

TMD fragmentation within groomed Jets: Factorization and Resummation

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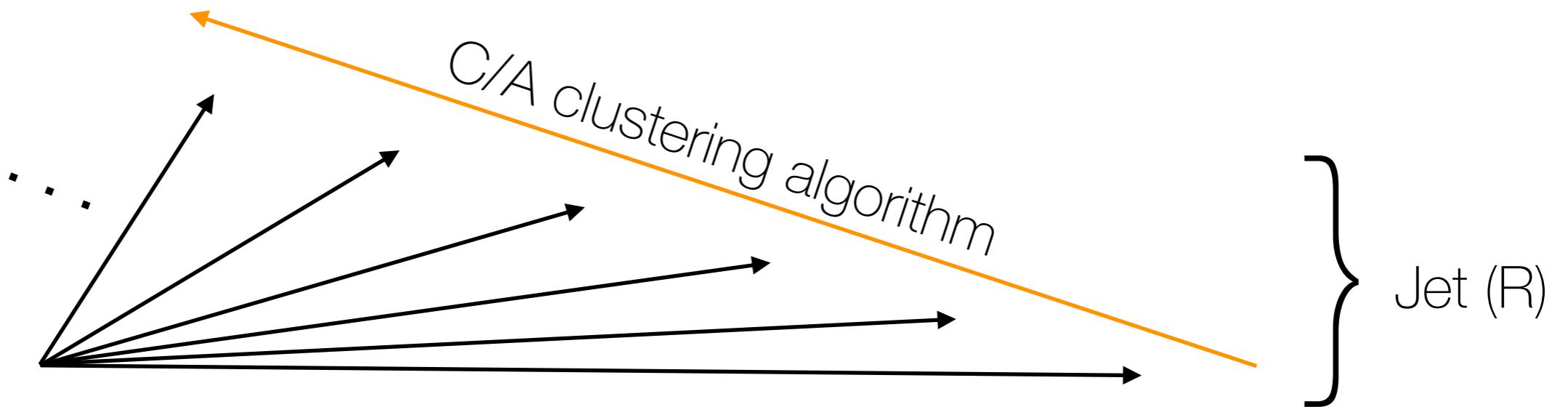
Based on: arXiv:1712.07653

TMD fragmentation within groomed Jets: Factorization and Resummation

- grooming procedure (soft-drop)
- factorization with groomed jets
- measurement
- resummation in momentum space
- resummation in impact parameter (b -)space
- probing TMD evolution

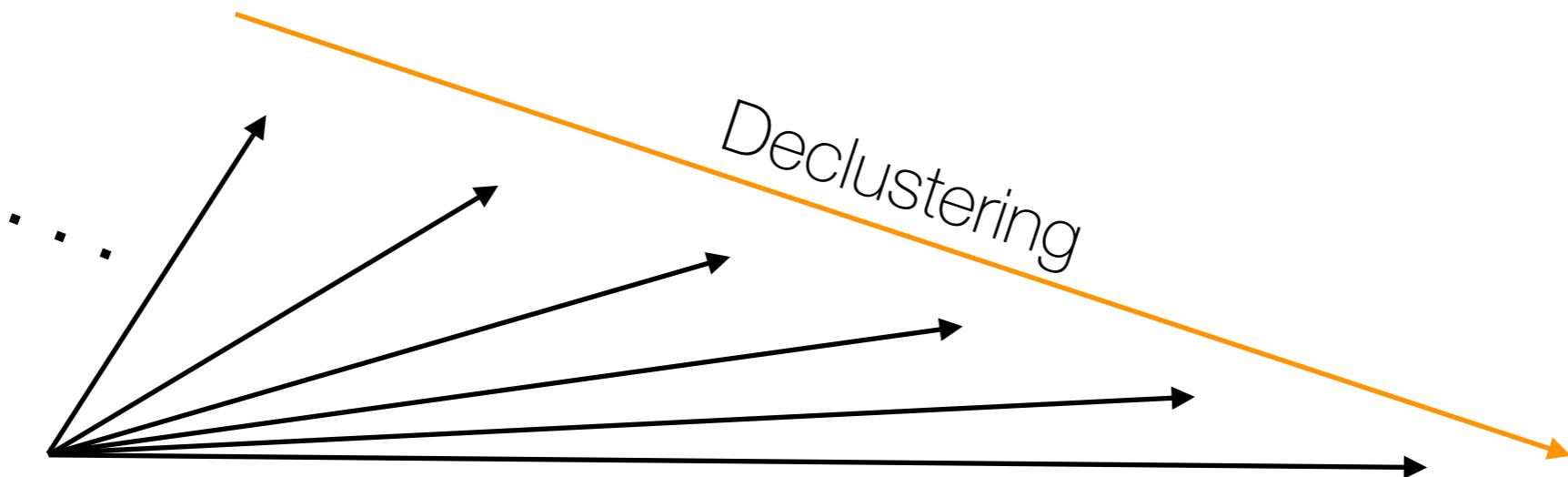
**universal for all TMDs
(e.g. proton TMDPDF)**

Grooming algorithm: mMDT/soft-drop ($\beta = 0$)



- The algorithm is imposed only on the jet constituents
- Particles closer in angle get clustered first
- Record clustering history in each step

Grooming algorithm: mMDT/soft-drop ($\beta = 0$)



"Declustering" with the reverse order:

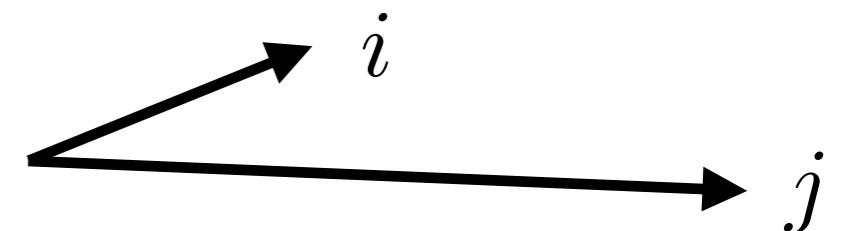
$$\frac{\min\{E_i, E_j\}}{E_i + E_j} > z_{\text{cut}}$$

True

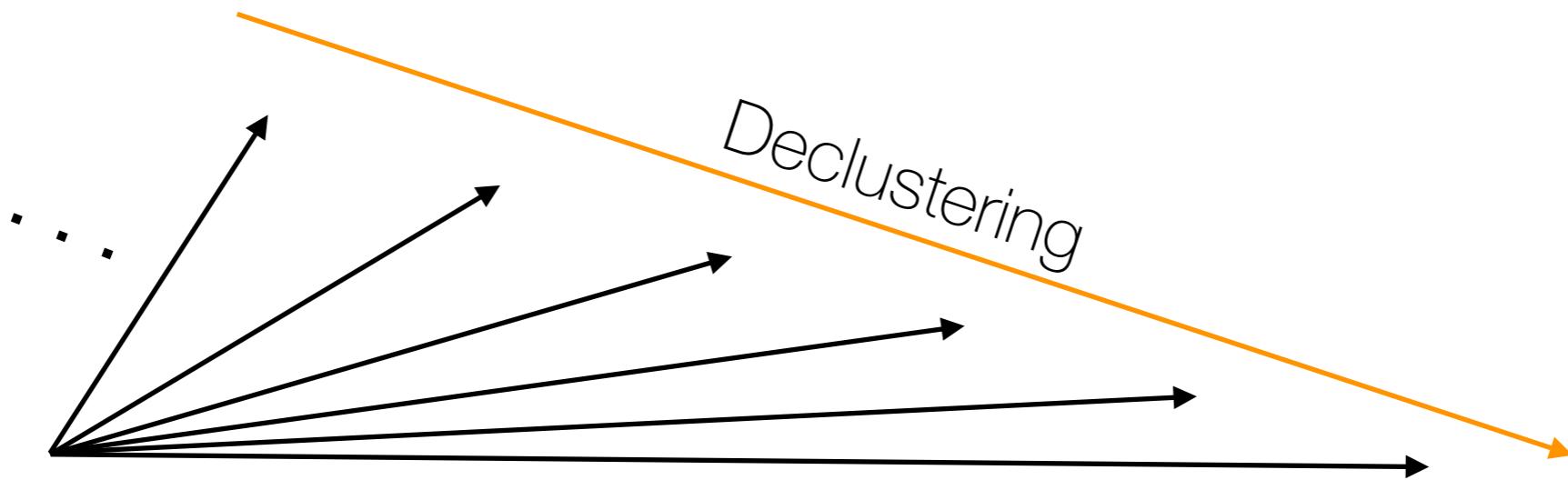
Stop. Remaining particles
consist groomed jet

False

Drop the softer of the two
branchings



Grooming algorithm: mMDT/soft-drop ($\beta = 0$)



- Removes soft wide angle radiation sensitive to the cone boundary and non-global effects (NGLs)
- Isolates collinear-energetic radiation near the center of the jet
- Smaller sensitivity to underlying event

Factorization with groomed jets

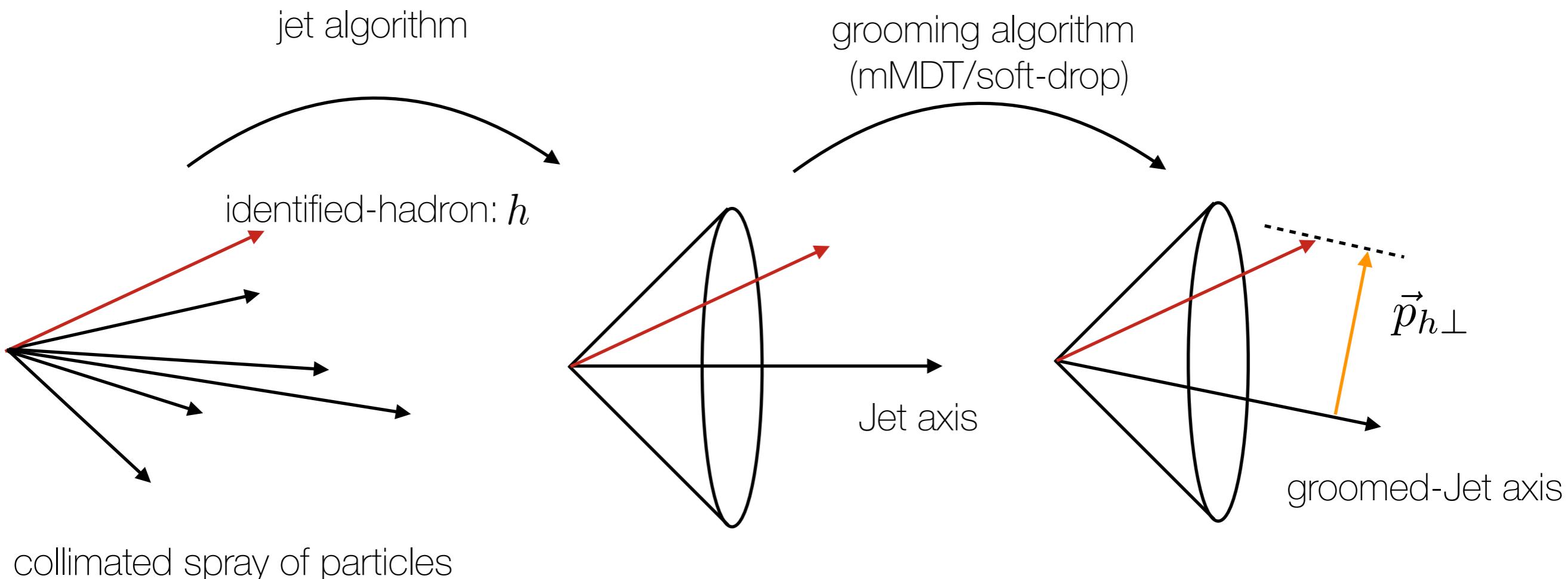
$$\frac{d\sigma}{d\vec{p}_J \, d\mathcal{M}} = \sum_{i=g,q} F_i(Q, R, z_{\text{cut}}, \vec{p}_J, \mathcal{C}) J_i(\mathcal{M}, z_{\text{cut}}, R, E_J)$$

Fraction of quark and gluon-initiated jets Collinear function describing
Independent of the measurement within radiation within groomed jet
the jet

\mathcal{C} : Additional phase space cuts (e.g. experimental constraints)

\mathcal{M} : Measurements on the jet constituents (e.g. jets invariant mass)

Measurement



Measurements: $\vec{p}_{h\perp}$, $z = \frac{E_h}{E_J}$

Only the particles that pass the grooming process will determine the direction of the groomed-jet axis

Factorization with groomed jet + identified hadron

$$\frac{d\sigma}{d\vec{p}_J d\vec{k}_\perp dz_h} = \sum_{i=g,q} F_i(Q, R, z_{\text{cut}}, \vec{p}_J, \mathcal{C}) \mathcal{G}_{i/h}(z_h, \vec{k}_\perp, z_{\text{cut}}, R, E_J)$$

Fraction of quark and gluon-initiated jets groomed TMD Fragmenting
Jet Function (TMDFJF)

\vec{k}_\perp is the transverse momentum of jet with respect to hadron

See also:

$$\vec{k}_\perp = -\frac{\vec{p}_{h\perp}}{z_h}$$

- TMDFJF (measurement along the jet axis)

arXiv:1610.06508 (Reggie Bain, YM, Thomas Mehen)

- TMDFJF (measurement along the winner-take-all axis)

arXiv:1612.04817 (Duff Neill, Ignazio Scimemi, Wouter J. Waalewijn)

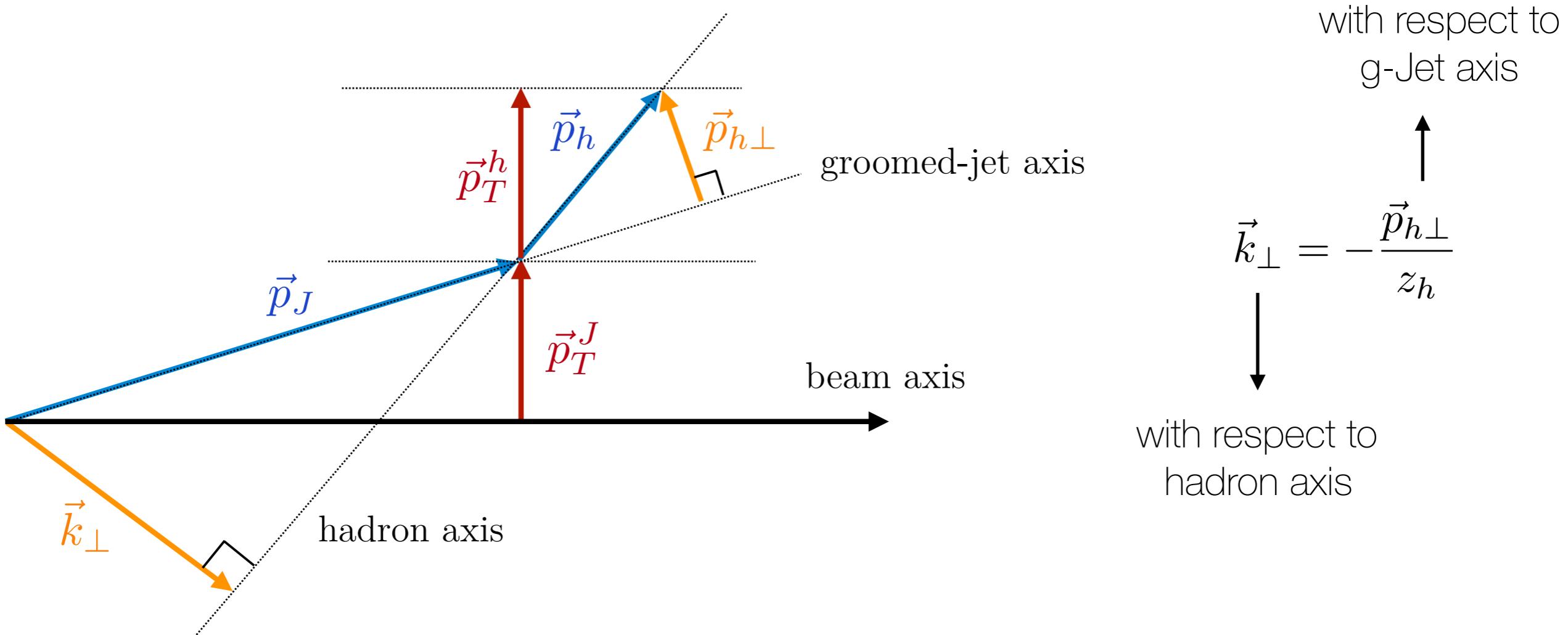
- TMDFJF (semi-inclusive)

arXiv:1705.08443 (Zhong-Bo Kang, Xiaohui Liu, Felix Ringer, Hongxi Xing)

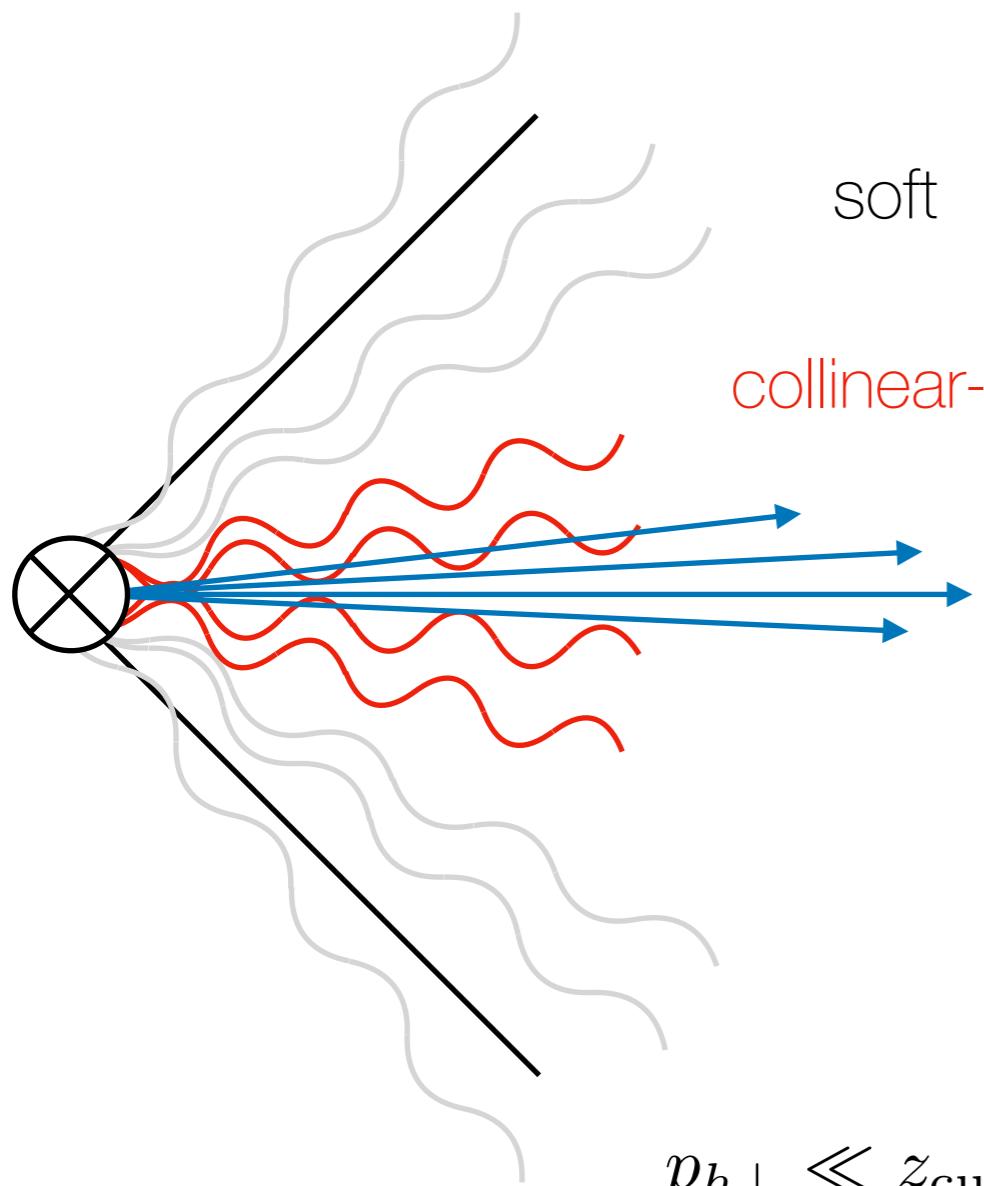
Groomed TMD fragmenting jet function

$$\mathcal{G}_{q/h}(z_h, \vec{k}_\perp, z_{\text{cut}}, R, E_J)$$

$$= z_h \sum_{X \in \text{Jet}(R)} \frac{1}{2N_c} \delta(2E_J - p_X^- - p_h^-) \text{tr} \left[\frac{\not{h}}{2} \langle 0 | \delta^{(2)}(\vec{k}_\perp + \vec{\mathcal{P}}_\perp^{SD}) \chi_n(0) | Xh \rangle \langle Xh | \bar{\chi}_n(0) | 0 \rangle \right] \vec{p}_{h\perp} = \vec{0}$$



Factorization of the TMD (FJF) in SCET



soft $p_s^\mu \sim z_{\text{cut}} Q(1, 1, 1)$

collinear-soft

$$p_{sc}^\mu \sim z_{\text{cut}} Q(\lambda_{sc}^2, 1, \lambda_{sc}) \quad \lambda_{sc} = \frac{p_{h\perp}}{z_{\text{cut}} Q}$$

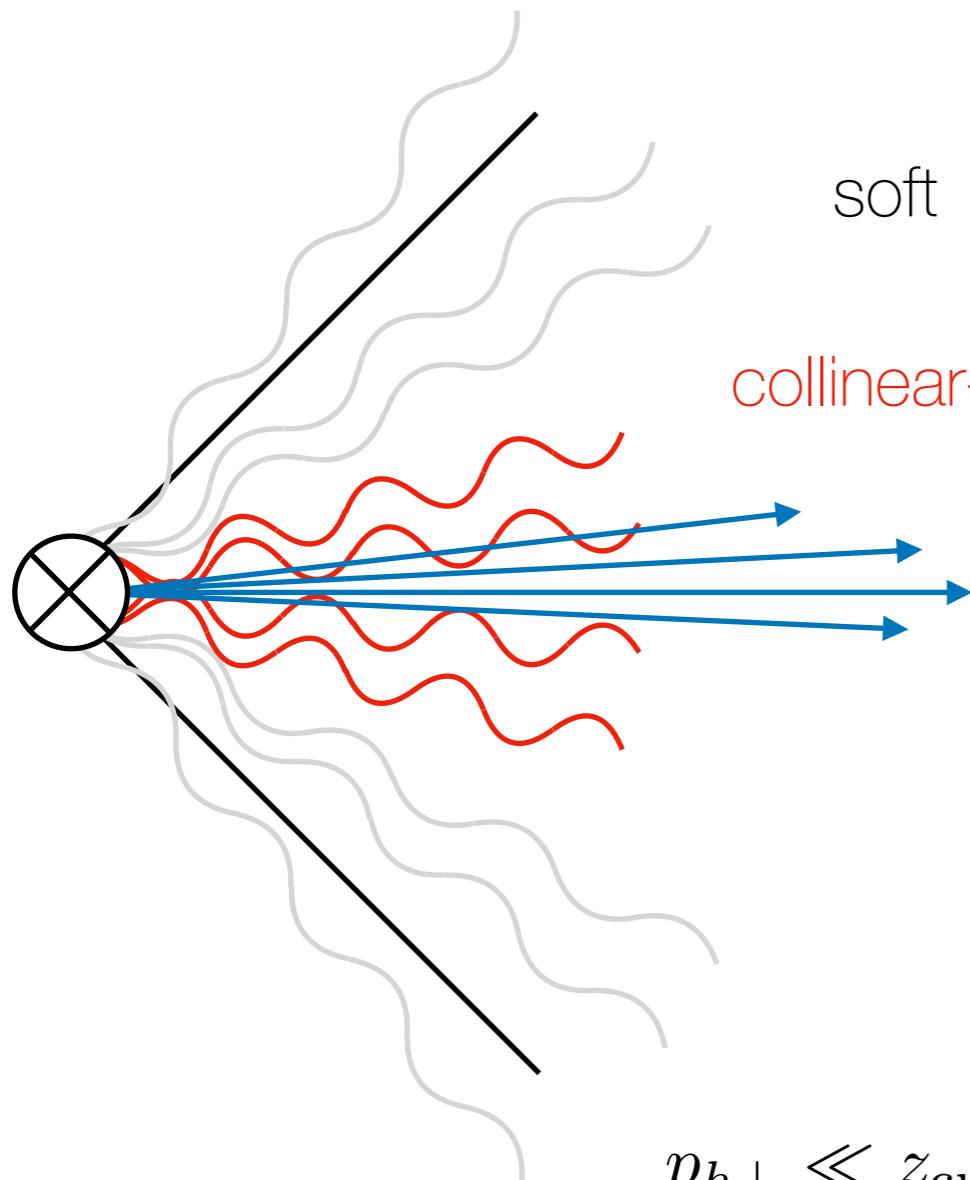
collinear

$$p_c^\mu \sim Q(\lambda_c^2, 1, \lambda_c) \quad \lambda_c = \frac{p_{h\perp}}{Q}$$

$$p_{h\perp} \ll z_{\text{cut}} Q, \quad z_{\text{cut}} \ll R \sim 1$$

Factorization of the TMD (FJF) in SCET

$$\mathcal{G}_{i/h}(z_h, \vec{k}_{\perp}, E_J, z_{\text{cut}}; \mu_L) = \int d^2 \vec{k}_{c\perp} \int d^2 \vec{k}_{s\perp} \delta^2(\vec{k}_{\perp} + \vec{k}_{c\perp} + \vec{k}_{s\perp}) S_i^\perp(\vec{k}_{s\perp}, z_{\text{cut}}) \mathcal{D}_{i/h}^\perp(z_h, \vec{k}_{c\perp})$$



soft $p_s^\mu \sim z_{\text{cut}} Q(1, 1, 1)$

collinear-soft $p_{sc}^\mu \sim z_{\text{cut}} Q(\lambda_{sc}^2, 1, \lambda_{sc})$ $\lambda_{sc} = \frac{p_{h\perp}}{z_{\text{cut}} Q}$

collinear $p_c^\mu \sim Q(\lambda_c^2, 1, \lambda_c)$ $\lambda_c = \frac{p_{h\perp}}{Q}$

$$p_{h\perp} \ll z_{\text{cut}} Q, z_{\text{cut}} \ll R \sim 1$$

Factorization of the TMD (FJF) in SCET

$$\mathcal{G}_{i/h}(z_h, \vec{k}_{c\perp}, E_J, z_{\text{cut}}; \mu_L) = \int d^2 \vec{k}_{c\perp} \int d^2 \vec{k}_{s\perp} \delta^2(\vec{k}_{\perp} + \vec{k}_{c\perp} + \vec{k}_{s\perp}) S_i^\perp(\vec{k}_{s\perp}, z_{\text{cut}}) \mathcal{D}_{i/h}^\perp(z_h, \vec{k}_{c\perp})$$

$$\mathcal{D}_{q/h}^\perp(z_h, \vec{k}_{c\perp}, E_J) = \sum_X \frac{z_h}{2N_c} \delta(2E_J - p_{Xh}^-) \text{tr} \left[\frac{\not{p}_h}{2} \langle 0 | \delta^{(2)}(\vec{k}_{c\perp} - \vec{\mathcal{P}}_\perp) \chi_n(0) | Xh \rangle \langle Xh | \bar{\chi}_n(0) | 0 \rangle \right]_{\vec{p}_{h\perp}=0}$$

- collinear modes are energetic and always pass the grooming constraint
 - independent of the cutoff parameter (z_{cut})
- contains the non-perturbative information of the fragmentation process

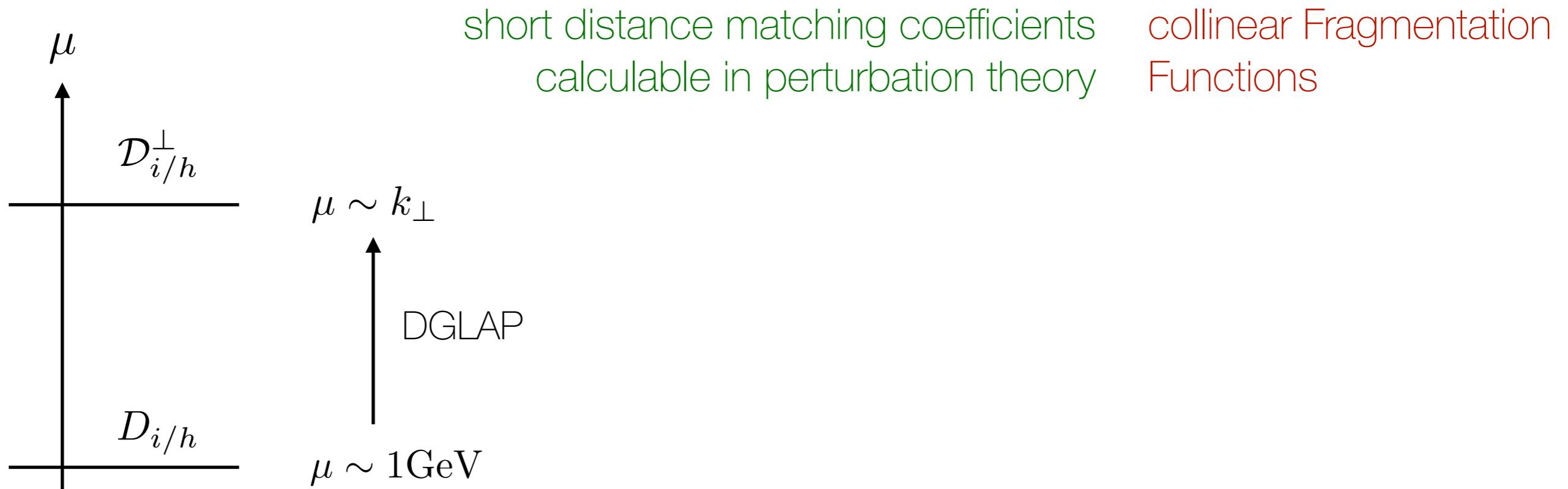
$$S_i^\perp(\vec{k}_{s\perp}, E_J, z_{\text{cut}}) = \frac{1}{N_i} \text{tr} \left[\langle 0 | T\{S_n^i S_{\bar{n}}^i\}(0) \delta^{(2)}(\vec{k}_{s\perp} - \vec{\mathcal{P}}_\perp^{SD}) \bar{T}\{S_n^i S_{\bar{n}}^i\}(0) | 0 \rangle \right]$$

- describes collinear-soft radiation that can pass the grooming constraint
- universal to all light hadrons → independent of hadron's energy fraction (z_h)

Matching onto collinear Fragmentation Functions

Although $\mathcal{D}_{q/h}^\perp(z_h, \vec{k}_{c\perp}, E_J)$ is a fundamentally non-perturbative object for $k_\perp \gg \Lambda_{\text{QCD}}$ can be matched onto the collinear Fragmentation Functions:

$$\mathcal{D}_{i/h}^\perp(z_h, \vec{k}_{c\perp}, E_J) = \int_{z_h}^1 \frac{dx}{x} \mathcal{J}_{ij}^\perp(x, \vec{k}_{c\perp}, E_J) D_{j/h}\left(\frac{z_h}{x}\right)$$



Renormalization Group and Resummation

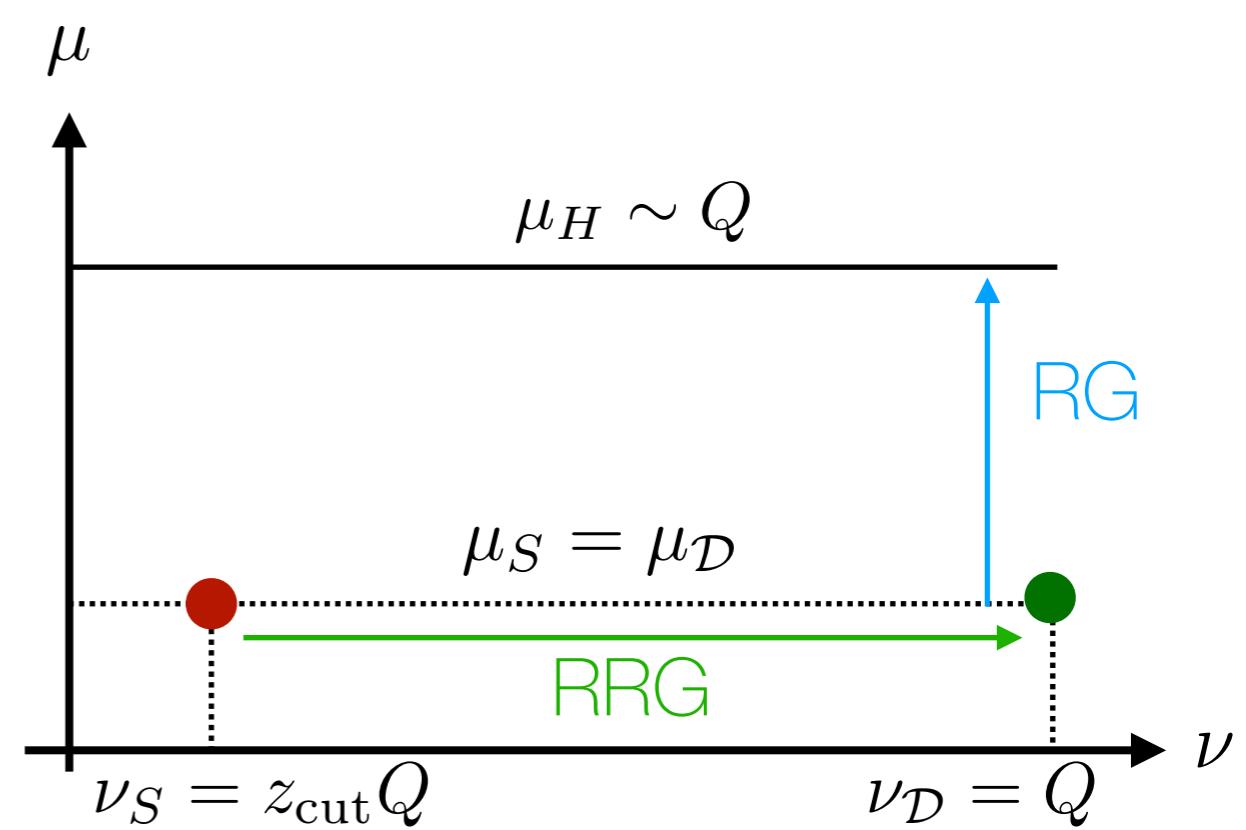
Rapidity regulator and Rapidity Renormalization Group (RRG):

J.-Y. Chiu, A. Jain, D. Neill and I. Z. Rothstein [arXiv:1202.0814](https://arxiv.org/abs/1202.0814)

$$\nu \frac{d}{d\nu} G(\mu, \nu) = \gamma_\nu G(\mu, \nu)$$

Virtuality Renormalization Group (RG):

$$\mu \frac{d}{d\mu} G(\mu, \nu) = \gamma_\mu G(\mu, \nu)$$



NLL-Resummation in momentum space

Fourier Transform → Solve RGE → Inverse Fourier Tranform → Fix Scales

$$\mathcal{G}_{j/h}^{\text{NLL}}(z_h, \vec{k}_\perp, z_{\text{cut}}; \mu) = \mathcal{V}(\vec{k}_\perp, z_{\text{cut}}, \mu_0) \mathcal{U}(\mu, \mu_0) D_{j/h}(z_h, \mu_0) \Big|_{\mu_0=k_\perp}$$

$$\mathcal{U}(\mu, \mu_0) = \exp \left[2\pi \frac{\gamma^{D \otimes S(\mu, z_{\text{cut}})}}{\beta_0 \alpha_s(\mu)} \ln(\alpha_s(\mu_0)/\alpha_s(\mu)) \right]$$

$$\mathcal{V}(\vec{k}_\perp, z_{\text{cut}}, \mu) = \frac{\exp(-2\gamma_E \omega_S)}{\pi} \frac{\Gamma(1 - \omega_S)}{\Gamma(\omega_S)} \frac{1}{\mu^2} \left(\frac{\mu^2}{k_\perp^2} \right)^{1 - \omega_S}$$

$$\omega_S = \frac{\alpha(\mu) C_i}{\pi} \ln \left(\frac{\nu_D}{\nu_S} \right)$$

traditional TMDs

groomed TMDFJF

$$-\frac{\alpha(k_\perp) C_i}{\pi} \ln \left(\frac{k_\perp}{Q} \right)$$

$$-\frac{\alpha(k_\perp) C_i}{\pi} \ln(z_{\text{cut}})$$

NLL-Resummation in momentum space

$$\omega_S = \frac{\alpha(\mu)C_i}{\pi} \ln\left(\frac{\nu_{\mathcal{D}}}{\nu_S}\right)$$

traditional TMDs

$$-\frac{\alpha(k_\perp)C_i}{\pi} \ln\left(\frac{k_\perp}{Q}\right)$$

groomed TMDFJF

$$-\frac{\alpha(k_\perp)C_i}{\pi} \ln(z_{\text{cut}})$$

Solution:

Fix scales in coordinate space
and take Fourier transform numerically

Also:

D. Kang, C. Lee, V. Vaidya
arXiv:1710.00078

M. A. Ebert, F. J. Tackmann
arXiv:1611.08610

In the perturbative region ω_S is small, therefore we can fix the scales momentum space directly.

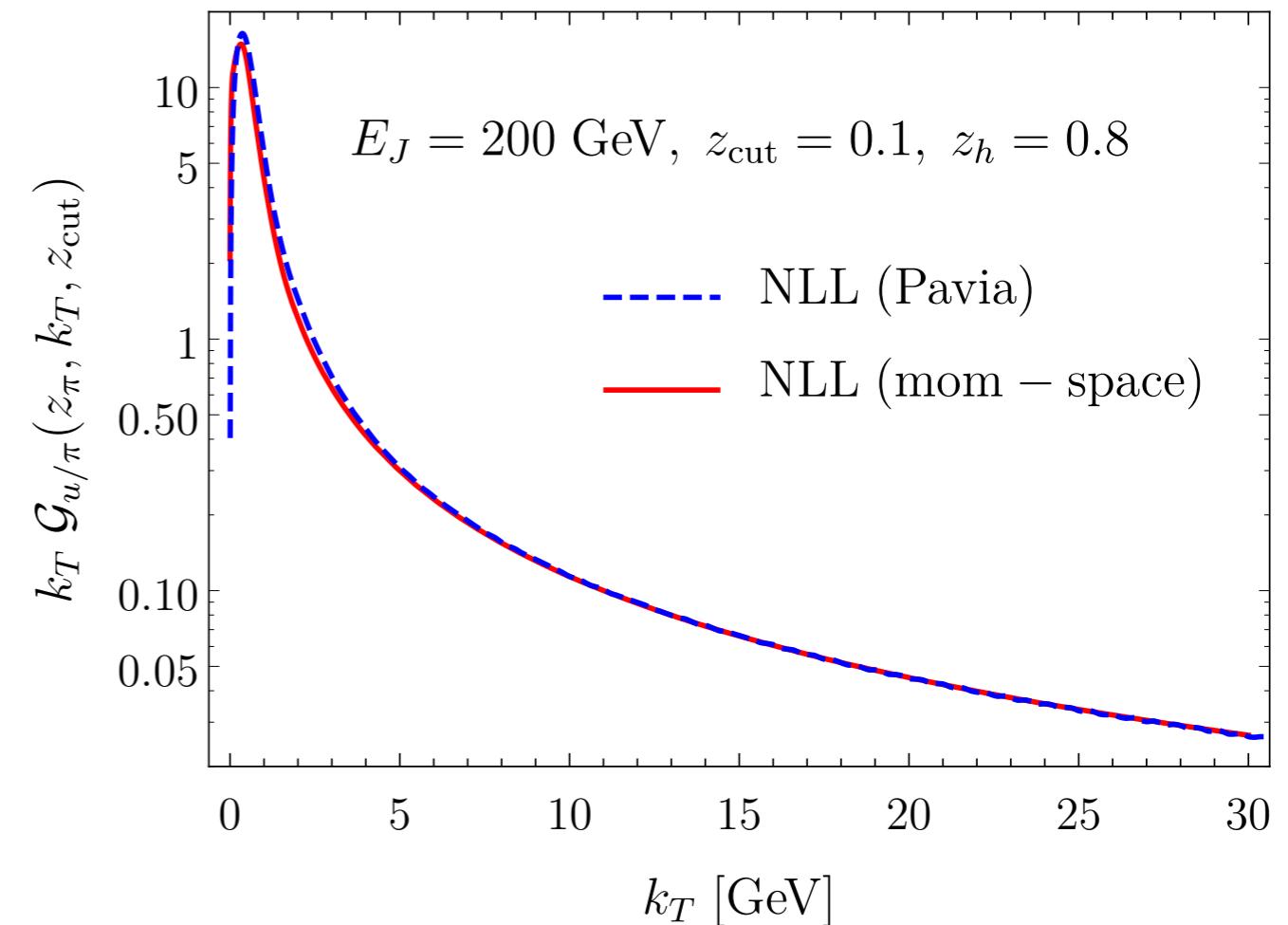
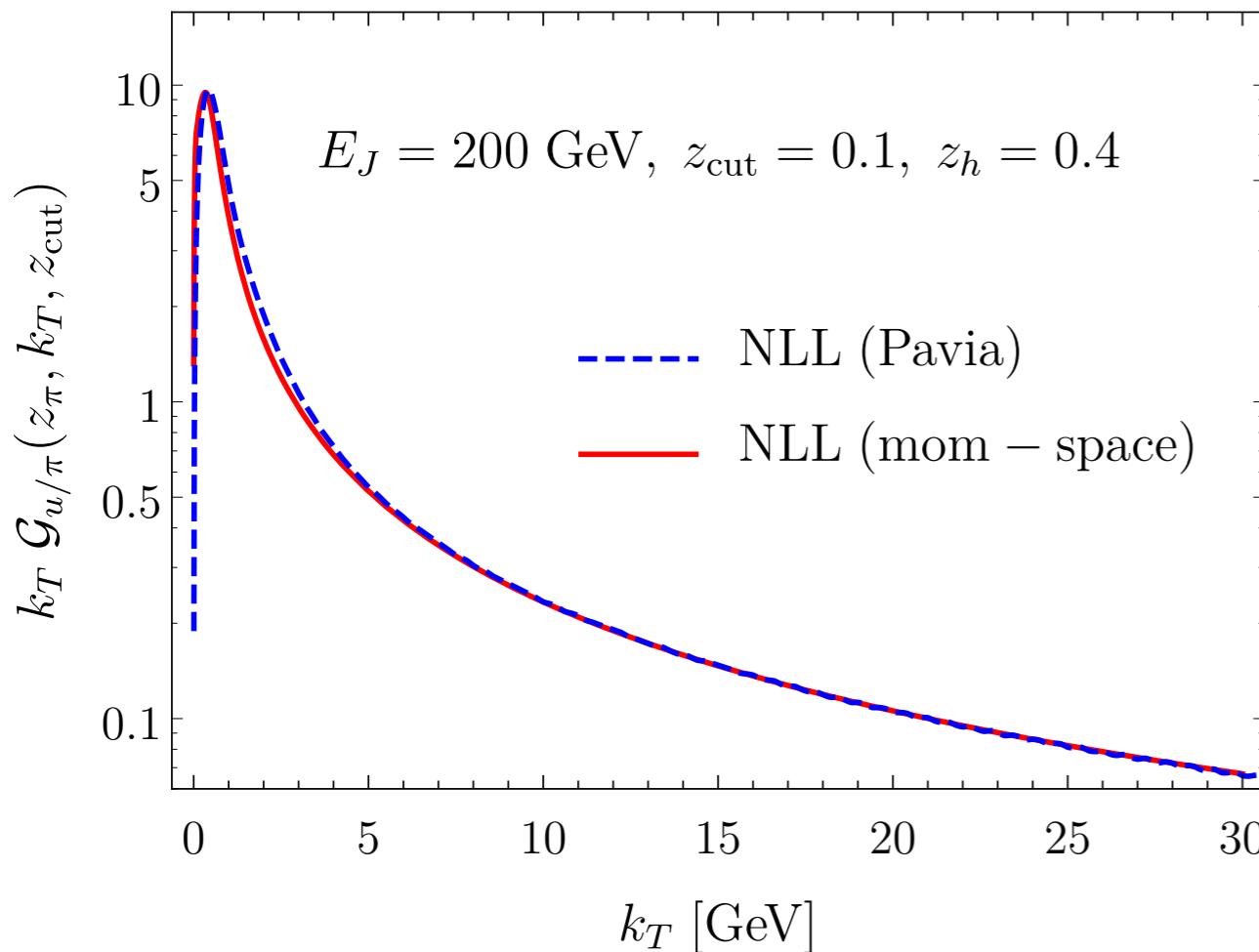
Common choice: $z_{\text{cut}} = 0.1$

$\omega_S \sim 1$: Only in the non-perturbative regime

NLL-Resummation in momentum space

$$\mathcal{N} \frac{d\sigma}{dk_{\perp}}(e^+e^- \rightarrow jet + jet(\pi))$$

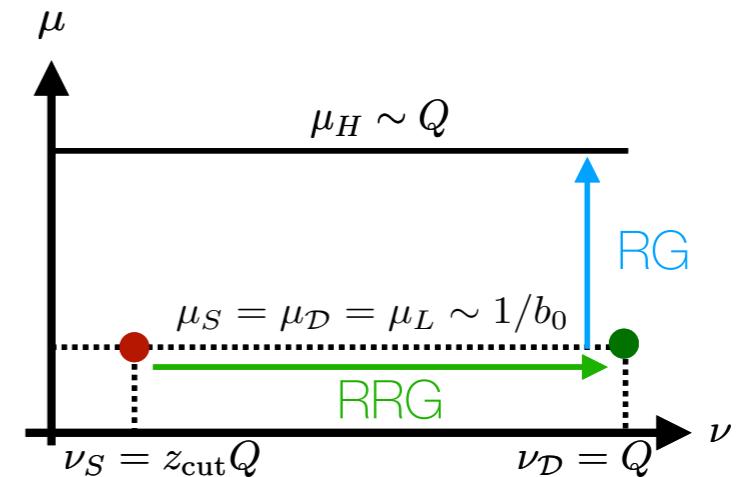
arXiv:1712.07653: YM, Duff Neill, and Varun Vaidya



Resummation in b-space

Fourier Transform → Solve RGE → Fix Scales → Inverse Fourier Tranform

$$\gamma_{\nu,i}^S(\mu) = -2 \int_{1/b_0}^{\mu} d \ln \mu' \Gamma_{\text{cusp}}^i[\alpha_s(\mu')] + \gamma^r(1/b_0)$$



$$\mathcal{D}_{i/h}^\perp(\mu_H, \nu = 2E_J) S_i^\perp(\mu_H, \nu = 2E_J) = U_i(\mu_L, \mu_H) \times \left[\mathcal{D}_{i/h}^\perp(\mu_L, \nu = 2E_J) S_i^\perp(\mu_L, \nu = 2E_J z_{\text{cut}}) \right]$$

$$U_i(\mu_L, \mu_H) \equiv \text{Exp} \left[- \int_{\mu_L}^{\mu_H} d \ln \mu \gamma_i^F[\alpha_s(\mu)] + 2 \ln(z_{\text{cut}}) \left(\int_{1/b_0}^{\mu_H} d \ln \mu \Gamma_{\text{cusp}}^i[\alpha_s(\mu)] + \gamma^r(1/b_0) \right) \right]$$

Rapidity anomalous dimension

Resummation in b-space

$$\gamma_{\nu,i}^S(\mu) \rightarrow \gamma_{\nu,i}^S(\mu) \Big|_{b \rightarrow b_*} - g_K(b; b_{\max})$$

$$b_* = \frac{b}{\sqrt{1 + (b/b_{\max})^2}}$$

$$g_K(b; b_{\max}) \xrightarrow[b \rightarrow 0]{} 0$$



Universal component of TMD observables:

- Groomed OR Un-groomed
- Fragmentation or PDFs

$$U_i(\mu_L, \mu_H) \equiv \text{Exp} \left[- \int_{\mu_L}^{\mu_H} d \ln \mu \gamma_i^F[\alpha_s(\mu)] + 2 \ln(z_{\text{cut}}) \left(\int_{1/b_0}^{\mu_H} d \ln \mu \Gamma_{\text{cusp}}^i[\alpha_s(\mu)] + \gamma^r(1/b_0) \right) \right]$$

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↑

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Rapidity anomalous dimension

variations of the cutoff parameter give as direct access to the rapidity anomalous dimension:

$$\frac{d}{d \ln z_{\text{cut}}} \left[\mathcal{N}(z_{\text{cut}}) \frac{d\sigma}{dp_{h\perp}} \right]$$

Normalized cross section

Non-perturbative TMD evolution

$$\gamma_{\nu,i}^S(\mu) \rightarrow \gamma_{\nu,i}^S(\mu) \Big|_{b \rightarrow b_*} - g_K(b; b_{\max}) \quad b_* = \frac{b}{\sqrt{1 + (b/b_{\max})^2}} \quad g_K(b; b_{\max}) \xrightarrow[b \rightarrow 0]{} 0$$

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Model:Fits	g_2	b_{\max} [GeV $^{-1}$]	b_{NP} [GeV $^{-1}$]
CSS:BNLY 2003	0.68	0.5	n.a.
CSS:KN 2006	0.18	1.5	n.a.
CSS:Pavia 2016	0.12	1.123	n.a.
AFGR: n.a.	0.10	0.5	2.0

CSS:

$$g_K(b; b_{\max}) = \frac{1}{2} g_2(b_{\max}) b^2$$

AFGR:

$$g_K(b; b_{\max}) = \frac{g_2(b_{\max}) b_{\text{NP}}^2}{2} \ln \left(1 + \frac{b^2}{b_{\text{NP}}^2} \right)$$

BNLY: [arXiv:0212159](https://arxiv.org/abs/0212159) F. Landry, R. Brock, P.M. Nadolsky, C.-P. Yuan

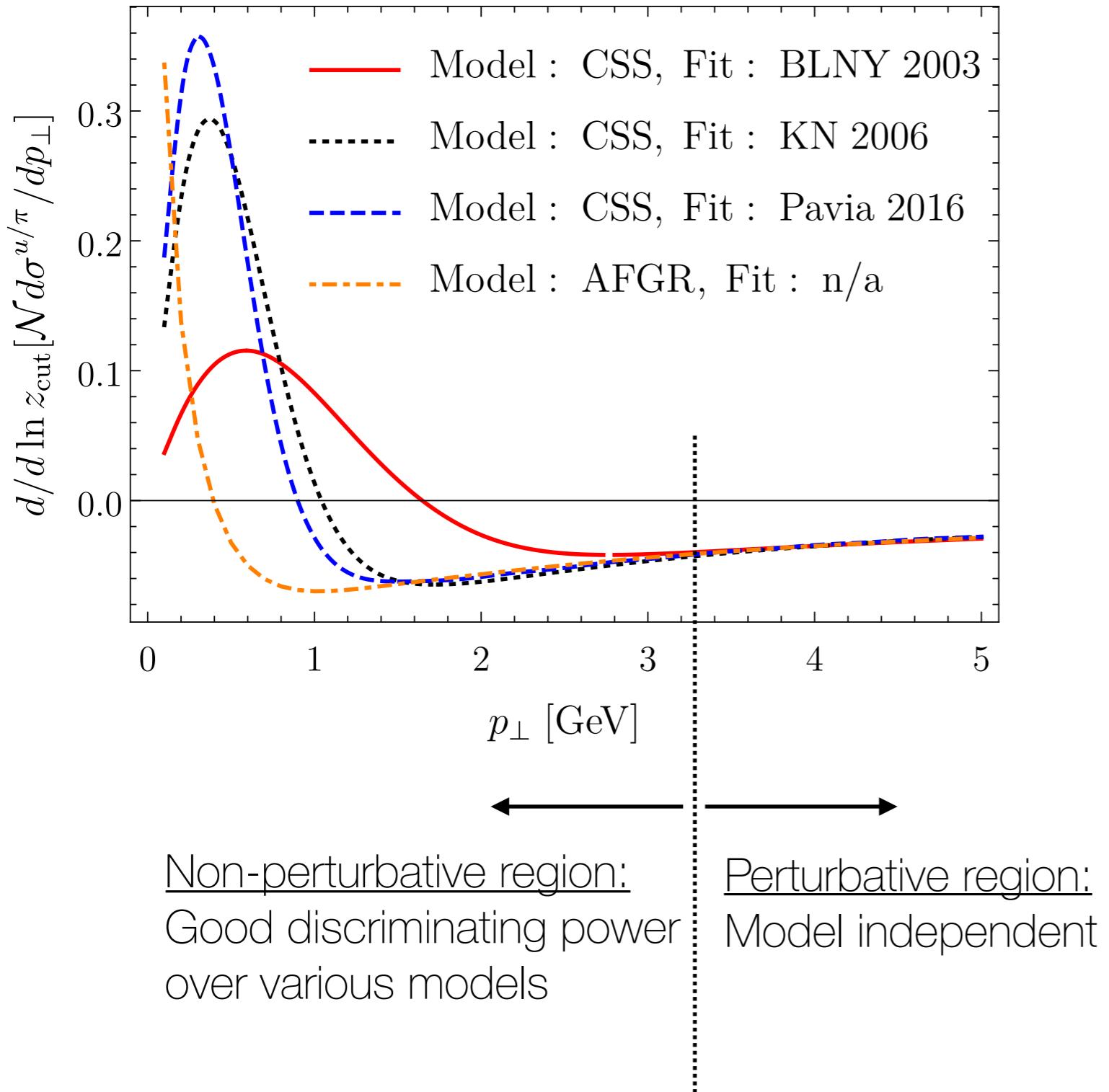
KN: [arXiv:0506225](https://arxiv.org/abs/0506225) A. V. Konychev, P. M. Nadolsky

Pavia: [arXiv:1703.10157](https://arxiv.org/abs/1703.10157) A. Bacchetta, F. Delcarro, C. Pisano, M. Radici, A Signori

AFGR: [arXiv:1401.2654](https://arxiv.org/abs/1401.2654) C. A. Aidala, B. Field, L. P. Gumberg, T. C. Rogers

Proposed observable

arXiv:1712.07653: YM, Duff Neill, and Varun Vaidya



$$\frac{d}{d \ln z_{\text{cut}}} \left[\mathcal{N}(z_{\text{cut}}) \frac{d\sigma}{dp_{h\perp}} \right]$$

Summary

Study fragmentation within groomed jets:

- Use EFT (SCET) for factorization and resummation of large logarithms
- No logarithmic enhancements from boundary effects (NGLs)
- Can easily extended for hadrons + jet substructure (e.g. jet mass, in preparation)

In the perturbative regime:

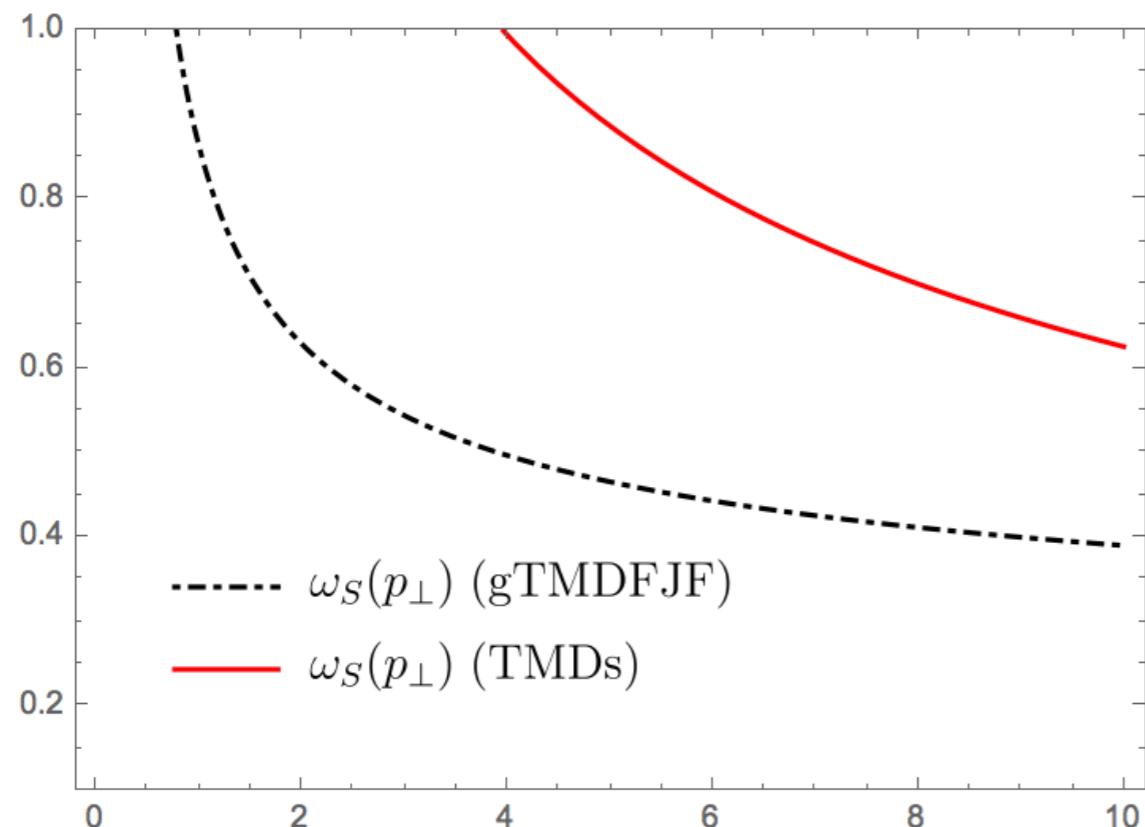
- Groomed TMD fragmentation can be studied directly in momentum space

In the non-perturbative regime:

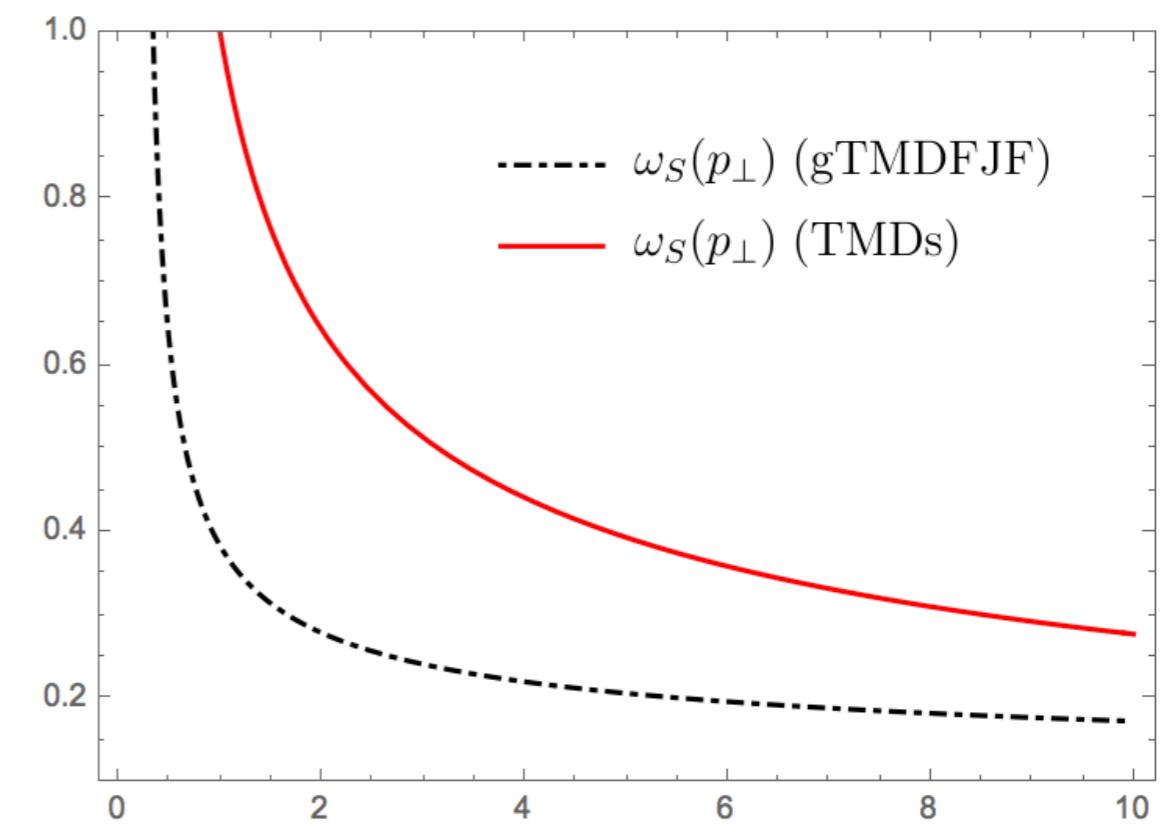
- Good discriminating observable for extracting non-perturbative TMD evolution

NLL-Resummation in momentum space

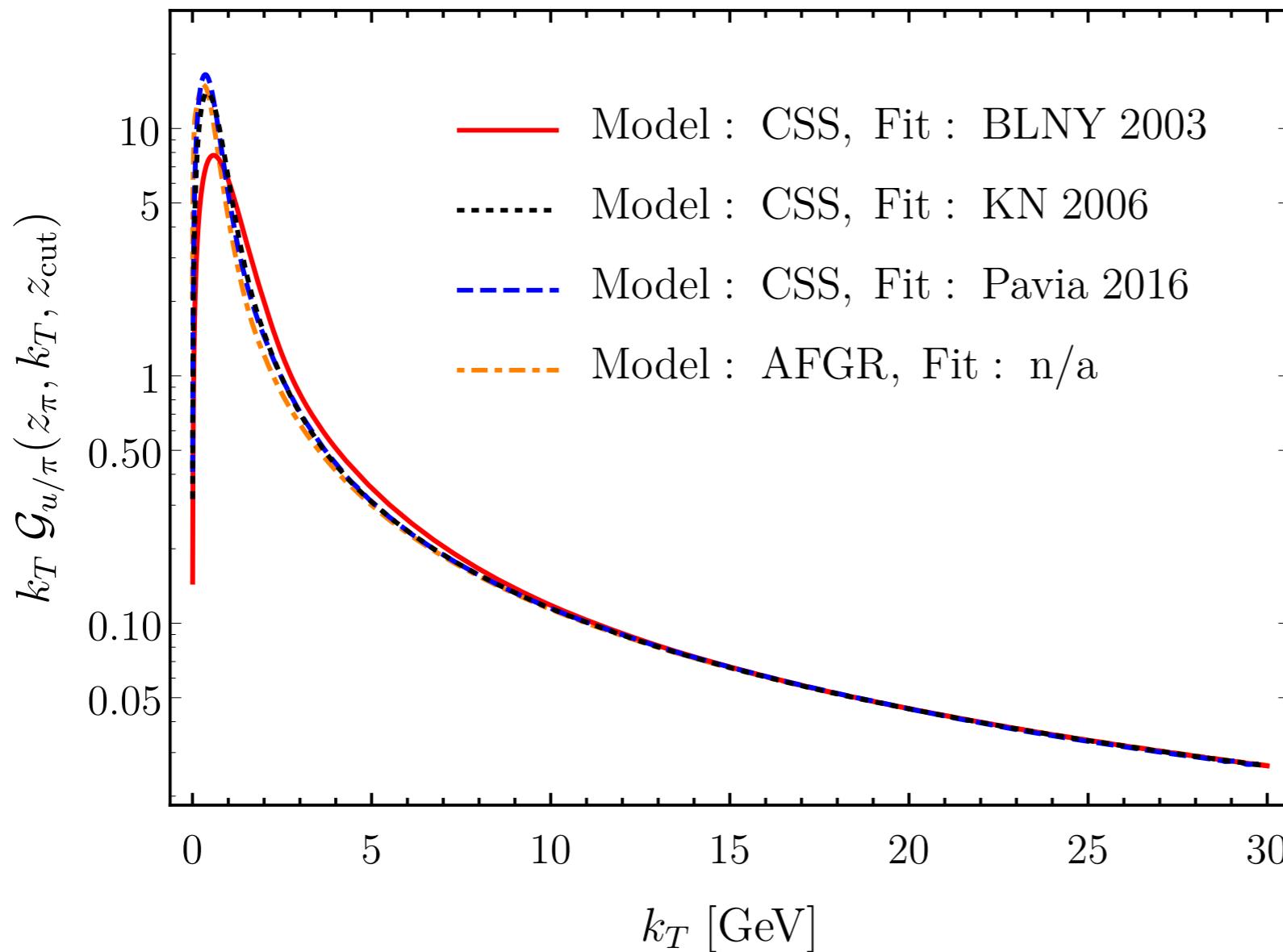
gluon-initiated Jet



quark-initiated Jet



NLL-Resummation in b-space vs momentum space



Probing non-perturbative TMD evolution

