$\mathcal{N} = 1$ Supersymmetric $SU(3)$ Gauge Theory - Towards simulations of Super-QCD

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Supersymmetric QCD ...

- describes the strong interaction in the MSSM.
- models the interaction of gluons and quarks with their superpartners, the gluinos and squarks.
- reduces to $\mathcal{N} = 1$ SYM theory for infinitely heavy quarks and squarks.

Interesting non-perturbative aspects are ...

- the mass spectrum and the formation of supermultiplets.
- the phase diagram at finite temperature.
- the (N_c, N_f) phase diagram of the massless theory.

Supersymmetry is broken by the lattice regularization

$\Rightarrow$ Fine-tune set of operators to restore susy in the continuum limit

1. Supersymmetric QCD in the continuum
2. One-loop effective potential
3. (Preliminary) Lattice results

Supersymmetric QCD in the continuum

$\mathcal{N} = 1$ $SU(N)$ SQCD

$$\mathcal{L} = \mathrm{tr}\left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\mathrm{i}}{2}\bar{\lambda}\gamma^{\mu}D_{\mu}\lambda\right) + \bar{\psi}\left(\mathrm{i}\gamma^{\mu}D_{\mu} + m\right)\psi$$
$$-D_{\mu}\phi^{\dagger}D_{\mu}\phi - m^2\phi^{\dagger}\phi - \frac{1}{2}g^2\left(\phi^{\dagger}T\sigma_3\phi\right)^2 - \mathrm{i}\sqrt{2}g\left(\phi^{\dagger}\bar{\lambda}P\psi - \bar{\psi}\bar{P}\lambda\phi\right)$$

with $\phi = \begin{pmatrix}\phi_+\\ \phi_-\end{pmatrix}$, $P = \begin{pmatrix}P_+\\ P_-\end{pmatrix}$, $\bar{P} = \gamma_0 P \gamma_0$ and $SU(N)$ generators T

- Gluons, real vector field A_μ in the adjoint representation (A)
- Gluinos, Majorana fermions λ in (A)
- N_f Quarks, Dirac fermions ψ in the fundamental representation (F)
- $2N_\mathrm{f}$ Squarks, complex scalars ϕ in the (F)
- Quark-Squark-Gluino Yukawa interaction

SQCD = SYM + QCD + Scalars + interaction

Preserved symmetries on the lattice

- Gauge symmetry
- Parity: $x = (t, \boldsymbol{x}) \to x' = (t, -\boldsymbol{x})$

$$\psi(x) \to \gamma_0 \psi(x')\,, \quad \lambda(x) \to \gamma_0 \lambda(x') \quad \text{and} \quad \phi_\pm(x) \to -\phi_\mp(x).$$

- Baryon number conservation:

$$U(1)_\mathrm{B}: \quad \psi \to e^{\mathrm{i}\alpha}\psi\,, \quad \phi \to e^{\mathrm{i}\alpha}\phi$$

- $\pm$ - exchange symmetry

- Restrict number of operators that need to be fine-tuned

Explicitly broken on the lattice

- Chiral symmetry:

$$\psi \to e^{i\alpha\gamma_5}\psi\,, \quad \phi \to e^{i\alpha\sigma_3}\phi$$

$$U(1)_A \xrightarrow{\text{anomaly}} \mathbb{Z}_{2N_f} \xrightarrow{\text{spont.}} \mathbb{Z}_2$$

- R symmetry (also in $\mathcal{N} = 1$ SYM):

$$\psi \to e^{-i\alpha\gamma_5}\psi\,, \quad \lambda \to e^{i\alpha\gamma_5}\lambda$$

$$U(1)_R \xrightarrow{\text{anomaly}} \mathbb{Z}_{2(N_c - N_f)} \xrightarrow{\text{spont.}} \mathbb{Z}_2$$

anomaly free subgroup

$$U(1)_A \otimes U(1)_R \xrightarrow{\text{anomaly}} U(1)_{AF}$$

- Supersymmetry ($\mathcal{N} = 1$ SYM):

$$\delta_\epsilon \phi_+ = -\sqrt{2}\bar{\epsilon} P \psi\,, \quad \delta_\epsilon \psi = \mathrm{i}\sqrt{2}(D_\mu \phi) P \gamma^\mu \epsilon - \sqrt{2} m P \sigma_1 \phi\, \epsilon\,,$$
$$\delta_\epsilon A_\mu^\alpha = -\mathrm{i}\bar{\epsilon}\gamma^\mu \lambda^\alpha\,, \quad \delta_\epsilon \lambda^\alpha = -\mathrm{i}\sigma^{\mu\nu} F_{\mu\nu}^\alpha \epsilon - 2\mathrm{i} g \gamma_5 \epsilon\, \phi^\dagger T^\alpha \sigma_3 \phi$$

Supersymmetric Ward identities $0 = \langle \bar{\mathcal{Q}} \mathcal{O} \rangle + \langle \mathcal{O} \bar{\mathcal{Q}} S \rangle$

$$\left\langle -\frac{1}{4} \operatorname{tr} F_{\mu\nu} F^{\mu\nu} \right\rangle = \frac{3}{8} \left\langle \frac{\mathrm{i}}{2} \operatorname{tr} \bar{\lambda} \gamma_\mu D^\mu \lambda \right\rangle \overset{SYM}{=} \frac{3}{2} \left(N_c^2 - 1 \right)$$

$$\frac{2}{3} \left\langle -\frac{1}{4} \operatorname{tr} F_{\mu\nu} F^{\mu\nu} \right\rangle + \langle D_\mu \phi^\dagger D^\mu \phi + m^2 \phi^\dagger \phi \rangle = N_c^2 - 1 + 2 N_c N_f$$

Konishi anomaly

$$\langle \bar{\lambda} \lambda \rangle = 32 \pi^2 m \, \langle \phi^\dagger \sigma_3 \phi \rangle\,.$$

Fine-tuning

- need to fine-tune a set of operators $\mathcal{O}$ to restore susy in the continuum limit
- $\mathcal{N} = 1$ SYM: only gluino mass $\bar{\lambda}\lambda$
- SQCD: all relevant (marginal) operators $\mathcal{O}$ with

$$[\mathcal{O}] \leq d = 4$$

- Mass dimension of constituent fields:

$$[A] = [\phi] = 1\,, \quad [\psi] = [\lambda] = \frac{3}{2}$$

$$\Downarrow$$

quadratic, cubic, and quartic interactions

quadratic interactions

- $SU(N)$ gauge invariance and baryon number conservation:

$$\bar{F} \otimes F \text{ and } A \otimes A$$

Quark and gluino masses

$$\bar{\psi}\,\Gamma\,\psi \text{ and } \bar{\lambda}\,\Gamma\,\lambda \quad \text{with } \Gamma = \{\mathbb{1}, \gamma_5\}$$

2 squark masses

$$M_1 = \operatorname{tr}\Phi \text{ and } M_2 = \operatorname{tr}(\Phi\sigma_1) \quad \text{with } \Phi_{rs} = \phi_r^\dagger \phi_s$$

- $\bar{\lambda}\gamma_5\lambda$ breaks parity:
  Twist can be used to improve the extrapolation to the chiral limit
  $\Rightarrow$ next talk by Marc Steinhauser

cubic interactions

- $SU(N)$ gauge invariance and baryon number conservation:

$$\bar{F} \otimes A \otimes F$$

- only operator compatible with all symmetries is

$$\mathrm{i}\left(\phi^{\dagger}\bar{\lambda}\, P\, \psi - \bar{\psi}\, \bar{P}\, \lambda\, \phi\right).$$

- appears already in the Lagrange function
- coupling can be absorbed in a rescaling of the scalar field

quartic interactions

- only scalar fields possible
- $SU(N)$ gauge invariance and baryon number conservation:

$$1 \otimes 1\,, \quad A \otimes A\,, \quad F \otimes \bar{F}\,, \quad R \otimes \bar{R}$$

- use $SU(N)$ Fierz identities to reduce this 4 types to $1 \otimes 1$
- 5 independent operators for $N_\mathrm{f} = 1$

$$\begin{aligned} V_1 =& \Phi_{++}^2 + \Phi_{--}^2\,, \quad V_2 = \Phi_{+-}^2 + \Phi_{-+}^2\,, \quad V_3 = \Phi_{++}\Phi_{--}\,, \\ V_4 =& \Phi_{+-}\Phi_{-+}\,, \quad V_5 = (\Phi_{+-} + \Phi_{-+})(\Phi_{++} + \Phi_{--})\,. \end{aligned}$$

- V_2 and V_5 break chiral symmetry

General Euclidean Lagrange function

$$\mathcal{L} = \frac{1}{g^2}\left(\frac{1}{4}\,\mathrm{tr}\,F_{\mu\nu}^2 + Z_\phi\, D_\mu\phi^\dagger D_\mu\phi + Z_\phi\, m_i^2\, M_i + Z_\phi^2\,\lambda_i\, V_i\right) + \frac{1}{2}\,\mathrm{tr}\,\bar{\lambda}\,(\gamma_\mu D_\mu - m_g)\,\lambda + \bar{\psi}\,(\gamma_\mu D_\mu - m_q)\,\psi + \mathrm{i}\sqrt{2}\,(\phi^\dagger\bar{\lambda}\,P\,\psi - \bar{\psi}\,\bar{P}\,\lambda\,\phi)$$

9 fine-tuning parameters for $N_\mathrm{f} = 1$ and fixed quark mass m_q

- bare gluino mass m_g
- 2 squark masses m_i
- squark wave function renormalization Z_ϕ (Yukawa interaction)
- 5 squark quartic couplings λ_i

Supersymmetric continuum action:

$\bar{\lambda}\lambda$	$\bar{\psi}\psi$	Z_ϕ	M_1	M_2	V_1	V_2	V_3	V_4	V_5
0	m	1	m^2	0	$(N_\mathrm{c}-1)/N_\mathrm{c}$	0	$1/N_\mathrm{c}$	-1	0

One-loop effective potential

Fine-tuning of squark potential guided by lattice perturbation theory

- Integration over fermions

$$S = S_{\text{Gauge}} + S_{\text{Squark}} - \operatorname{tr}\ln\left(\not{D}_\psi - m\right) - \frac{1}{2}\operatorname{tr}\ln \not{D}_\lambda - \frac{1}{2}\operatorname{tr}\ln\left(\mathbb{1} - \Delta_\lambda Y \Delta_\xi \bar{Y}^\dagger\right)$$

- Calculate 1-loop effective squark potential

$$V_{\text{eff}}^{\text{1-loop}}(\phi) = V(\phi) + \underbrace{\frac{1}{2}\operatorname{tr}\ln\left(\left.\frac{\delta^2 S[\varphi]}{\delta\varphi(x)\delta\varphi(y)}\right|_{\varphi=\phi}\right)}_{\text{Bosonic contribution}} \underbrace{- \frac{1}{2}\operatorname{tr}\ln\left(\frac{\delta^2 S[\phi]}{\delta\Psi(x)\delta\Psi(y)}\right)}_{\text{Fermionic contribution}}$$

Continuum

$$V_{\mathrm{B}}(p) = C_{\mathrm{F}}\,(2\Delta_\phi + 6\Delta_G)\; M_1$$
$$+ b_1(\Delta_\phi^2, \Delta_G^2, N_{\mathrm{c}})\; V_1 + b_3(\Delta_\phi^2, \Delta_G^2, N_{\mathrm{c}})\; V_3 + b_4(\Delta_\phi^2, \Delta_G^2, N_{\mathrm{c}})\; V_4$$

$$V_{\mathrm{F}}(p) = -8\,C_{\mathrm{F}}\Delta_\phi\; M_1$$
$$+ f_1(\Delta_\phi^2, \Delta_G^2, m, N_{\mathrm{c}})\; V_1 + f_3(\Delta_\phi^2, \Delta_G^2, m, N_{\mathrm{c}})\; V_3 + f_4(\Delta_\phi^2, \Delta_G^2, m, N_{\mathrm{c}})\; V_4$$

Propagators $\Delta_{\mathrm{G}} = \frac{1}{p^2}$ and $\Delta_\phi = \frac{1}{p^2+m^2}$

- Only operators of the tree-level potential appear at 1-loop:

$$M_1\,, \quad V_1\,, \quad V_3\,, \quad V_4$$

- Quadratic divergences cancel as expected
- Quartic couplings b_i and f_i contain logarithmic divergences

Lattice

- Wilson mass and discrete lattice momenta break supersymmetry
- Replace continuum propagators by lattice propagators
- All operators are generated at 1-loop

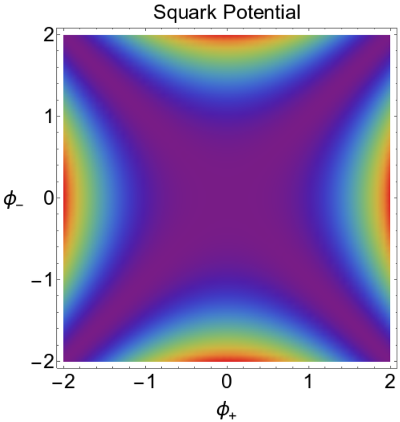
$$V_{\text{eff}}^{\text{continuum}} = \sum_k \lambda_i(k)\,\mathcal{O}_i \quad \text{and} \quad V_{\text{eff}}^{\text{lattice}} = \sum_k \hat{\lambda}_i(k)\,\mathcal{O}_i$$

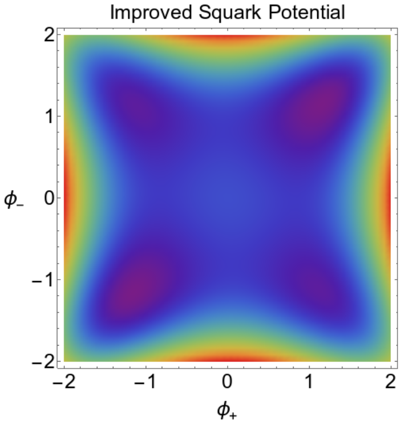
Improved lattice action

$$S_{\text{Improved}}(m,g) = S(m,g|m_{\text{q}},m_{\text{g}},Z_\phi) + V_{\text{counter}}(m)$$

$$V_{\text{counter}}(m) = \sum_{i=1}^{7} \Delta\lambda_i(m,V,N_{\text{c}})\,\mathcal{O}_i \quad \text{with} \quad \Delta\lambda_i = \sum_k (\lambda_i(k) - \hat{\lambda}_i(k))$$

L	M_1	M_2	V_1	V_2	V_3	V_4	V_5
8	-1.3793	-0.63618	0.30573	-0.05689	0.25293	0.21088	0.07710
16	-1.3785	-0.63665	0.31012	-0.05722	0.25888	0.22181	0.08316
32	-1.3784	-0.63666	0.31081	-0.05723	0.25983	0.22434	0.08436
64	-1.3784	-0.63666	0.31095	-0.05723	0.26003	0.22495	0.08464
128	-1.3784	-0.63666	0.31099	-0.05723	0.26008	0.22510	0.08471
256	-1.3784	-0.63666	0.31100	-0.05723	0.26009	0.22514	0.08473

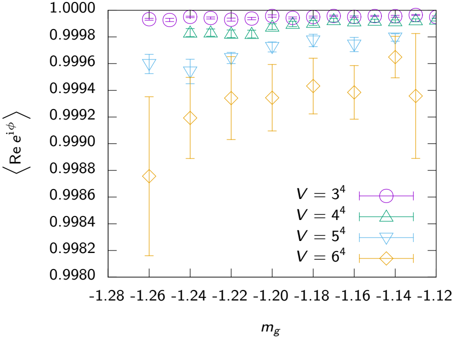


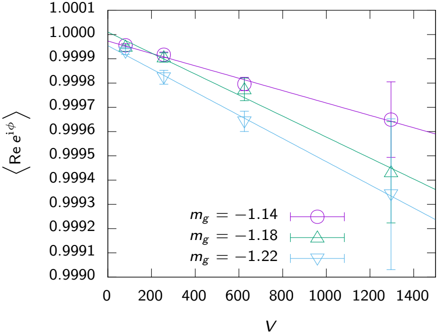


(Preliminary) Lattice results

Lattice setup and phase of the Pfaffian

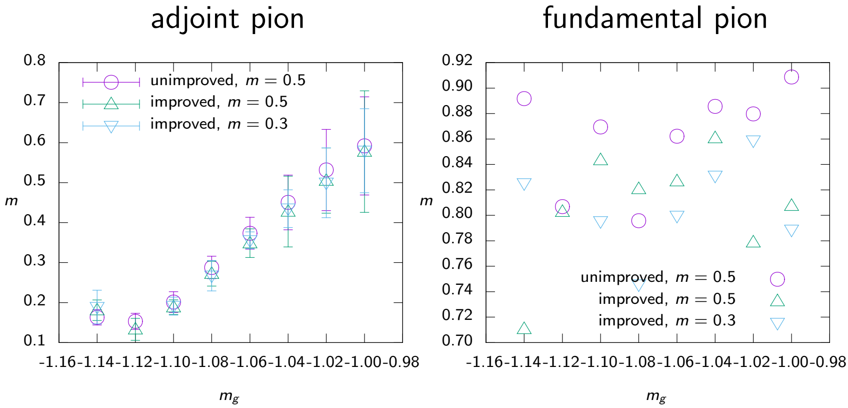
- Wilson gauge action with $\beta = 6.3 \ldots 9.9$, $m = 0.5$, $m_q = -0.4$ and volume $V = 8^3 \times 16$
- Wilson fermions / rational-HMC algorithm





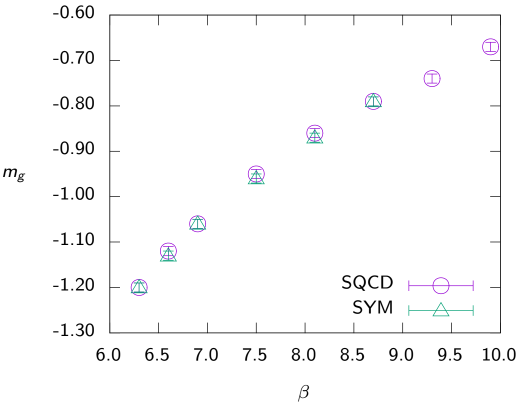
- Extrapolation: $0.996 \ldots 0.998$ @ $8^3 \times 16$ and $0.94 \ldots 0.97$ @ $16^3 \times 32$

Fine-tuning of m_g



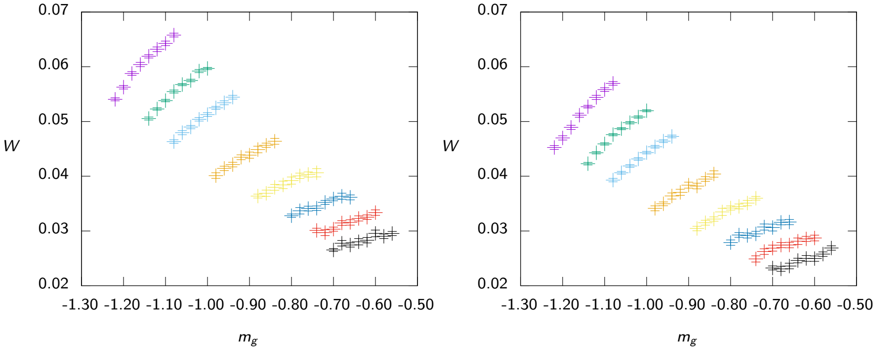
- we use the adjoint pion mass to fine-tune m_g
- fundamental pion mass almost independent of gluino mass and squark mass

SQCD compared to SYM



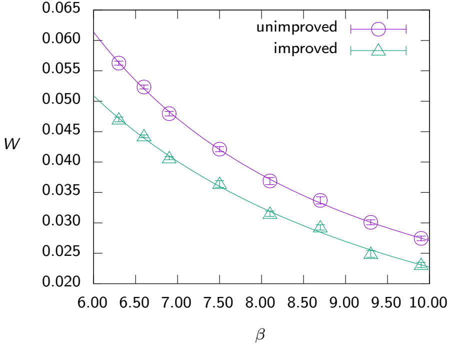
- massless gluino critical lines of SQCD and SYM coincide
- almost independent of squark /quark mass and scalar potential

Unimproved vs improved Ward identity



$$\frac{2}{3}\langle S_W\rangle + \left\langle D_\mu\phi^\dagger D^\mu\phi + m^2\phi^\dagger\phi\right\rangle = N_c^2 - 1 + 2N_cN_f = 14$$

Unimproved vs improved Ward identity



- Fit to $W(\beta) = W_\infty + a\beta^b$

$$W_\infty^{\text{unimproved}} = 0.008(3) \quad \text{and} \quad W_\infty^{\text{improved}} = -0.003(11)$$

Conclusions and Outlook

- No severe sign problem in lattice simulations
- One-loop improved lattice action
- Preliminary lattice results for Ward identities, the chiral critical line, fundamental / adjoint pion mass
- Fine-tuning of the Yukawa interaction
- Gluino-glue and Quark-Squark bound states

Literature

M. J. Strassler: *QCD, supersymmetric QCD, lattice QCD and string theory: Synthesis on the horizon?*
Ian Affleck, Michael Dine, and Nathan Seiberg: *Dynamical Supersymmetry Breaking in Supersymmetric QCD*
Joel Giedt: *Progress in four-dimensional lattice supersymmetry*
M. Costa and H. Panagopoulos: *Supersymmetric QCD on the Lattice: An Exploratory Study*

- Coupling between Dirac- and Majorana fermions
- Rewrite fermion Lagrange function

$$\mathcal{L}_\mathrm{f} = \frac{1}{2}\bar{\Psi}\begin{pmatrix} \not{D} & \mathrm{i}\,Y \\ -\mathrm{i}\,\bar{Y}^\dagger & \not{D}_\xi - m \end{pmatrix}\Psi = \frac{1}{2}\Psi^\mathsf{T}\underbrace{\begin{pmatrix} C\not{D} & \mathrm{i}\,CY \\ -\mathrm{i}\,(CY)^\mathsf{T} & C(\not{D}_\xi - m) \end{pmatrix}}_{M}\Psi$$

$$Y = \hat{\phi}^\dagger P e^\mathsf{T} - \hat{\phi}^\mathsf{T}\bar{P}e^\dagger\,, \quad \bar{Y}^\dagger = e^*\bar{P}\hat{\phi} - eP\hat{\phi}^*$$

- Majorana spinor Ψ and $M = -M^\mathsf{T}$ $\Rightarrow$ Pfaffian

Bosonic effective action

$$S_\mathrm{eff} = S_\mathrm{Gauge} + S_\mathrm{Squark} - \mathrm{tr}\ln\left(\not{D}_\psi - m\right) - \frac{1}{2}\mathrm{tr}\ln\not{D}_\lambda - \frac{1}{2}\mathrm{tr}\ln\left(\mathbb{1} - \Delta_\lambda Y \Delta_\xi \bar{Y}^\dagger\right)$$