Stress Analysis of High-Pressure Test Vessel

Introduction and Summary

The ASME Boiler and Pressure Vessel Code, Section VIII, Div. I (2017), is used to verify the design of a 2 atmosphere (30 psig) test vessel. It is a cylinder and a 0.315-inch and 0.15-inch thick flat heads are attached on each end of the cylinder.

The Code calculations show that the required cylindrical shell thickness is 0.00294 inches; the actual minimum shell thickness is 0.2565 inches. The calculations also show that the required flat circular head thickness is 0.0897 inches; the actual head thickness is 0.15 inches. The 0.787 in diameter penetration in the 0.315-inch flat head is shown to require no additional reinforcement, other than that inherent in the 0.315-inch thick head. The same is true of the 0.787-inch blind holes on the both flat heads, and the 6-32 blind holes to bolt the pipe flanges to the both flat heads.

The 0.4-inch hole itself, even if a through-hole, would require no reinforcement per UG-36(c)(3)(a). The only structural issue is whether the relatively thin disk of material is in danger of being ejected at the operating pressure of 30 psi. The results of the assessment show that the blind hole is sound, and presents no special structural issue.

A 3-D finite element model was created to check the Code calculations, and to look at stresses associated with support. The stresses in the center of the flat heads do not exceed 9000 psi; the hoop stresses in the cylindrical shell do not exceed 5000 psi. These maximum stresses are well below the allowable stress for SS316 steel of 18,800 psi.

This analysis does not address the piping and flanges that bolt to the heads, except as they affect the vessel stresses through their support function. The analysis shows that the contribution of support reactions to the head stress is negligible.

Cylindrical Shell

From the ASME Boiler and Pressure Vessel Code, Section VIII, Div. I, UG-27, the thickness of a cylindrical shell under internal pressure must be no less than

$$t = PR/(SE - 0.6P)$$

where t = thickness of shell
     R = inside radius of shell = 1.8425 in
     P = internal pressure = 30 psig
     S = allowable stress = 18,800 psi (SA-249 welded SS316 tube, per Div. II Part D)

E = weld efficiency = 1 (welded tube allowable stress is used)

Substituting,

$$t = \frac{30(1.8425)}{18800(1) - 0.6(30)} = 0.00294 \; in.$$

The shell is perforated by 12 6-32 bolts. Although its maximum thickness is 0.4025 inches, its minimum thickness (in a plane through a bolt hole) is only 0.2565 inches. This is still larger than the minimum requirement of 0.00294 inches. Therefore, the shell meets Div. I criterion.

Flat Head

From Section VIII, Div. 1, UG-34(c)(2), Formula (1), the minimum required thickness of flat unstayed circular heads must be no less than

$$t = d\left(\frac{CP}{SE} + 1.9Wh_g/SEd^3\right)^{1/2}$$

where t = thickness of flat head

C = factor for attachment = 0.25 (from Fig. UG-34(p))
P = pressure = 30 psig
S = allowable stress = 18,800 psi (SA-249 welded SS316 tube, per Div. II Part D)
E = weld efficiency = 1 (no welds)
d = diameter of head = 10.5 in (from Fig. UG-34(p))
W = total bolt load (calculated by Appendix 2; unneeded since h_g = 0)
h_g = gasket moment arm = 0 (gasket symmetry about bolt)

Substituting,
$$t = 4.49(0.25(30)/18800)^{1/2} = 0.0897 \; in.$$

Actual minimum flat head thickness = 0.15 inches. Therefore, the head meets Div. I criterion.

Reinforcement of Penetration and Blind holes

There are three 0.787-inch diameter penetrations through the 0.315-inch thick head plate. From UG-36(c)(3)(-a)(-1), openings less than 3 ½ inches in diameter in shells or heads greater than 0.00294 inches thick require no reinforcement other than that inherent in the design.

No reinforcement is required for the 0.787-inch penetration, or for the blind bolt holes used to fasten the pipe flanges to the head.

The 0.4-inch blind hole is located on the exterior of the 0.315 inch and 0.15 inch thick flat heads. These holes extend toward the outer pressure boundary to within 0.04 inches. The 0.4 hole itself,

even if a through-hole, would require no reinforcement per UG-36(c)(3)(a)(-1), openings less than 3 ½ inches in diameter in shells or heads greater than 0.00294 inches thick require. The only structural issue is whether the relatively thin disk of material is in danger of being ejected at the operating pressure of 30 psig. The evaluation is shown in following section.
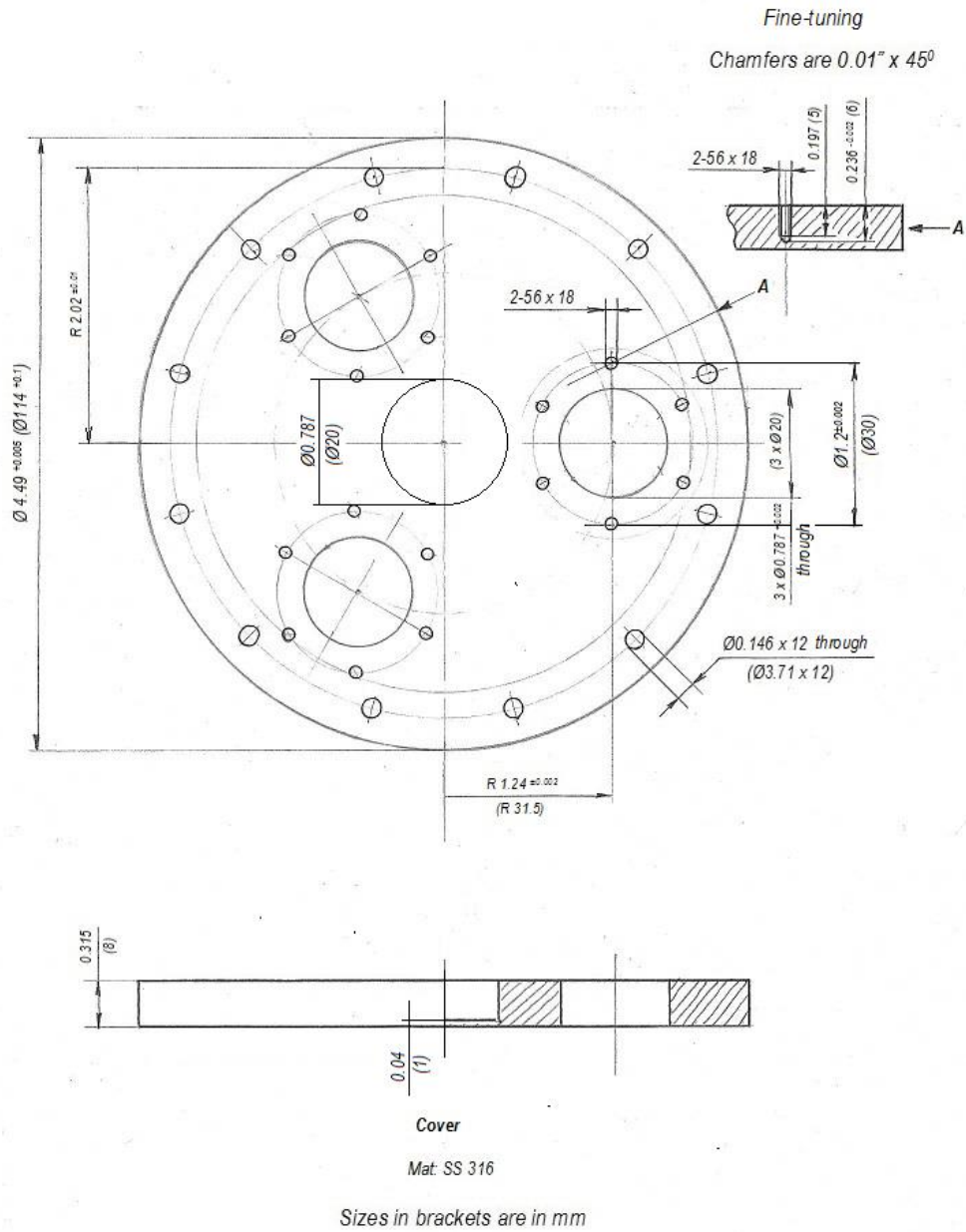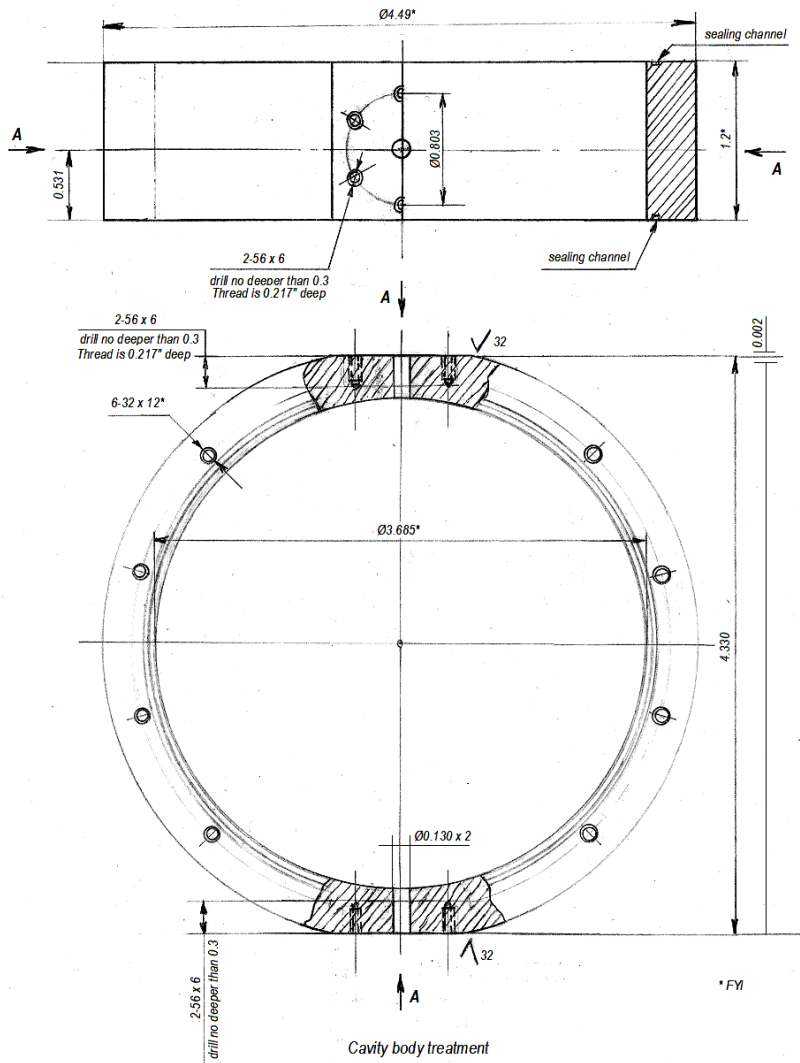

Figure 1: 0.315-inch thick head plate.

Ø4.49"

sealing channel

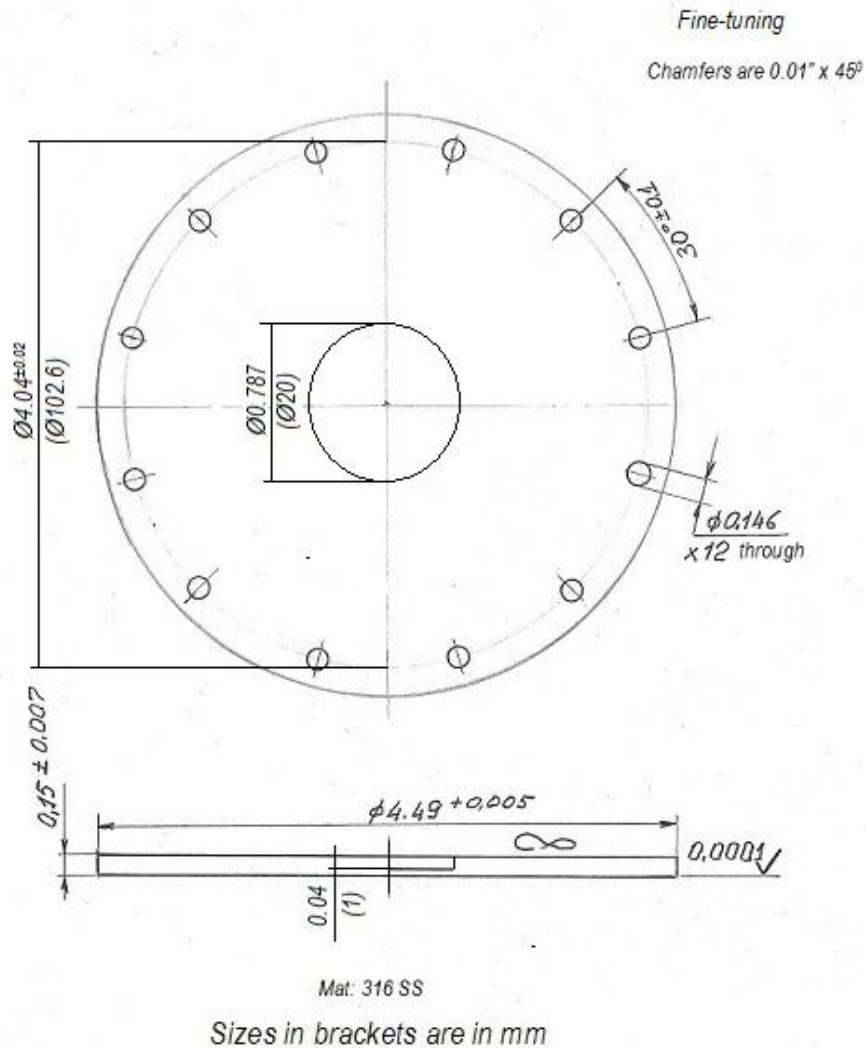1.2"

0.531

Ø0.803

2-56 x 6
drill no deeper than 0.3
Thread is 0.217" deep

sealing channel

A

2-56 x 6
drill no deeper than 0.3
Thread is 0.217" deep

32

0.002

6-32 x 12"

Ø3.685"

4.330

Ø0.130 x 2

32

2-56 x 6
drill no deeper than 0.3

A

* FYI

Cavity body treatment

Figure 2: Cylinder shell wall

Fine-tuning

Chamfers are 0.01" x 45°

$\varnothing 4.04^{\pm 0.02}$
$(\varnothing 102.6)$

$\varnothing 0.787$
$(\varnothing 20)$

$30 \pm 0.4°$

$\varnothing 0.146$
x 12 through

0.15 ± 0.007

$\varnothing 4.49 ^{+0.005}$

0.0001

0.04
(1)

Mat: 316 SS

Sizes in brackets are in mm

Figure 3: 0.15-inch thick head plate.

0.4-inch blind hole

Material properties

The head is made of SS316, with the material properties and allowable stress listed in the main body of this report. Specifically, the maximum allowable membrane stress for the head material is $S_m = 15$ ksi. Allowable stress induces the 0.8 Fermi derating factor.

Approach

Two calculations are performed.

1. Plug shear – this calculation looks at the uniform shear stress that will develop at the circumference of the disk under the operating pressure. This stress must be limited to 0.5 S_m = 7.5 ksi.
2. Minimum thickness of flat head – this calculation is performed per UG-34(c)(2), and determines the minimum thickness of a flat head the same diameter as the disk under the operating pressure.

Plug Shear

The plug shear stress is

$$\tau = \frac{PA}{2\pi r t}$$

where t = thickness of disk = 0.04 in
A = surface area of disk = 0.122 in^2
P = operating pressure = 30 psi
r = radius of hole = 0.19685 in

Substituting,

$$\tau = \frac{30(0.122)}{2\,3.14\,0.19685\,(0.04)} = 73.9\ psi.$$

This is well below the allowable of 7500 psi.

Minimum Thickness of Flat Head

From UG-34(c)(2), for heads not attached by a means that produces edge moments, the minimum required thickness is

$$t = d\sqrt{\frac{CP}{SE}}$$

where t = thickness of flat head
C = factor for attachment = 0.25 (from Fig. UG-34(p))
P = pressure = 30 psi
S = allowable stress = 18800 psi (SA-249 welded SS316 tube, per Div. II Part D)
E = weld efficiency = 1 (no welds)

Substituting,

$$t = 4.49(0.25(30)/18800)^{1/2} = 0.0897\ in.$$

Actual minimum flat head thickness = 0.15 inches. Therefore, the head meets Div. I criterion.

Finite Element Analysis Model

The Code calculations are based on thin shell/plate theory; the test vessel components are relatively thick, and their actual stresses are not well predicted by the Code equations. A 3-D finite element model was created to verify the design. For the purposes of the simulation the bolt(s) were tightened to 10.1 in-lbf. An internal pressure of 2 atmospheres gauge (29.4 psi, 101350 Pa) was applied to the model.

For 2-atmospheres pressure in the vessel the figure 9 show the "hoop" stresses in the cylindrical portion of the vessel. Figures 7, 10 and 11 show the von Mises stress across the thin window portions of the vessel for the 2-atmospheres pressure simulation. Figures 12 and 13 show the deflection of those parts. All stresses are less than predicted by the Code calculations, and less than the 18,800 psi (129.6 MPa) allowable for SS316.

	Property	Name	Value	Unit	Property group
✔	Density	rho	8000[kg/m^3]	kg/m³	Basic
✔	Young's modulus	E	1.93e11	Pa	Young's modulus and Poisson's ratio
✔	Poisson's ratio	nu	0.28	1	Young's modulus and Poisson's ratio
	Relative permeability	mur	1	1	Basic
	Electrical conductivity	sigma	1.351e6[S/m]	S/m	Basic
	Coefficient of thermal expansion	alpha	1.59e-5[1/K]	1/K	Basic
	Heat capacity at constant pressure	Cp	500[J/(kg*K)]	J/(kg·K)	Basic
	Relative permittivity	epsilonr	1	1	Basic
	Thermal conductivity	k	16.2[W/(m*K)]	W/(m·K)	Basic

Table 1 - parameters used for the SS material of the cavity
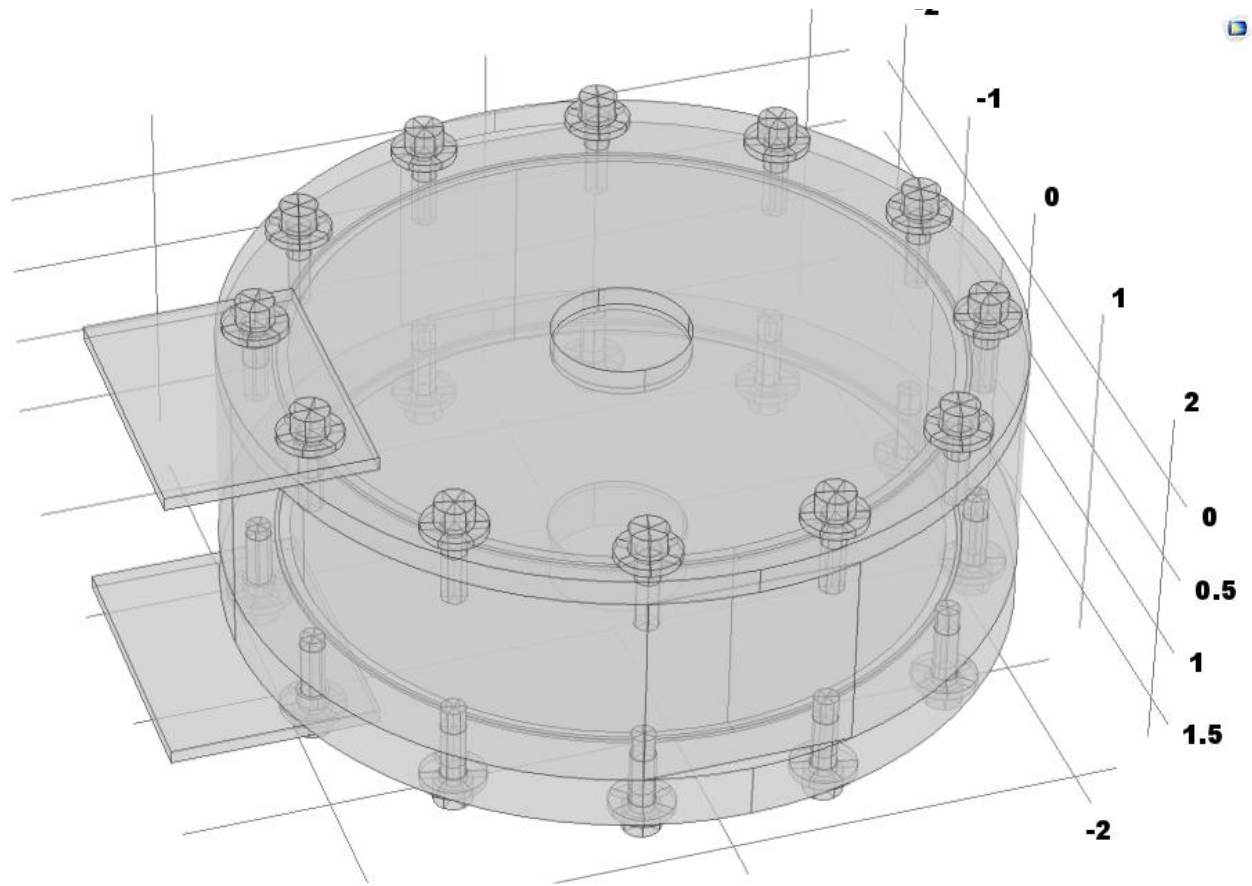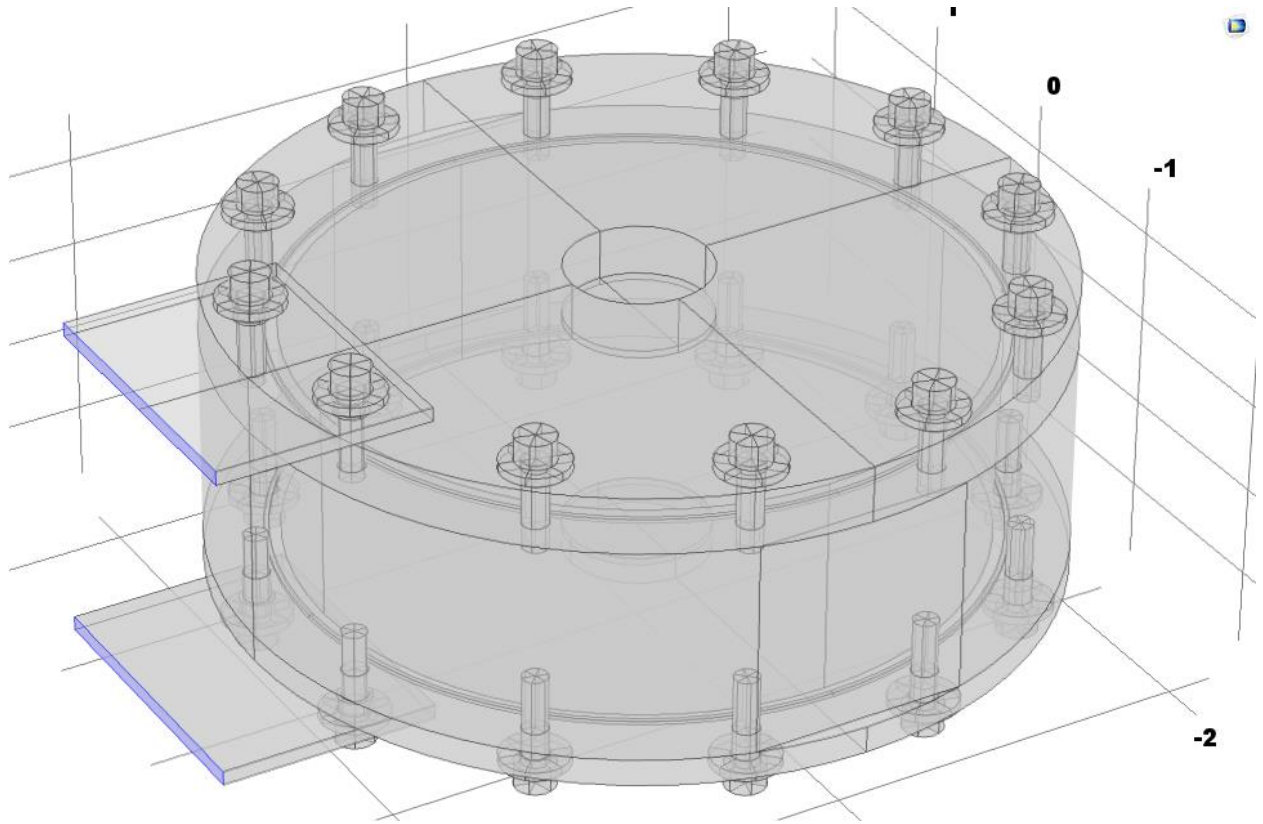
Figure 4. - Graphic of model in Comsol

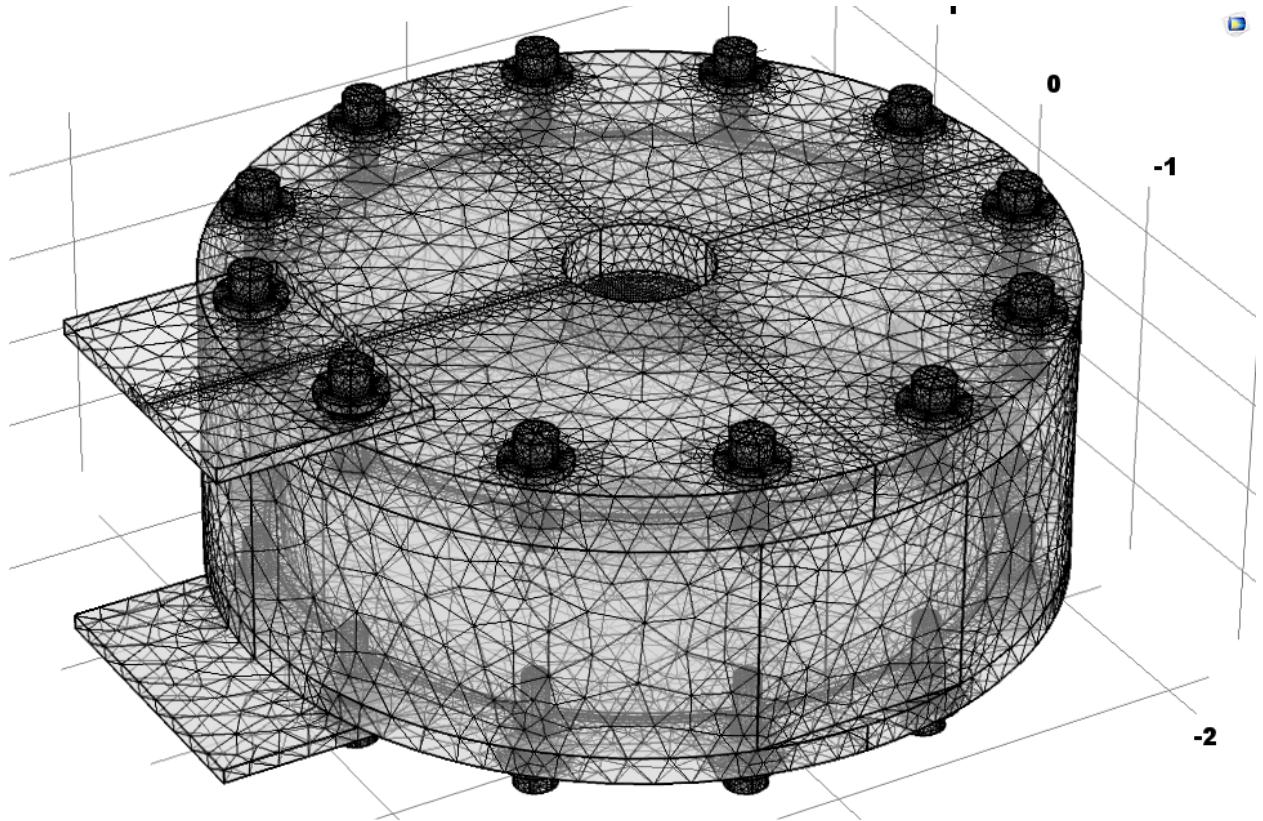**Figure 5 - Graphic showing "fixed" (unmoving) surfaces in model.**

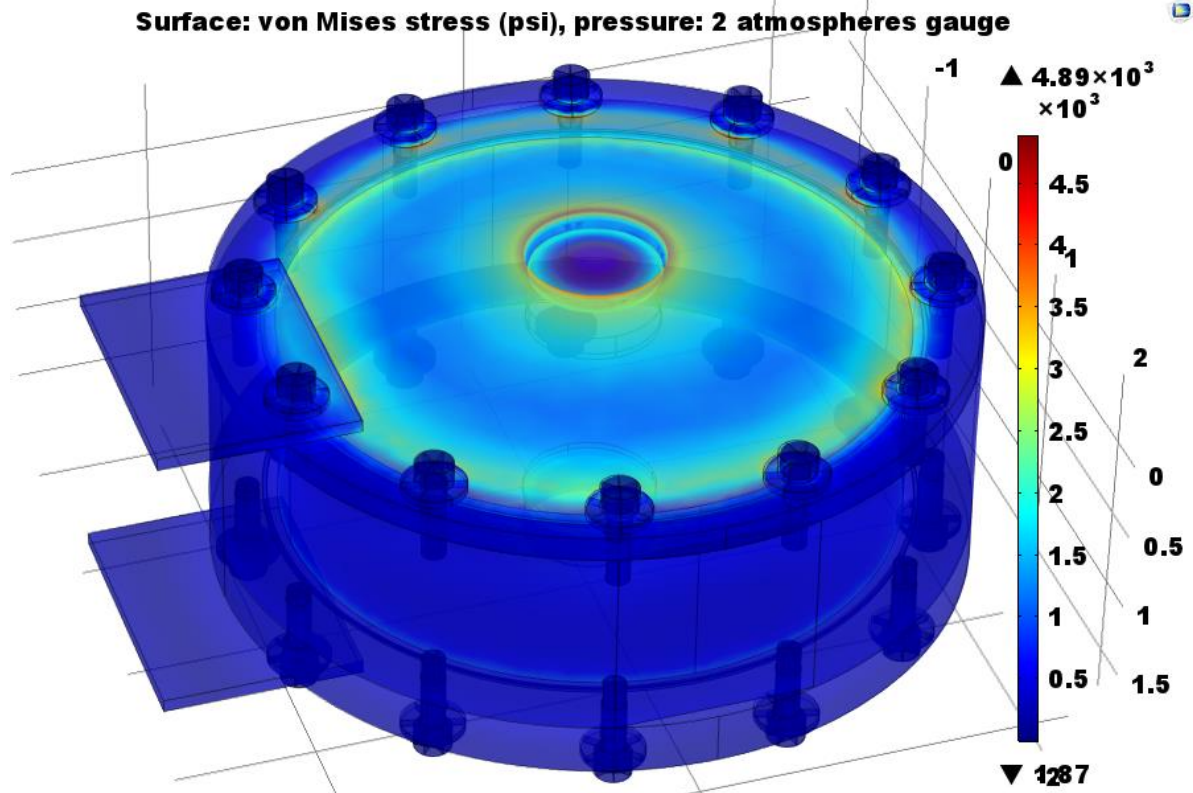**Figure 6 - Graphic showing meshing of the model. (~410k meshes used)**

Surface: von Mises stress (psi), pressure: 2 atmospheres gauge

Figure 7 - von Mises Stress (psi) showing "thin side" (0.150" thick) end plate.

Surface: von Mises stress (psi), pressure: 2 atmospheres gauge
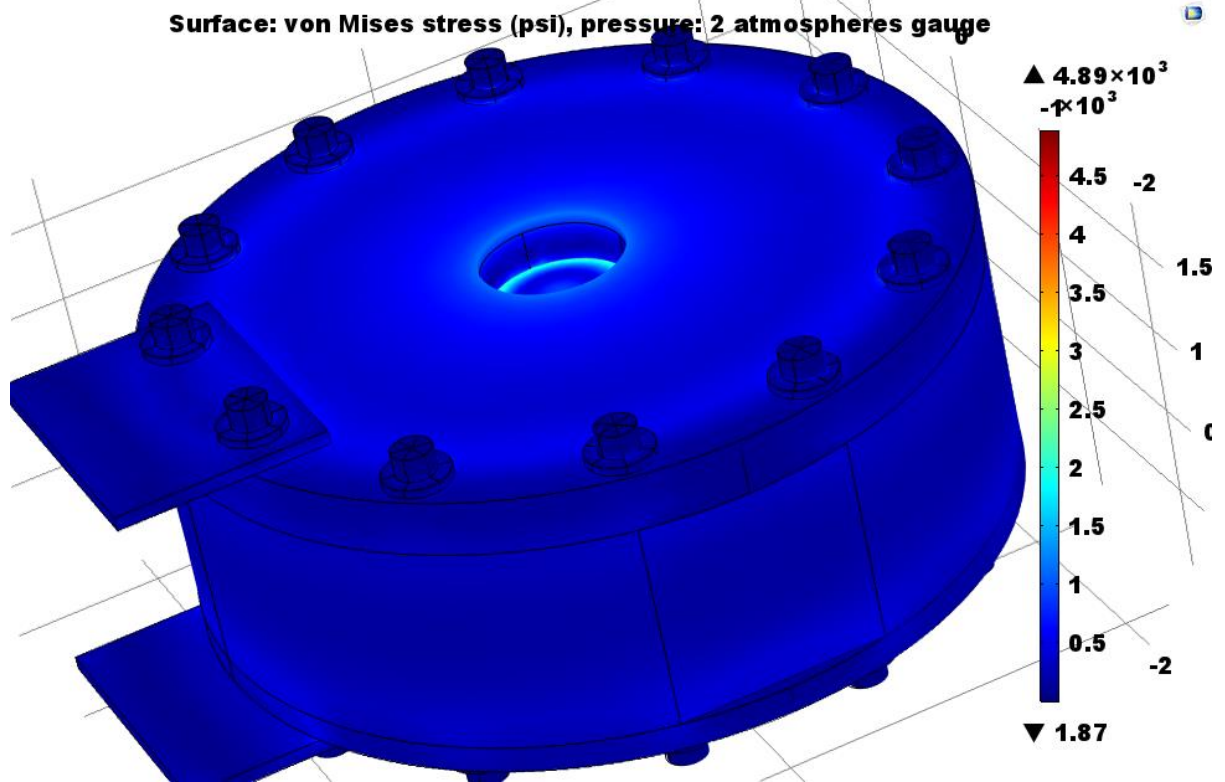
Figure 8 - von Mises Stress (psi) showing "thick side" (0.315" thick) end plate.

1: press_mult=2, bolt_ptf=10.1 Surface: von Mises stress (psi)
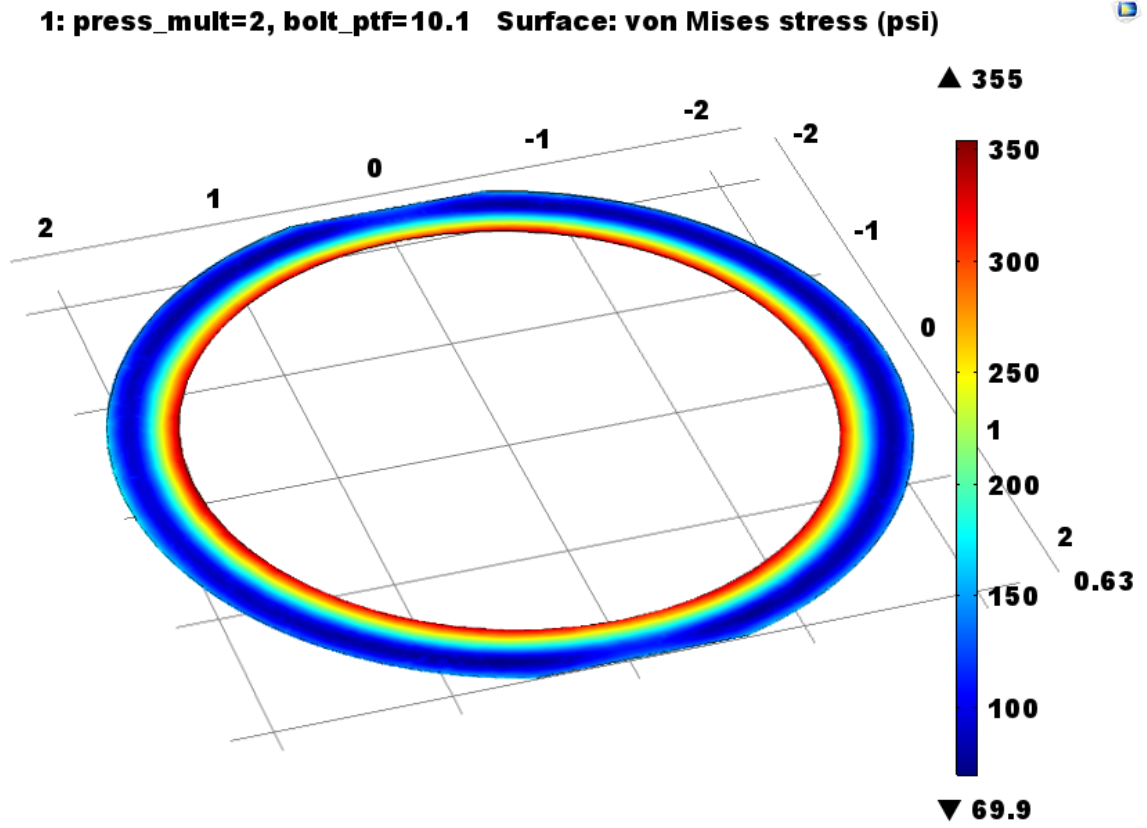
Figure 9 - Graphic showing the "hoop" stress at a plane located at the center-line of the cylindrical portion of the cavity.
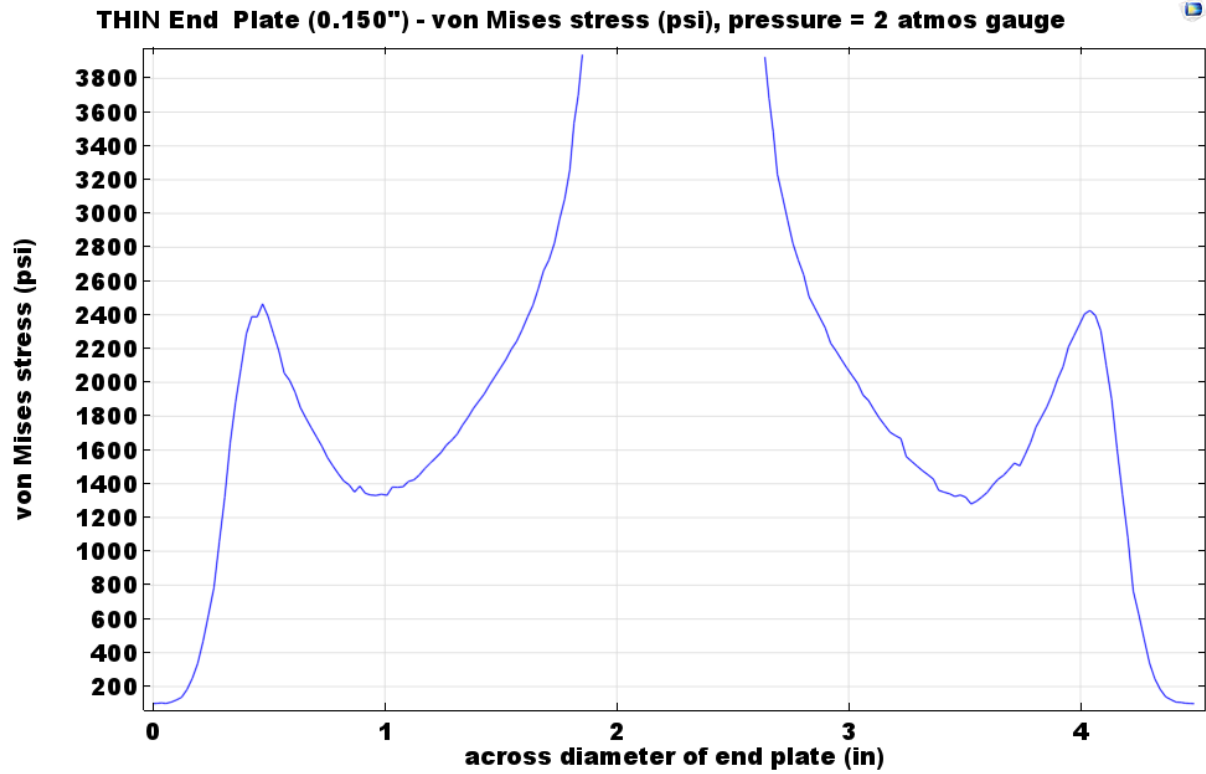
Figure 10 - von Mises Stress measured diametrically across the outer surface of the "thin" (0.150" thick) side. Center gap is the window location.
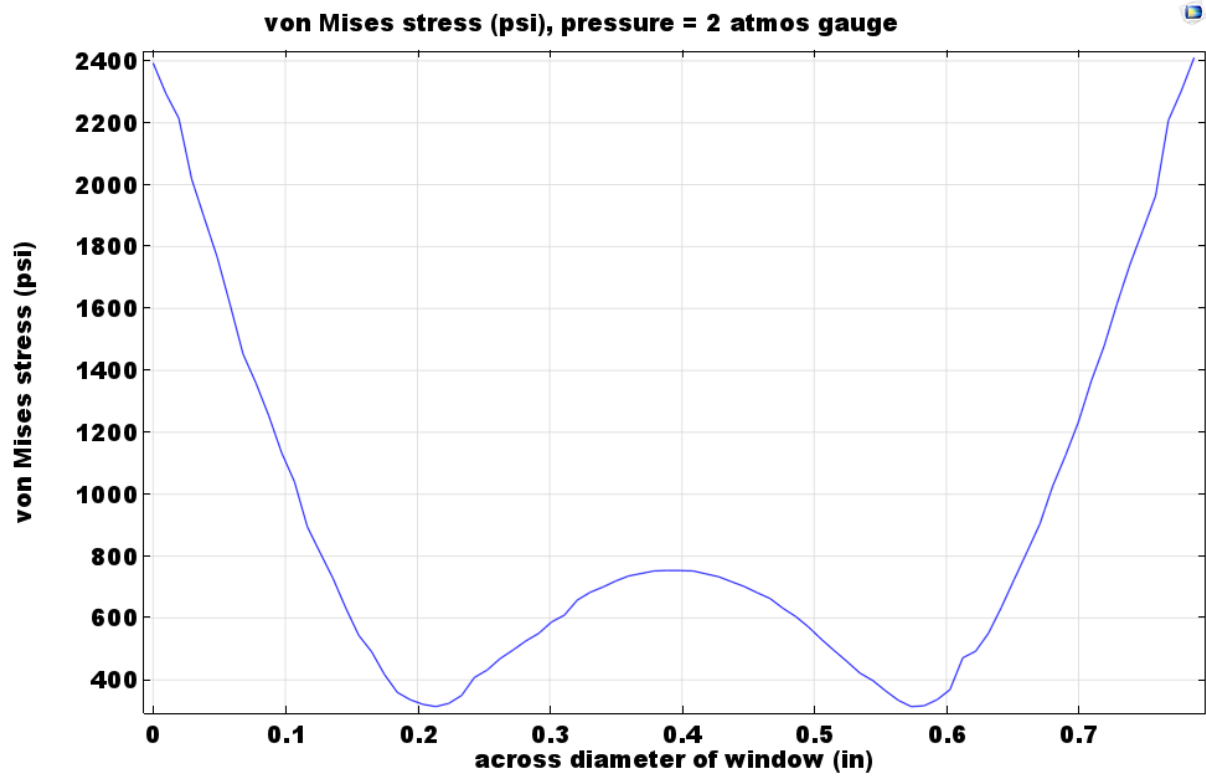
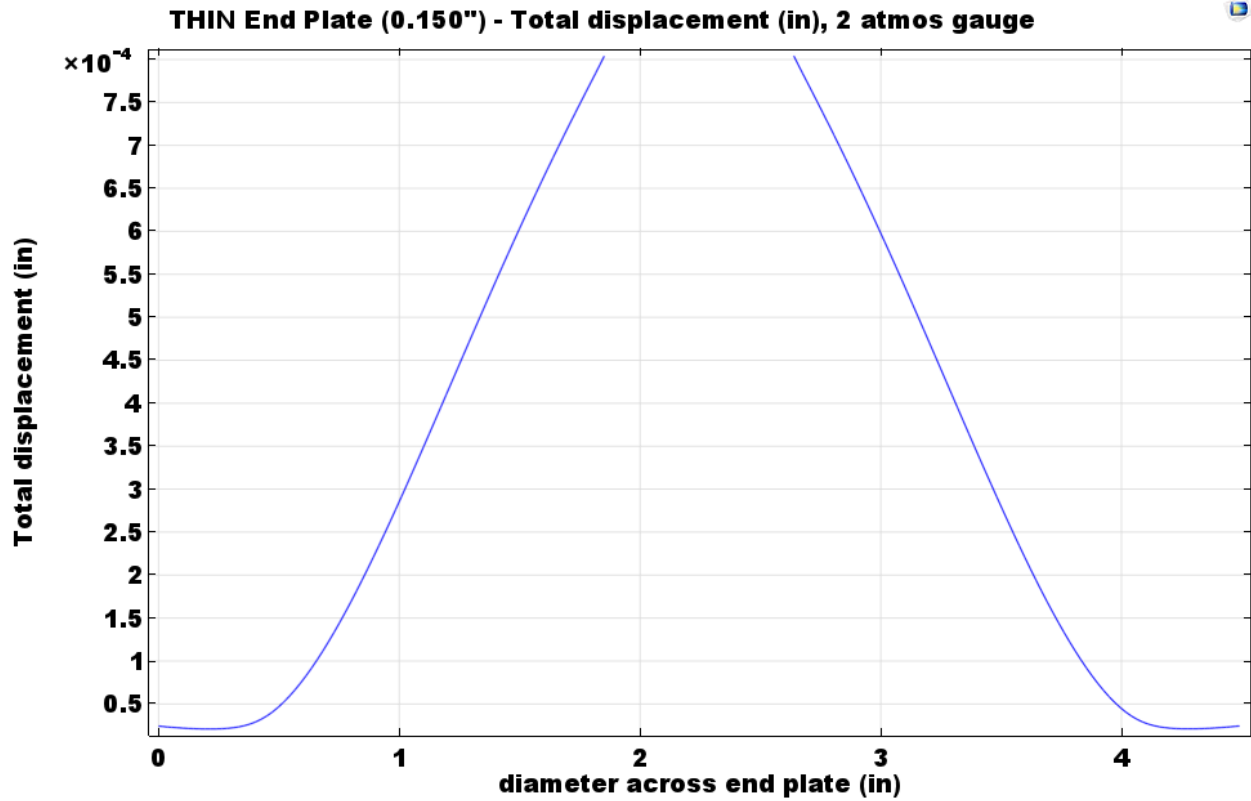**Figure 11 - von Mises Stress measured diametrically across the outer surface of the window on the "thin side".**

Figure 2 - "Thin side" end plate displacement measured diametrically across the outer surface of the end plate.
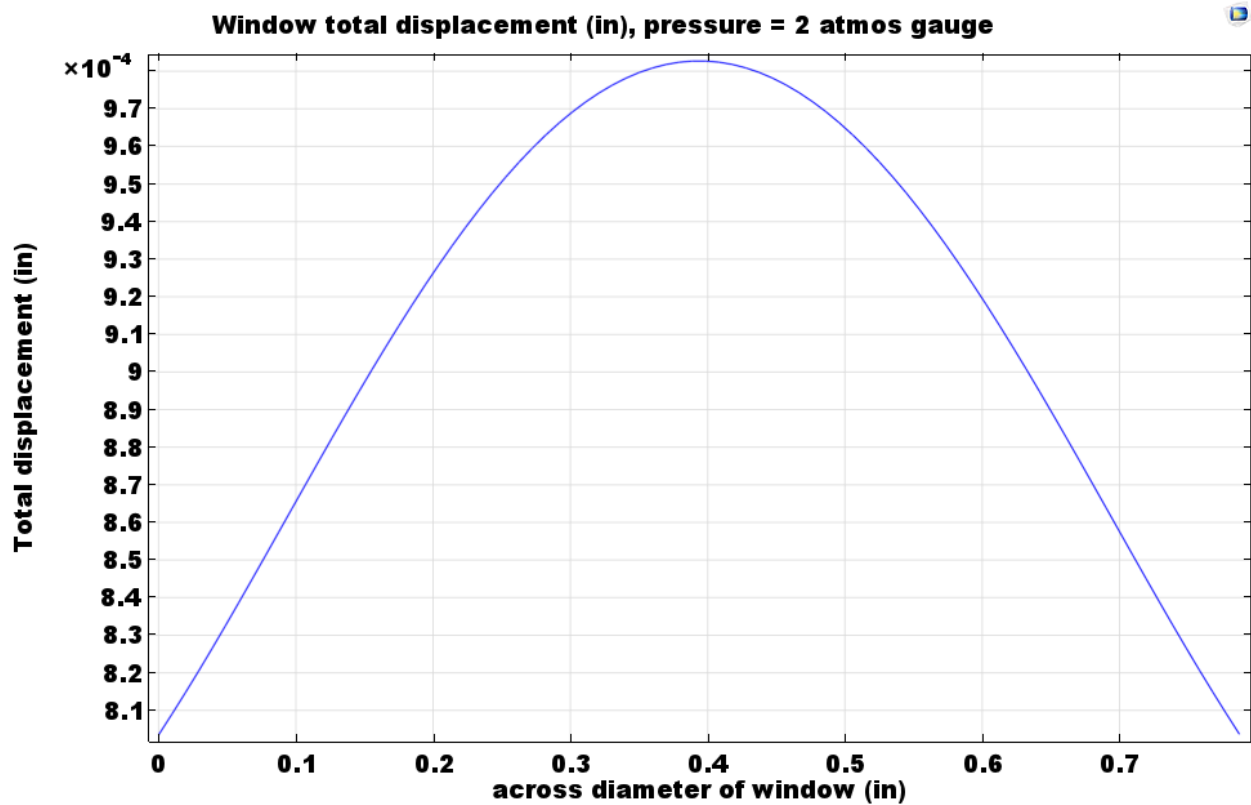

Figure 13 - "Thin side" window displacement measured diametrically across the outer surface of the window.
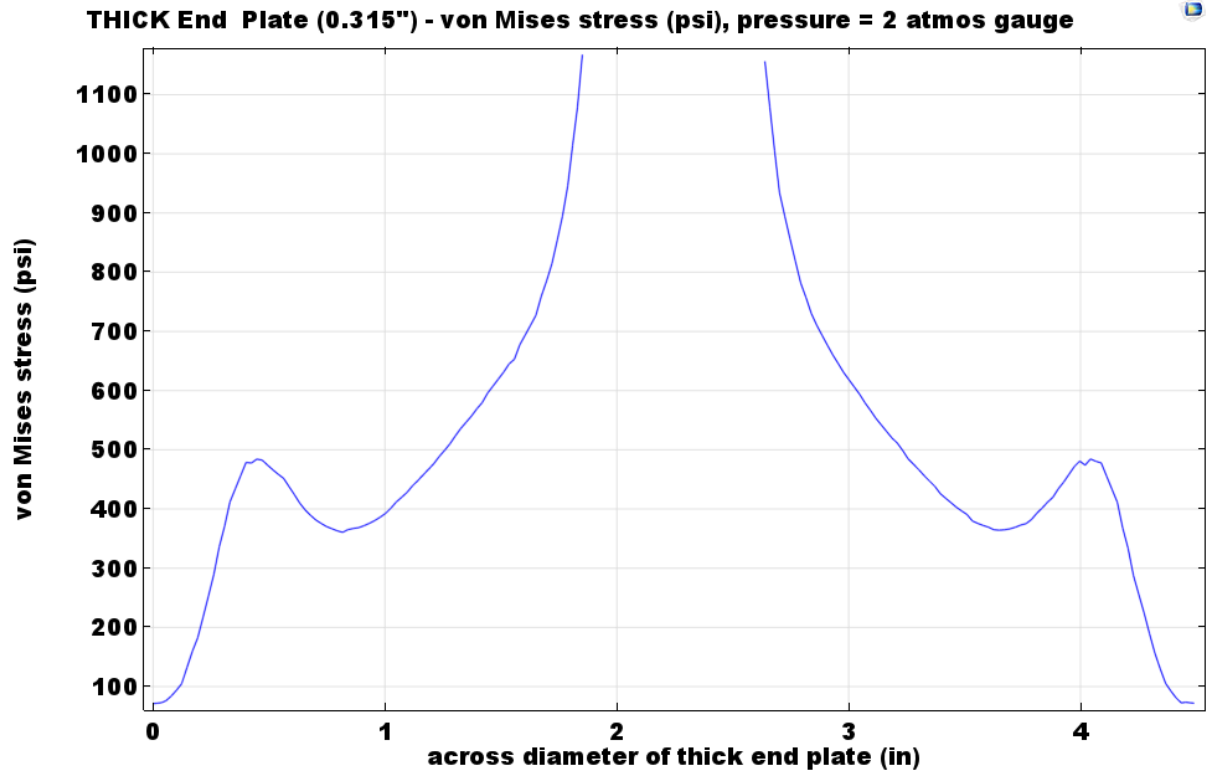
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Figure 14 - "Thick side" end plate (0.315" thick) von Mises Stress measured diametrically across the outer surface of the end plate.
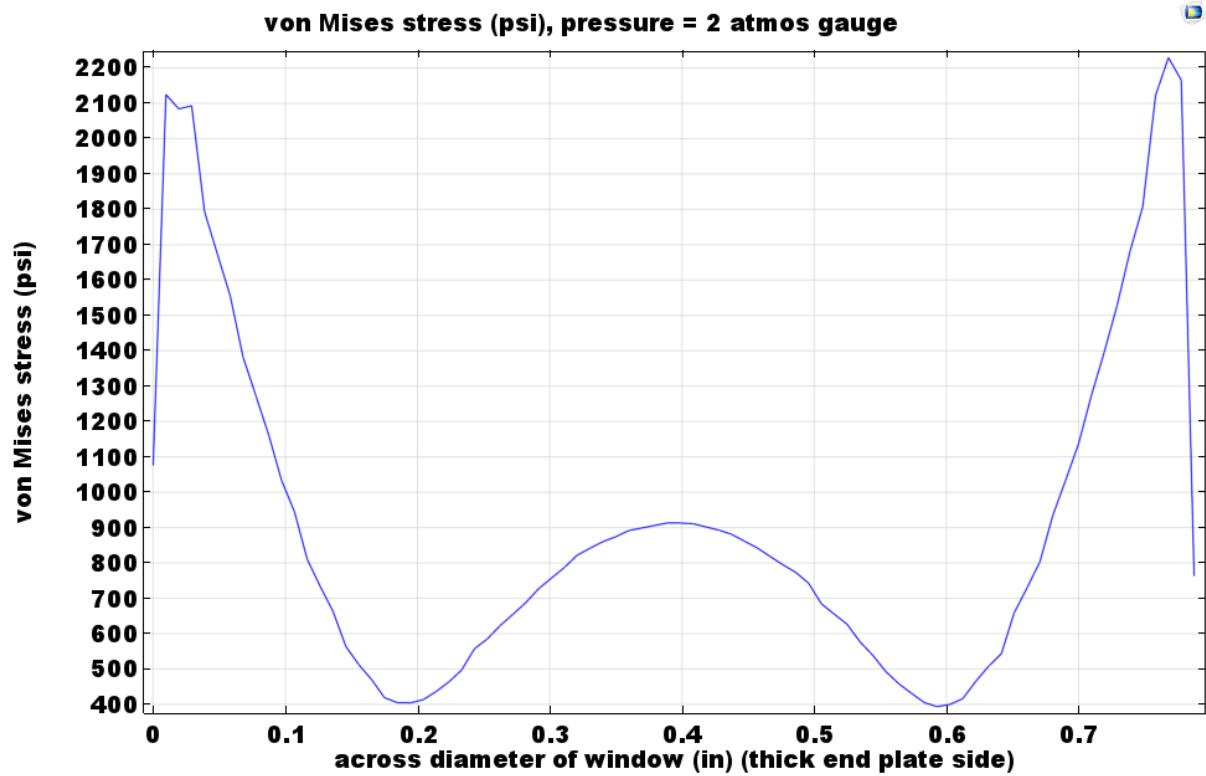

Figure 15 - "Thick side" window von Mises stress, measured diametrically across the outer surface of the window.
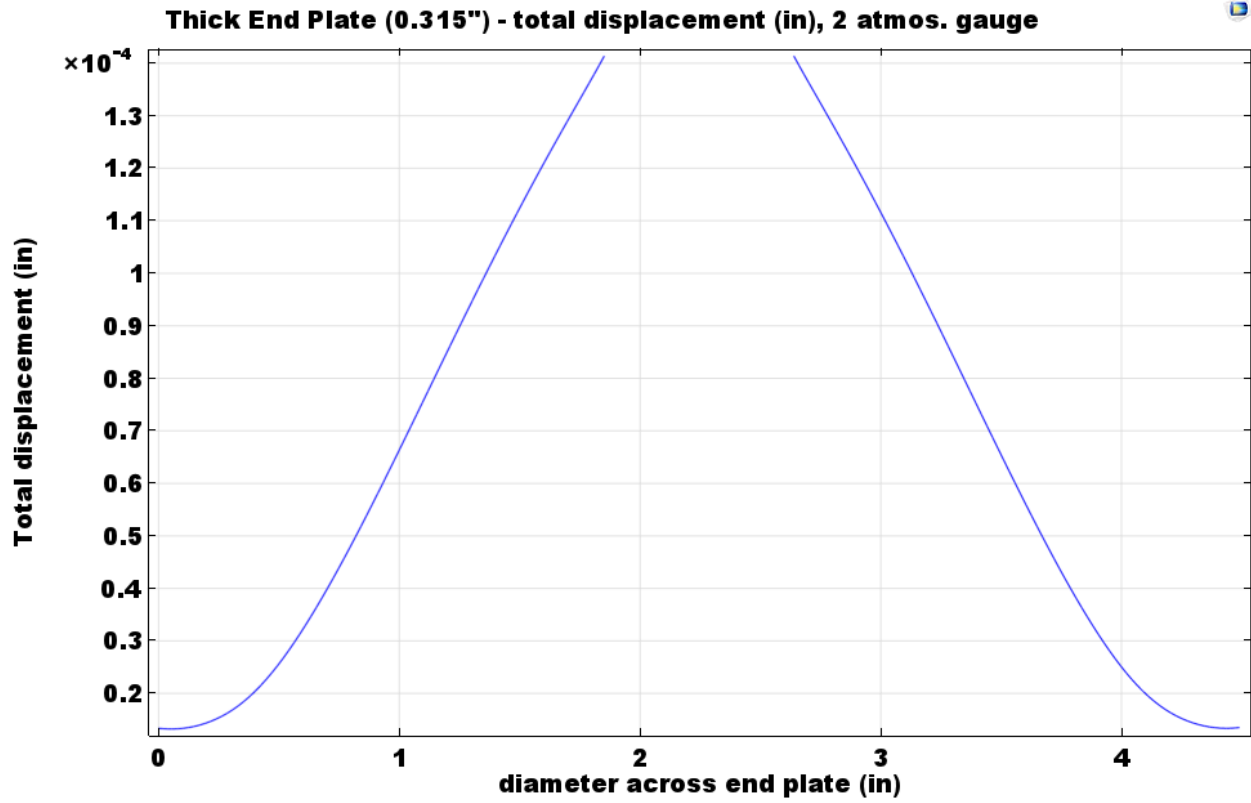
14

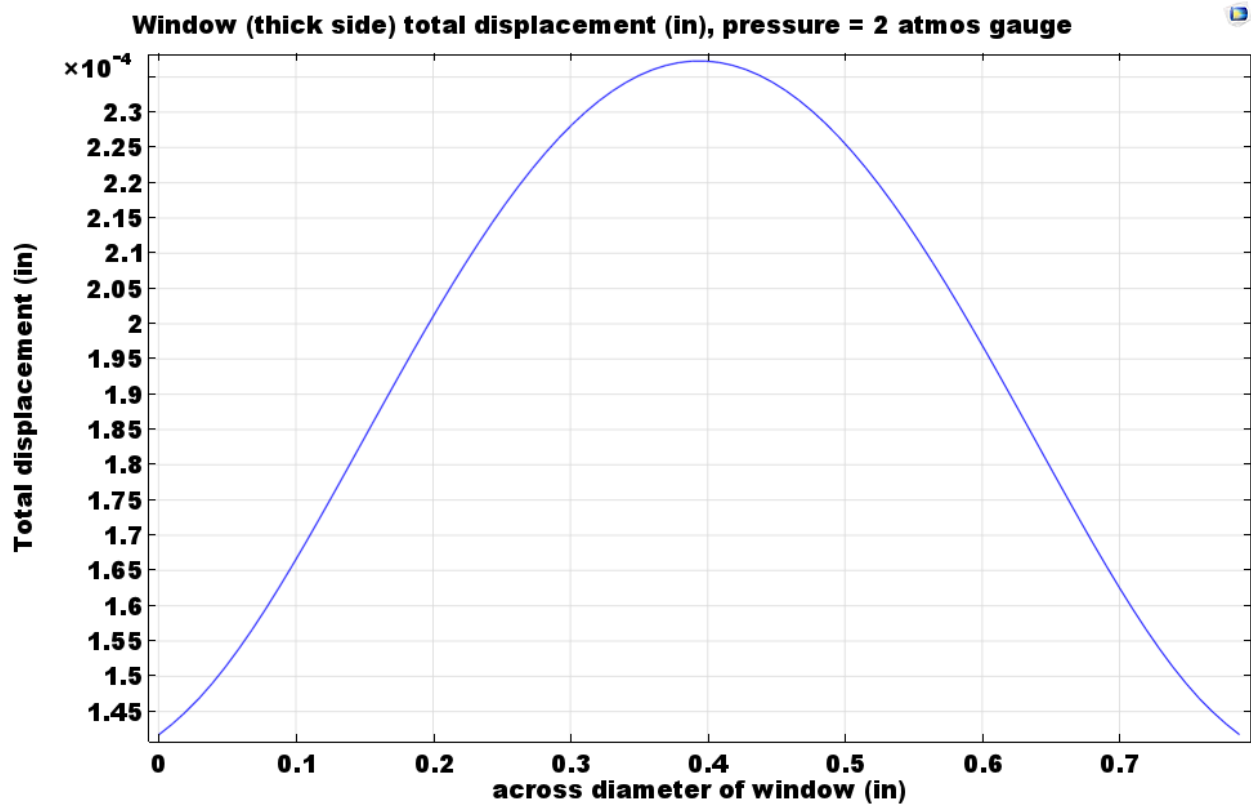**Figure 16 - "Thick side" end plate displacement.**

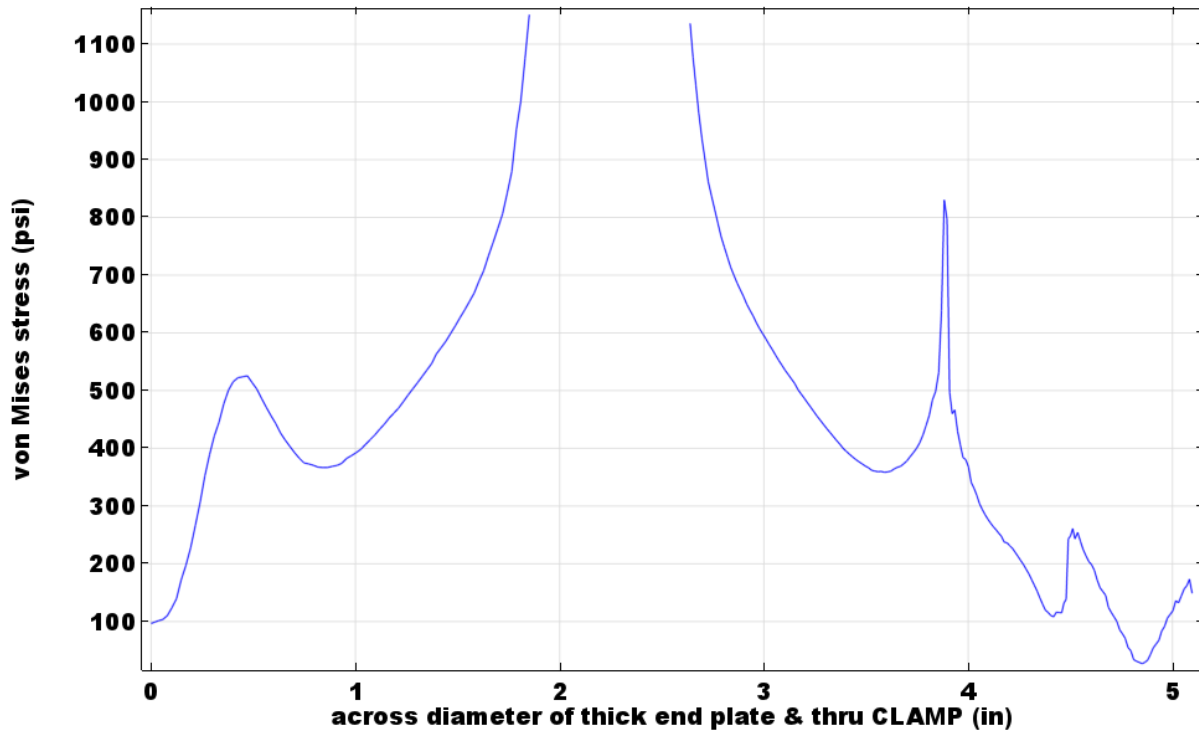**Figure 17 - Thick side" window displacement.**

Figure 18 - "Thick side" end plate (0.315" thick) von Mises Stress measured diametrically across the outer surface of the end plate in the direction which includes the CLAMP.
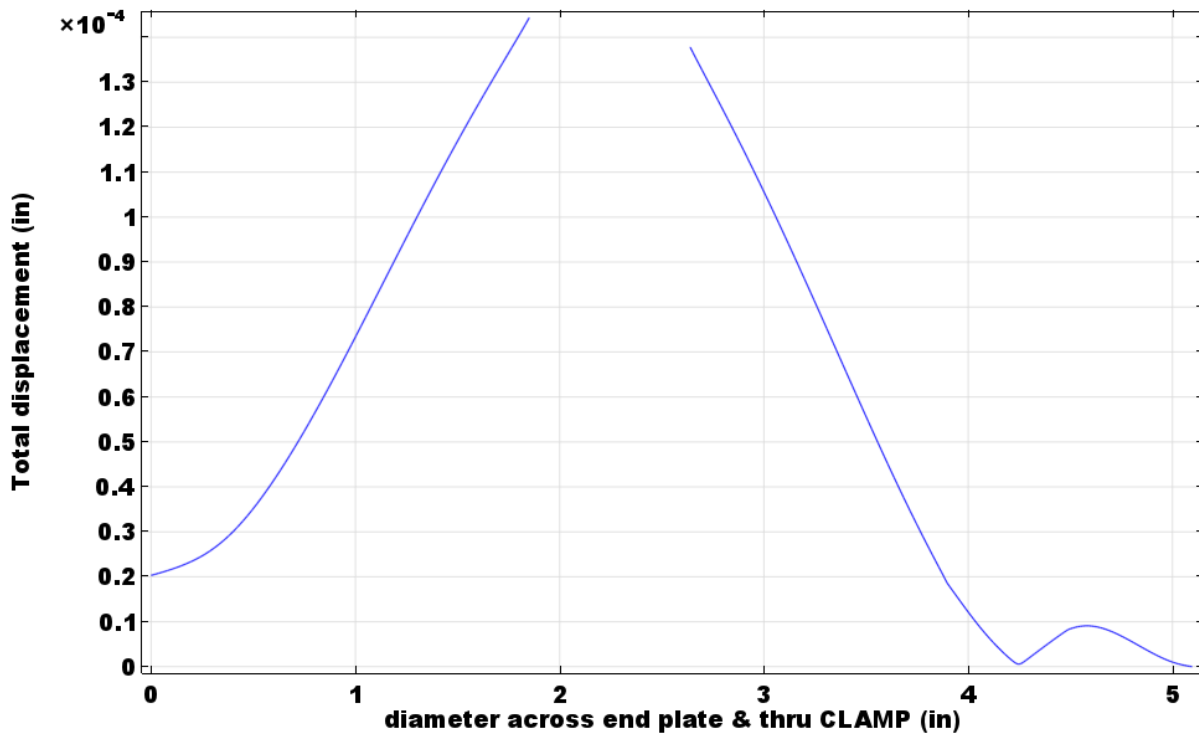

Figure 19 - "Thick side" end plate displacement. (includes Clamp portion)

Conclusion

This analysis verifies with both ASME Code calculations and a finite element model that stresses in the test vessel are well within the limits imposed by the ASME Code for SS316 stainless steel.

This analysis is confined to the test vessel proper, and does not cover the piping and pipe flanges, with bolt to the vessel, except as they affect the vessel in their support function. No high stresses were observed in these regions, but further work should be addressed toward assuring that the piping and flanges are adequately sized for 30 psi.

As relates to the vessel proper, the concept of support using the plates bolted to the heads is sound; the vessel weight of 2.5 lbs produces additional stresses in the head of less than 500 psi negligible compared with the pressure stresses.

Additional Test Cell Venting Requirements

Summary

The gas pressure increment rate is evaluated in the worst case, in which all RF energy and the beam energy deposition goes into heating the gas. The temperature increment due to the RF power heating is 2.5E-2 degrees K/minute while that due to the beam energy deposition is 3.3oE-2 degrees K/minute. The overall temperature increment is 5.8E-2 degrees K/minute. Thus the overall pressure relief is 5.6E-3 psi/minute. This value is within a measurement error of a pressure gauge.

Pressure relief required by RF heating

The peak RF energy in the cavity is 150 Watt per RF pulse. Each pulse could be as long as 10 μs. The cavity runs 0.1 Hz (1 RF pulse in 10 seconds). Thus, the average RF power is 150 x 10E-5 x 0.1 = 150 μW. Normally, this power is dissipated in the walls of the test cavity. Thus, when operating at the full repetition rate at room temperature the test cavity can be cooled with a convection flow of air of the circumference.

Nevertheless, we are asked the question, "if all of the RF power went into heating the gas, how fast would the pressure rise?" According to calculations shown in the heating calculation, the pressure increment rate due to the RF power is 2.4E-3 psi/minute.

Pressure relief required by beam energy deposition

The beam energy deposition in a 3-cm thick 2-atm air is 24 keV per single 120 GeV/c proton. The full beam intensity will be 3E12 protons per pulse. Thus, the total energy deposition is 3E12 x 24E3 x 1.6E-19 = 1.2E-2 Joules. The beam hits on the test cavity every minute. The average beam power deposition is 1.2E-2/60 = 0.2 mW. According to calculations shown in the heating calculation, the pressure increment rate due to the beam energy deposition is 3.2E-3 psi/minute.

Heating

The amount of energy needed to increase the temperature of a mass with specific heat C is $dE=dT\,Cm$ or $dE/dt = mCdT/dt$. Where E is in Joules, C in Joules/(gram Kelvin), and the mass, m = (w = molecular weight) PV/22.4. Using PV = nRT or dP/P = dT/T at constant V and compressibility, we have dP/dt = [22.4/CwVT]dE/dt. Here we evaluate gas heating in two cases shown below.

At 300 K, a specific heat of Air is 1.01 J/g/K. The test cavity volume is 140 cc = 0.14 liters. The total weight of 2 atm air in the cavity is 2*0.14/22.4 *28.8 = 0.36 gram. To raise 0.36 gram of air 1 degree K requires 0.36*1.01 = 0.364 Joules. The maximum RF power deposition in the gas is 150 μJ/s. The temperature rises in a minute, if the air were isolated is 60*150E-6/0.364 = 2.5E-2 degrees K/minute. The fractional pressure rise is proportional to the fractional temperature rise at constant volume such that the pressure rise would be dP=dT/T P = 0.025/300 2 = 1.65E-4 atm/minute = 2.4E-3 psi/minute. The increment is lower than a precision of a pressure detector. Note this result is independent of the initial pressure since the heating rate is inversely

proportional to the pressure. For ideal gases it only depends on the molecular weight and the heat capacity of the gas.

In case of the beam energy deposition, a gain average heating power is 0.2 mW. The temperature rises in one minute, if the air were isolated is 60*0.2E-3/0.364 = 3.3E-2 degrees K/minute. The fractional pressure rise is dP=dT/T P = 3.3E-2/300 2 = 2.2E-4 atm/minute = 3.2E-3 psi/minute.

Note on Flow Rates of Compressible Gases through Pipes and Valves

These calculations are for the 30 psi operating condition, See Chapter 3 from RSB-570 (appended) for background and more detailed discussions at 30 psi MAWP.

Case 1) Determine the initial flow rate in the vent line from the open relief valve through a 3 ft tube to the outside exhaust pipe.

Number of moles of gas and density at 30 psig

$$\left(P + \frac{an^2}{V^2}\right)(V - nb) = nRT$$

where
T = 293 K
V = 0.14 liters (L)
P = 30 psi = 2 atm
R = 0.08205746 L atm/K/mol
a = 1.33 L^2 atm/mol^2
b = 0.0366 L/mol

Rearranging the equation and solving for "n" the number of moles, yields
n = 0.011 moles or 0.327 grams.
Then the density (ρ_{int}) of air inside the test cavity 293 K is calculated to be
ρ_{int} = 2.34E-3 g/cm^3 = 0.145 lbs/ft^3.

Flow rate through the tube:

$$q'_h = 24700 \frac{Yd^2}{S_g} \sqrt{\frac{\Delta P \rho_{int}}{K}}$$

where
Y = 0.71
K = 5
ΔP = (30-14.7)*0.725 = 11.1 psi

d = 0.305 in
S_g = 0.070
q_h' = 13.2E3 ft³/hr = 220 scfm.

Published data for the Circle Seal 5159T-3MP relief valve show the flow rate to be about 5 scfm at the set pressure. Thus, for case 1) the tube does not restrict the flow from the relief valve.

Case 2) and 3) do not appear to substantially change with the lower set pressure.