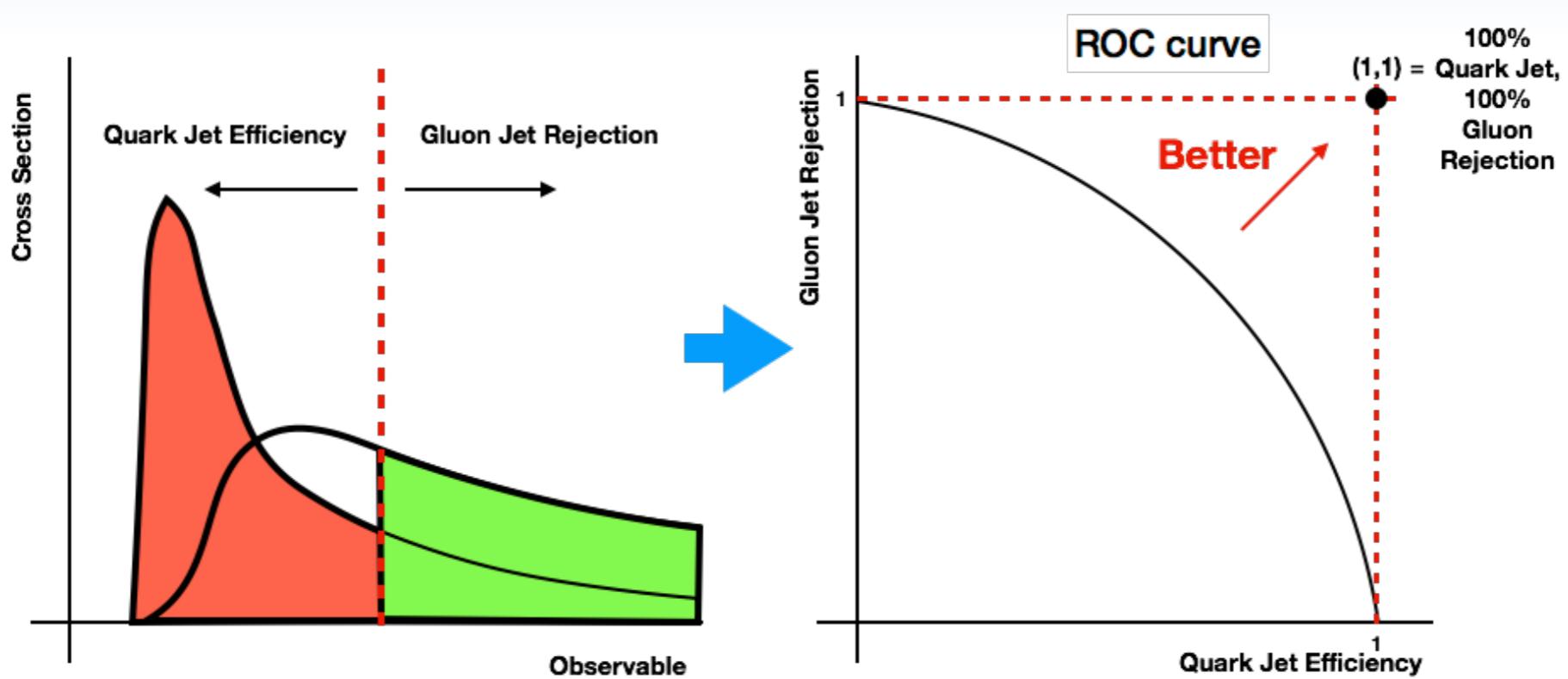


Relation to inclusive jet function

- □ poles except ones associated with DGLAP cancel.
- □ logs except ones associated with the jet scale cancel.
- Inclusive jet reproduced. Hard-collinear-soft structure collapse to Hard-collinear factorization.

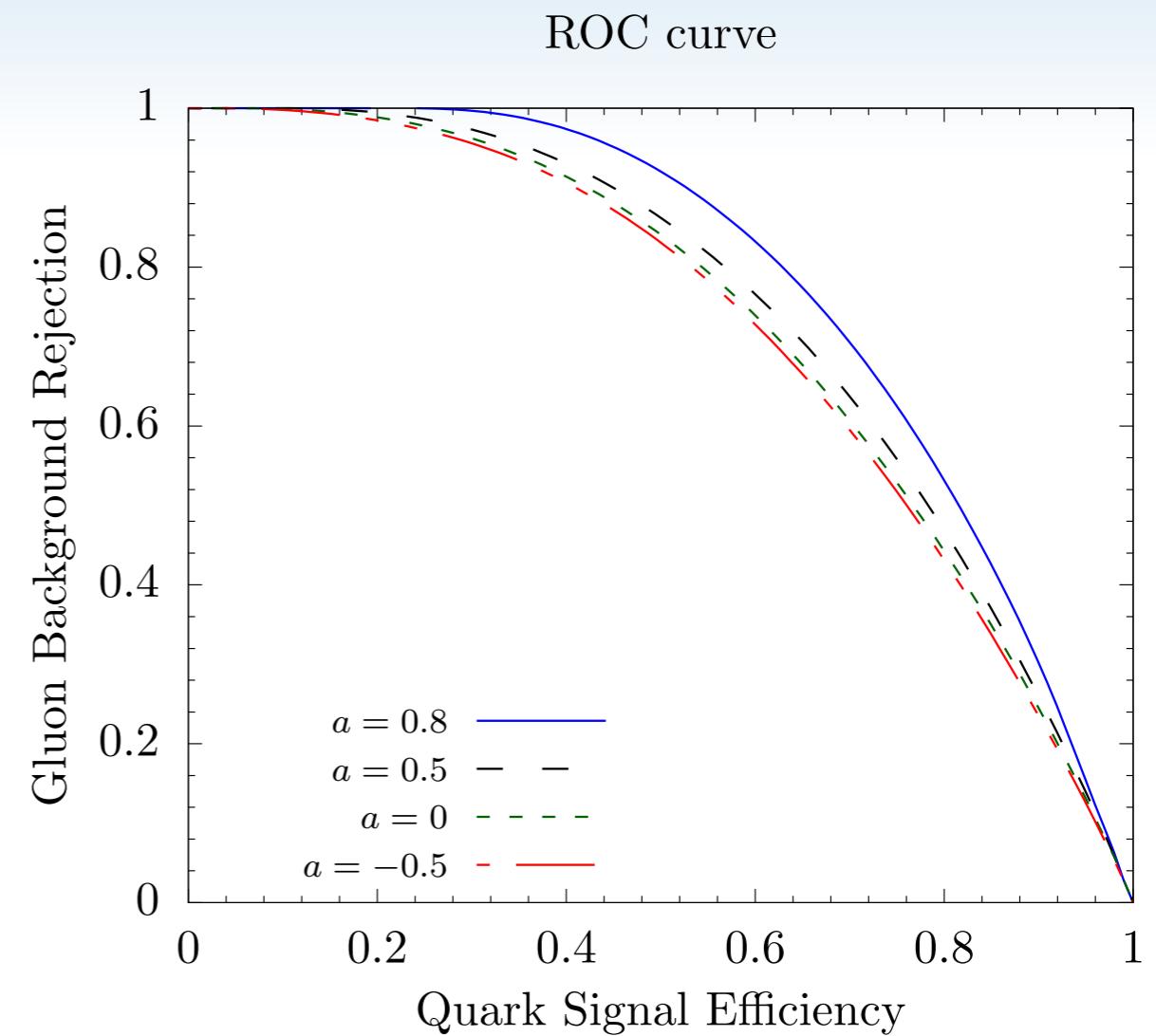
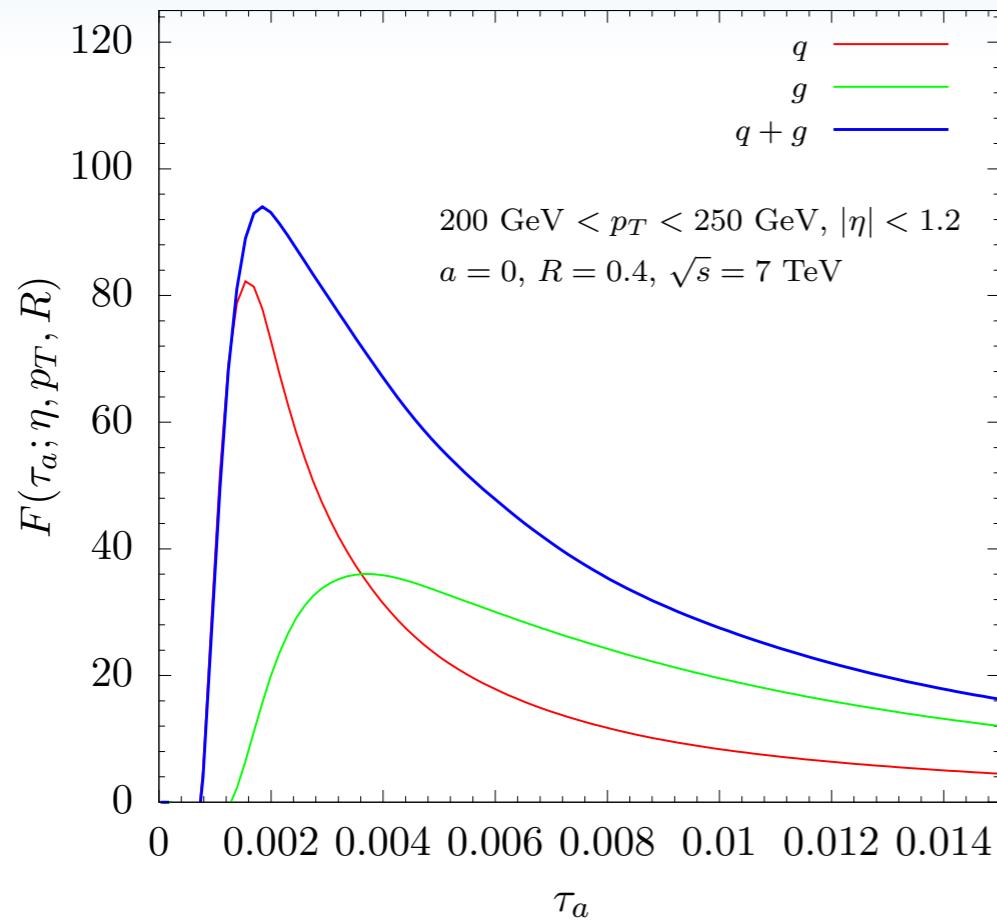


Quark and gluon discrimination



- We can study how well angularity discriminates between quark and gluon jet as a continuous function of 'a'.

Quark and gluon discrimination



- We can study how well angularity discriminates between quark and gluon jet as a continuous function of 'a'.
- As 'a' increases, better discrimination but more sensitive to non-perturbative effects.

Non-perturbative Model

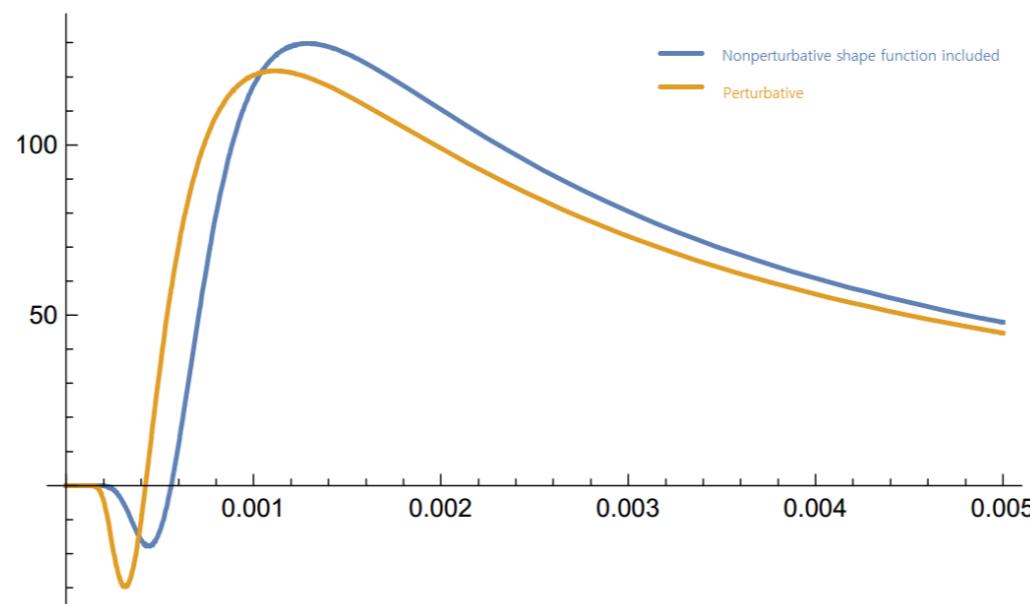
- As τ_a gets smaller, $\mu_S \sim \frac{p_T \tau_a}{R^{1-a}}$ can approach Λ_{QCD} :

$$S_i(\tau_a, \mu) \rightarrow \int d\tau'_a S_i(\tau_a - \tau'_a, \mu) S^{\text{NP}}(\tau'_a)$$

where

$$S^{\text{NP}}(\tau_a) = \frac{\mathcal{N}(A, B, \Lambda)}{\Lambda} \left(\frac{\frac{p_T \tau_a}{R^{1-a}}}{\Lambda} \right)^{A-1} \exp \left(- \left(\frac{\frac{p_T \tau_a}{R^{1-a}} - B}{\Lambda} \right)^2 \right)$$

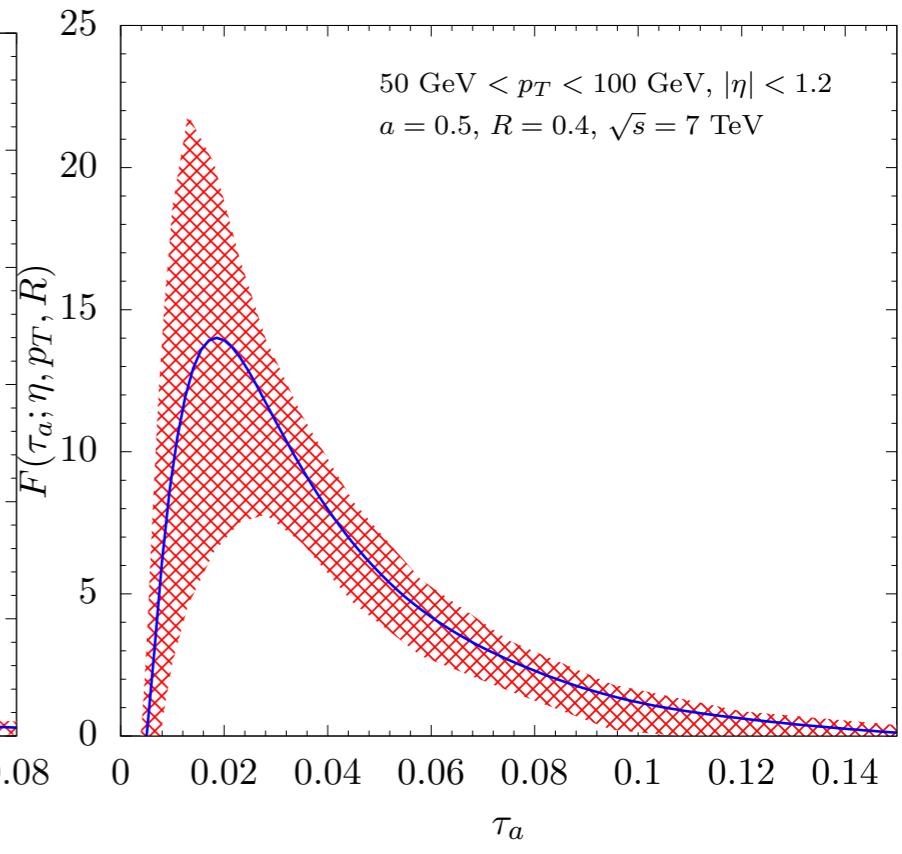
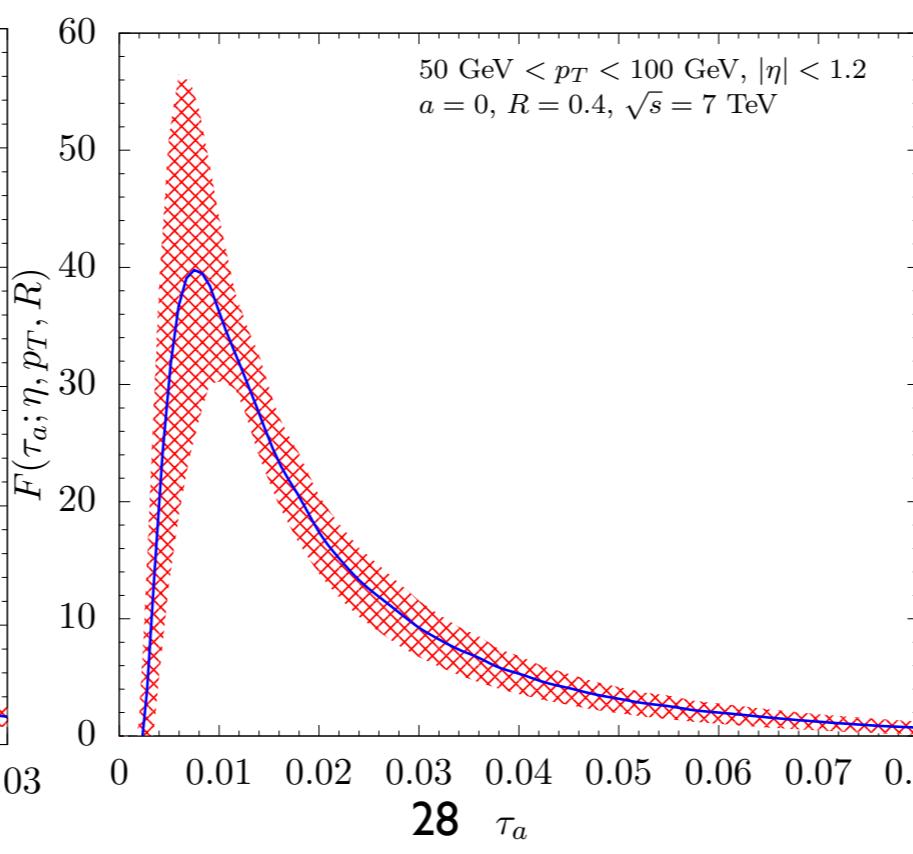
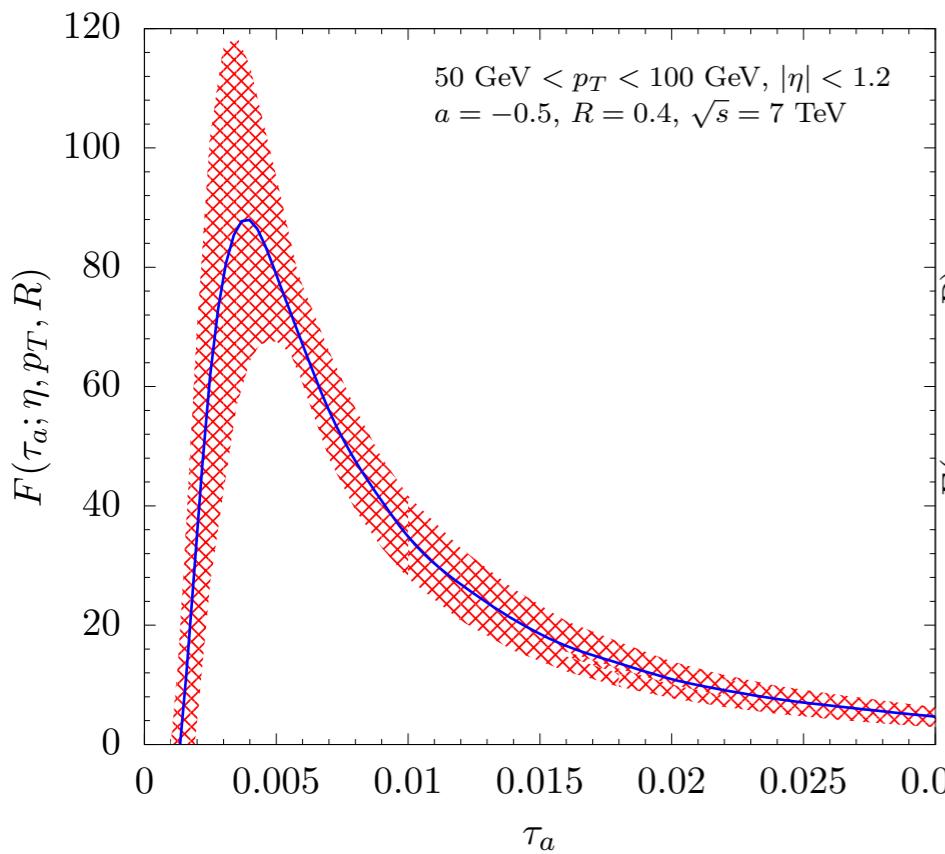
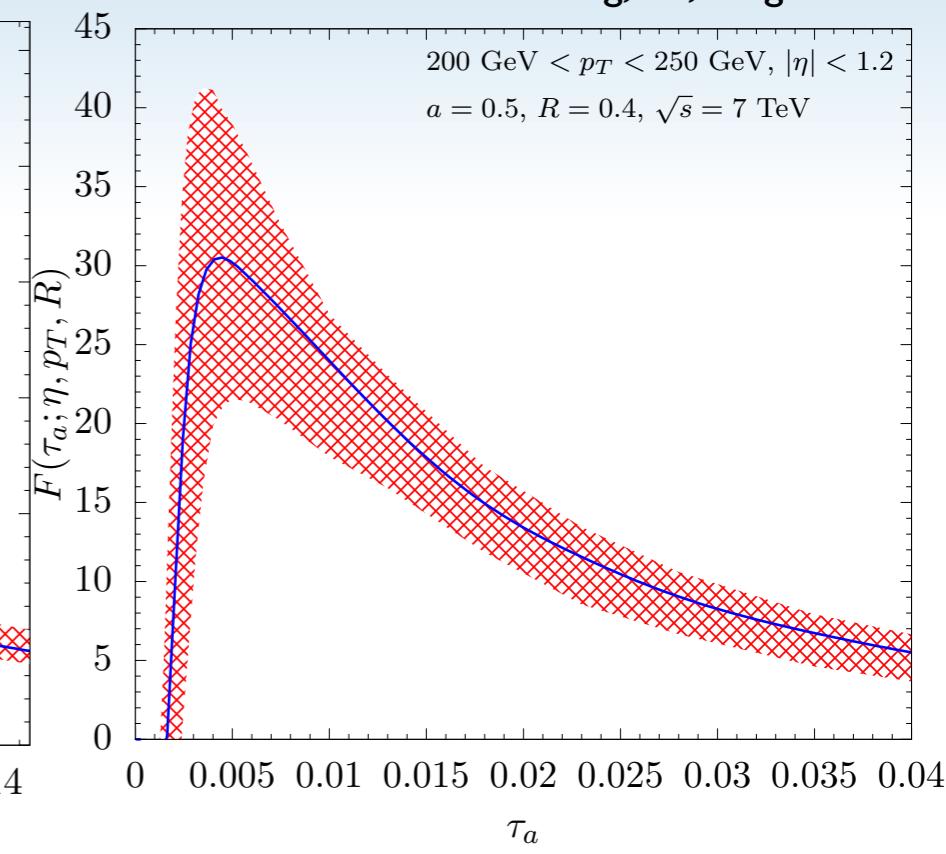
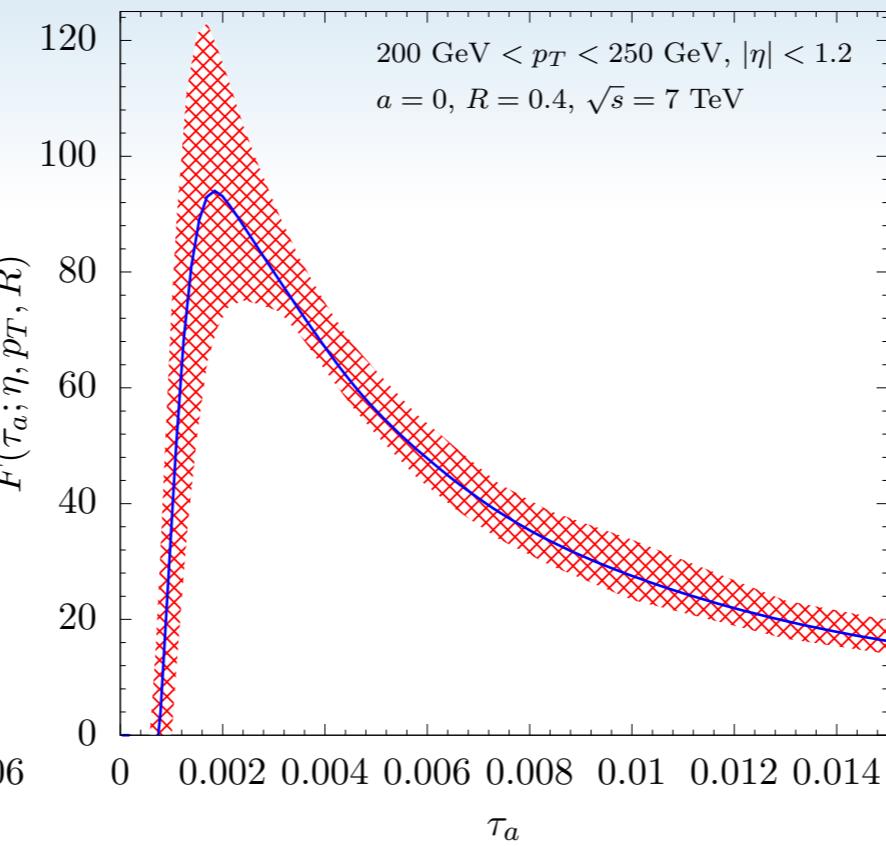
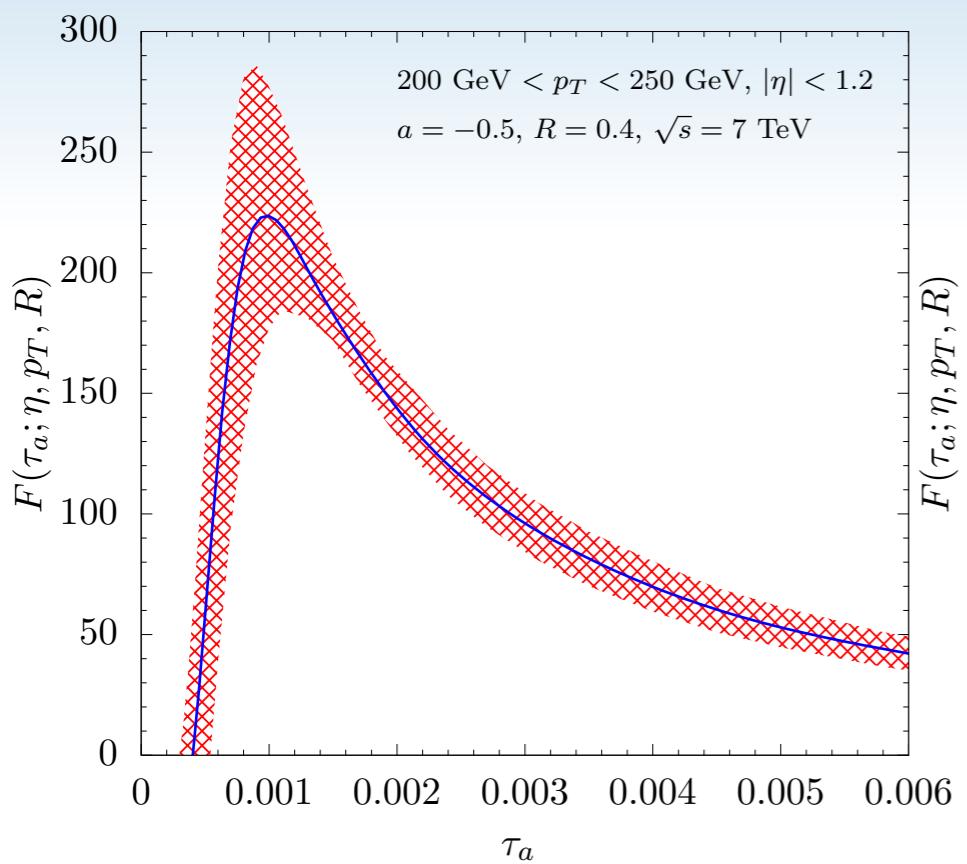
- Since we are sensitive to non-perturbative effects at small values of τ_a , the distribution is unaffected at large values of τ_a .



- Profile function is included to prevent soft scale from reaching landau pole and scales involved are varied up and down by factor 2 with respect to the canonical scale.

Phenomenology

Kang, KL, Ringer '18



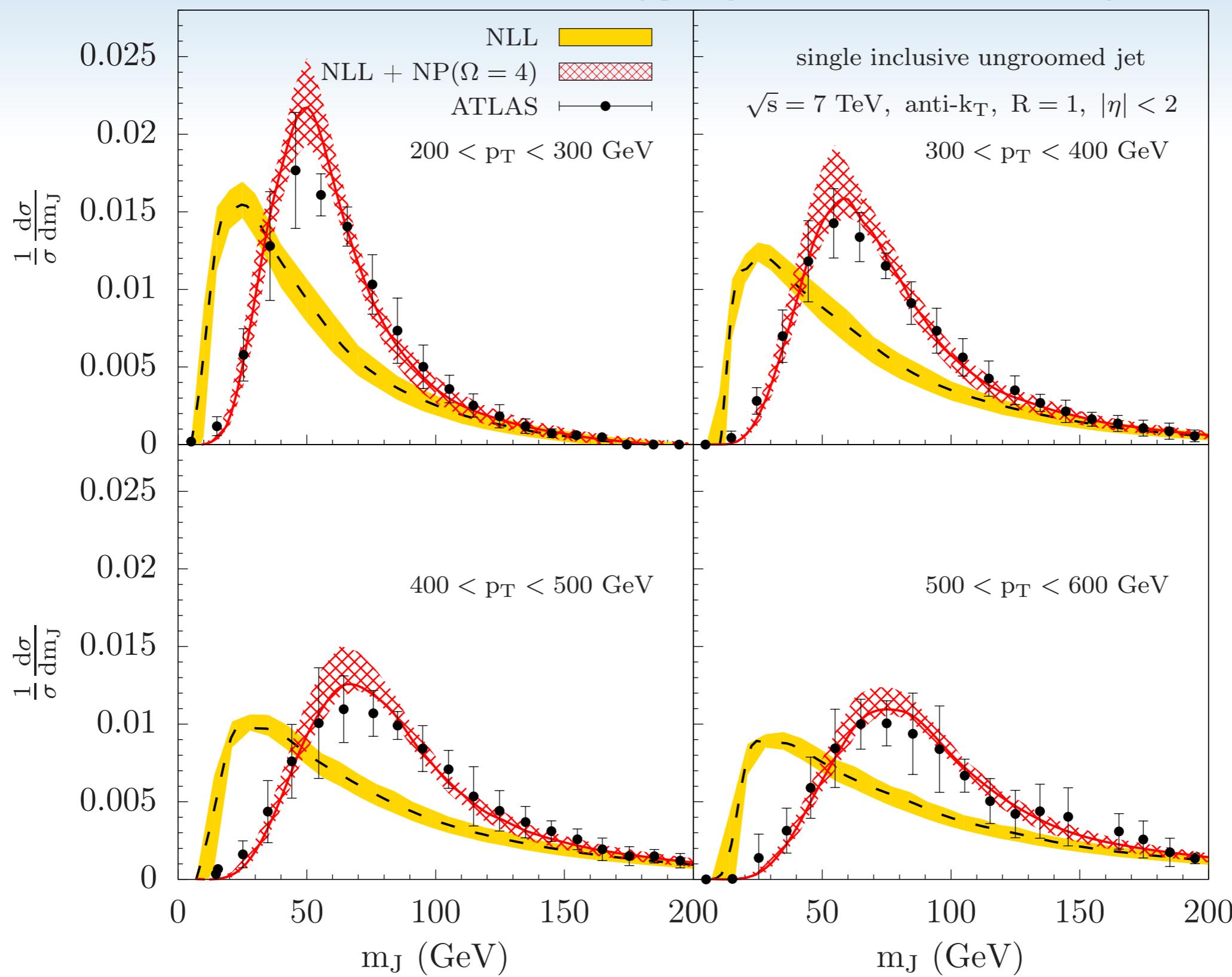
Phenomenology ($a=0$, jet mass)

- As pointed out yesterday (Prof. Stewart's talk), NP (both hadronization and MPI) effects in jet mass is well-represented by just shifting first-moments.
- Single parameter NP soft function from *Stewart, Tackmann, Waalewijn '15* :

$$S_{\kappa}^{NP}(k) = \left(\frac{4k}{\Omega_{\kappa}^2} \right) \exp \left(-\frac{2k}{\Omega_{\kappa}} \right)$$

- The parameter Ω_{κ} is associated with the amount of shift.

Phenomenology ($a=0$, jet mass)



Conclusions

- Formalism for studying semi-inclusive jet production with or without substructure measurements were introduced.
- From μ_J to μ_H , the semi-inclusive jet production follows DGLAP evolution.
- Demonstrated how angularity measured case integrates to inclusive jet case.
- Continuous parameter dependence on quark and gluon discrimination power was considered.
- We now have a consistent baseline calculation for jet mass in pp.
Extend to jet mass in heavy ion collisions!