A Quantum Many-Body Approach to QGP and Heavy Flavor

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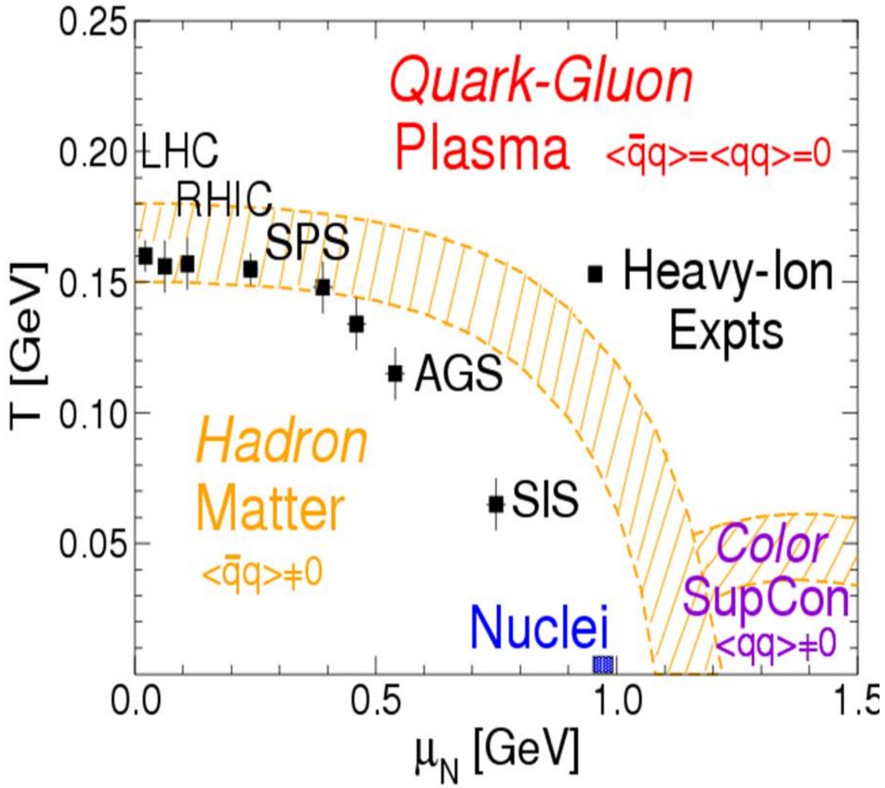
Outline

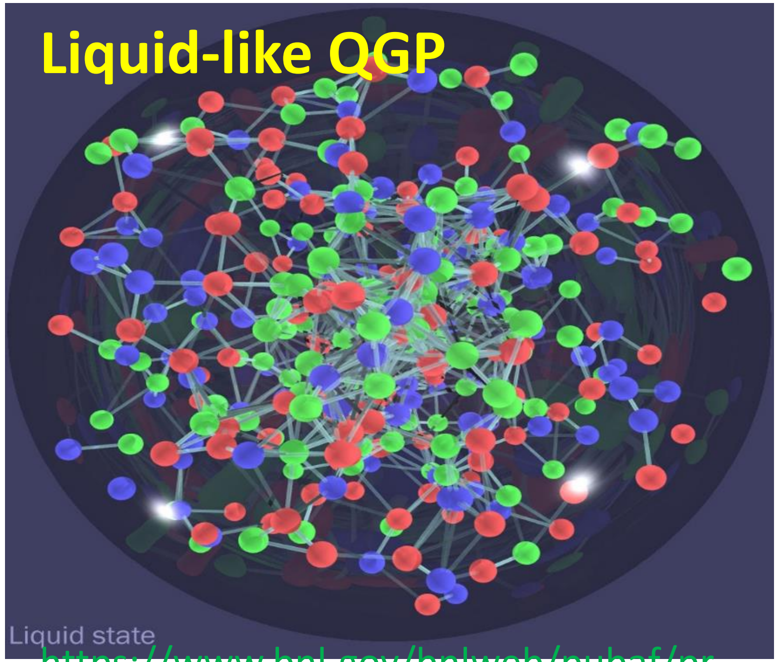
1) **Background and Motivation**
2) **Thermodynamic *T*-matrix Approach**
3) **Applications to QGP & Heavy Flavors**
   - Benchmark with lattice QCD
   - Spectral properties of QGP & heavy flavors
   - Transport properties of QGP & heavy flavors
4) **Conclusion and Perspective**

More is Different

- Many-body physics of QED: properties of crystals, superconductivity in metals, etc.

Many-Body Physics of QCD





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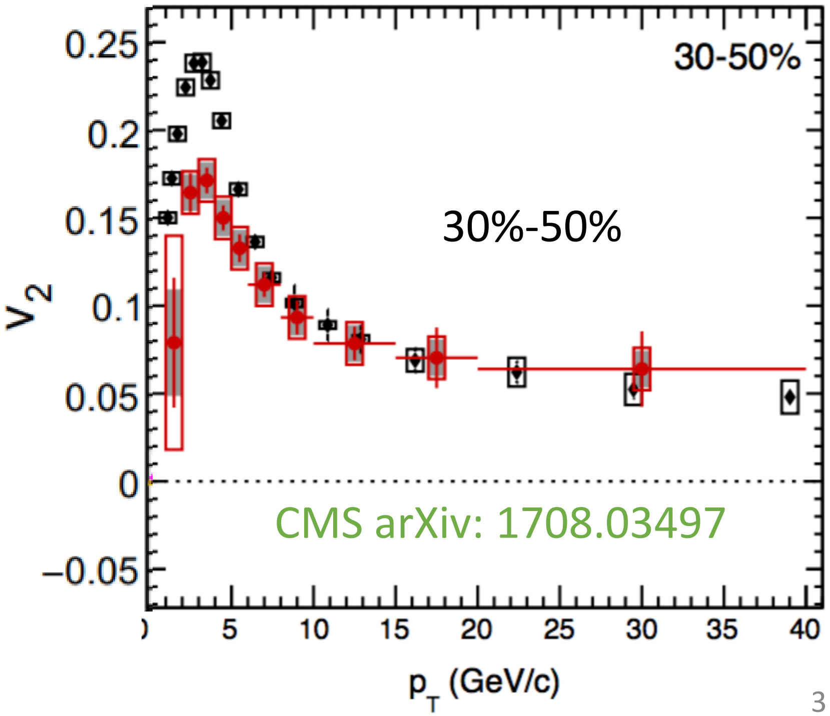

CMS arXiv: 1708.03497

How First Principle QCD Helps Us

- pQCD: at high momentum transfer
  - Energy loss for high energy jet, electromagnetic properties of QGP at high temperature.
- Lattice QCD: non-perturbative, at imaginary time and near zero chemical potential
  - QCD equation of state (EoS), heavy quark free energy, Euclidean time correlator, quark number susceptibilities, etc.
- Several phenomena still challenging for first principle calculations
  - In medium color force, degrees of freedom, spectral functions , transport properties, physics at high density, etc.

QCD Inspired Models

- Various models for different problems:
  - Quasi-particle approach: EoS
  - Potential Model for Quarkonium: free energy, Euclidean correlator, dissociation and regeneration of the quarkonium
  - AdS-CFT: EoS, transport coefficients
  - NJL model: chiral symmetry breaking, EoS, phase structure
- Could we develop a single framework to benchmark a wide variety of information and apply to a wide variety of phenomena?

Our attempts: *T*-matrix Approach

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Model Assumption and Why it is possible

- Assumption
  - There is an effective theory in non-perturbative region of QCD whose degrees of freedom are partons with effective masses (Input) and interactions approximately characterized by color potential (Input)
- Why these assumptions
  - Justified in heavy quark limit
  - Empirically, constituent quark model (using potential) approximately works for both heavy and light hadron spectroscopy
  - Practically, this approach incorporates a wide variety of physics and is still calculable with reasonable computing resources

A Unified Hamiltonian as Starting Point

- In-medium Hamiltonian

$$H = \sum \varepsilon_i(\mathbf{p})\psi_i^\dagger(\mathbf{p})\psi_i(\mathbf{p}) + \psi_i^\dagger(\frac{\mathbf{P}}{2} - \mathbf{p})\psi_j^\dagger(\frac{\mathbf{P}}{2} + \mathbf{p})V_{ij}^a\psi_j(\frac{\mathbf{P}}{2} + \mathbf{p}')\psi_i(\frac{\mathbf{P}}{2} - \mathbf{p}')$$

effective in-medium mass $\quad \varepsilon_i(\mathbf{p}) = \sqrt{M_i^2 + \mathbf{p}^2}$

- Interaction ansatz: Cornell potential with relativistic corrections

$$V_{ij}^a(\mathbf{p}, \mathbf{p}') = \mathcal{R}_{ij}^{\mathcal{C}}\mathcal{F}_a^{\mathcal{C}}V_{\mathcal{C}}(\mathbf{p} - \mathbf{p}') + \mathcal{R}_{ij}^{\mathcal{S}}\mathcal{F}_a^{\mathcal{S}}V_{\mathcal{S}}(\mathbf{p} - \mathbf{p}')$$

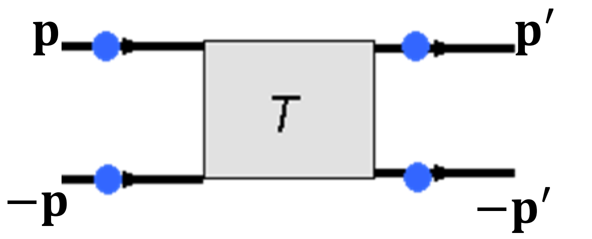
- color-Coulomb and string (“confining”) interaction

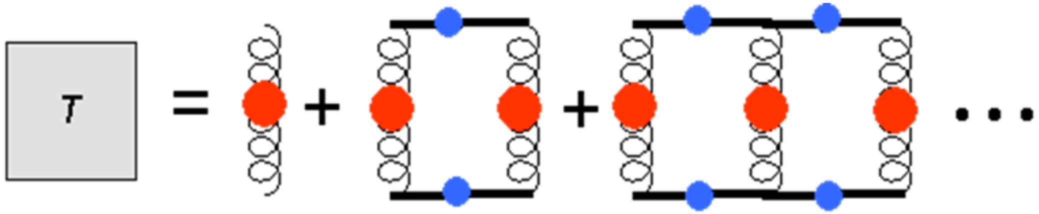
Starting point for many-body approach

[Liu+RR ’16 and 17]

Thermodynamic T-matrix Approach

$T(E, \mathbf{p}, \mathbf{p}')$





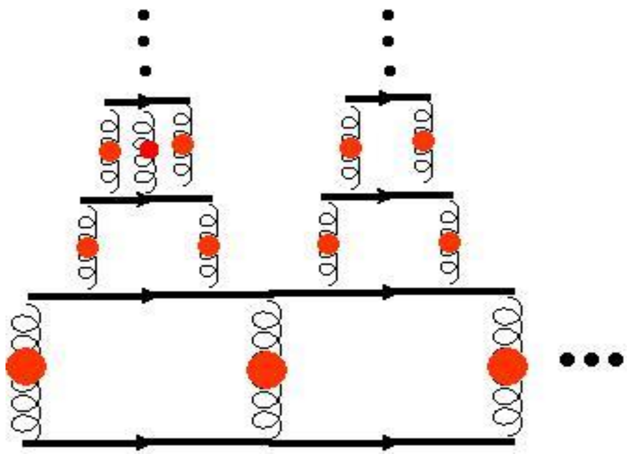
- T-matrix in CM frame

$$T(z, \mathbf{p}, \mathbf{p}') = V(\boldsymbol{p}, \boldsymbol{p}') + \int \frac{d^3p}{(2\pi)^4} V(\mathbf{p}, \mathbf{k}) \hat{G}(z, \mathbf{k}) T(z, \mathbf{p}, \mathbf{p}')$$

- Self-consistent integral equation and medium effects

$$\hat{G}(iE_n, \mathbf{k}) = -\beta^{-1} \sum G(iE_n - i\omega_n) G(i\omega_n)$$

$$G(i\omega_n, \mathbf{p}) = \frac{1}{i\omega_n - \varepsilon(\mathbf{p}) - \Sigma(i\omega_n, p)} \qquad \Sigma = \sum_{s,c,f} \int d\widetilde{^4k}\, T(\Sigma)\, G(\Sigma)$$



$\boxed{\text{Inputs } V \text{ and } M_i \text{ need to be constrained}}$

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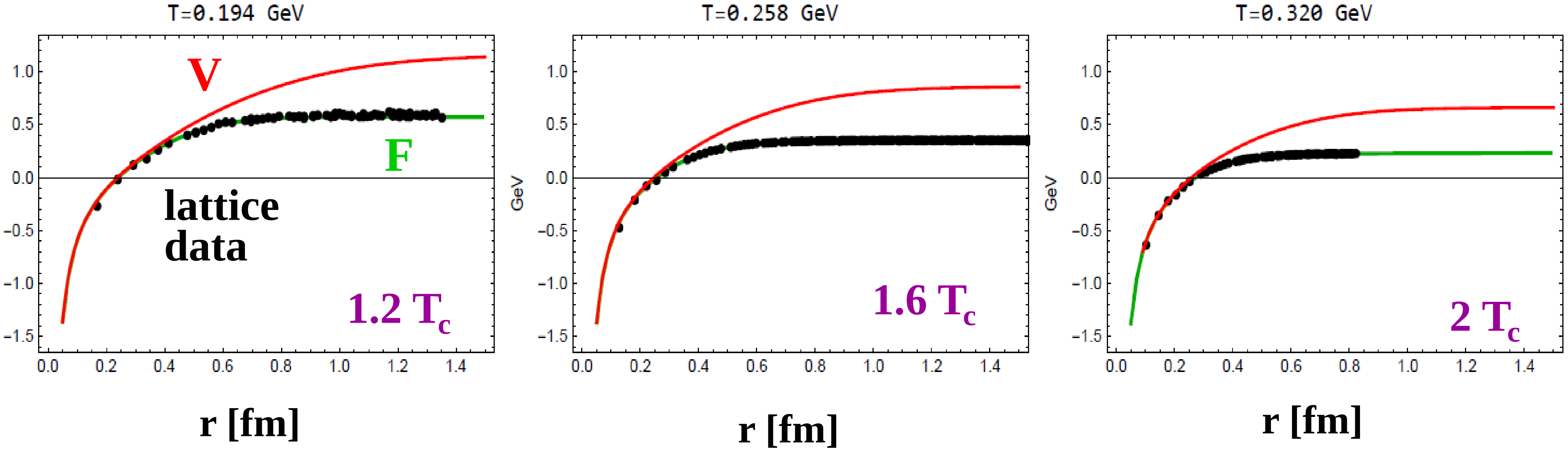
Heavy-Quark Free Energy and Potential

- $Q\bar{Q}$ free energy $$F_{Q\bar{Q}}(T,r) = -T\ln\left(\int_{-\infty}^{\infty} dE\ \rho_{Q\bar{Q}}(E,r)e^{-\beta E}\right)$$
- Spectral functions $$\rho_{Q\bar{Q}}(E,r) = \frac{-1}{\pi}\mathrm{Im}\left[\frac{1}{E - V(r) - \Sigma_{Q\bar{Q}}(E+i\epsilon,r)}\right]$$
- Potential ansatz $$V(r) = -\frac{4}{3}\alpha_s\frac{e^{-m_D r}}{r} - \frac{4}{3}\alpha_s m_D - \sigma\left(\frac{e^{-m_s r-(c_b m_s r)^2}}{m_s} - \frac{1}{m_s}\right)$$



- remnant of long-range "confining" force in QGP S.Liu+RR' 15 and 17

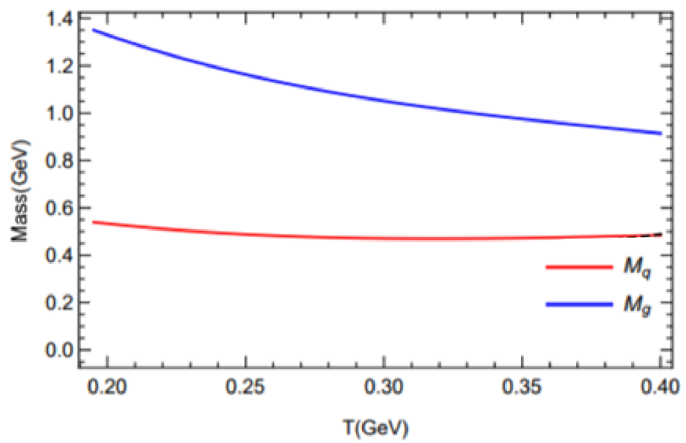
Equation of State for QGP and Parton Mass

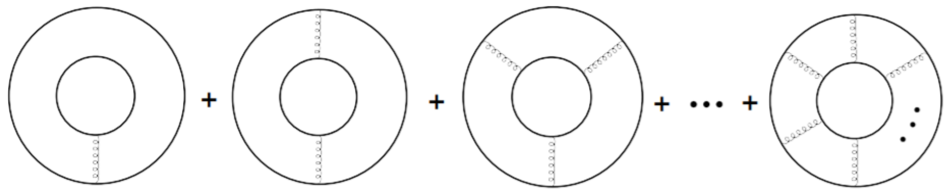
Thermodynamic Potential

$$\Omega = \mp \frac{-1}{\beta} \sum_n \mathrm{Tr}\{\ln(-G^{-1}) + (G_0^{-1} - G^{-1})G\} \pm \Phi$$

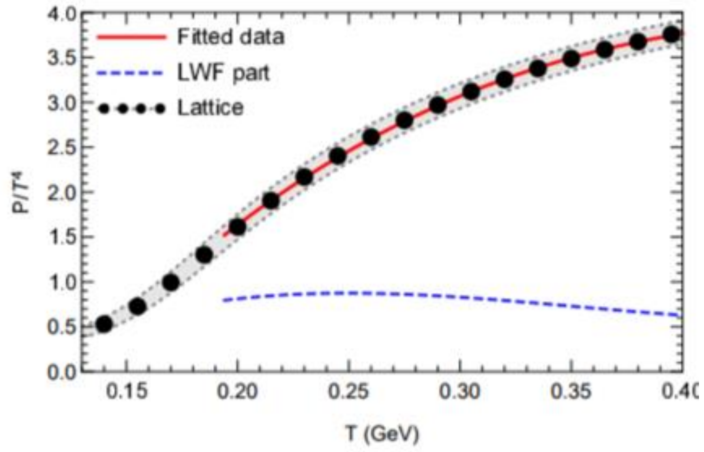
"Skeleton" Diagrams

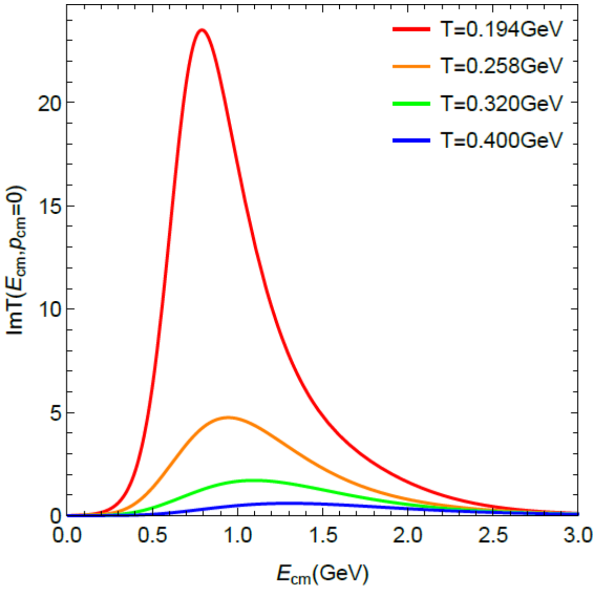
$$\Phi = \sum_{n,v} \mathrm{Tr}\left\{\frac{1}{2v}\Sigma_v G\right\} \quad \boxed{= \frac{1}{2}\sum_n \mathrm{Tr}\{-\ln(1 - GGV)\}}$$





- **2PI Resumed Φ with Matrix-Log technique**

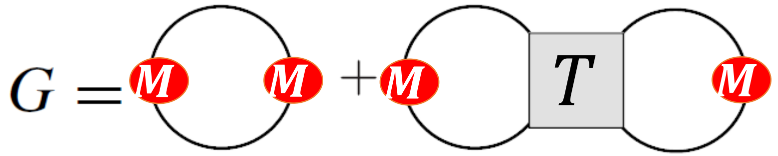




- **Broad hadronic resonances emerge near T_{pc}, dominate EoS**

[S.Liu+RR '16 & 17]

Quarkonium Euclidean Correlator

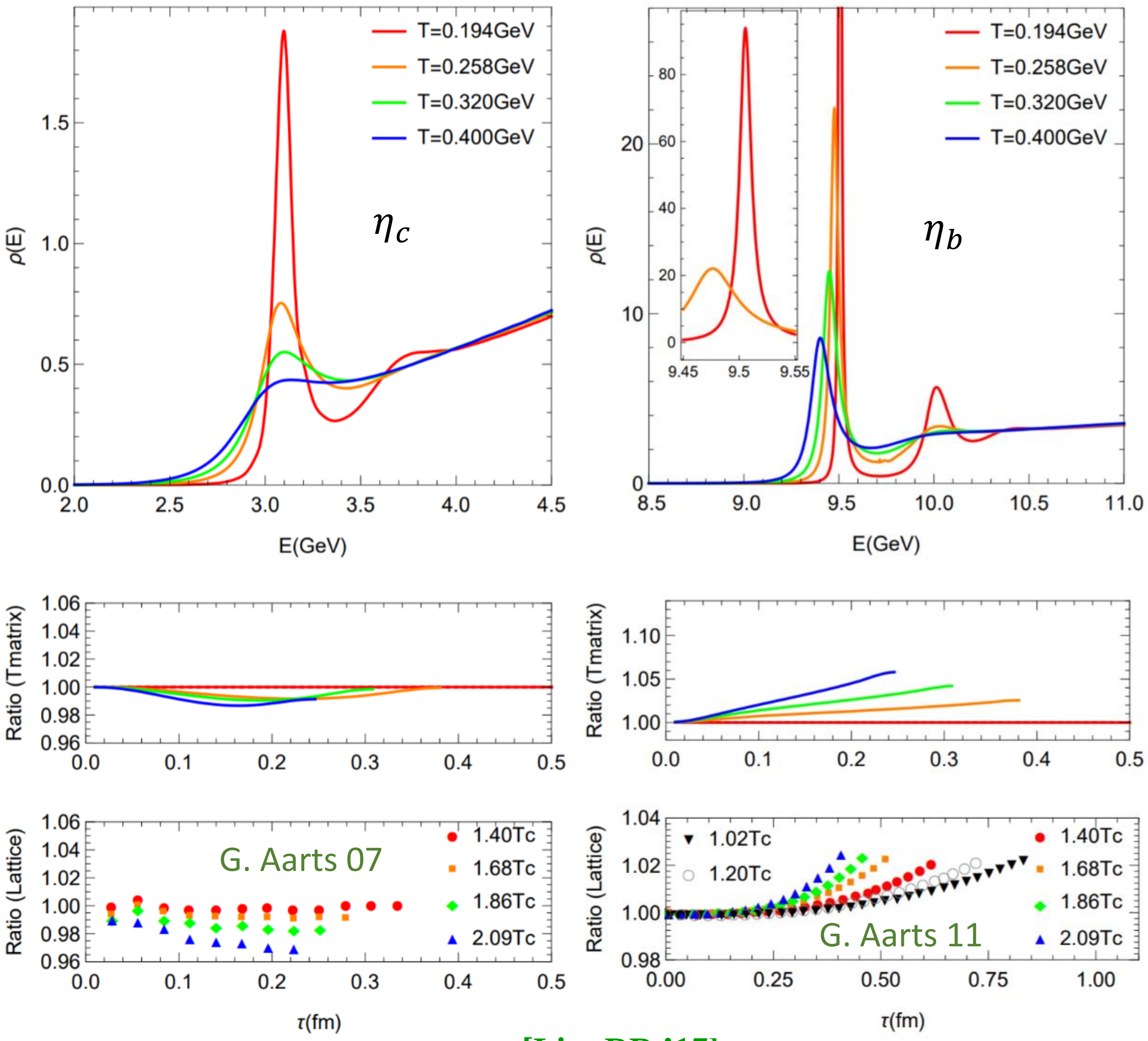


- dissociation/regeneration rate for quarkonium

- Sequential dissociation/regeneration

$$R(T_{\text{ref}}, T) = \frac{\int dE\, \rho(E,T)\mathcal{K}(\tau,E,T)}{\int dE\, \rho(E,T_{\text{ref}})\mathcal{K}(\tau,E,T)}$$

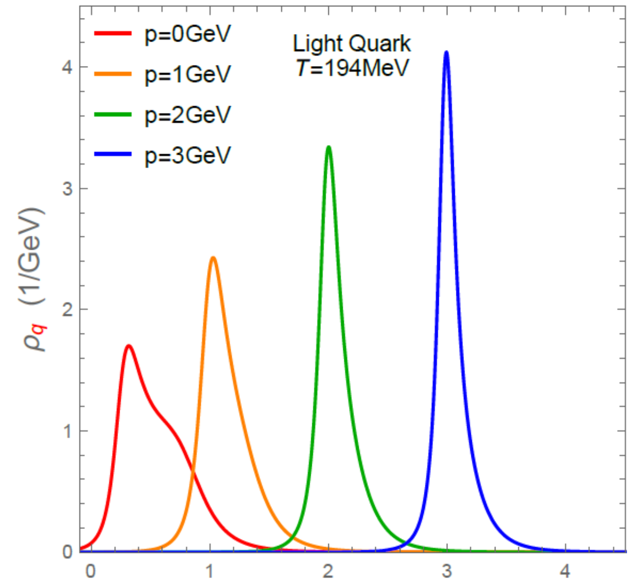
- No strong temperature dependence of α_s

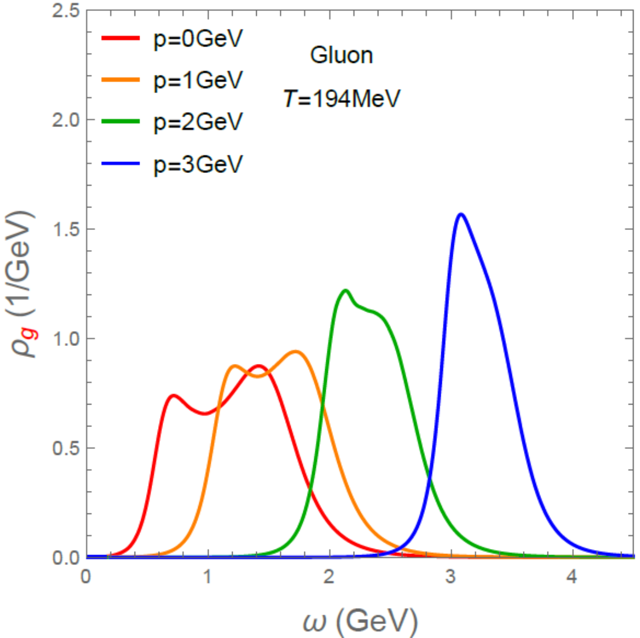


[Liu+RR '17]

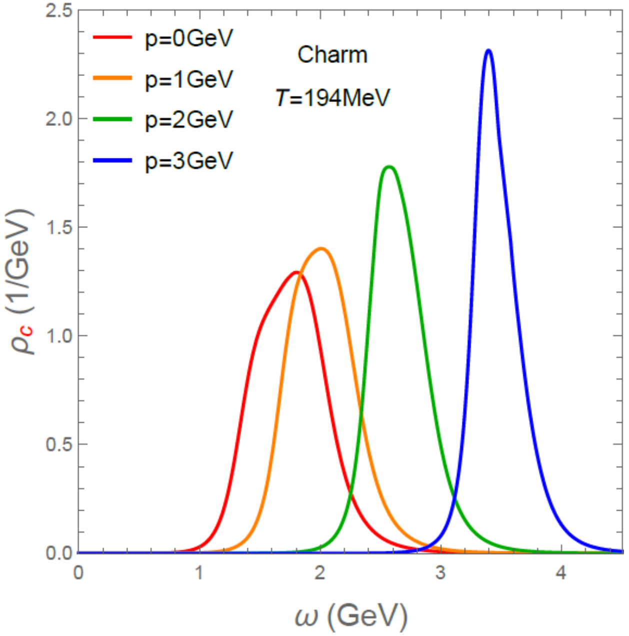
Parton Spectral Functions in QGP

Up/Down + Gluon

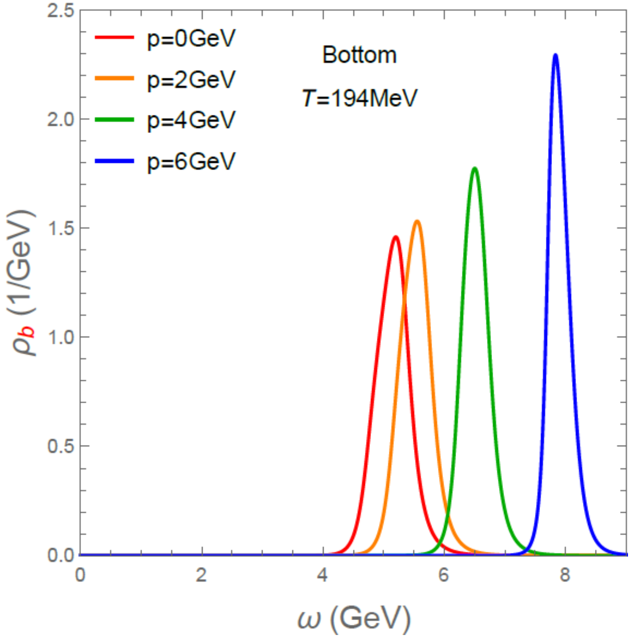




Charm



Bottom



[Liu+RR '16 and 17]

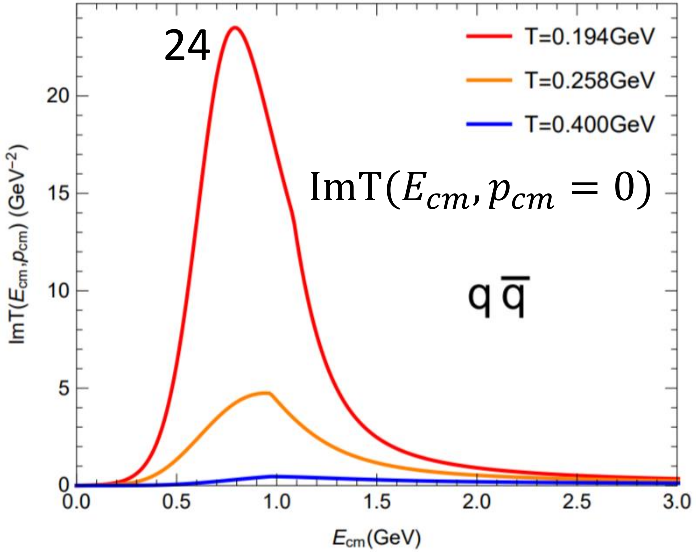
- QGP structure changes with resolution scale
- Soft parton quasi-particles dissolve at low ***T***, re-emerge with increasing $\boldsymbol{M_q}$, ***p***, ***T***

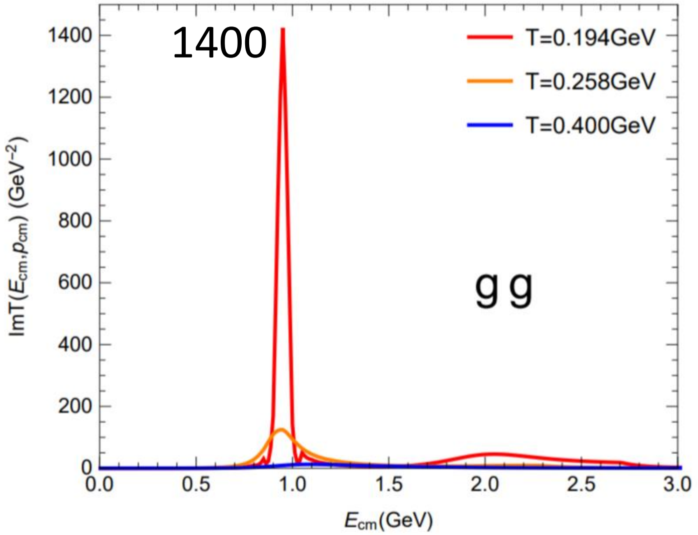
Two-body *T*-matrix Amplitude

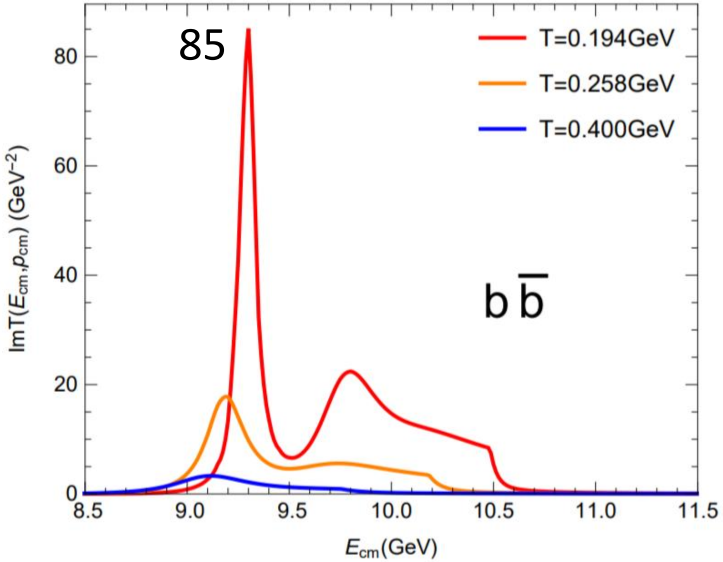
Examples in 60 (C, S, F, L) channels each *T*

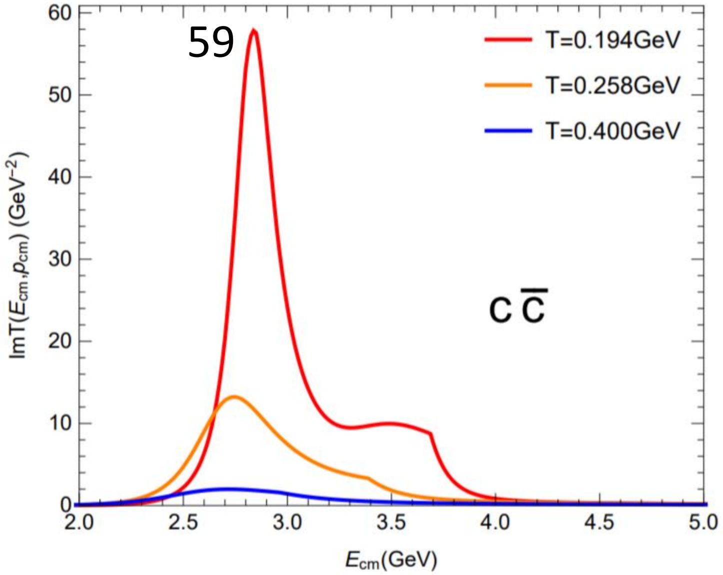
- Transit to hadronic states through a strong broader resonance at low temperature

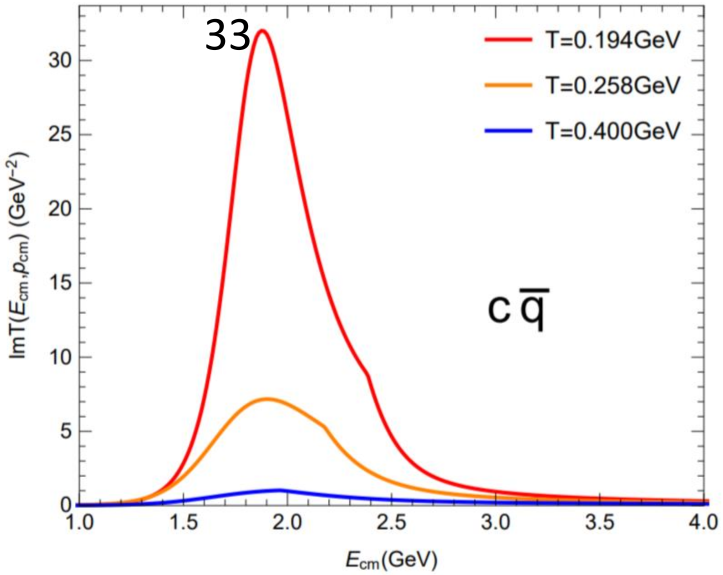
- Sequential dissociation/regeneration



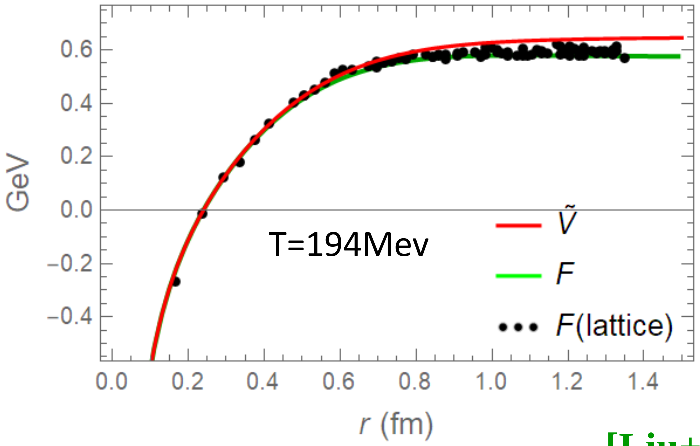


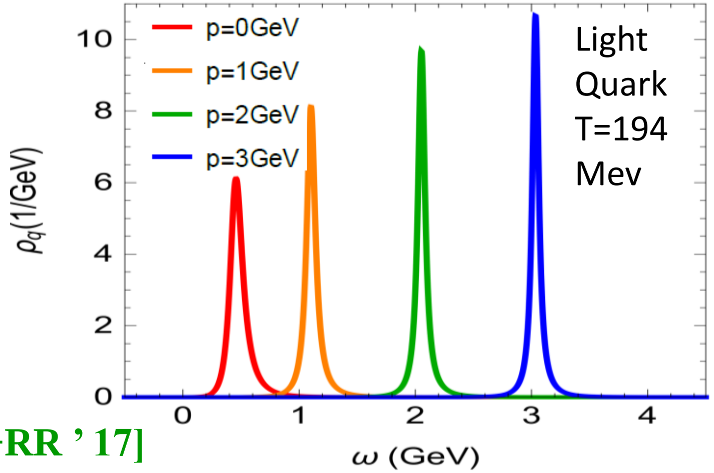




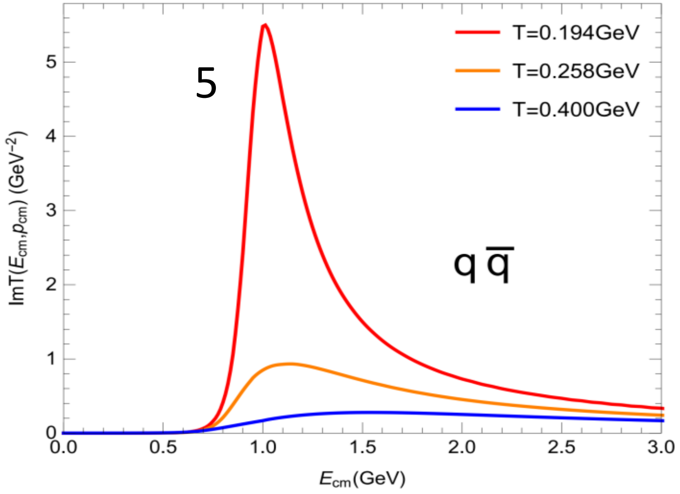


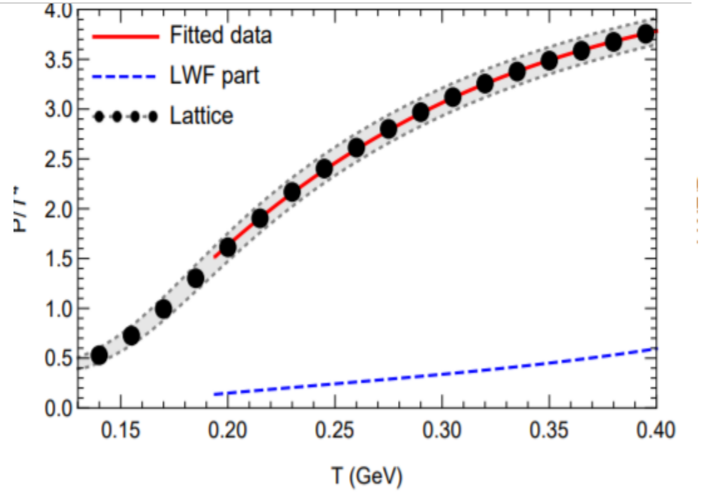
Fit is not Unique: Weakly Coupled Solution (WCS)

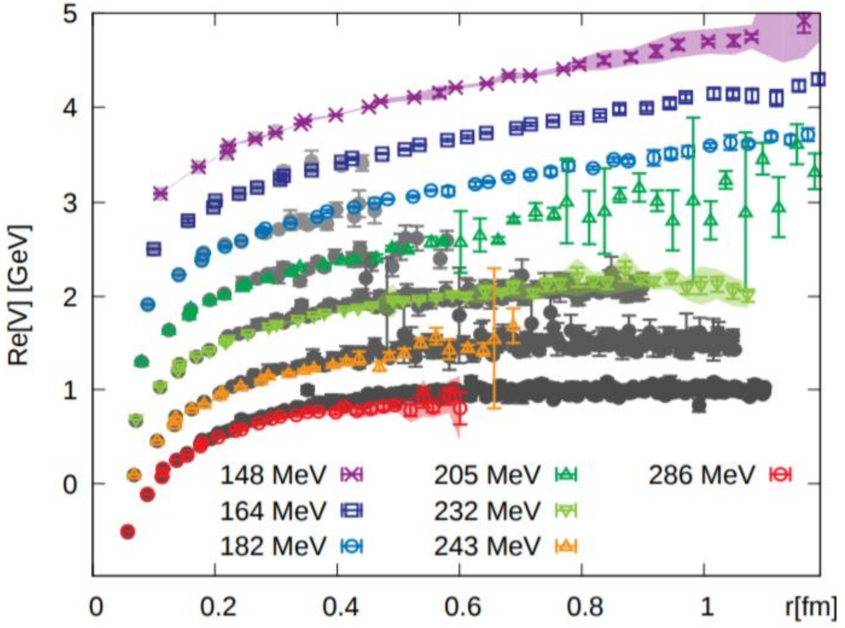




[Liu+RR ' 17]







[Burnier et al ’14]

Bayesian Analysis of lattice Data

Weakly Coupled Solution: *V* close to F, quasi particle spectral function, weak resonance, no transition of degrees of freedom

Transport Coefficients (Formalism)

- HQ relaxation rate → Determine the phenomenology of the HQ Langevin dynamics

$$\Gamma(p) \approx A(p) = \left\langle \left(1 - \frac{\mathbf{p}\cdot\mathbf{p}'}{p^2}\right) \rho_i \rho_i \rho_c \right\rangle$$

- Spatial diffusion coefficients for QGP

$$D_s = \frac{T}{M_Q A(0)}$$

Conjecture to have $D_s(2\pi T) \geq 1$

- Viscosity for QGP → Determine the phenomenology of hydrodynamics

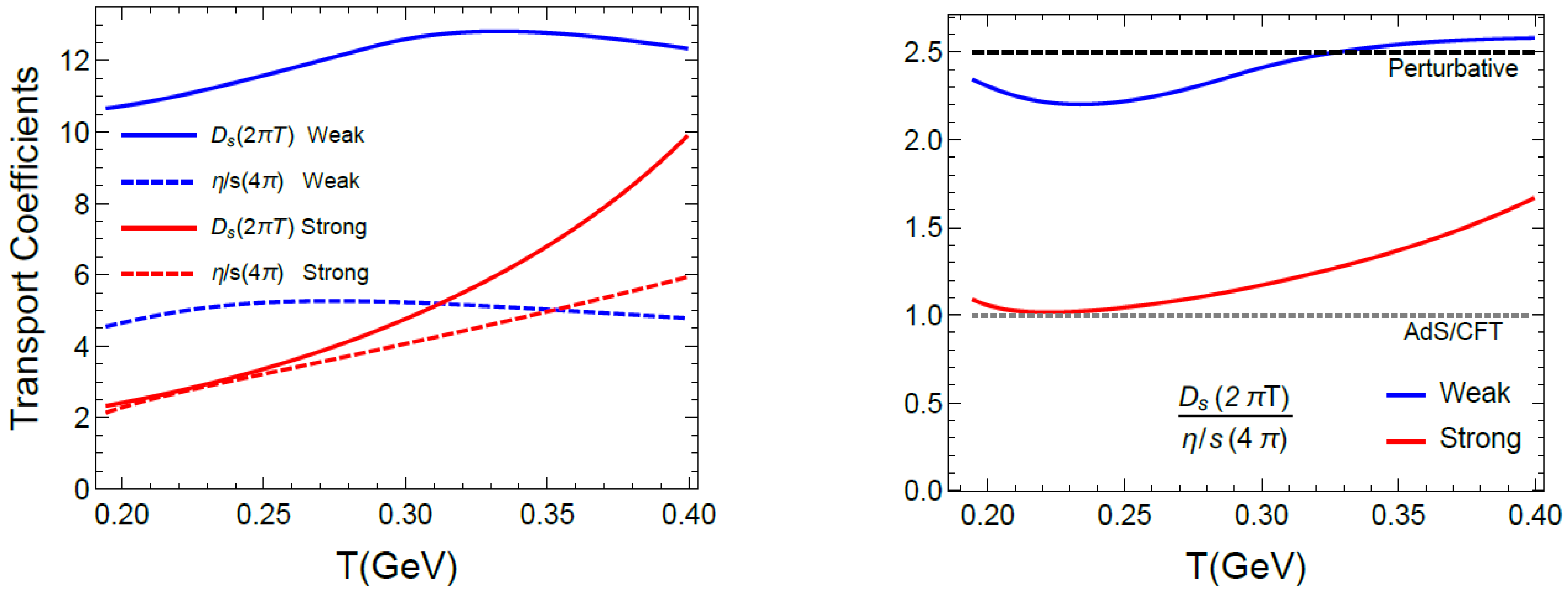
$$\eta = -\sum_i \frac{\pi d_i}{\omega} \int \frac{d^3\mathbf{p}\, d\lambda}{(2\pi)^3} \frac{p_x^2 p_y^2}{\varepsilon_i(p)} \rho_i(\lambda+\omega, p)\rho_i(\omega, p)\big(n(\lambda+\omega) - n(\omega)\big)$$

Conjecture to have $\eta/s\,(4\pi) \geq 1$

Experiemental observables such as R_{AA} and v_2 for various species of hadrons

Viscosity and Heavy-Quark Diffusion

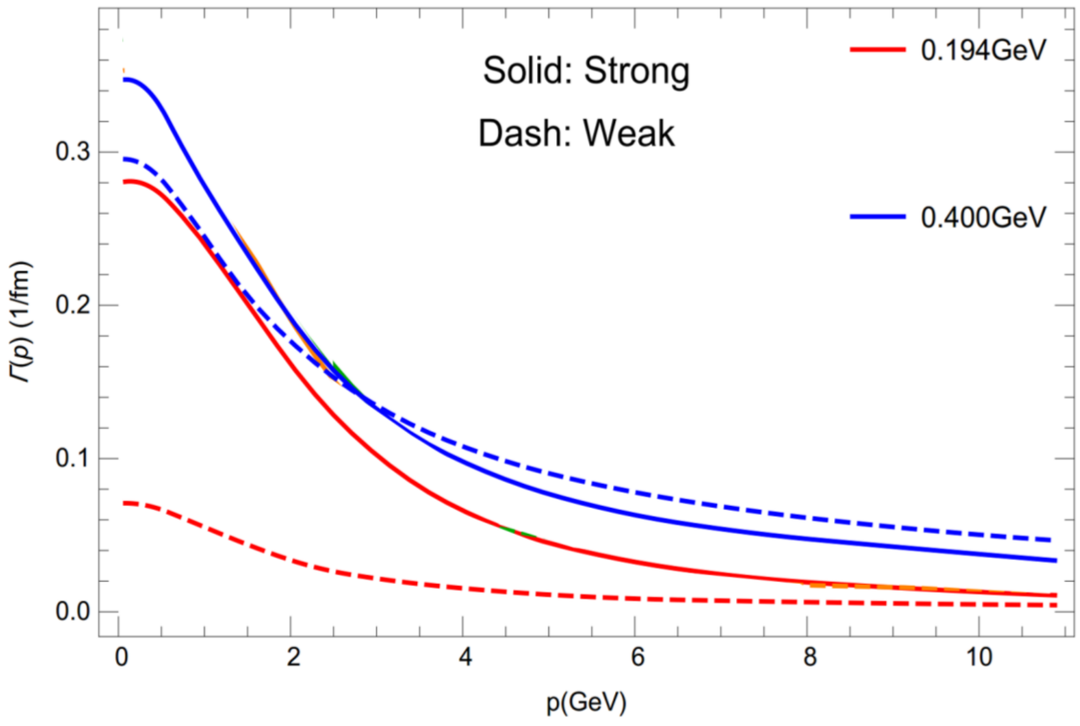
Liu & Rapp arXiv:1711.11559

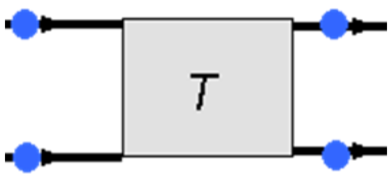


- Strongly coupled: $4\pi(\eta/s) \sim (2\pi T)D_s$
- Perturbative: $4\pi(\eta/s) \sim 2/5(2\pi T)D_s$
- Transition as ***T*** increases

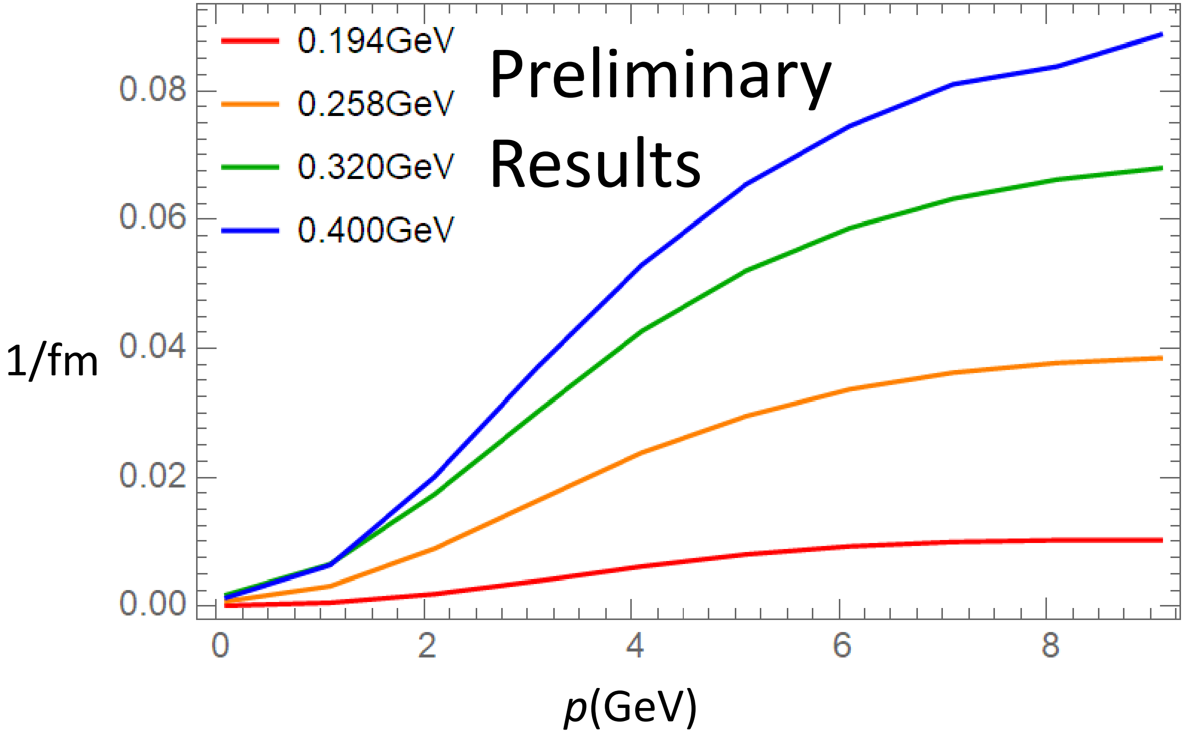
Relaxation Rate

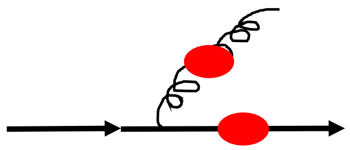
Collision induced Relaxation Rate $\Gamma(p)$





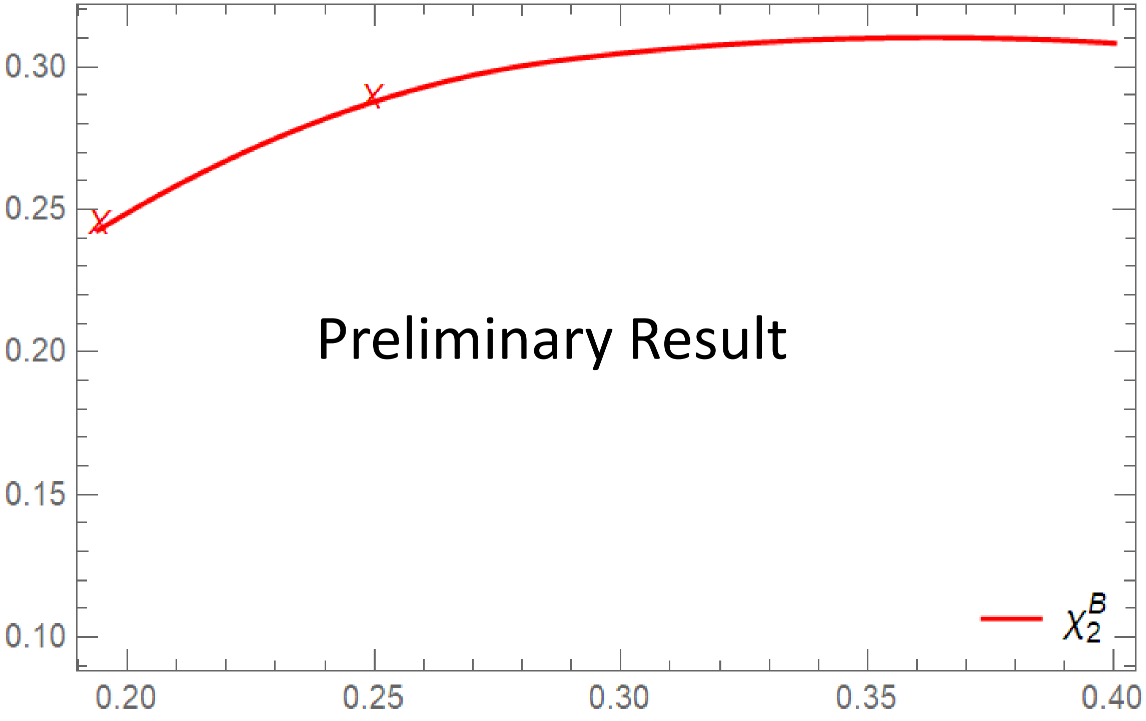
Radiative Induced Relaxation Rate $\Gamma(p)$





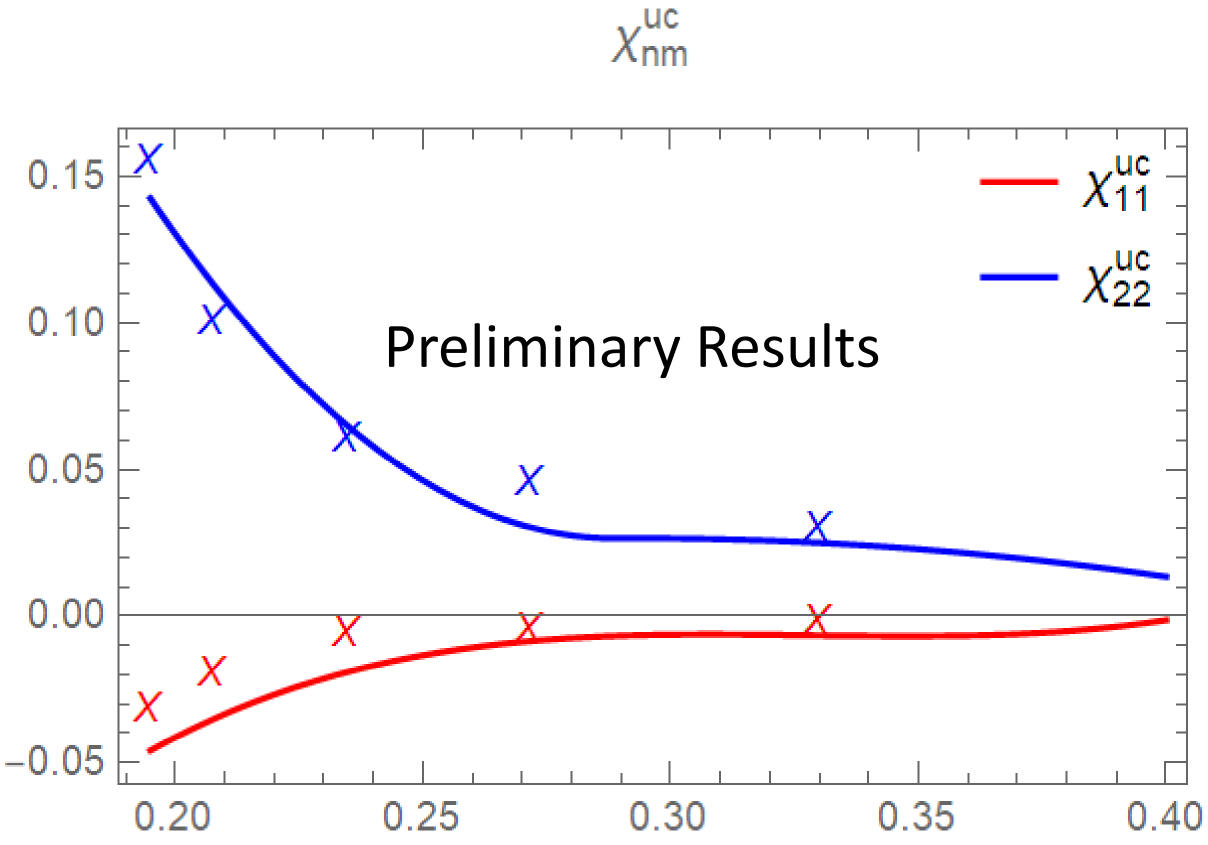
Preliminary for Quark Number Susceptibilities

- Light quark susceptibilities



T(GeV)

- Heavy light susceptibilities



T(GeV)

Conclusion and Perspective

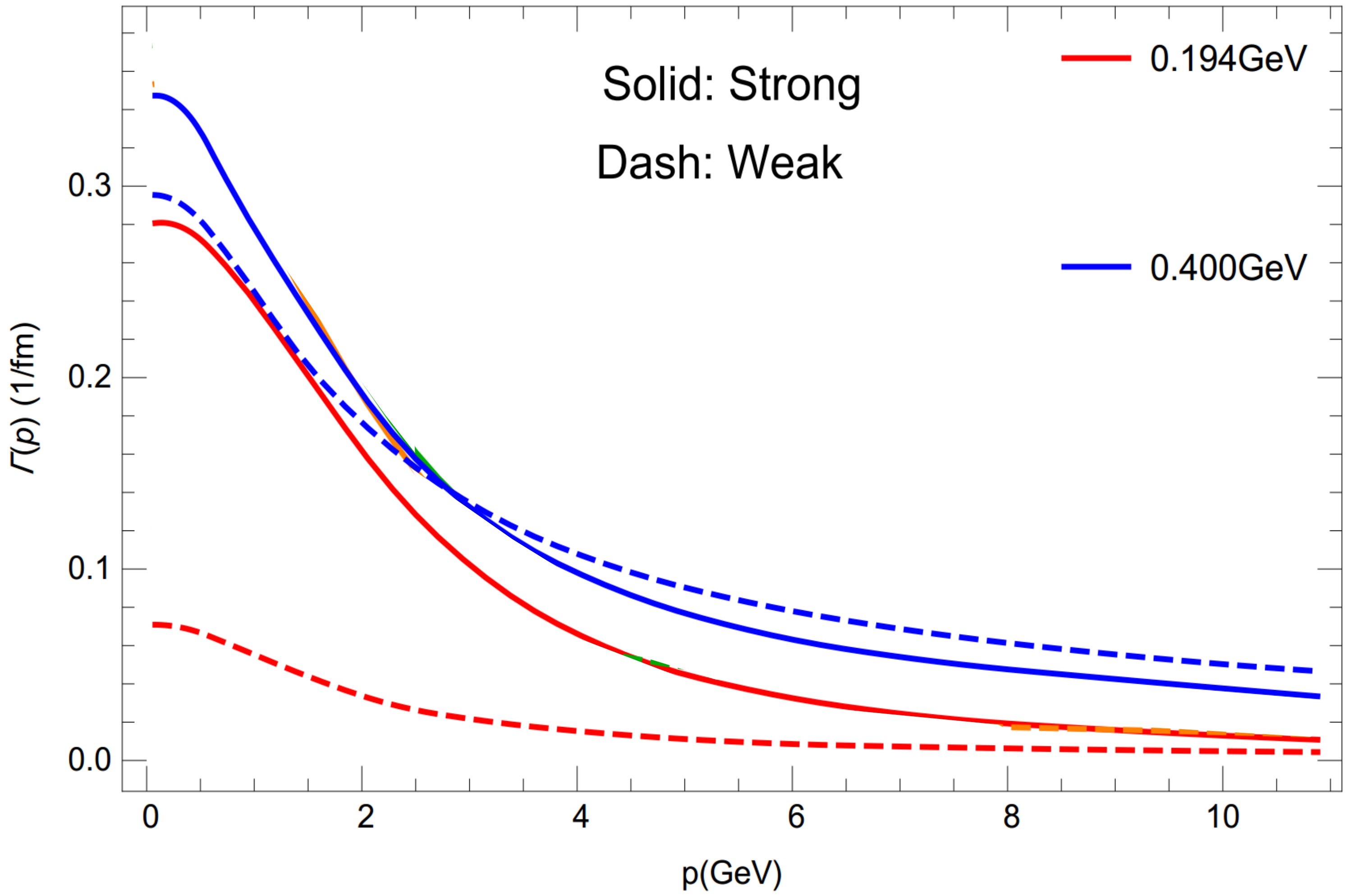
- We found a Solution in T-matrix approach consistent with:
  - Three lattice QCD data (EoS, Free energy, Quarkonium correlator)
  - Transport properties of the QGP (small η/s, small D_s) which lead to large v_2 of the D meson observed at experiments
- This solution predicts:
  - A strong force with large remnant string term not far above T_c
  - Broad non-quasi-particle spectral functions at low temperature
  - Degrees of freedom transit to broad resonance states at low temperature

Could we develop a single framework to benchmark a wide variety of information and apply to a wide variety of physics?

Maybe …

If true, how to derive from QCD?

Heavy-Quark Relaxation Rate and Langevin Dynamics



Liu, He, Rapp, inprep

He, Fries, Rapp, PLB14

- Relativistic Langevin Equation

$$dx = \frac{\mathbf{p}}{E}dt$$

$$dp = \Gamma(p)\,\mathbf{p}dt + \sqrt{D(\mathbf{p}+d\mathbf{p})dt}\,\rho$$

Friction force term

Random force term where $D(\mathbf{p}) = \Gamma(p)E\,T$ and ρ is Gaussian noises

D Meson Observables

Brownian Motion of Heavy Quark In QGP

Large friction force

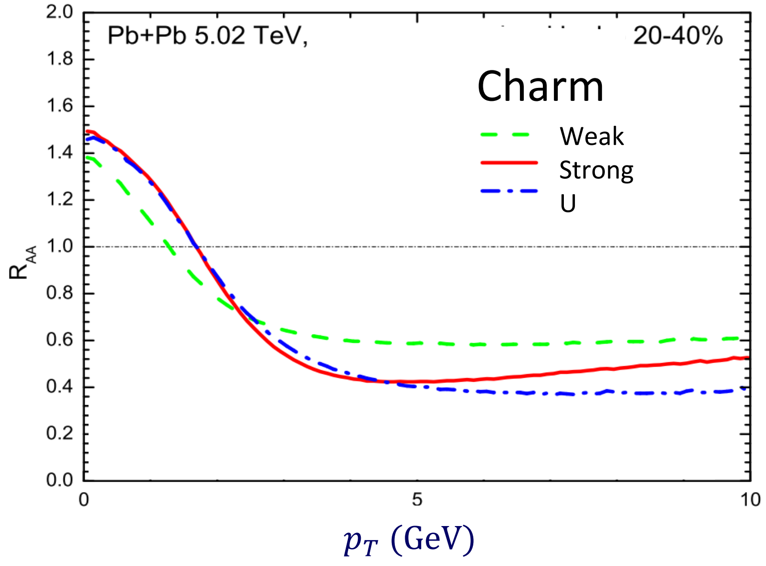
↓

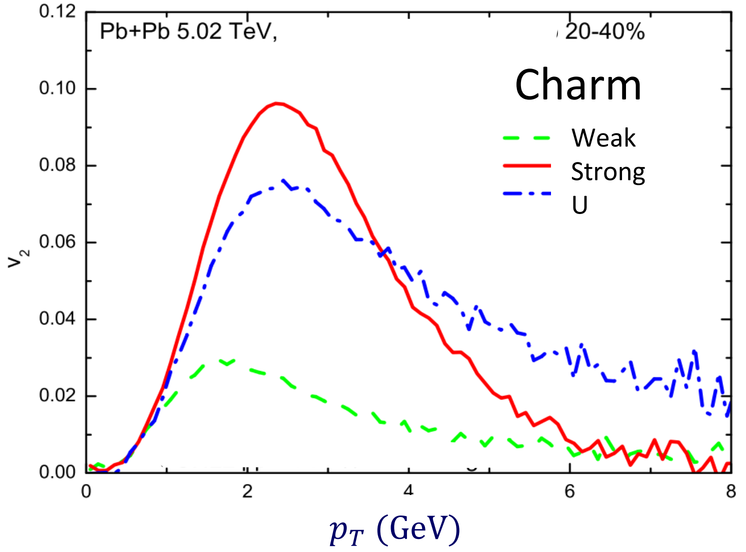
Quarks tend to move together in the medium

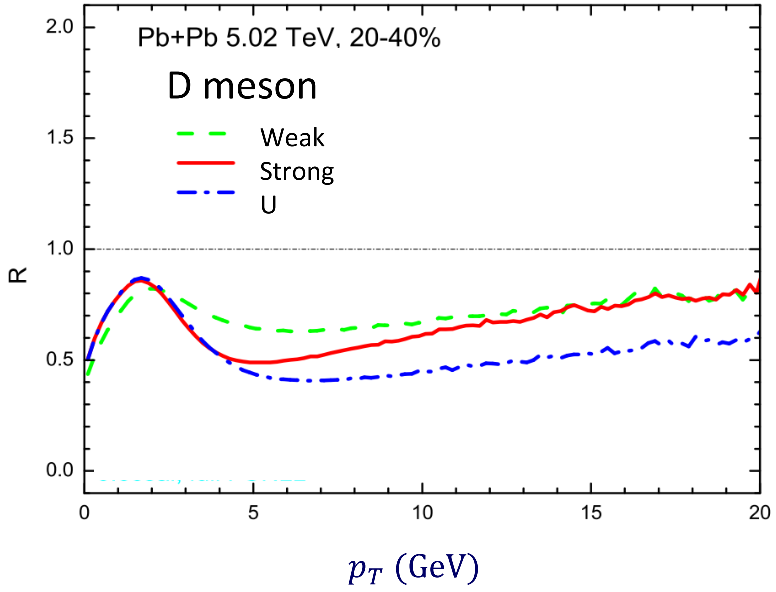
↓

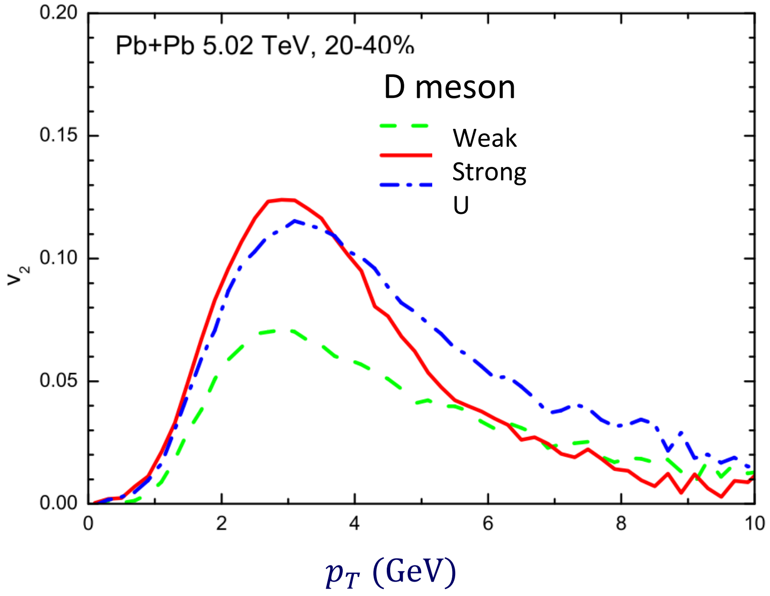
Large flow (v_2)

Heavy Meson v_2 measure the ***Color Force*** in QGP!









Back Up

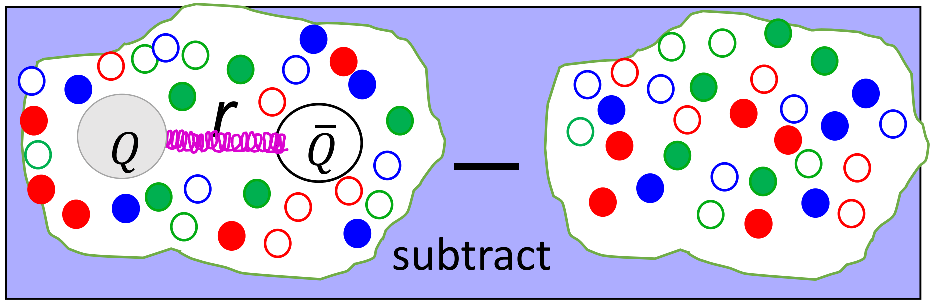
Heavy-Quark (HQ) Free Energy

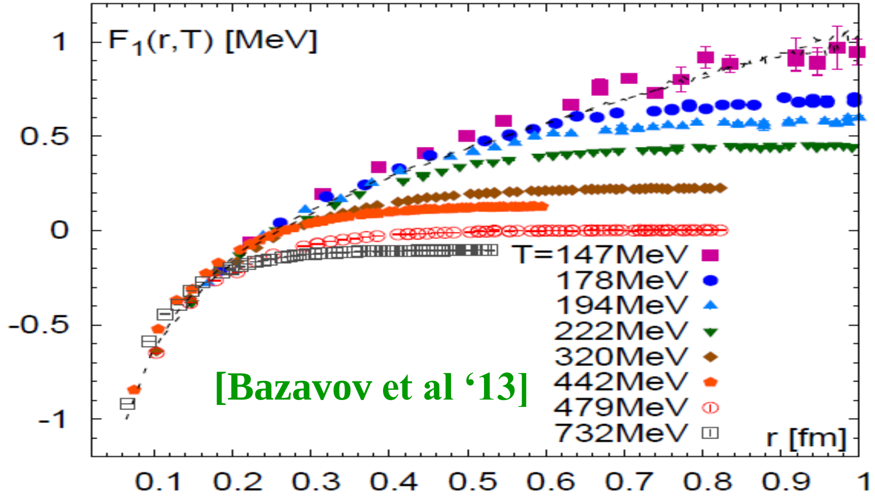
- In medium: $F_{Q\bar{Q}}(T,r)$ = change in free energy (lattice)

$$F_{Q\bar{Q}}(T,r) = -T\ln\left(G^{>}(-i\tau,r)\right)|_{\tau=\beta}$$

4-point $Q\bar{Q}$ Green function: $G^{>}(-i\beta,r) \leftrightarrow G(E,r)$

$$G(E,r) = \hat{G}(E) + \hat{G}(E)T(E,r)\hat{G}(E)$$





- Solve *T*-matrix in static limit analytically in energy space

$$G(E,r) = \frac{1}{E + i\epsilon - \tilde{V}(r) - \Sigma(E+i\epsilon,r)}$$

- Euclidean time space using spectral representation

$$\boxed{F_{Q\bar{Q}}(T,r) = -T\ln\left(\int_{-\infty}^{\infty} dE\, \frac{-1}{\pi}\mathrm{Im}\left[\frac{1}{E + i\epsilon - \tilde{V}(r) - \Sigma(E+i\epsilon,r)}\right] e^{-\beta E}\right)}$$

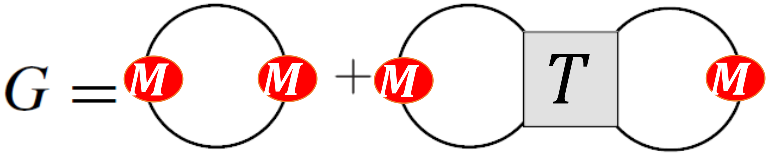
S.YF Liu + Rapp, NPA 941 S.YF Liu + Rapp, 2017 arXiv:1711.03282

Quarkonium Euclidean Correlator

- Current-current correlator

$$G^{>}(-i\tau) = \int \frac{d^3\mathbf{r}}{(2\pi)^3} \left\langle J_M(-i\tau,\mathbf{r}) J_M^{\dagger}(0,0) \right\rangle \qquad J_M(-i\tau,\mathbf{r}) = \psi^{\dagger}(-i\tau,r)\Gamma_M \psi(-i\tau,r)$$

- Calculated by *T*-matrix in energy space



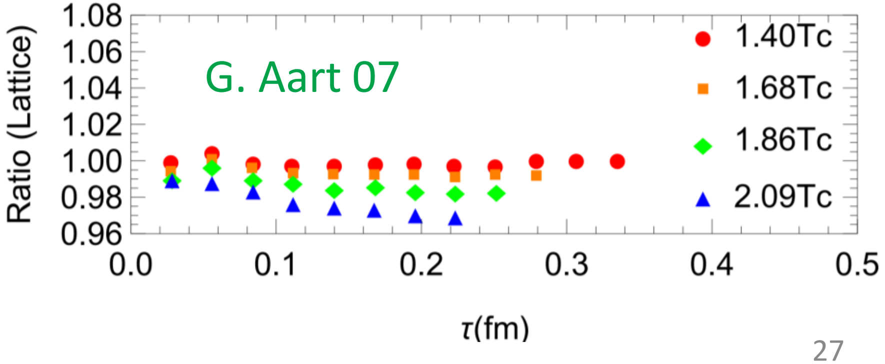
R. Felix, R. Rapp NJP 13

$$G(E) = \int \frac{d^3\mathbf{p}}{(2\pi)^3} \hat{G}(E,p) + \int \frac{d^3\mathbf{p}}{(2\pi)^3} \int \frac{d^3\mathbf{p}'}{(2\pi)^3} \hat{G}(E,p) T(E,\mathbf{p},\mathbf{p}') \hat{G}(E,p')$$

- Euclidean time space

$$G^{>}(-i\tau, T_{\mathrm{ref}}, T) = \int dE\, \rho(E,T_{\mathrm{ref}}) \mathcal{K}(\tau,E,T)\,, \rho(E,T) = -\frac{1}{\pi}\,\mathrm{Im}[G(E)]\,, \mathcal{K}(\tau,E,T) = \frac{\cosh[E(\tau-\beta/2)]}{\sinh[\beta/2]}$$

$$R(T_{\mathrm{ref}},T) = \frac{G^{>}(-i\tau,T,T)}{G^{>}(-i\tau,T_{ref},T)} = \frac{\int dE\, \rho(E,T)\mathcal{K}(\tau,E,T)}{\int dE\, \rho(E,T_{\mathrm{ref}})\mathcal{K}(\tau,E,T)}$$



EoS, Free Energy and Potential and (SCS)

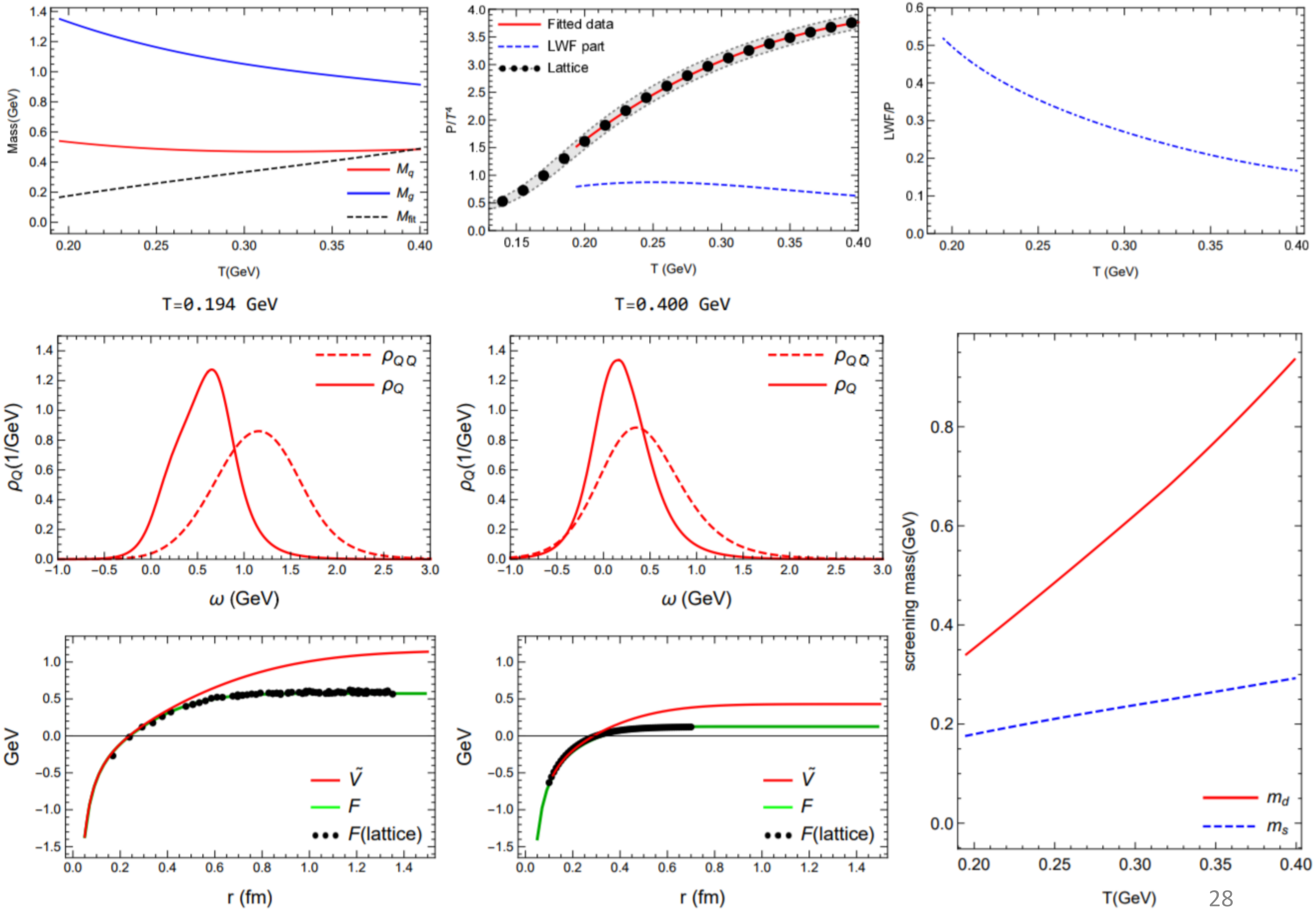
- Transition of freedom indicated by LWF

- Large width

- Width slightly decreases with temperature

- *V* much larger than *F*

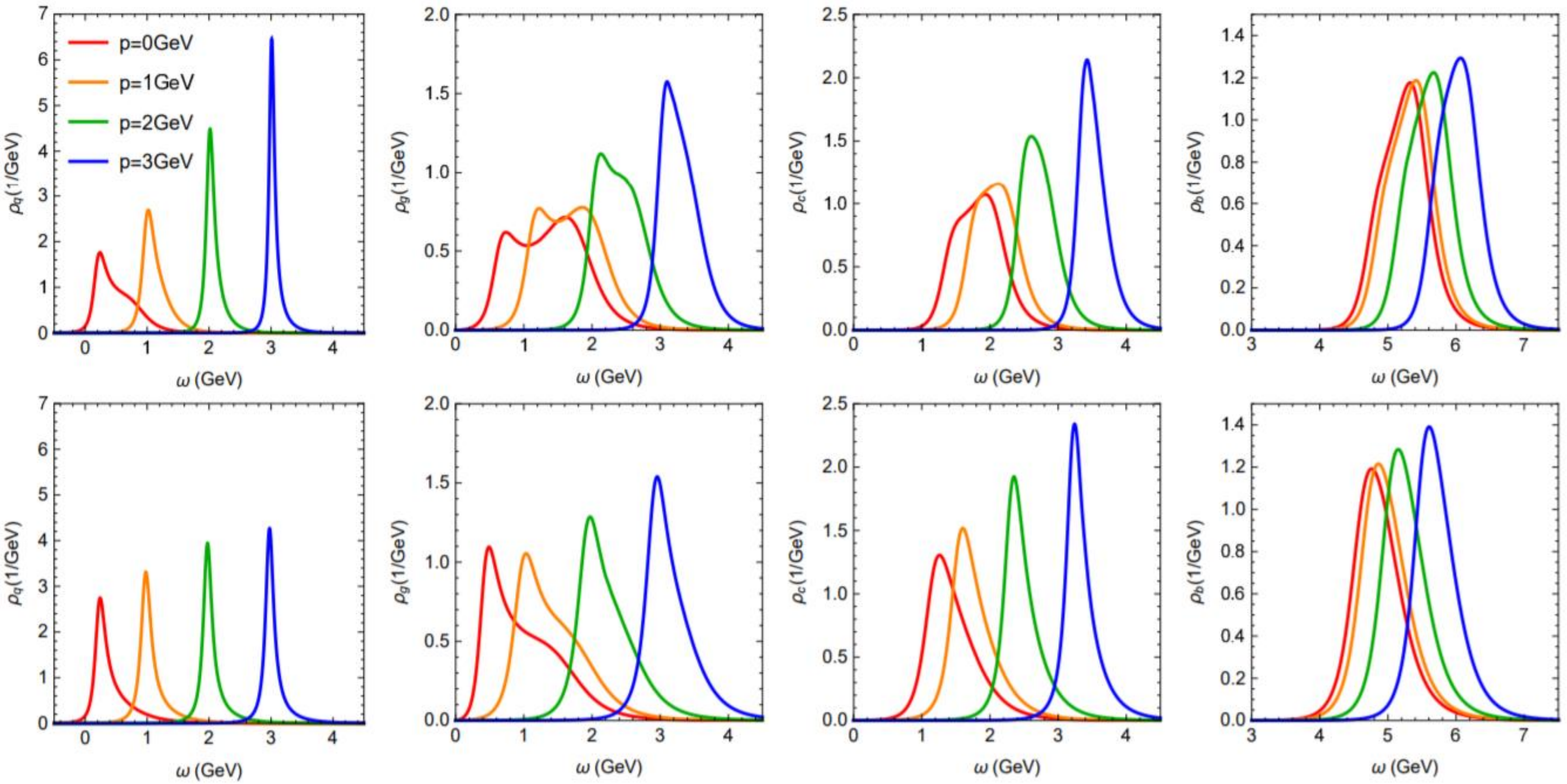
- Small screening mass for string term



Spectral Functions of Partons (SCS)

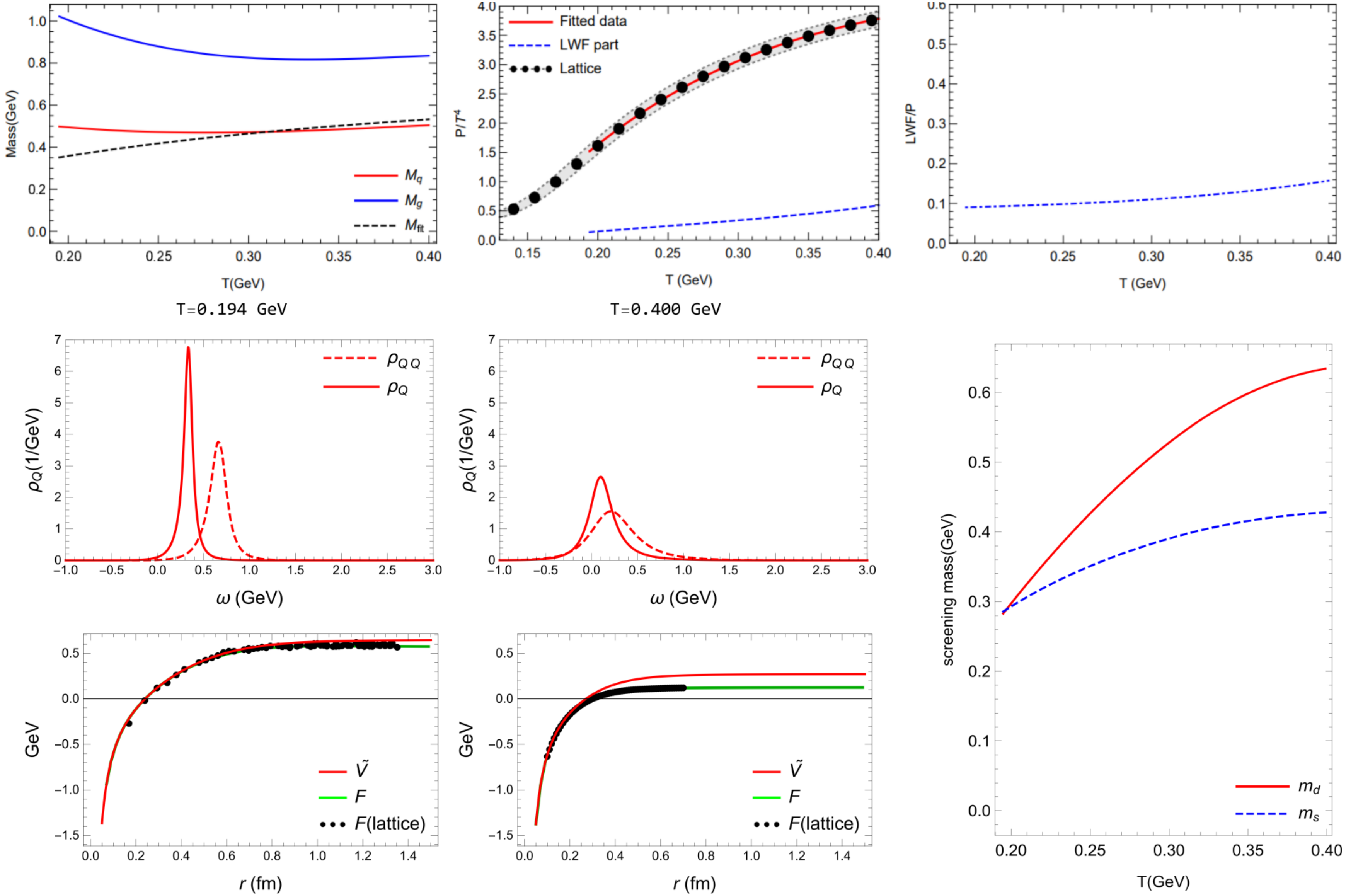
- Broad at low momentum and low temperature

- Sharp at high momentum and high temperature



EoS, Free Energy and Potential (WCS)

- No transition of freedom indicated by LWF
- Small width
- Width linearly increases with temperature
- *V* much closer to *F*
- Similar screening masses for string and Coulomb terms

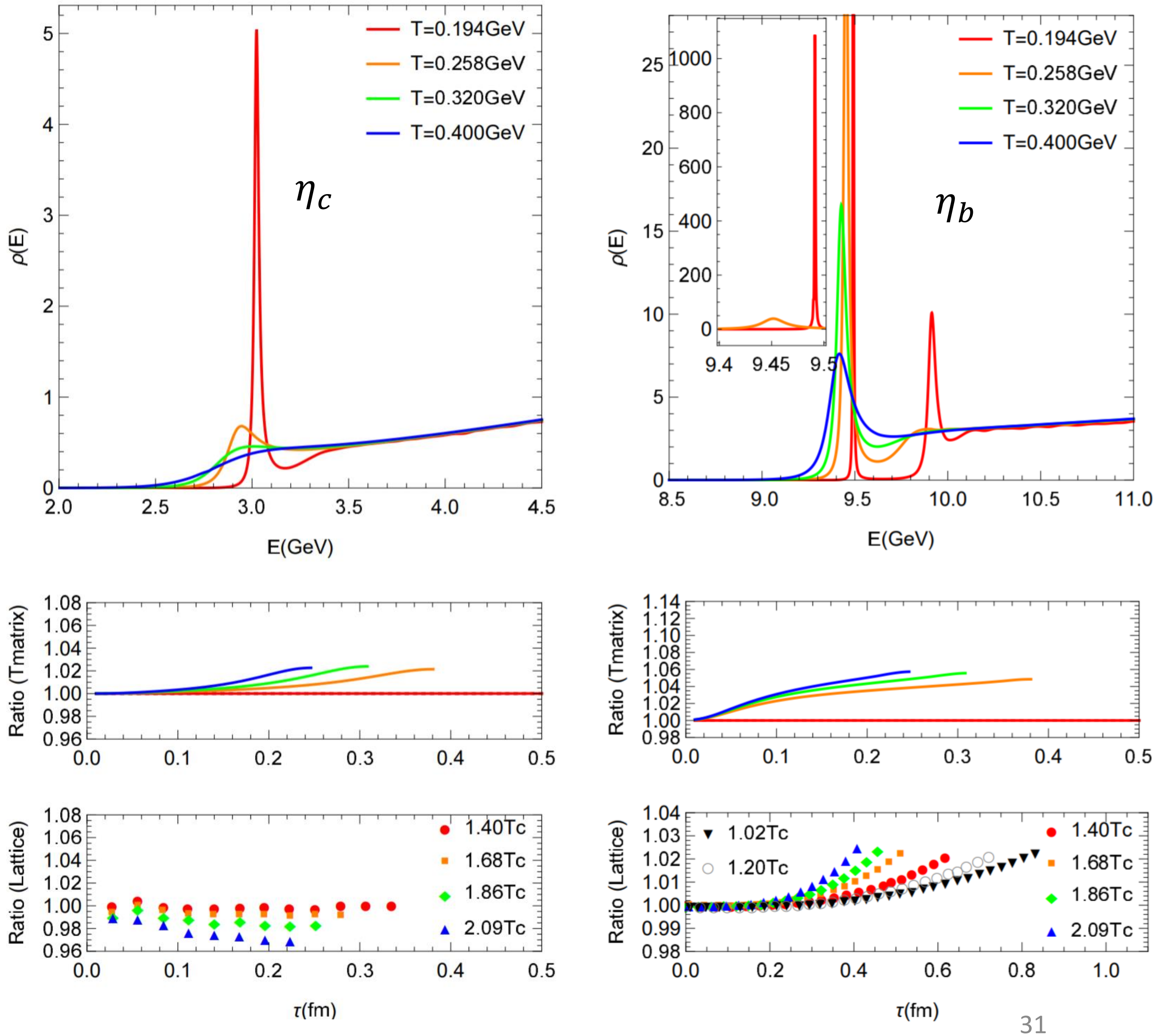


Quarkonium Euclidean Correlator (WCS)

- Small dissociation/regeneration rate for quarkonium
- Sequential dissociation

$$R(T_{\mathrm{ref}}, T) = \frac{\int dE\, \rho(E,T)\mathcal{K}(\tau,E,T)}{\int dE\, \rho(E,T_{\mathrm{ref}})\mathcal{K}(\tau,E,T)}$$

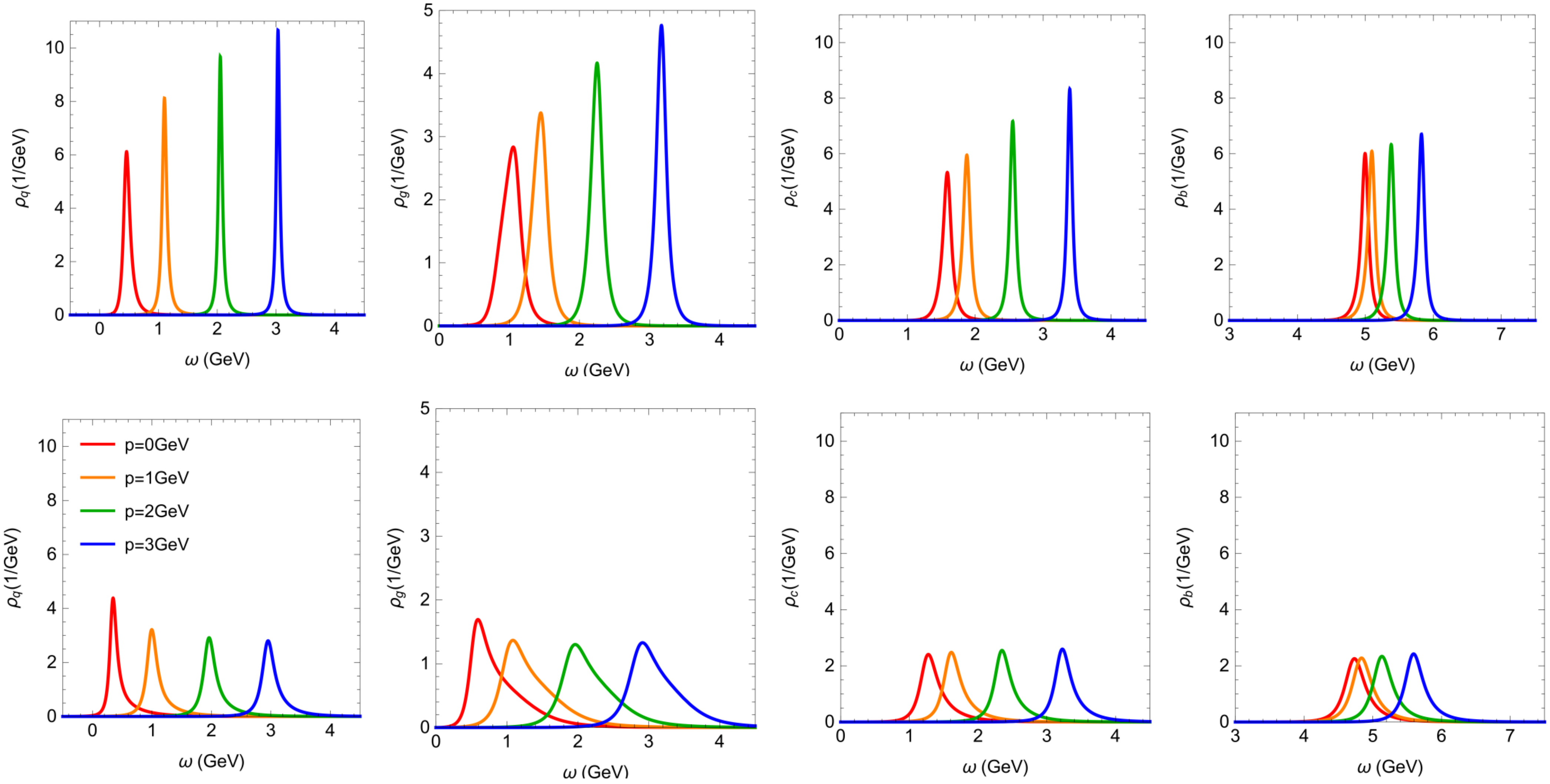
- No strong temperature dependence of α_s



Spectral Functions of Partons (WCS)

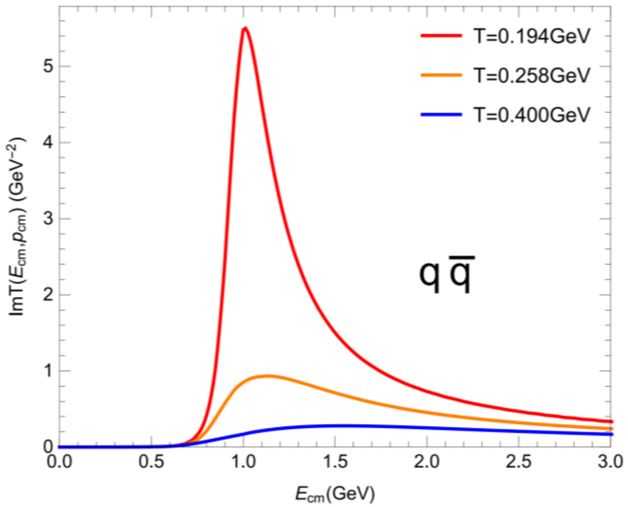
- Sharp at low momentum and low temperature

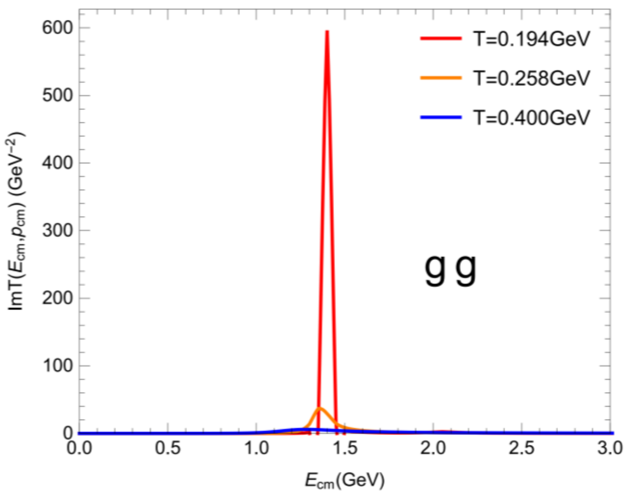
- Larger width at high temperature

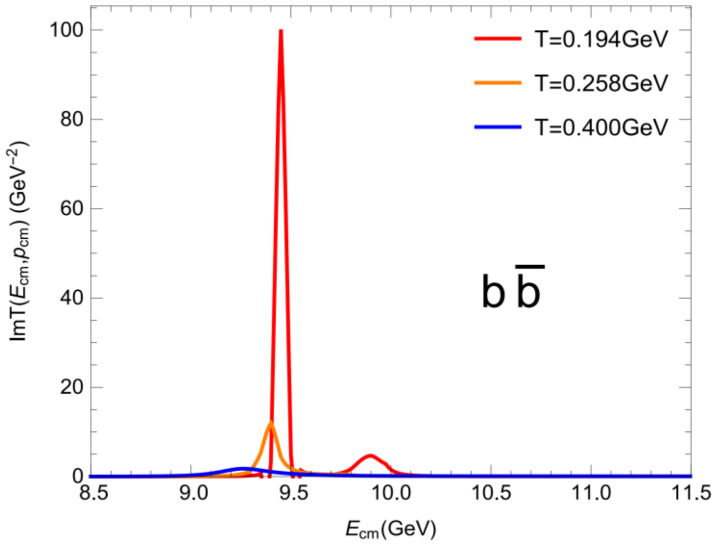


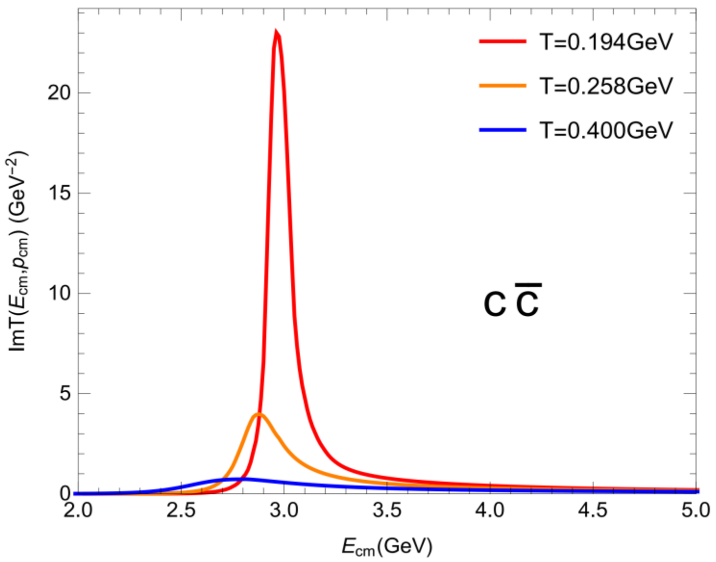
Two-body *T*-matrix Amplitude (WCS)

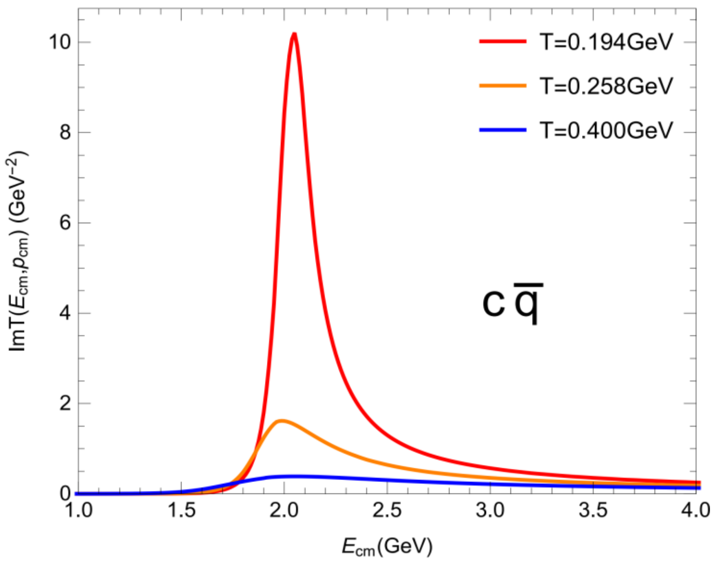
- Weak sharper resonance at low temperature
- Sequential dissociation/regeneration











Equation of State (EoS)

- Luttinger-Ward-Baym formalism for many-body system:

$$\Omega(T) = \sum_{s,c,f} \int d\widetilde{^4p} \pm \left[[\ln(-G^{-1}) + \Sigma\, G] - \Phi(G) \right] \qquad \Phi(G) = \sum_{v} \frac{1}{2v} \Sigma_v\, G$$

- Self-energy from *T*-matrix:

$\hat{G}$: two body propagator

$$\Sigma(\omega_n, p) = \sum_{s,c,f} \int d\widetilde{^4p}\, T\, G = \sum_{s,c,f} \int d\widetilde{^4p}\, \{V + V\hat{G}V + \cdots + V\hat{G}V\hat{G} \ldots . V\}\, G \;=\; \sum_{s,c,f,n} \int d\widetilde{^4p}\, V\left(1 - \hat{G}V\right)^{-1} G$$

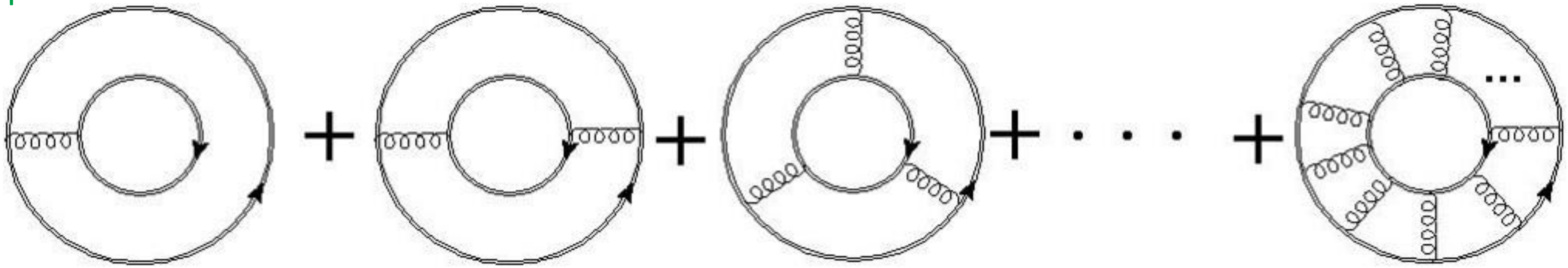
Matrix Inverse

- Matrix Log to sum up the skeleton expansion:

$$\Phi(G) = \sum_{v} \frac{1}{2v} \Sigma_v \mathrm{G} = \frac{1}{2} \sum_{s,c,f} \int d\widetilde{^4p} \left\{ V + \frac{1}{2} V\hat{G}V + \cdots + \frac{1}{v} V\hat{G}V\hat{G} \ldots . V \right\} \hat{\mathrm{G}} \;=\; \boxed{\frac{1}{2} \sum_{s,c,f} \int d\widetilde{^4p}\, \ln\left(1 - \hat{G}V\right)}$$

Matrix Log

S.YF Liu + Rapp arXiv:1609.04877
S.YF Liu + Rapp arXiv:1612.09138



Resonance in QGP

Examples in 60 color, spin, flavor, angular momentum channels
Each T

Sequential dissociation

(Notice Scale)

Melt as the T Increase