A tale of two strange metals

Daniel Ben-Zion, UCSD

based on work done with John McGreevy [1711.02686, to appear in PRB]

also featured: work done by Gazit et al. [Nature Physics volume 13, pages 484-490 (2017)]

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2018/03/19

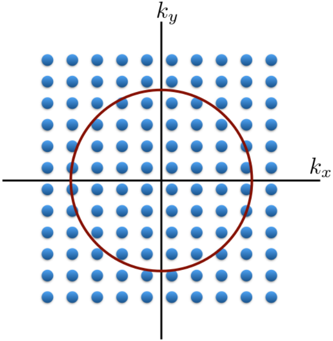
Outline

1. The physics of strongly correlated electrons
    1.1 The RG view on condensed matter fermion systems
    1.2 Challenges to “conventional” physics
2. Analytical approaches: renormalization group and effective field theories
3. The numerical technique: Matrix Product States
    3.1 DMRG Algorithm
    3.2 Large scale computations with OSG

Noninteracting fermions

Exclusion principle: no two fermions can occupy the same state

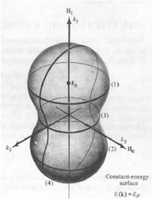
Fermions with kinetic energy $p^2/2m$ (in a box) and no interaction or potential

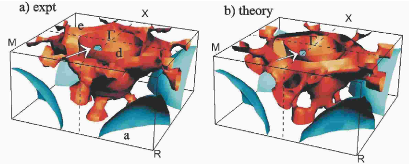


If we have a finite density of fermions, fill the energy states one by one in order of increasing energy. Defines a 'Fermi surface'

Fermi liquids are pretty stable

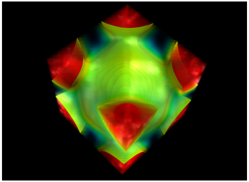
Noninteracting fermions form a fermi sea of filled states (and interacting fermions often do too)

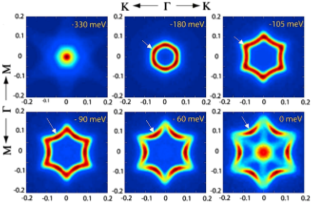




Experimental and calculated (via LDA f-core) FSs for CeIn3, shown in half BZ. Orange: holelike FS. Light blue: R-centred electronlike FS

Ashcroft and Mermin (MoO_2 or LiFeP), Biasini





Weber et al 1304.5363, Wehling et al. 1405.5774

Fermi liquids are pretty stable

[Shankar] cond-mat/9307009 and [Polchinski] hep-th/9210046, and [Benfatto-Gallivotti] Phys. Rev. B 42, 9967

Action linearized around an isotropic fermi surface $S = S_0 + S_{int}$:

$$S_0 = \int đ\omega \, đ^d k \, \overline{\psi}(k\omega)(\mathbf{i}\omega - v_f k)\psi(k\omega)$$

$$S_{int} = -\int \prod_{i=1}^{4} đ\omega_i \, đ^d k_i \;\; V_{1234}\overline{\psi}(1)\overline{\psi}(2)\psi(3)\psi(4)\delta(1+2-3-4)$$

Under an RG transformation, scale towards the fermi surface. Naive scaling of V shows that the interaction is irrelevant

A more careful accounting of the fermi surface kinematics shows that the interaction is irrelevant everywhere except the cases of forward scattering ($k_2 = k_3, \; k_1 = k_4$) and back scattering ($k_1 = k_3, \; k_2 = k_4$) where it is marginal.

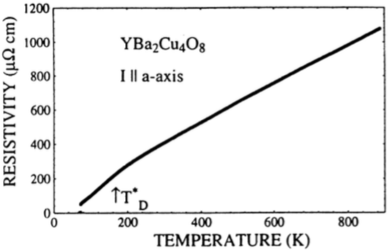
The system can be described in terms of weakly interacting quasiparticles. This is the basis of Landau Fermi Liquid theory.

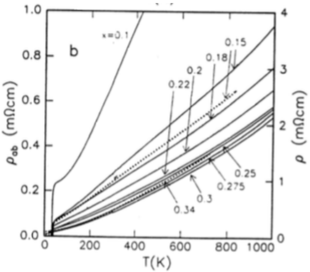
superconductivity is an exception

How to get something other than a Fermi liquid?

If the only effect of the four fermion term is to dress the electrons into quasiparticles, how do you get something else?

Motivation: linear in T resistivity of high T_c superconductors.





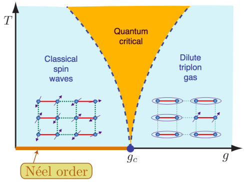
[Timusk and Statt] cond-mat/9905219v1

How to get something other than a Fermi liquid?

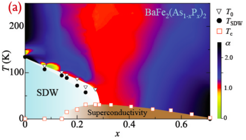
If the only effect of the four fermion term is to dress the electrons into quasiparticles, how do you get something else?

Requires strong correlation effects. One popular scenario is quantum criticality many reviews, one which has a focus on relation to strange metals is [1102.4628]

In the quantum criticality picture, a zero temperature quantum critical point controls the physics in a so-called 'quantum critical fan' at finite temperature. Explains the observation of 'universal' behavior over a large range of temperatures in many materials



from Sachdev [1102.4628]



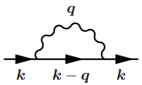
[S. Kasahara et al., Phys. Rev. B 81, 184519]

How to get something other than a Fermi liquid?

One way is by proximity to a quantum critical point

[Holstein et al, Baym et al, ... Halperin-Lee-Read, Polchinski, Altshuler-Ioffe-Millis, Nayak-Wilczek, Schafer-Schwenzer, Chubukov et al, Y-B Kim et al, Fradkin et al, Lawler et al, Metzner et al, S-S Lee, Metlitski-Sachdev, Mross et al]

Schematically $\mathcal{L} = \overline{\psi}(\omega - v_f k)\psi + \overline{\psi}\psi a + L(a)$



Bosonic mode produces branch cut of fermion greens function at fermi surface
i.e. in the Ising nematic example: $\Sigma(\omega) \sim \omega^{2/3}$

Residue of the greens function at the fermi surface goes to zero $\rightarrow$ quasiparticles are killed

But! The scattering which is responsible for destroying quasiparticles is primarily small angle scattering. Hence this mechanism does not lead to the transport phenomenology we're looking for.

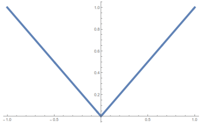
How to get something other than a Fermi liquid which also doesn't have the transport properties of a fermi liquid?

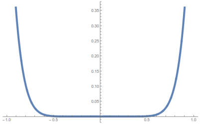
Coupling to a critical bosonic mode affects the quasiparticle lifetime, but this is distinct from the *transport* lifetime, which is what governs the current.

Need a source for scattering processes with large momentum transfer at low energy.

critical fermions with dynamical exponent z:

$$\omega \sim k^z$$





A $z = \infty$ quantum critical system

SYK model Sachdev-Ye, Sachdev-Parcollet-Georges, Kitaev: N ($\gg 1$) fermions with quenched kinetic energy and long ranged random interactions

$$\mathbf{H}_{syk} = \sum_{i<j,k<l} J_{ijkl} c_i^\dagger c_j^\dagger c_k c_l$$

c are canonical fermions: $\{c_i^\dagger, c_j\} = \delta_{ij}$

The couplings J are gaussianly distributed random variables normalized as $\overline{|J_{ijkl}|^2} = \frac{J^2}{2N^3}$

This model is, miraculously, solvable!

More precisely, diagrams in perturbation series are proportional to powers of the vanishingly small quantity $1/N$, and it is possible to resum the leading contributions.

note for the future: also possible to generalize to q-fold interaction rather than fourfold as written here

SYK: disorder averaged diagrammatics

Near $\omega \to 0$, greens function is dominated by self energy term, which is itself given by a product of greens functions

$$\mathcal{G}(\omega) \approx -\frac{1}{\Sigma(\omega)} \Rightarrow \mathcal{G}(\omega)\Sigma(\omega) = -1, \qquad \Sigma(\tau) = J^2 \mathcal{G}(\tau)^2 G(-\tau)$$

these equations have an emergent reparametrization symmetry $\tau \to f(\tau)$

because we dropped the free term $\mathbf{i}\omega$ in the low energy approximation

$$\mathcal{G}(f(\tau), f(\tau')) = [f'(\tau) f'(\tau')]^{-1/4} \mathcal{G}(\tau, \tau')$$

$$\Sigma(f(\tau), f(\tau')) = [f'(\tau) f'(\tau')]^{3/4} \Sigma(\tau, \tau')$$

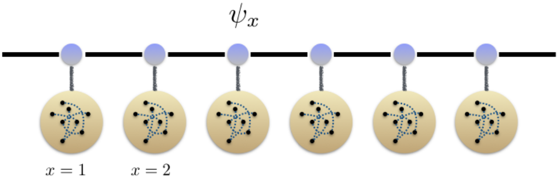
A power law form for $\mathcal{G}$ solves the coupled equations: find

$$\mathcal{G} \sim |\omega|^{-1/2} \mathrm{sgn}\,(\omega)$$

for general q, find $G(\tau) \sim \tau^{-2/q}$ in time domain, $\omega^{(2-q)/q}$ in frequency

Couple to a fermi surface

Consider a lattice of SYK dots, and another species of fermion ψ with Hamiltonian $\mathbf{H}_\psi = \sum_k (\epsilon(k) - \mu)\psi_k^\dagger \psi_k$



Couple the fermions by an onsite hybridization

$$\mathbf{H}_{hyb} = \sum_{x,i} g_{x,i}\psi^\dagger(x) c_i(x) + h.c. \qquad \overline{|g_{xi}|^2} = g^2/N$$

$\langle \psi^\dagger \psi \rangle$ propagator will acquire anomalous exponent. There is a fermi surface, but divergence is non analytic.

and, importantly, SYK properties are protected by the number imbalance

Couple to a fermi surface: Renormalization group analysis

- dimensional analysis shows g is relevant for $q > 2$ and marginal at $q = 2$. RG calculation at $q = 2 + \epsilon$!

In the effective field theory, coupling g is renormalized by the term

$$\delta g^2 \sim \int_a d\tau \, \langle c^\dagger(\tau) c(0) \rangle \langle \psi^\dagger(\tau) \psi(0) \rangle + h.c.$$

contribution from lower bound of integral (this is what we're rescaling)

$$\sim \frac{1}{2} C(J) K_d k_F^{d-1} v_F^{-1} a^{-2/q}$$

now write the β function including first nontrivial contribution

$$\beta(g^2) \equiv \frac{d}{d \log a} g^2 = \underbrace{(2 - 2\Delta(q))}_{=-1+\epsilon} g^2 + \frac{1}{2} C(J) K_d k_F^{d-1} v_F^{-1} \left(\frac{-2}{q} \right) g^4$$

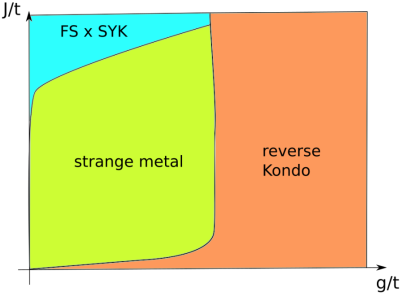
Setting $\beta(g^2) = 0$ obtains a fixed point $g_\star^2 = \frac{2 v_F}{C(J) K_d k_F^{d-1}} \epsilon + O(\epsilon^2)$

Key results

- ψ fermions form a fermi surface, but the Green's function has a power law singularity rather than simple pole

- This means the quasiparticle weight vanishes, and there are no long lived quasiparticle excitations.

- Conductivity can be calculated from a Kubo formula, has a scaling form $\sigma \sim T^{1/2}$ addresses the attempts to connect with phenomenology of strange metal
- Cartoon phase diagram



Numerical studies of one dimensional quantum systems

The Arena: states of a quantum many body system are described by vectors in Hilbert space.

- for simplicity, suppose we consider a chain of spin 1/2 degrees of freedom
- each site has a 2 dimensional HS spanned by the states $|\uparrow\rangle$ and $|\downarrow\rangle$

A basis can be obtained by enumerating all possible configurations of spins being up or down.

$$|\uparrow\uparrow\uparrow\rangle, |\uparrow\uparrow\downarrow\rangle, |\uparrow\downarrow\uparrow\rangle, |\downarrow\uparrow\uparrow\rangle, |\uparrow\downarrow\downarrow\rangle, |\downarrow\uparrow\downarrow\rangle, |\downarrow\downarrow\uparrow\rangle, |\downarrow\downarrow\downarrow\rangle$$

The goal: calculate the ground state of some quantum hamiltonian

if you succeed, try to calculate some correlation functions

The basic problem: # of states grows exponentially with the number of sites 2^L!

Hilbert space is too large

The basic problem: # of states grows exponentially with the number of sites q^L!

$$|\psi\rangle = \sum_{\{\sigma_i\}} \psi_{\sigma_1\sigma_2...\sigma_N}|\sigma_1\sigma_2...\sigma_N\rangle$$

$\psi_{\sigma_1\sigma_2...\sigma_N}$ is a list of 2^L numbers.

Not only is this list of coefficients practically impossible to obtain (i.e. by exact diagonalization of some Hamiltonian), it is also completely useless.

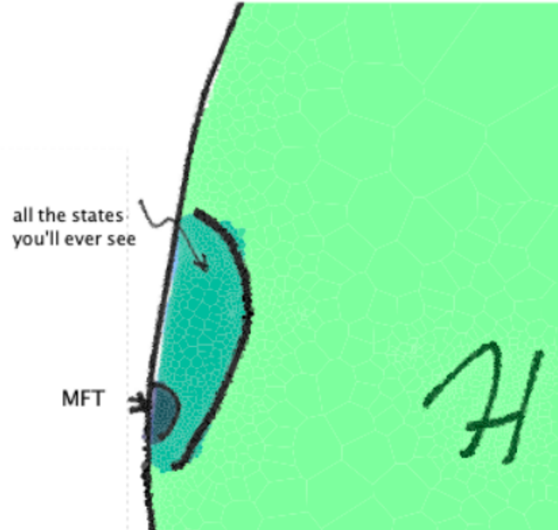
This exponential complexity of the quantum many body problem has led to some dramatic statements by famous scientists.

"In general, the many-electron wavefunction $\Psi(r_1, ..., r_N)$ for a system of N electrons is not a legitimate scientific concept [for large N]" - Kohn 1998 Nobel Lecture

Hilbert space is an illusion

Nature does not explore the full Hilbert space, and ground states of physical hamiltonians live in a small corner which has a special property.

thanks to John McGreevy for the image



By physical, mean

- It is local
- It is extensive: $H_{A \bigcup B} \approx H_A + H_B$

The special property these ground states share is that they are low entanglement

Finding the right language to ask our questions

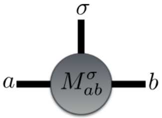
Nature does not explore the full Hilbert space, restrict our considerations to the small corner of states with low entanglement.

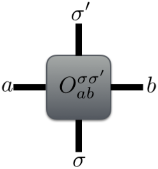
The natural way to describe states of one dimensional quantum systems seems to be in terms of Matrix Product States [White, and many more]

Matrix product states are states which are of the form

$$\psi_{\sigma_1\sigma_2\ldots\sigma_N} = M^{\sigma_1}_{i_1} M^{\sigma_2}_{i_1 i_2} M^{\sigma_3}_{i_2 i_3} \ldots M^{\sigma_N}_{i_{N-1}} = \mathrm{tr} M^{\sigma_1} M^{\sigma_2} \ldots M^{\sigma_N}$$

They have a graphical representation (operators too)

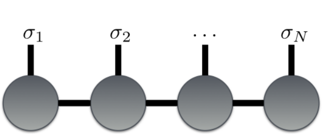


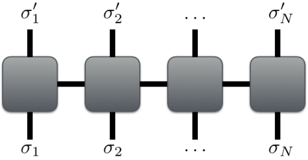


the simplest matrix product state is just a product state: $|\psi\rangle = \otimes_i |\psi_i\rangle$

Computations with matrix product states

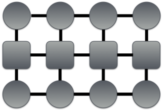
We can use matrix product state technology to compute efficiently!



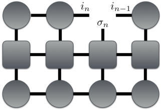


What actually needs to be done at the computational level? Lanczos methods, and lots of tensor products

$$\langle\psi|\hat{H}|\psi\rangle =$$



$$\frac{\partial}{\partial M^{\sigma_n}_{i_{n-1} i_n}} \langle\psi|\hat{H}|\psi\rangle =$$



Efficient contraction of the network is essential. Reduces computational time from exponential to $\sim L^3$.

Large scale computations with OSG

Our Hamiltonian has many coupling constants $\hat{H} = \hat{H}(g_{x,i}, J_{x,ijk})$.

- The coupling constants g and J are taken as random variables from some distribution
- Interested in calculating disorder averaged quantities (quenched vs annealed)
- Allows us to study finite N; analytical results correspond to limit $N \to \infty$

Calculate physical quantity (entanglement entropy, or correlation function) for a *particular* disorder realization, then average over many disorder realizations.

DMRG is able to find ground state to a high degree of accuracy in $4 \sim 8$ hours. Using OSG, able to perform disorder averaging (sometimes on order of 500 disorder realizations) over parameter space varying g and J.

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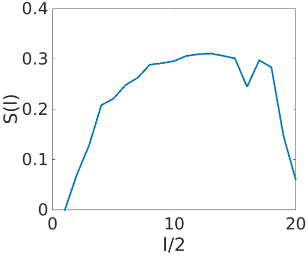
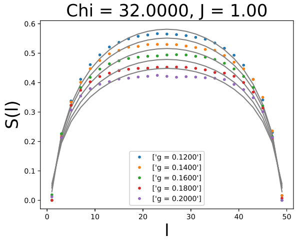


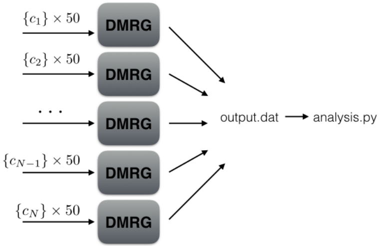
$\longrightarrow$



Large scale computations with OSG

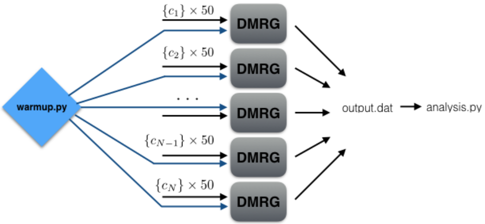
- two workflow setups

1. standard parameter sweep, running each job x times to account for randomness



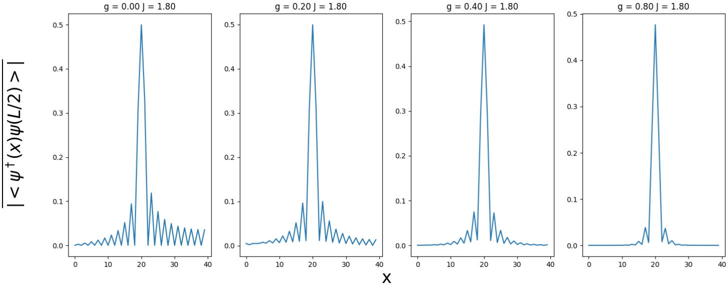
2. divide computation into two parts, and pre-solve part of it before sending to OSG

   offloading more memory intensive work allows to increase scope of numerics while keeping all jobs running on OSG $\lesssim$ 2 GB of memory



Large scale computations with OSG

Correlation functions shows binding of mobile fermions into local singlets

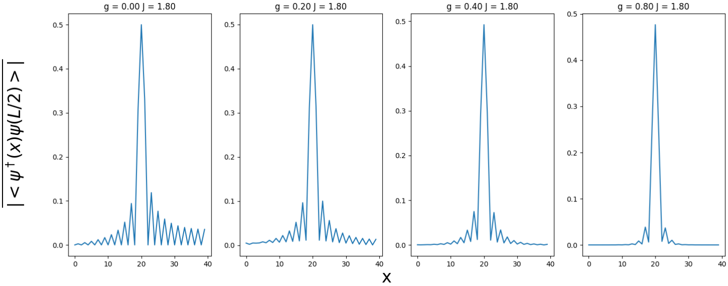


At small values of g, we can fit a power law of form $\frac{\sin(k_f(x-y))}{\pi|x-y|^\alpha}$

$\alpha = 1$ is the free fermion value, obtains at $g = 0$

Large scale computations with OSG

Correlation functions shows binding of mobile fermions into local singlets



Limiting case of localization at large g is good sanity check. Small values of g looks like power law correlators. Becomes difficult to confidently distinguish $x^{-\text{large }\#}$ vs exponential. $g \sim 0.2$ looks extended (?) but $g \sim 0.4$ already looks localized

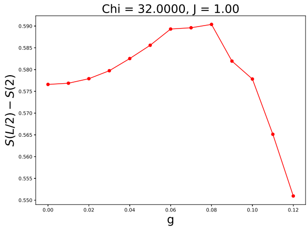
power laws with power other than one can be found in luttinger liquids, but it's interesting to obtain it in this disordered system

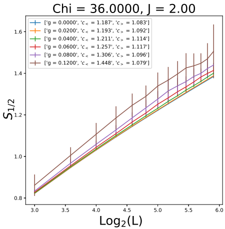
Large scale computations with OSG

The EE is easily accessible in the MPS formalism, and it's scaling behavior in critical systems contains universal information about the physics

$$S_{half}(L) \sim \frac{c}{6} \log L$$

At large g, $S_{half} \sim const$. Question is does anything interesting happen in between, at finite N





Difficult to study numerically even with DMRG for $N \gtrsim 7$. Results seem to support existence of intermediate phase, but not conclusive. Computations with random *quadratic* clusters do not show same behavior, which is promising.

Work of Gazit et al. [Nature Physics volume 13, pages 484-490 (2017)]

They study a model of itinerant fermions coupled to a fluctuating gauge field, a type of lattice gauge theory

Gauge field mediates attractive interaction between fermions which drives superconductivity

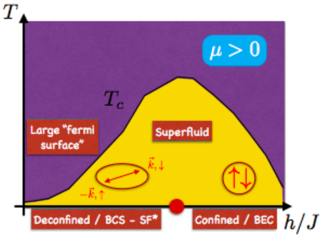
Ising lattice gauge theory undergoes confinement/deconfinement transition

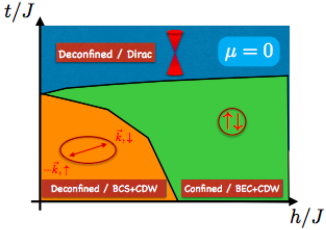
On deconfined side, pairing is of the BCS type. On confined side, a topological superfluid obtains.

Precisely at half filling, new deconfined phase with emergent Dirac excitations

Using quantum monte carlo, find a single continuous transition where confinement and symmetry breaking occur simultaneously.

images from Gazit et al.



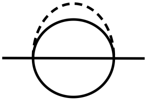


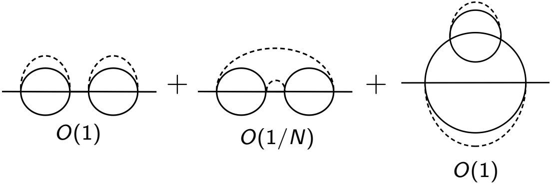
The End

Thanks to collaborators, and especially to OSG for doing a great service to the scientific community.

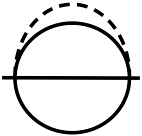
SYK: disorder averaged diagrammatics

Kitaev, Stanford-Maldacena The free Hamiltonian is 0: $\mathcal{G}_0(\omega) = (\mathbf{i}\omega)^{-1}$

leading order: 

next order: 

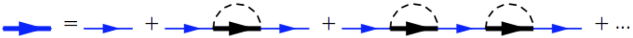
the rule is interaction vertices need to be paired up or else the coupling will average to zero

$\Sigma =$ 

Diagrammatics for landau quasiparticle

$$\mathbf{H} = \sum_k (\epsilon(k) - \mu)\psi_k^\dagger \psi_k + \sum_{x,i} g_{x,i}\psi^\dagger(x) c_i(x) + h.c.$$

Propagator of the SYK fermion (black) is the self energy of the itinerant fermion (blue)



$$\Sigma_\psi(k,\omega) \sim |\omega|^{-1/2}$$

weird! coupling to the 'bath field' which has exponent $2\nu = +1/2$ gives a more reasonable looking dispersion near fs

smallness of $1/N$ suppresses backreaction of ψ onto the syk state