Parton Distributions from Large-Momentum Effective Theory

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Outline

- 1. Difficulties of calculating parton distributions
- 2. Large momentum effective theory (LaMET)
- 3. Applications of the LaMET approach

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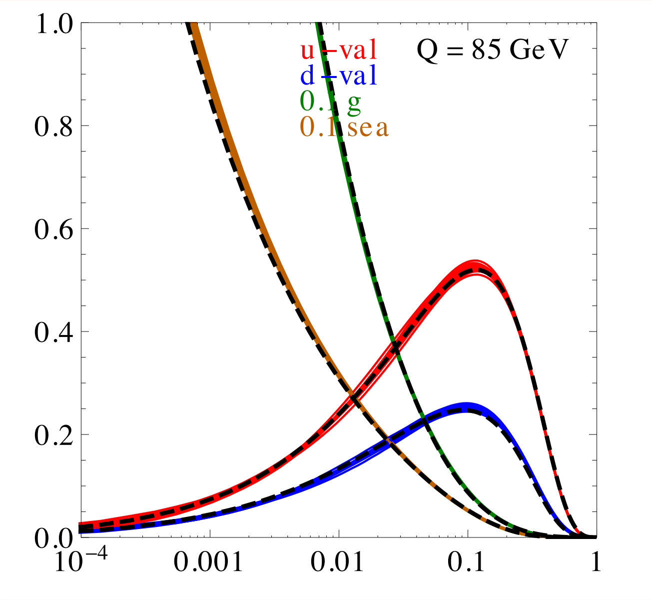
PDFs from global data analysis

- Currently our best knowledge of the PDFs comes from the global analysis of high-energy scattering data



CT10 NNLO PDF, CTEQ-TEA group, 2014

1. Extensive experimental analysis motivates a first principle calculation for comparison;

2. First principle calculation might be able to shed light on kinematic regions and flavor structures where experiments cannot constrain so precisely;

3. The cost of improving calculations seems to be much smaller than building larger experiments.

Operator definition of PDF

- Definition of PDFs in QCD factorization theorems:

$$q(x,\mu) = \int \frac{d\xi^-}{4\pi} e^{-ixP^+\xi^-} \langle P | \overline{\psi}(\xi^-) \gamma^+ U(\xi^-,0) \psi(0) | P \rangle$$

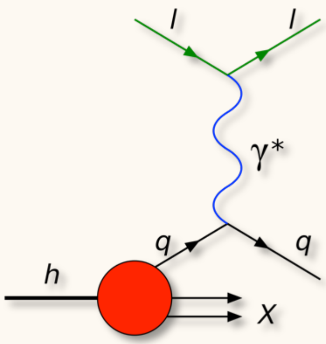
$$\sigma = \sum_{a,b} f_a(x_1) \otimes f_b(x_2) \otimes \sigma_{ab}$$

$$\xi^\pm = (t \pm z)/\sqrt{2} \quad U(\xi^-,0) = P\exp\left[-ig \int_0^{\xi^-} d\eta^- A^+(\eta^-) \right]$$

- Gauge-invariant and boost-invariant light-cone correlation;
- In the light-cone gauge $A^+=0$, has a clear interpretation as parton number density,

$$q(x) \sim \int dk^+ d^2k_\perp \delta(k^+ - xP^+) \langle P | \hat{n}(k^+, k_\perp) | P \rangle$$

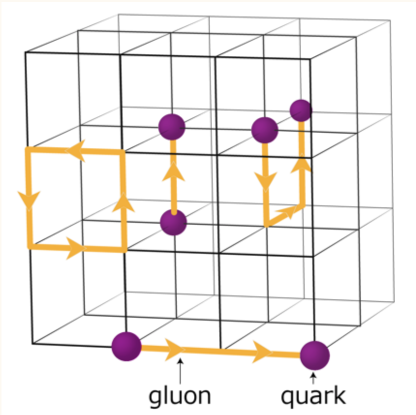
Lattice QCD is the only practical method to solve QCD nonperturbatively so far



Parton model:

- Minkowski space, real time
- Emerges in the infinite momentum frame (IMF), or, the proton as seen by an observer moving at the speed of light (on the light-cone)

$$\xi^{+}=(t+z)/\sqrt{2}=0$$



$$e^{iS}\rightarrow e^{-S}$$
$$\langle O\rangle=\int D\psi D\bar{\psi}DA\, O(x)e^{-S}$$

Lattice QCD:

- Euclidean space, imaginary time ($t=i\tau$)
- Nucleon static or at finite momentum
- Cannot calculate time-dependent quantities generally due to difficulty in analytical continuation in time

PDF not directly accessible from the lattice!

PDF from the Euclidean Lattice

- Computation of PDF moments:

$$n_\mu = (1,0,0,-1)/\sqrt{2}$$

$$\int dx\, x^{n-1} q(x,\mu) dx = a_n(\mu) = n_{\mu_1} n_{\mu_2} \cdots n_{\mu_n} \left\langle P \left| \bar{\psi}(0)\gamma^{\mu_1} i\overleftrightarrow{D}^{\mu_2} \cdots i\overleftrightarrow{D}^{\mu_n} \psi(0) \right| P \right\rangle$$

  - Moments are calculable as matrix elements of local gauge-invariant and frame-independent operators;
  - Fitting the PDF from the moments;
  - Operator mixing due to broken Lorentz symmetry limits computation for moments higher than 3.

n≤3, W. Detmold et al., EPJ 2001, PRD 2002;
D. Dolgov et al. (LHPC, TXL), PRD 2002;

Proposals in recent years

- **Restoration of rotational symmetry to calculate higher moments**

  n>3, Z. Davoudi and M. Savage, PRD 2012.

- **Fictitious heavy-to-light current-current correlator**

  D. Lin and W. Detmold, PRD 2006.

- **OPE of the Compton amplitude**

  A. J. Chambers et al. (QCDSF), PRL 2017

- **Direct computation of the physical hadronic tensor**

  K.F. Liu (et al.), 1994, 1999, 1998, 2000, 2017.

Proposals in recent years

- **Large momentum effective theory (LaMET)**

  X. Ji, PRL 2013; Sci.China Phys.Mech.Astron. 2014.

  Quasi-PDF (Large momentum factorization)

  Gradient flow method

  C. Monahan and K. Orginos, JHEP 2017.

  Pseudo-PDF (Small distance factorization)

  A. Radyushkin, PRD 2017;
  K. Orginos, A. Radyushkin, J. Karpie and S. Zafeiropoulos, 2017.

- **Lattice cross section**

  Y.-Q. Ma and J. Qiu, 2014, 2017.

- **Factorization of Euclidean correlations in coordinate space**

  V. M. Braun and D. Mueller, EPJ C 2008;
  G. S. Bali, V. M. Braun, A. Schaefer, et al., 2017.

Outline

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Parton model and the IMF

- Consider one starts from a static proton. The notion of parton does not exist as quarks and gluons are not free;
- Under a Lorentz boost along the z direction (dynamical transformation), the interacting quark or gluon can be transformed into an infinite number of particles, thus a longitudinal momentum density depends on the reference frame and is not physically meaningful;
- Nevertheless, when boosted to the IMF, all interaction effects are suppressed by powers of the infinite momentum, and the parton model emerges as the leading order approximation.

Large momentum effective theory

- If one knows the nucleon wavefunction in the IMF, then all parton physics can be solved, but this is highly nontrivial and unknown in an interacting theory like QCD;
- The good thing is that QCD has asymptotic freedom. If there is a large scale, one can formulate an effective theory defined by that scale, and use this effective theory to match full QCD to physics below the scale;
- For example, the heavy-quark effective theory where the heavy quark mass sets the scale.
- **In large-momentum effective theory, the nucleon momentum P^z sets the scale.**

Large momentum effective theory

Large momentum effective theory (LaMET) is a theory that expands in powers of $1/P^z$, where P^z is the proton momentum (Ji, PRL 2013, Sci. China Phys. Mech. Astro., 2014):

1. Construct a Euclidean quasi-observable $\tilde{O}$ which can be calculated in lattice QCD;

2. The IMF limit of $\tilde{O}$ is constructed to be a parton observable O at the operator level;

$$\boxed{\begin{array}{l} |P\neq 0\rangle = U(\Lambda(P))|P_0=0\rangle, \quad U(\Lambda(P=\infty))^{-1}\tilde{O}U(\Lambda(P=\infty)) = O \\ \langle P=\infty|\tilde{O}|P=\infty\rangle = \langle P_0=0|O|P_0=0\rangle \end{array}}$$

Recall that one does not know the proton wavefunction in the IMF!

Large momentum effective theory

3. At finite P^z, the matrix element of $\tilde{O}$ depends on the cut-off Λ of the theory (if not renormalized) and generally P^z, i.e., $\tilde{O}(P^z/\Lambda)$, while that of O depends on the renormalization scale μ (if in the MSbar scheme), i.e., $O(\mu)$;

$$\boxed{\begin{aligned}\tilde{O}(P^z/\Lambda) &= \left\langle P = P^z \middle| \tilde{O} \middle| P = P^z \right\rangle, \\ O(\mu) &= \left\langle P = \text{any} \middle| O \middle| P = \text{any} \right\rangle\end{aligned}}$$

4. Taking the $P^z \to \infty$ ($P^z >> \Lambda$) limit of $\tilde{O}(P^z/\Lambda)$ is generally ill-defined due to the singularities in quantum field theory,

$$\boxed{\lim_{P^z \gg \Lambda} \tilde{O}(P^z/\Lambda) = ?}$$

Large momentum effective theory

5. But it can be related to $O(\mu)$ through a factorization formula:

$$\boxed{\tilde{O}(P^z/\Lambda) = Z(P^z/\Lambda, \mu/\Lambda) \otimes O(\mu) + \frac{c_2}{P_z^2} + \frac{c_4}{P_z^4} + \ldots}$$

- P^z is much larger than Λ_{QCD} as well as the proton mass M to suppress the power corrections;

- One can regard as the $O(\mu)$ effective theory observable, and $\tilde{O}(P^z/\Lambda)$ as given by full QCD;

- $O(\mu)$ and $\tilde{O}(P^z/\Lambda)$ have the same infrared (IR) physics, and thus can be perturbatively matched to each other through the leading term.

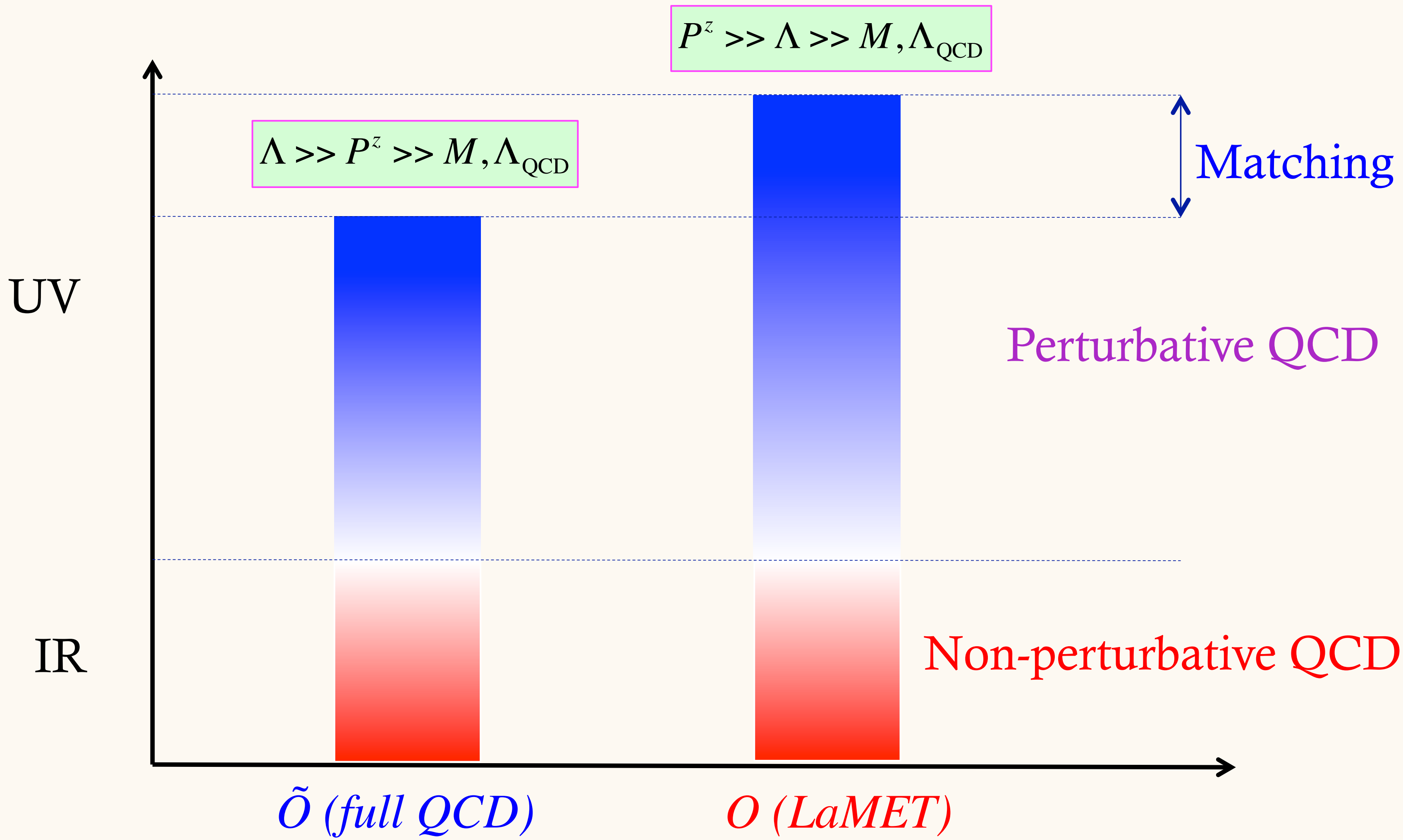
Large momentum effective theory

6. $\tilde{O}(P^z/\Lambda)$ satisfies a "renormalization group equation":

$$\boxed{\gamma(\alpha_s) = \frac{1}{Z}\frac{d\,Z}{d\ln P^z}}$$

- The parton observable $O(\mu)$ in the IMF is the "fixed point" of this RG equation;
- Physics near the "fixed point", i.e., $\tilde{O}(P^z/\Lambda)$ with different large P^z, are related by the RG equation.

How matching works



Large momentum effective theory

✧ Quasi-PDF:

X. Ji, PRL 2013; Sci.China Phys.Mech.Astron. 2014.

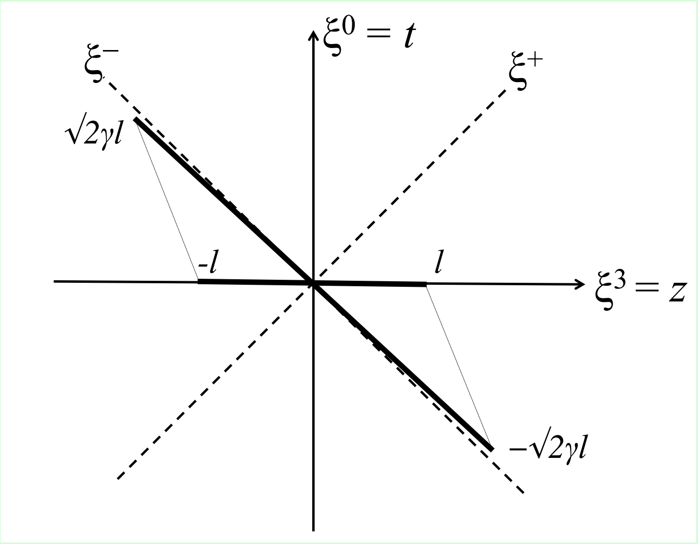
$$\tilde{q}(x,P^z,\Lambda=a^{-1})=\int\frac{dz}{4\pi}e^{ixP^z z}\left\langle P\middle|\bar{\psi}(z)\gamma^z U(z,0)\psi(0)\middle|P\right\rangle$$

$$z^\mu=(0,0,0,z)$$

$$U(z,0)=P\exp\left[-ig\int_0^z dz' A^z(z')\right]$$

- Time-independent correlation along the z direction, calculable in lattice QCD when $P^z \ll \Lambda$;

- Under an infinite Lorentz boost along the z direction ($P^z \gg \Lambda$), the spatial gauge link approaches the light-cone direction, and the quasi-PDF reduces to the (light-cone) PDF.



Large momentum effective theory

- The (renormalized) quasi PDF is related to the PDF through a factorization formula:

$$\tilde{q}_i(x, P^z, \tilde{\mu}) = \int_{-1}^{+1} \frac{dy}{|y|}\, C_{ij}\left(\frac{x}{y}, \frac{\tilde{\mu}}{P^z}, \frac{\mu}{P^z}\right) q_j(y, \mu) + \mathcal{O}\left(\frac{M^2}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{P_z^2}\right) ,$$

  - They have the same IR divergences;
  - C factor matches their UV difference, and can be calculated in perturbative QCD;
  - Higher-twist corrections suppressed by powers of P^z.

Procedure of Systematic Calculation

1. Simulation of the quasi PDF in lattice QCD

3. Subtraction of higher twist corrections

$$\tilde{q}_i(x, P^z, \tilde{\mu}) = \int_{-1}^{+1} \frac{dy}{|y|}\, C_{ij}\left(\frac{x}{y}, \frac{\tilde{\mu}}{P^z}, \frac{\mu}{P^z}\right) q_j(y, \mu) + \mathcal{O}\left(\frac{M^2}{P_z^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{P_z^2}\right),$$

2. Renormalization of the lattice quasi PDF, and then taking the continuum limit

4. Matching to the MSbar PDF.

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Current status

Collaborations actively working with the LaMET approach:

- LP3 Collaboration:

  J.W. Chen, T. Ishikawa, L. Jin, H.-W. Lin, Y.-S. Liu, Y.-B. Yang, J.-H. Zhang, R. Zhang, and Y.Z.

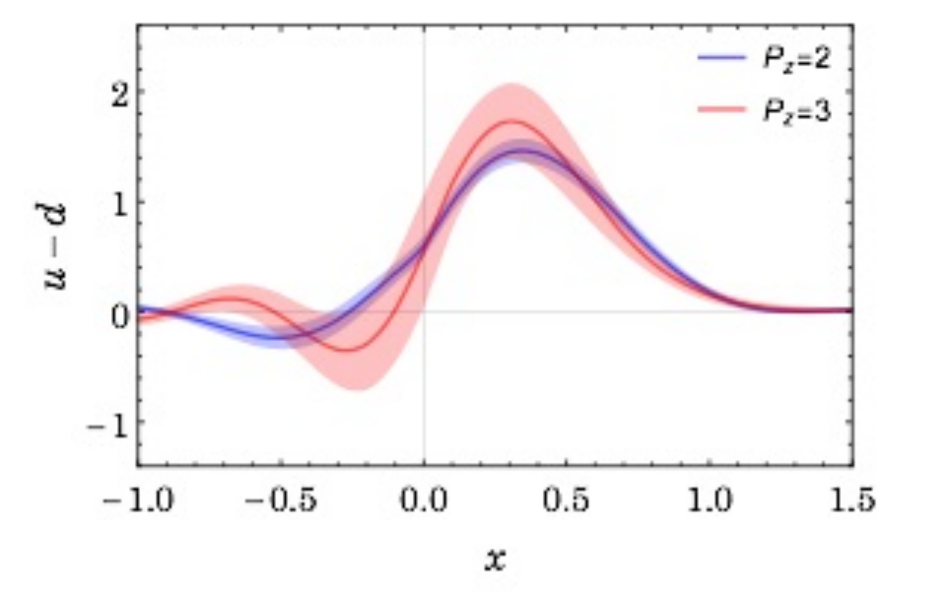
- European Twisted Mass Collaboration (ETMC).

  C. Alexandrou, M. Constantinou, K.Cichy, V. Drach, E. Garcia-Ramos, K. Hadjiyiannakou, K. Jansen, F. Steffens, C. Wiese et al.

- χ QCD Collaboration (Gluon polarization calculation):

  A. Alexandru, T. Drapper, M. Glatzmaier, K.F. Liu, R.S. Suffian, Y.-B. Yang, Y.Z., et al.

First complete analysis with nonperturbative lattice renormalization



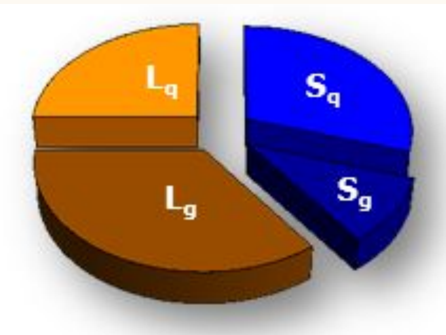
P^z={2,3}*0.43GeV

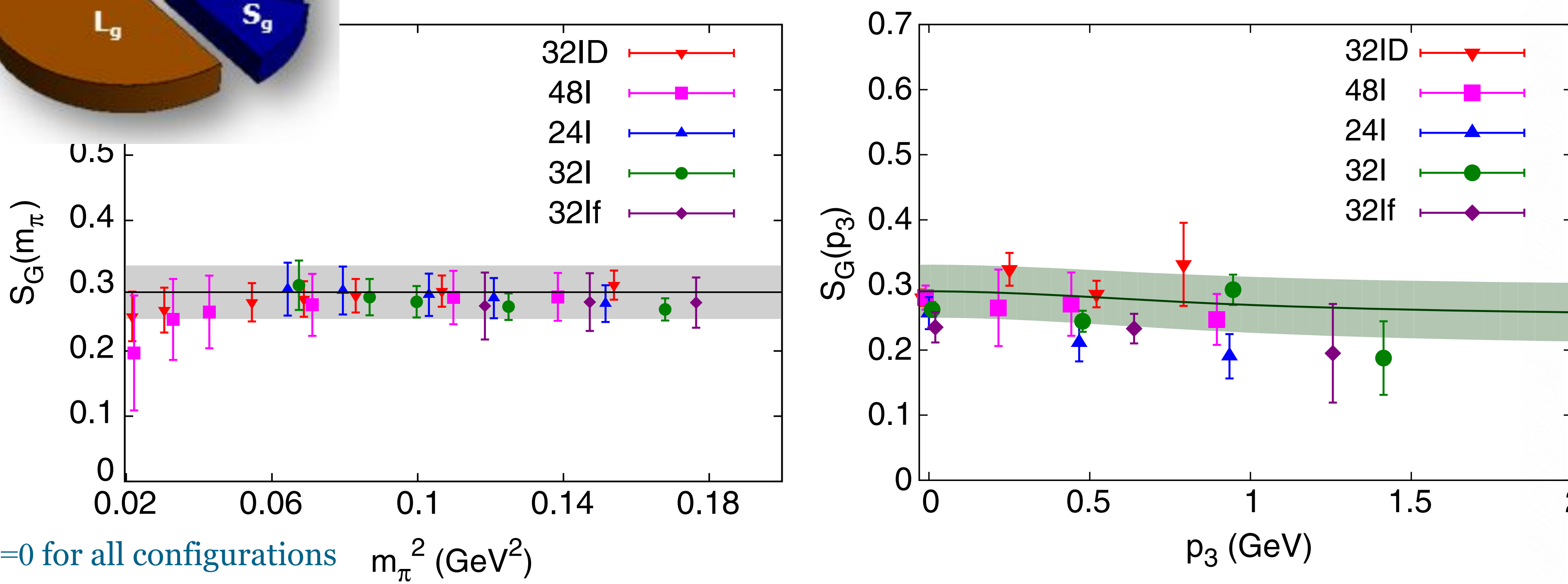
J.W. Chen, T. Ishikawa, L. Jin, H.-W. Lin, Y.-B. Yang, J.-H. Zhang, and Y.Z., (LP3), 2017

This is just an exploratory study. Improved results will come soon!

Gaussian-filter and derivative methods to reduce truncation error, by H.-W. Lin et al., 2017; A Gaussian re-weight method, by J.-H. Zhang et al. (LP3), 2017.

First calculation of gluon spin from lattice QCD





P=0 for all configurations

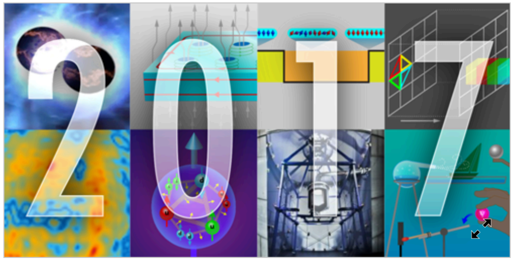
X. Ji, J.-H. Zhang, and Y.Z., PRL 2013, PLB 2015; Y. Hatta, X. Ji, and Y.Z., PRD 2014;
Y.-B. Yang, R. S. Sufian, Y.Z., et al (χ QCD collaboration)., PRL 2017

$$\Delta G(\mu^2 = 10\text{GeV}^2) \approx S_G(\infty, \mu^2 = 10\text{GeV}^2) = 0.251(47)(16)$$

Highlights of the Year

December 18, 2017 • *Physics* 10, 137

Physics looks back at its favorite stories from 2017.



Features

Highlights of the Year
Physics looks back at its favorite stories from 2017.

Research News: Editors' Choice
Electric tears, topological ocean waves, cosmic rays, and more in our monthly recap of papers from the literature.

Q&A: Inside a Cell's Engine Room
George Shubeita describes his studies of the tiny motors that transport vital cargoes within biological cells.

More Features »

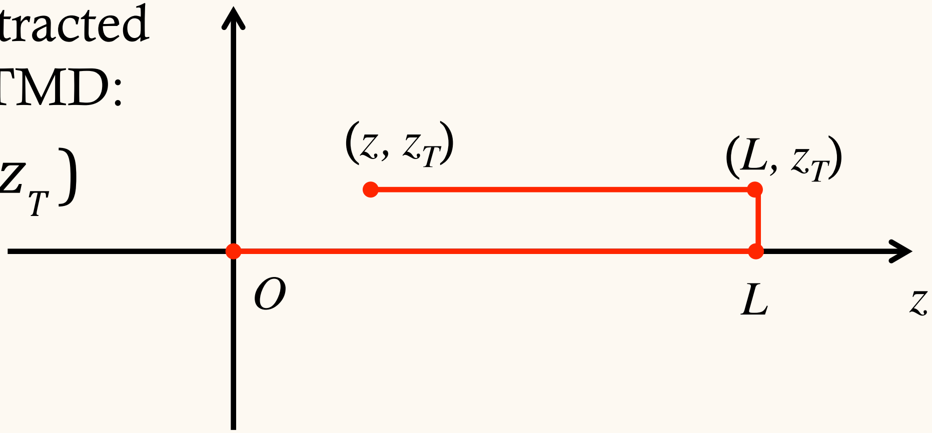
Gluons Provide Half of the Proton's Spin

The gluons that bind quarks together in nucleons provide a considerable chunk of the proton's total spin. That was the conclusion reached by Yi-Bo Yang from the University of Kentucky, Lexington, and colleagues (see Viewpoint: **Spinning Gluons in the Proton**). By running state-of-the-art computer simulations of quark-gluon dynamics on a so-called spacetime lattice, the researchers found that 50% of the proton's spin comes from its gluons. The result is in agreement with recent experiments and shows how such lattice simulations can now accurately predict an increasing number of particle properties. The simulations also indicate that, despite being substantial, the gluon spin contribution is too small to play a major part in "screening" the quark spin contribution—which according to experiments is only 30%—through a quantum effect called the axial anomaly. The remaining 20% of the proton spin is thought to come from the orbital angular momentum of quarks and gluons.

Transverse-momentum dependent distributions (TMDs)

Unsubtracted quasi-TMD:

$\tilde{q}(x, z_T)$



Subtracted quasi-TMD:

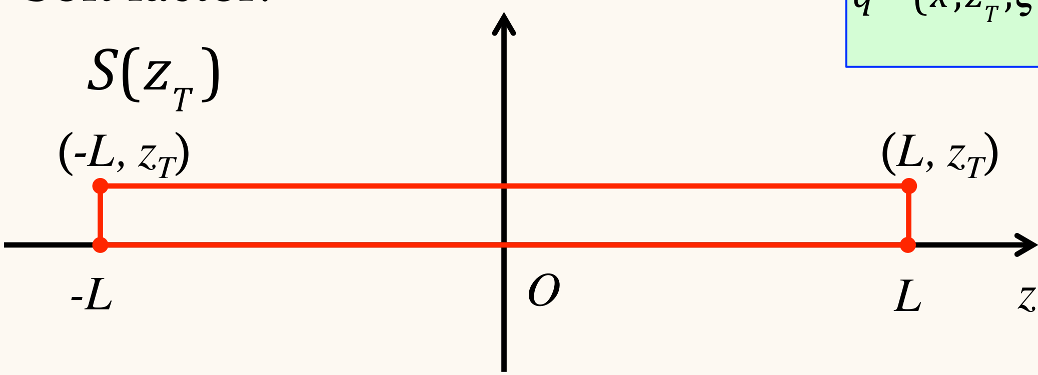
$$\boxed{\tilde{q}^{sub}(x, z_T) = \frac{\tilde{q}(x, z_T)}{\sqrt{S(z_T)}}}$$

LaMET matching:

$$\boxed{\tilde{q}^{sub}(x, z_T; \zeta^2) = e^{-\int_{\mu_b^2}^{\zeta^2} \frac{d\mu^2}{\mu^2}(-1)\frac{\alpha_S(\mu) C_F}{2\pi}} q^{sub}_{TMD}(x, z_T; \zeta^2)\left[1 + \frac{\alpha_S(\mu) C_F}{2\pi}(-2)\right]}$$

Soft factor:

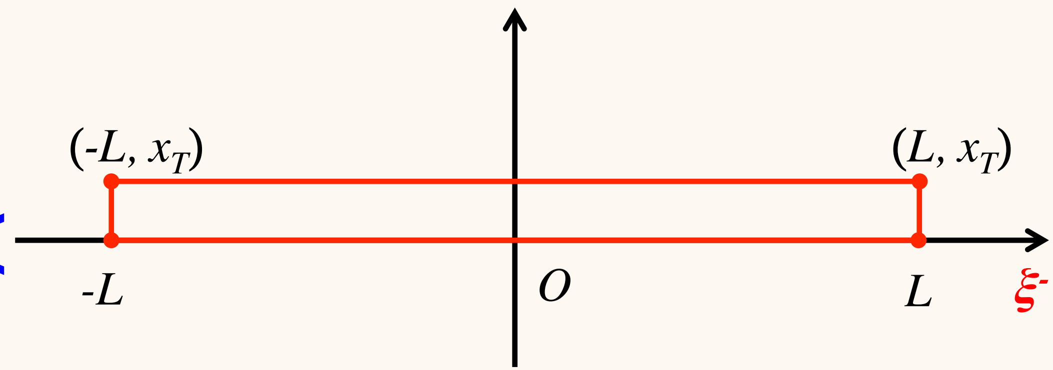
$S(z_T)$



X. Ji, L. Jin, F. Yuan, J. Zhang and Y.Z., arXiv: 1801.05930

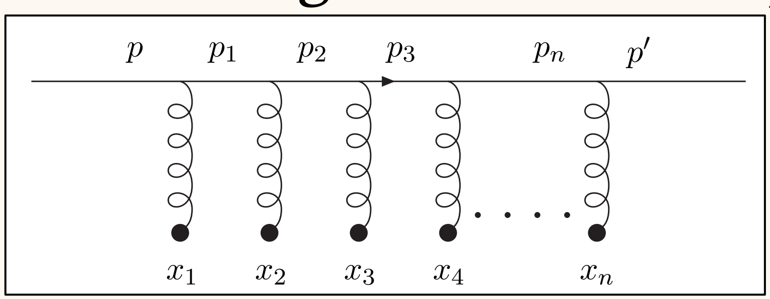
Jet quenching parameter ?

- Definition:

$r = |x_\perp|$ $\qquad V(r) \equiv -\lim_{L\to\infty} \frac{1}{L} \ln \Big\langle$  $\Big\rangle$

$$\hat{q} \equiv \int \frac{d^2 p_\perp}{(2\pi)^2}\, p_\perp^2 C(p_\perp), \quad V(r) = \int \frac{d^2 p_\perp}{(2\pi)^2} \left[1 - e^{i p_\perp \cdot x_\perp}\right] C(p_\perp)$$

- Physical picture: leading-order Eikonal approximation



Jet quenching parameter ?

- In reality, the parton is moving at a large finite momentum relative to the rest quark-gluon plasma (QGP). The Eikonal approximation is the IMF limit of this picture;

$$\lim_{P\to\infty} \text{off-the-light-cone correlation} = \text{light-cone correlation}$$

- The slightly-off-the-light-cone correlation in a rest thermal ensemble (QGP) is equal to the equal-time correlation in a boosted thermal ensemble; M. Panero, K. Rummukainen, A. Schaefer, 2013

$$\left\langle \text{(Slightly-off-the) light-cone correlation} \right\rangle_{\text{rest}} = \left\langle \text{equal-time correlation} \right\rangle_{\text{boosted}}$$

- The equal-time correlation has a non-trivial dependence on the QGP momentum (energy, or boost parameter), which can be related to the Eikonal dipole amplitude through LaMET.

Summary

- LaMET allows us to calculate the PDF from a Euclidean quasi-PDF on the lattice;
- A systematic procedure to calculate PDF from the lattice has been set up for precision calculations;
- The LaMET approach can be used to calculate other parton physics such as TMD and jet quenching parameter.