Phase structure of three flavor QCD in external magnetic fields using HISQ

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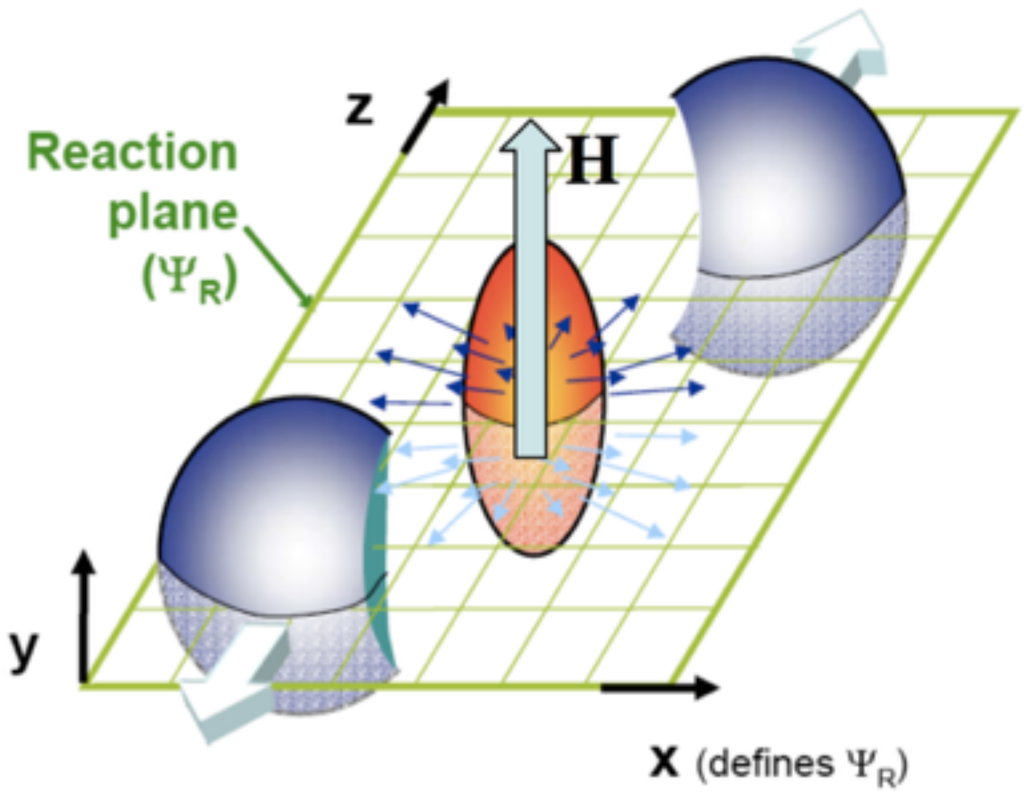
Our purpose 1/4

Relativistic Heavy Ion Collisions = QCD + External magnetic field

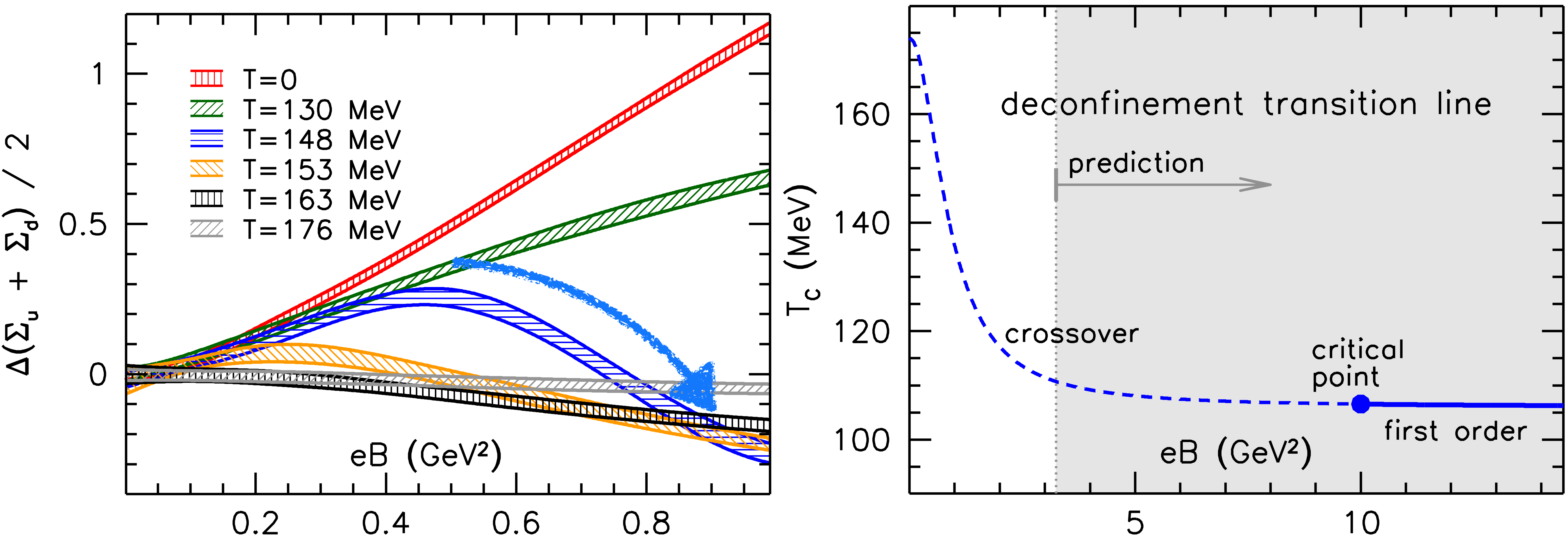


Non-central heavy-ion collisions produce huge magnetic field,
$eB \sim 10^{18}$ Gauss ($\sqrt{eB} \sim 442$ MeV)
(c.f. $eB^{lab} = 4.5\times10^{5}$ Gauss)
→ It might affect to QCD phase structure

Kharzeev, Dmitri E. Prog.Part.Nucl.Phys. 75 (2014) 133-151 arXiv:1312.3348 [hep-ph]

Our purpose 2/4

Why m_π = 320 & 80 MeV with HISQ in external magnetic fields?



Stout staggered fermions at the physical point

- **Inverse** magnetic catalysis at the critical temperature
- Tc decreases along with external field eB
- New critical endpoint for large eB is suggested by an effective model

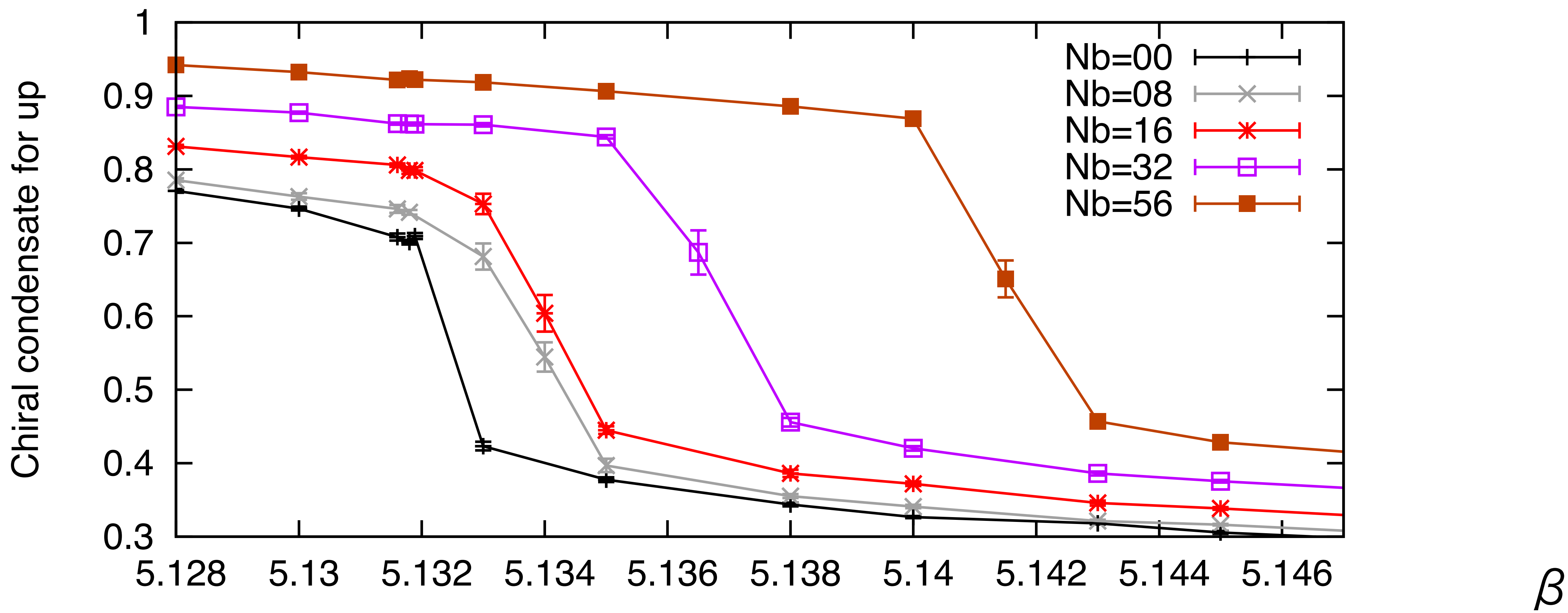
G. S. Bali *et al* 1206.4205

G. Endrodi 1504.08280

Our purpose 3/4

Why m_π = 320 & 80 MeV with HISQ in external magnetic fields?

Unimproved staggered fermions Nf =3 A.T. *et al* 1711.02884

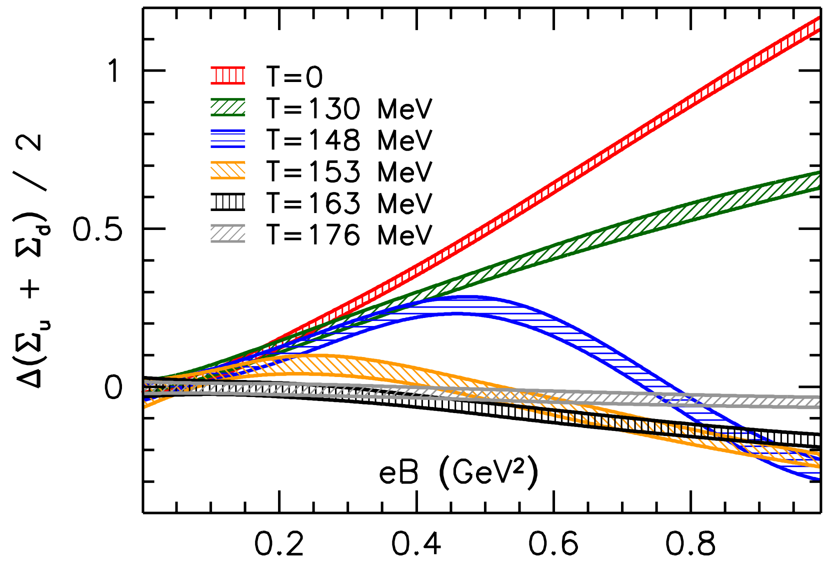


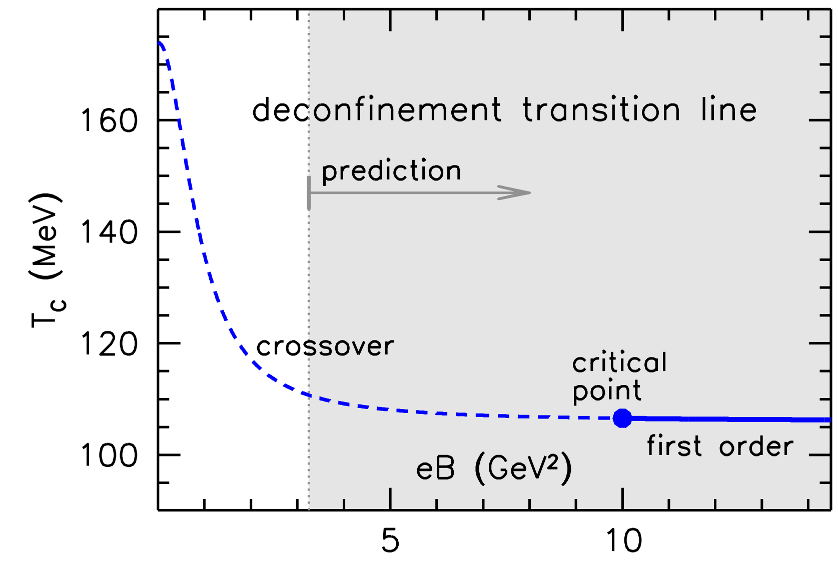
In our previous work, we found for m_π~300 MeV,

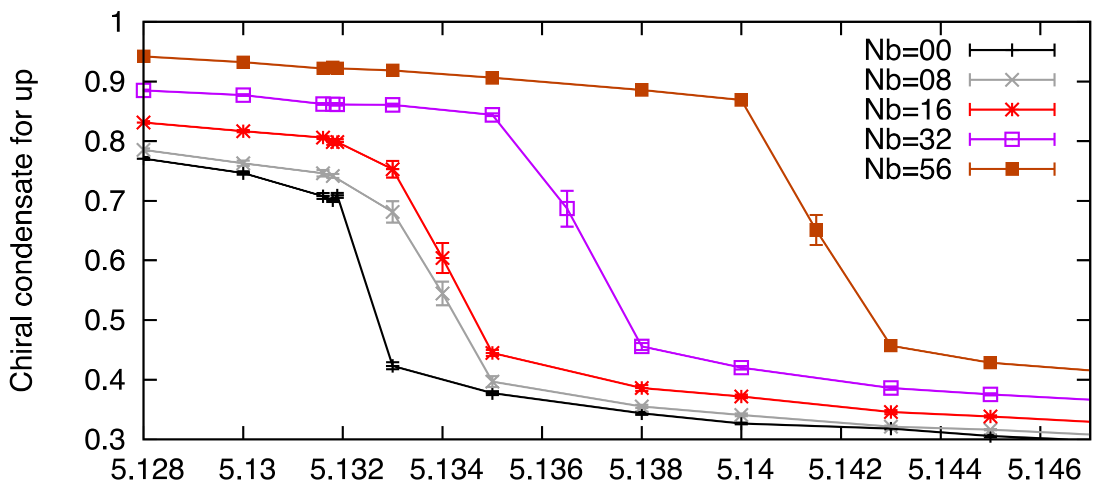
- **Normal** magnetic catalysis <u>for whole temperature</u>
- Tc increases along with external field eB (∝ Nb)
- The Binder cumulant indicates 1st order phase transition

Our purpose 4/4

Why m_π = 320 & 80 MeV with HISQ in external magnetic fields?





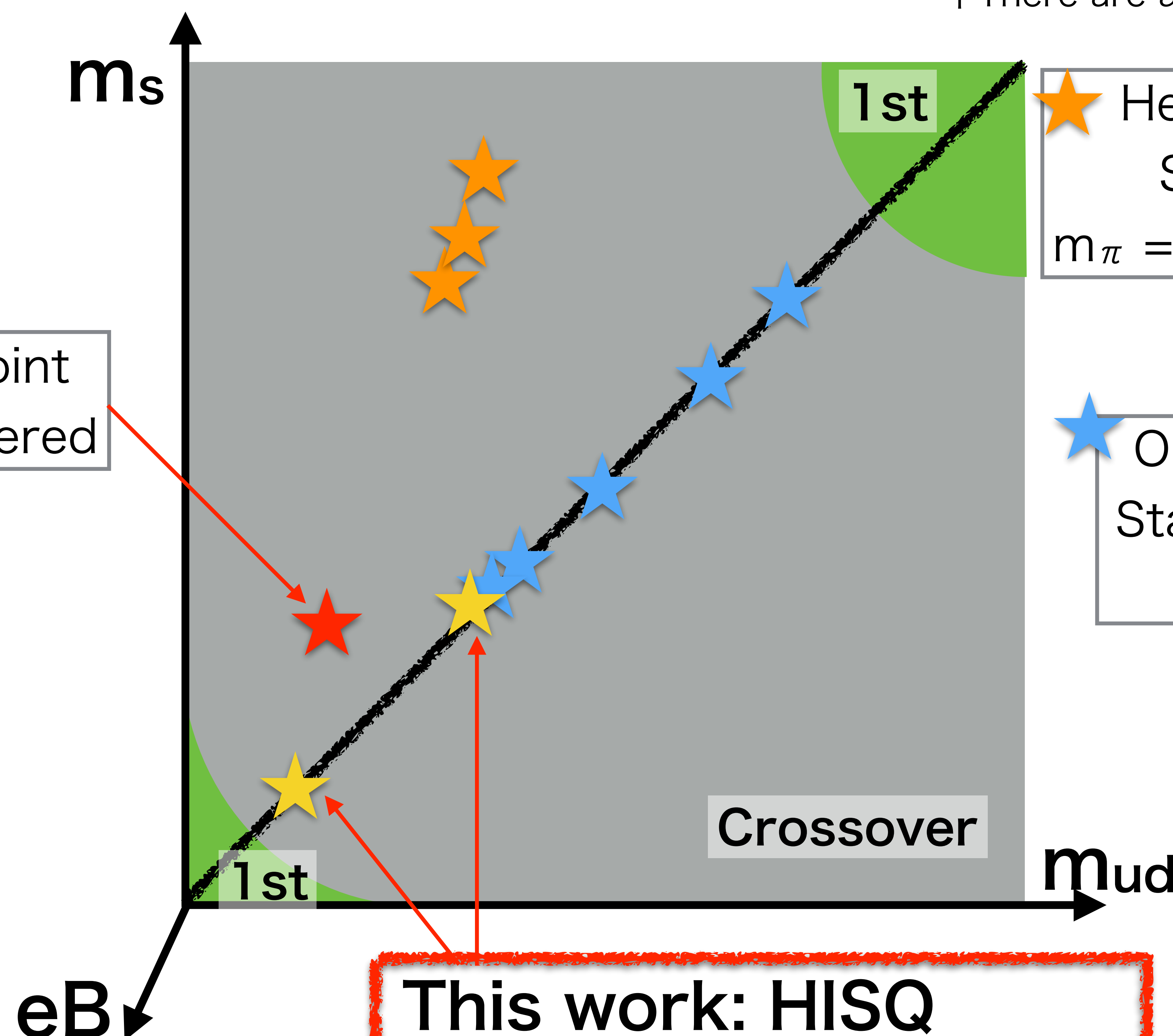


We employ **Highly Improved Staggered Quarks(HISQ):**
- To see cutoff effect and mass dependence on the catalysis
- To see Tc(eB) behavior. Does it go up or down?
- To find first order at large external field eB

Our target regime

Works with external magnetic field

†There are also Wilson & overlap works



Heavy mass regime
Stout staggered
m_π = 340, 440, 664 MeV

Physical point
Stout staggered

Our previous work
Standard staggered
m_π > 300 MeV

This work: HISQ
m_π = 80 & 320 MeV

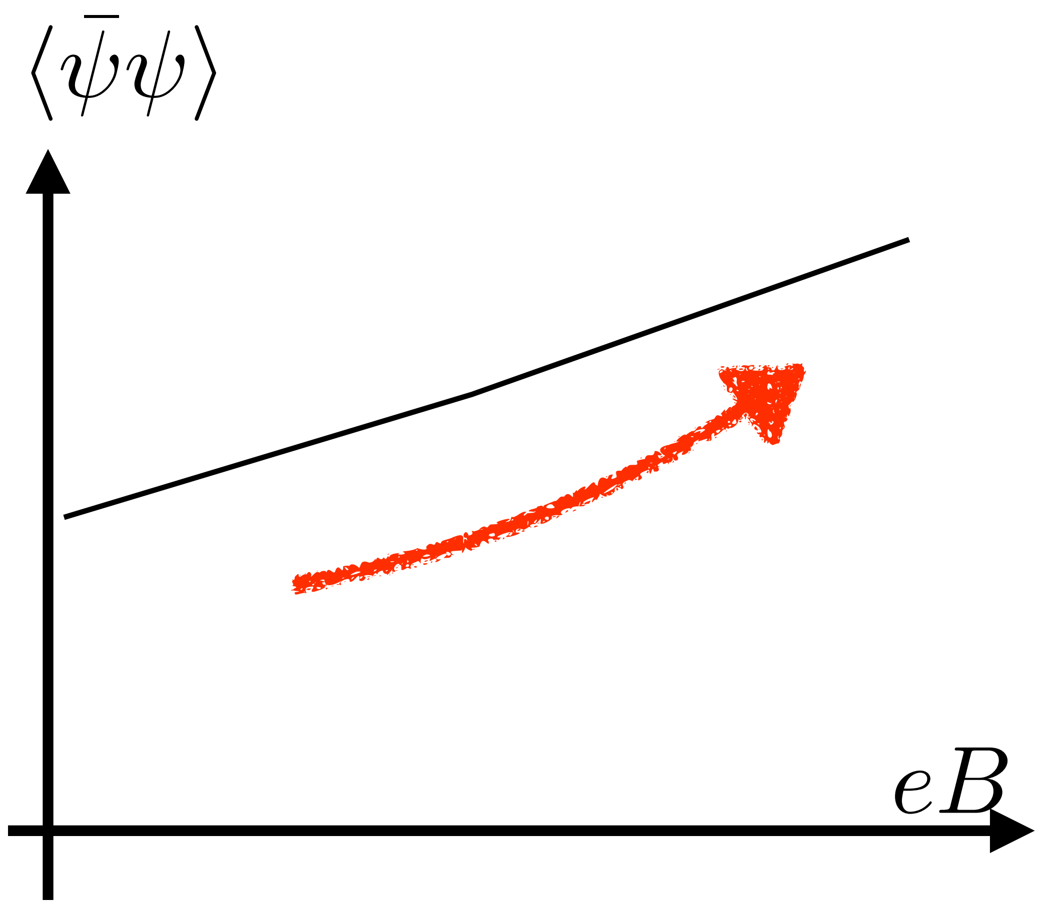
F.R. Brown, et al PRL65, 2491 (1990)

A.T. *et al* 1711.02884
G. Endrodi 1504.08280
M. D'Elia *et al* XQCD2018
and references therein

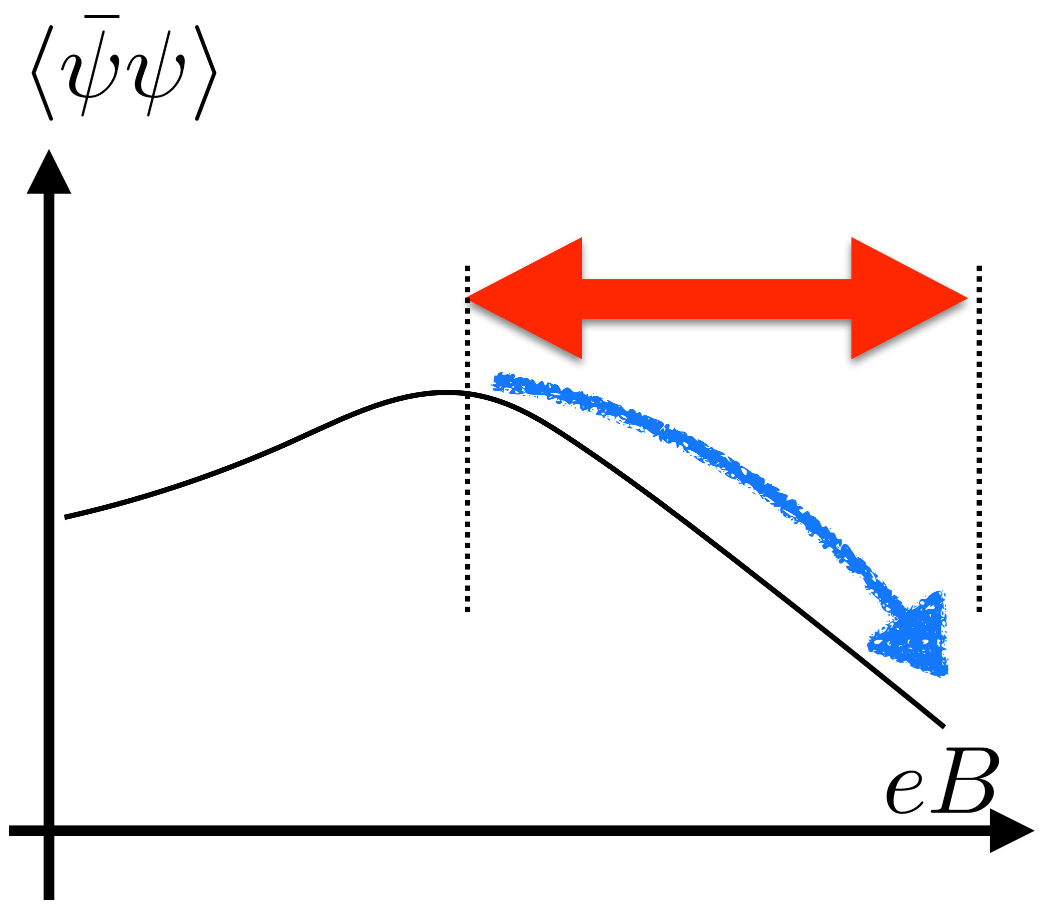
Terminology

External magnetic field (inversely) catalyzes the chiral condensate

(Normal)
Magnetic Catalysis



Inverse
Magnetic Catalysis



Our setup

m_π = 320 & 80 MeV with HISQ

Previous

3 degenerate flavors
Unimproved staggered quarks with RHMC
m_π > 300 MeV

This work

3 degenerate flavors with **HISQ** (RHMC)
m_π = 320 MeV (~ lightest in unimproved case)
& 80 MeV (near to SU(3) chiral limit)

The Dirac spectrum and topological susceptibility using the stochastic estimator.

[L. Giusti et al, 2009. G. Cossu et al 2016. P. d. Forcrand et al 2018.]

Our setup

m_π = 320 & 80 MeV with HISQ

N_f = 3 QCD with **HISQ** (m_π = 320 and 80 MeV)
Tree level Symanzik gauge action
$N_\sigma^3 x N_\tau$ = 16^3 x6 (a ~ 0.27 fm)
The number of configurations ~ O(1000)
Fermilab and CCNU GPU machines

$$N_b \propto a^2 eB$$: # of magnetic flux

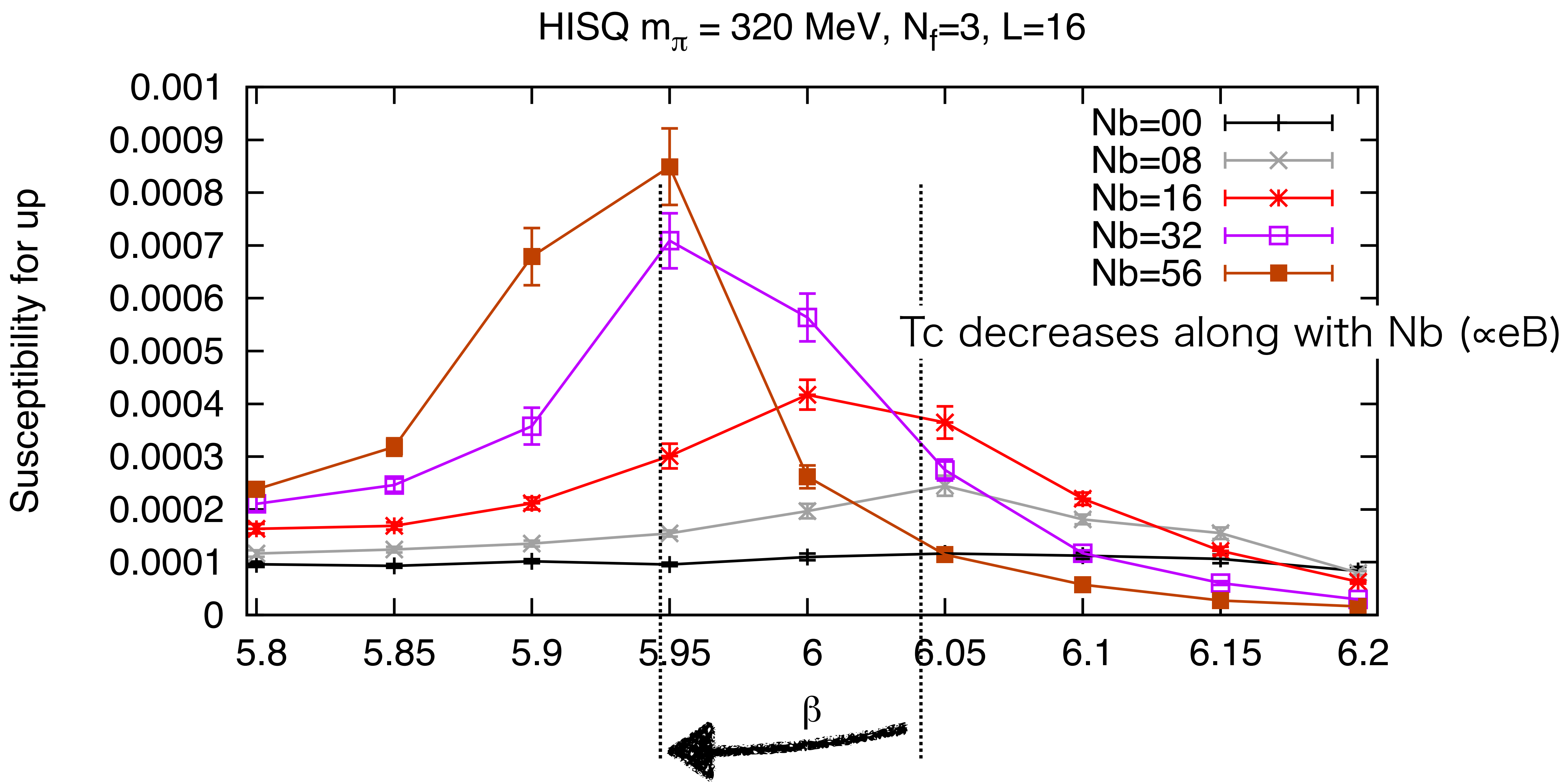
Nb(Nx=16)	0	8	16	32	56
√eB (MeV)	0	330	460	660	870

m	320 MeV	80 MeV	80 MeV
Nx=Ny	16	16	24
$m_\pi L$	7	1.7	2.6 (Not showed)

m_{π} = 320 MeV

Results with m_π = 320 MeV

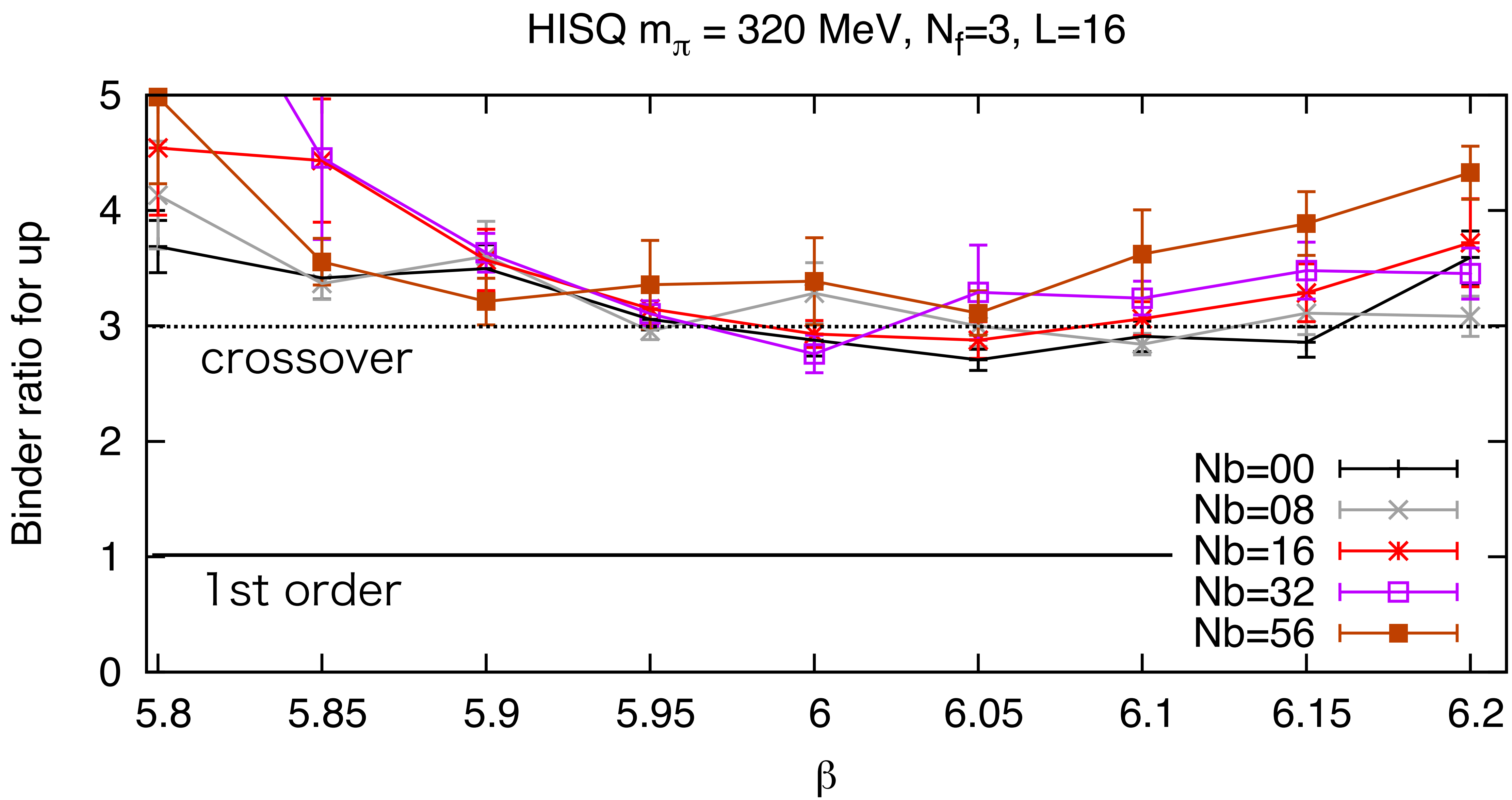
Susceptibility for heavier regime(~ lightest mass in unimproved staggered)



Consistent with results with physical point

Results with m_π = 320 MeV

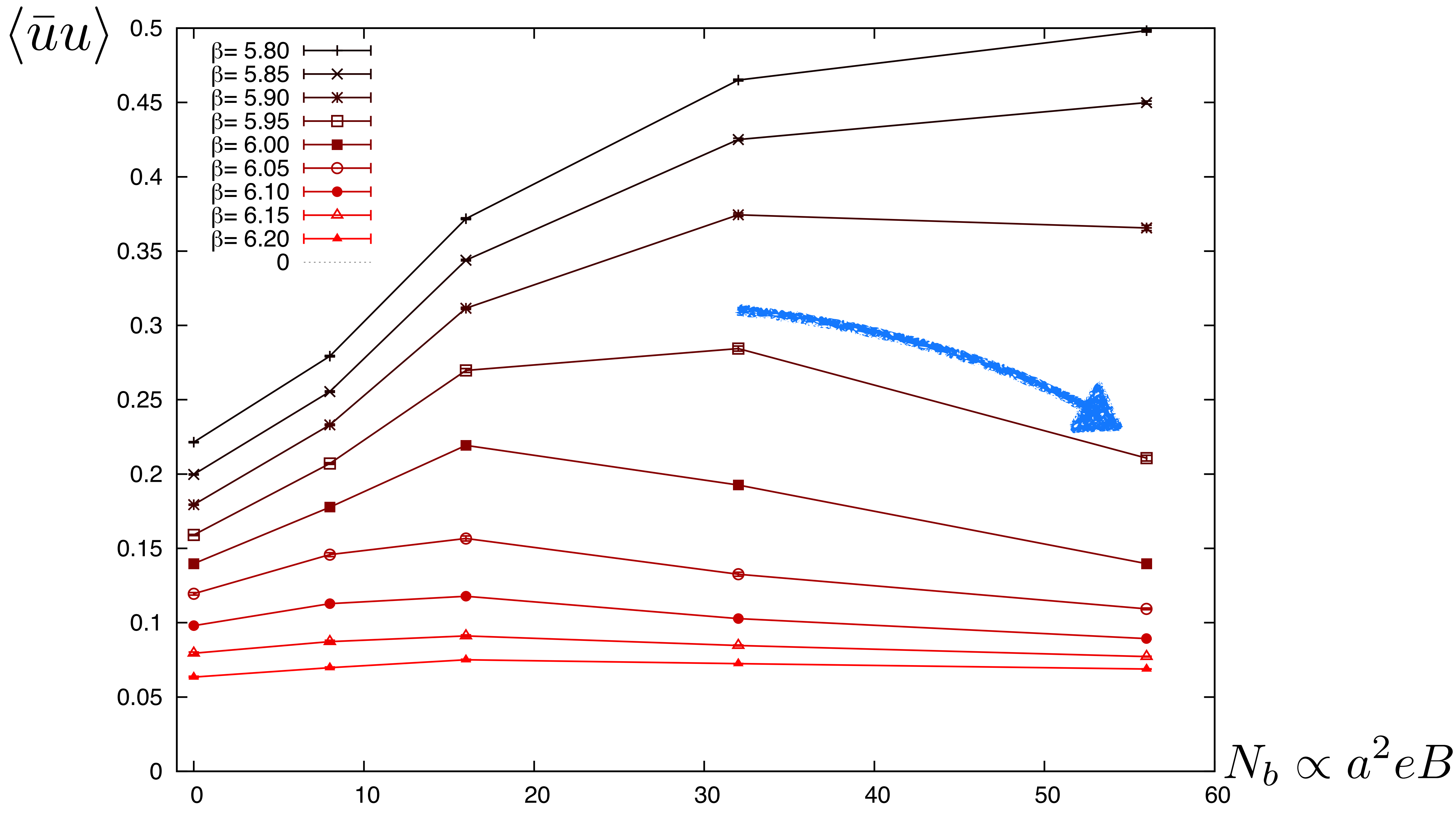
Binder cumulant for Heavier regime(~ lightest mass in unimproved staggered)



No indication of 1st order, different from unimproved staggered results

Results with m_π = 320 MeV

Chiral condensate for Heavier regime(~ lightest mass in unimproved staggered)



m_π = 320 MeV data near Tc shows inverse magnetic catalysis
-> Unimproved staggered results are probably artifacts

Results with m_π = 320 MeV

Decomposition of relative increase of the chiral condensate

Relative increase of the chiral condensate

$$r(B) = \frac{\langle\bar{\psi}\psi\rangle(B) - \langle\bar{\psi}\psi\rangle(0)}{\langle\bar{\psi}\psi\rangle(0)}$$

$$r^{\rm full}(B) = \underbrace{r^{\rm sea}(B)} + \underbrace{r^{\rm val}(B)} + O(B^4)$$

eB only in **sea** quarks

eB only in **valence** quarks

We can see sea and valence effects on normal/inverse magnetic catalysis

[M. D'Elia *et al* 1103.2080]

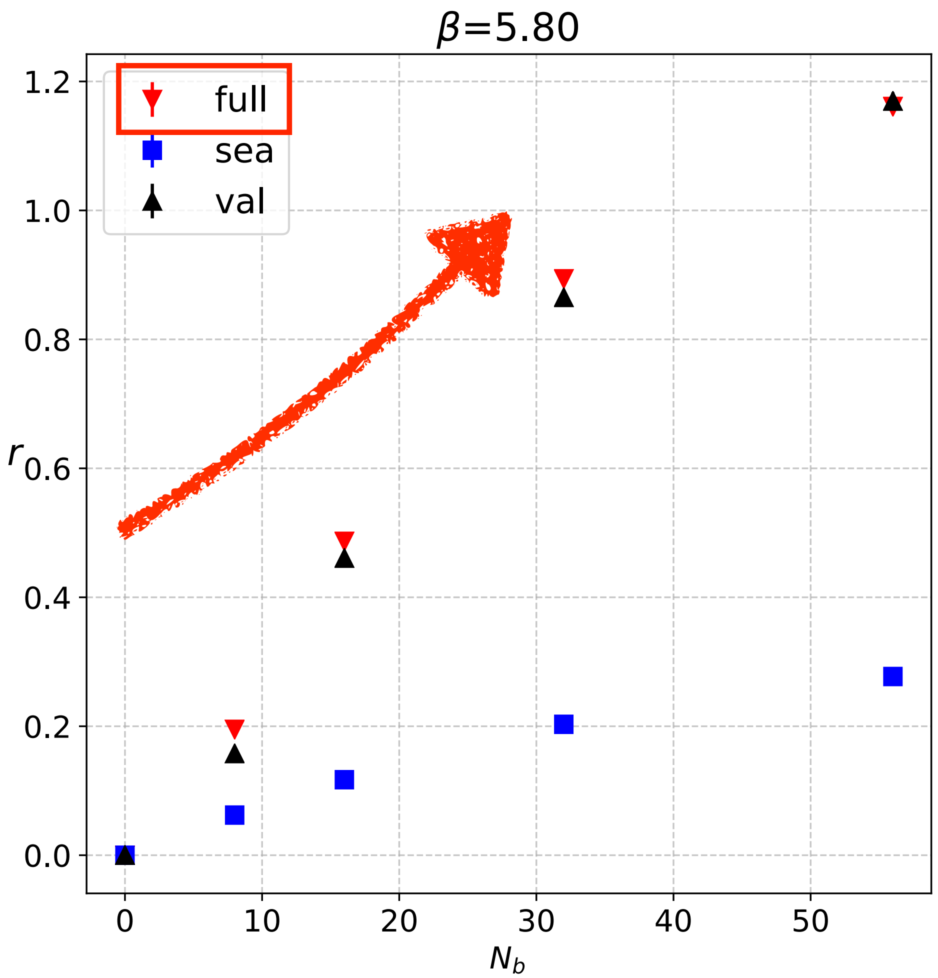
A. Tomiya

Results with m_π = 320 MeV

Heavier regime: Consistent with previous results at physical points

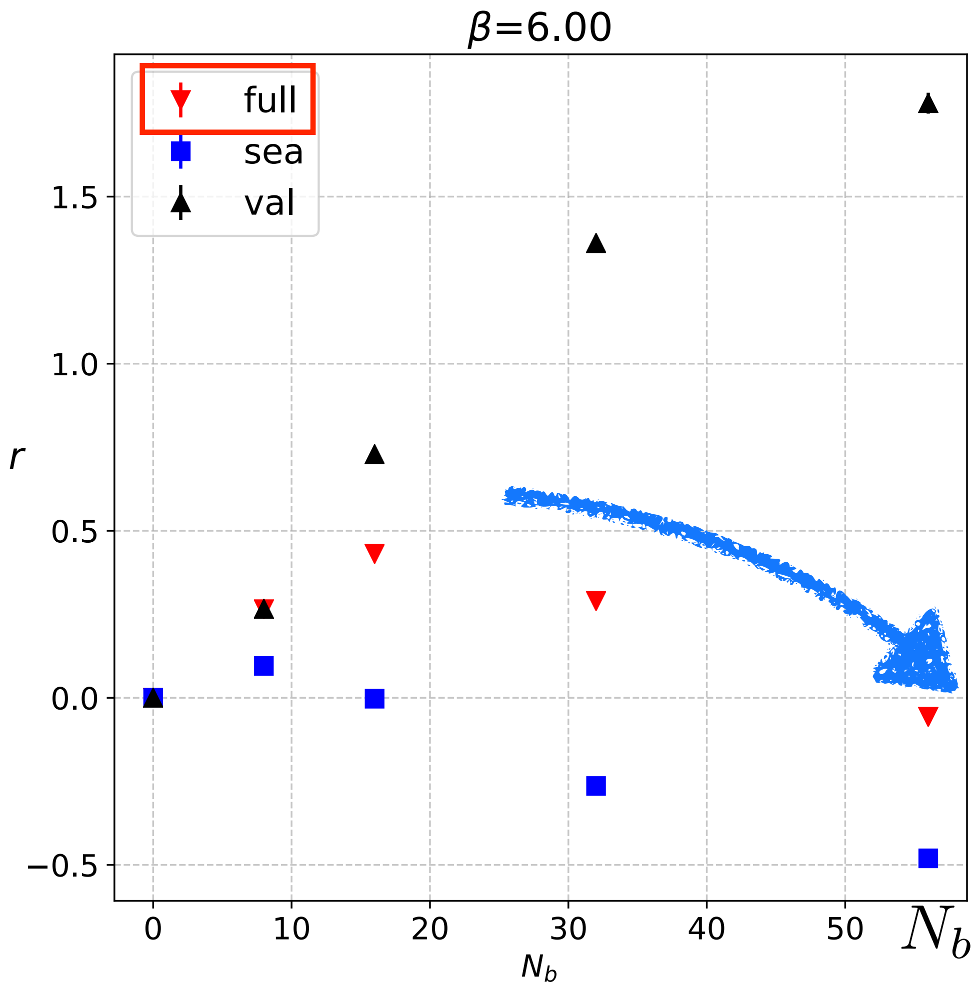
$$r(B) = \frac{\langle\bar{\psi}\psi\rangle(B) - \langle\bar{\psi}\psi\rangle(0)}{\langle\bar{\psi}\psi\rangle(0)}$$

T < Tc



(Normal) Magnetic Catalysis

T > Tc



$N_b \propto a^2 eB$

Inverse Magnetic Catalysis

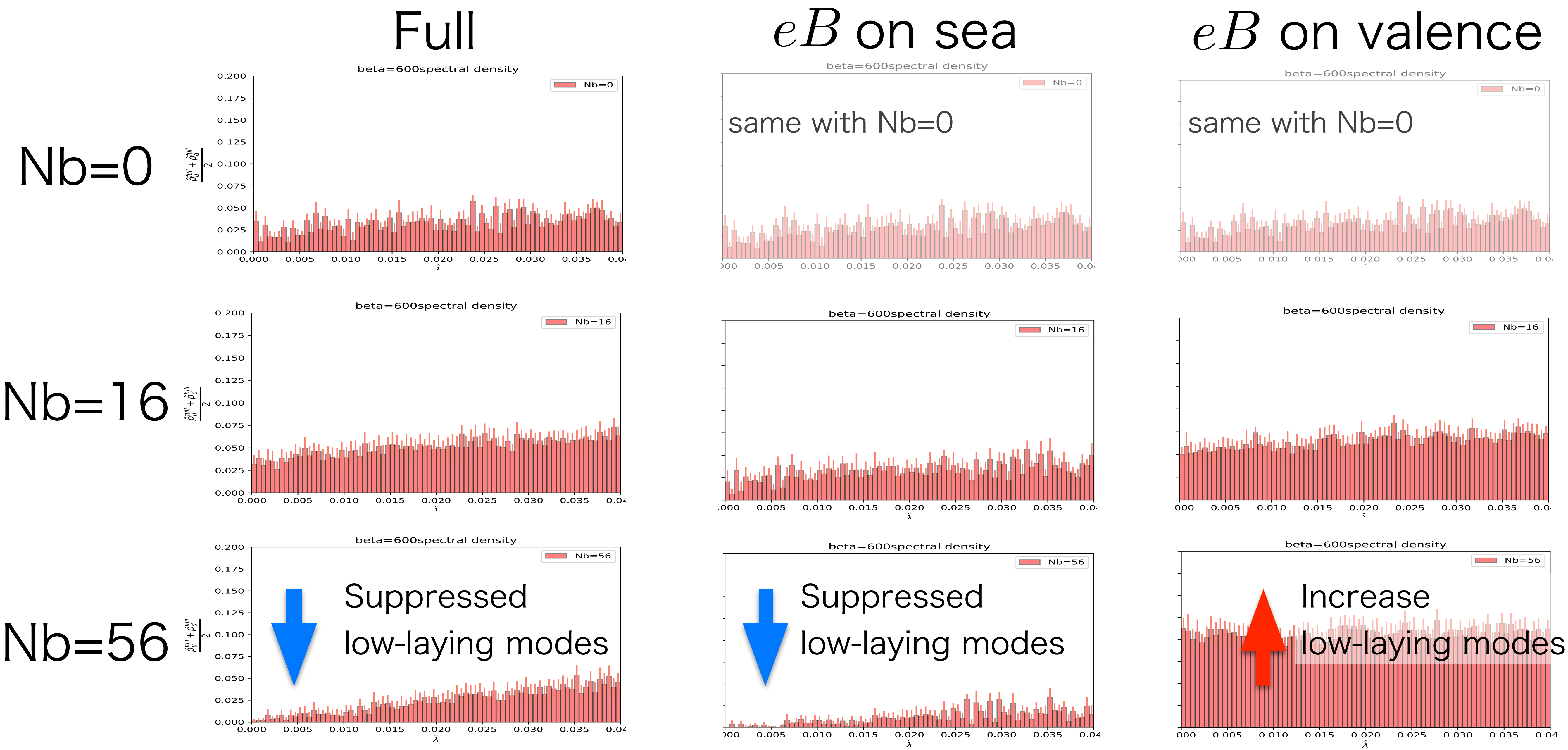
Sea quarks drive inverse catalysis
Similar to previous results at the physical point

Using stochastic estimator [L. Giusti et al, 2009. G. Cossu et al 2016. P. d. Forcrand et al 2018.]

Results with m_π = 320 MeV

Dirac spectrum: sea quarks drive inverse catalysis

$\beta = 6 (T \gtrsim T_c)$

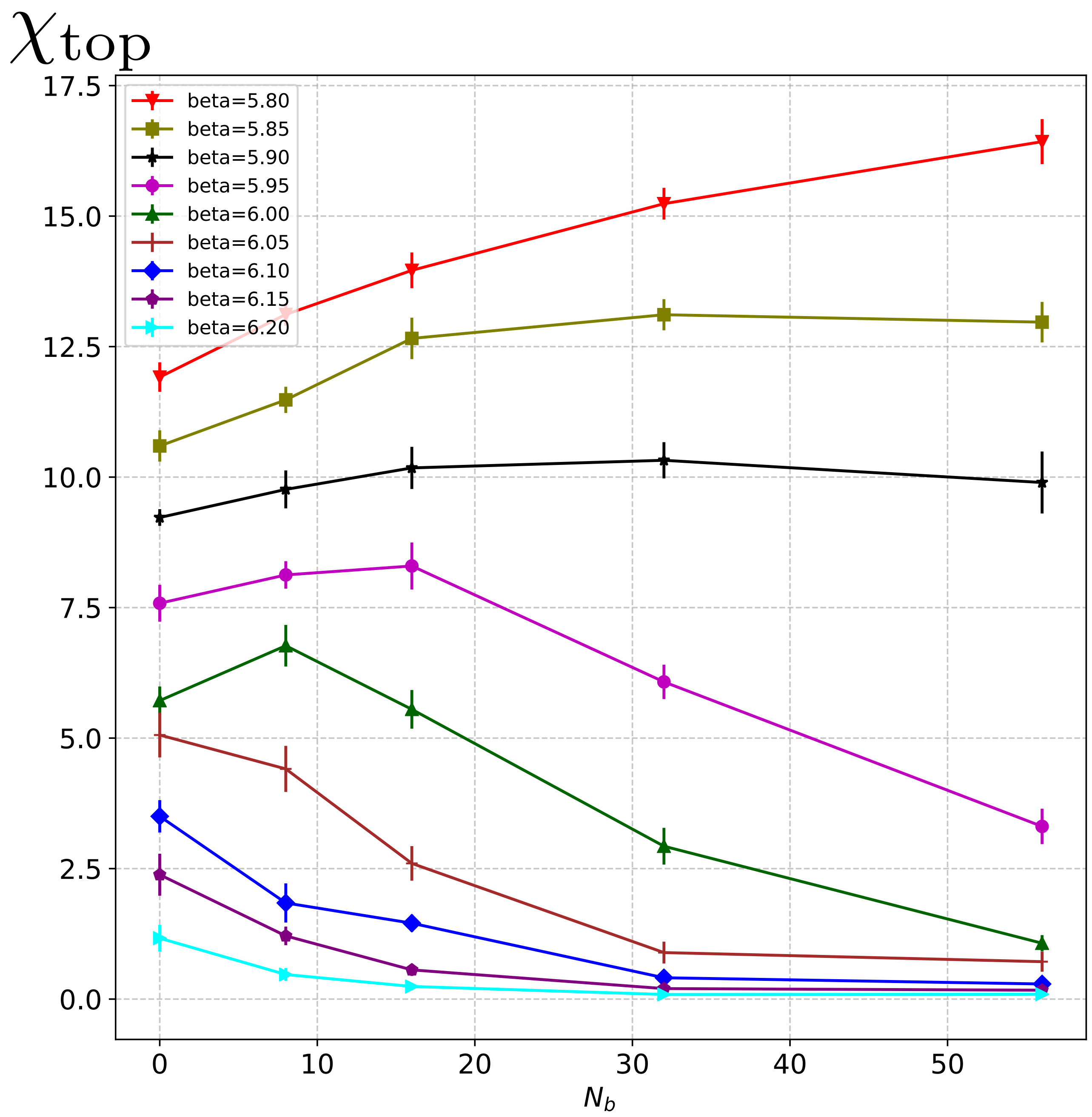


Valence quarks only contribute to normal catalysis

Using stochastic estimator [L. Giusti et al, 2009. G. Cossu et al 2016. P. d. Forcrand et al 2018.]

Results with m_π = 320 MeV

Topological susceptibility shows similar tendency to the chiral condensate



$$\langle\bar\psi\psi\rangle = \int_0^\infty d\lambda \frac{2m\rho(\lambda)}{\lambda^2+m^2}.$$

$$\chi_{\rm top} = m^2 \int_0^\infty d\lambda \frac{4m^2\rho(\lambda)}{(\lambda^2+m^2)^2}.$$

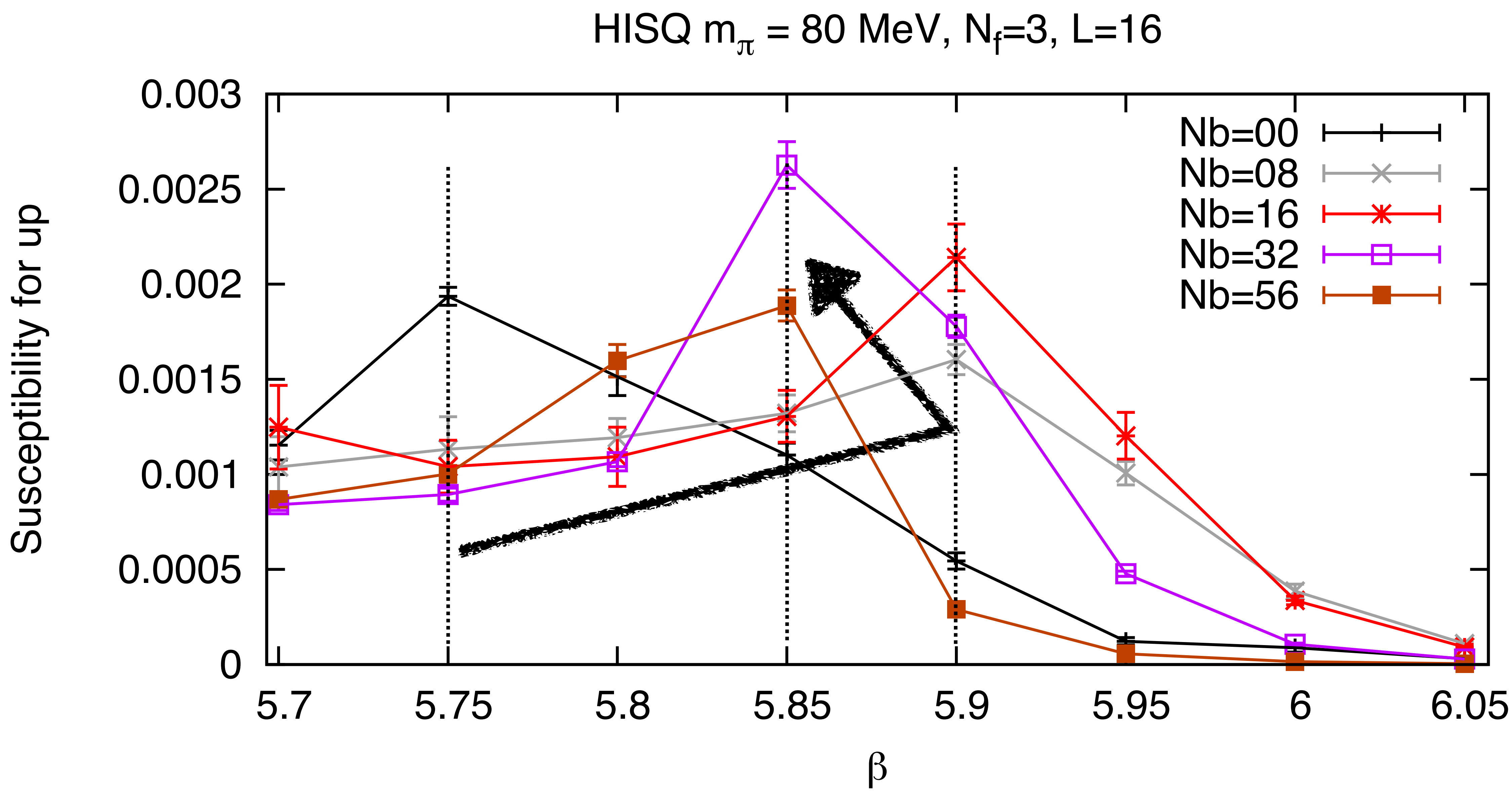
Similar tendency to the chiral condensate

Using stochastic estimator [L. Giusti et al, 2009. G. Cossu et al 2016. P. d. Forcrand et al 2018.]

$$m_\pi = 80 \text{ MeV}$$

Results with m_π = 80 MeV

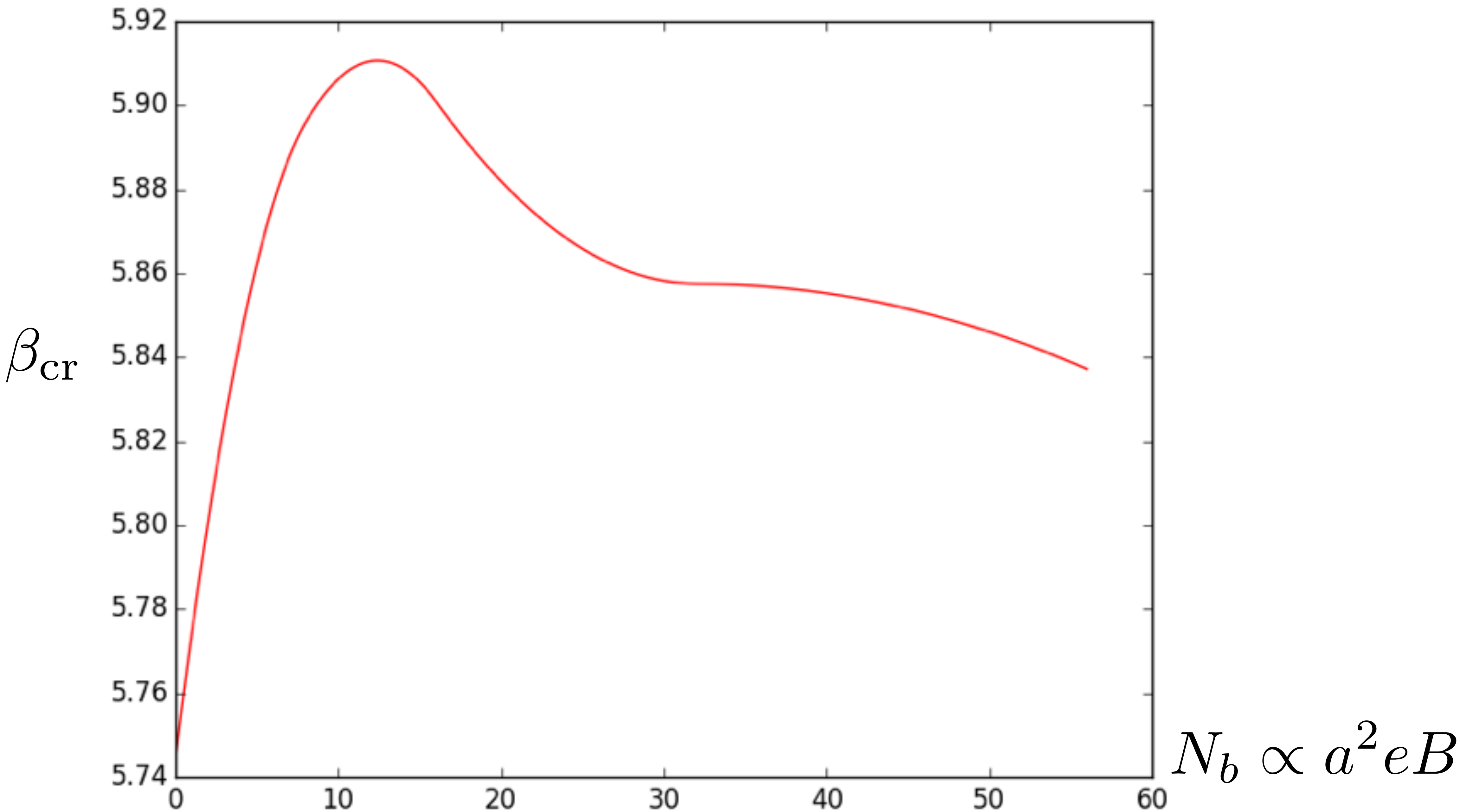
Susceptibility in light pion mass regime (near the SU(3) chiral limit)



Tc goes up and down along with Nb??

Results with m_π = 80 MeV

Susceptibility in light pion mass regime (near the SU(3) chiral limit)

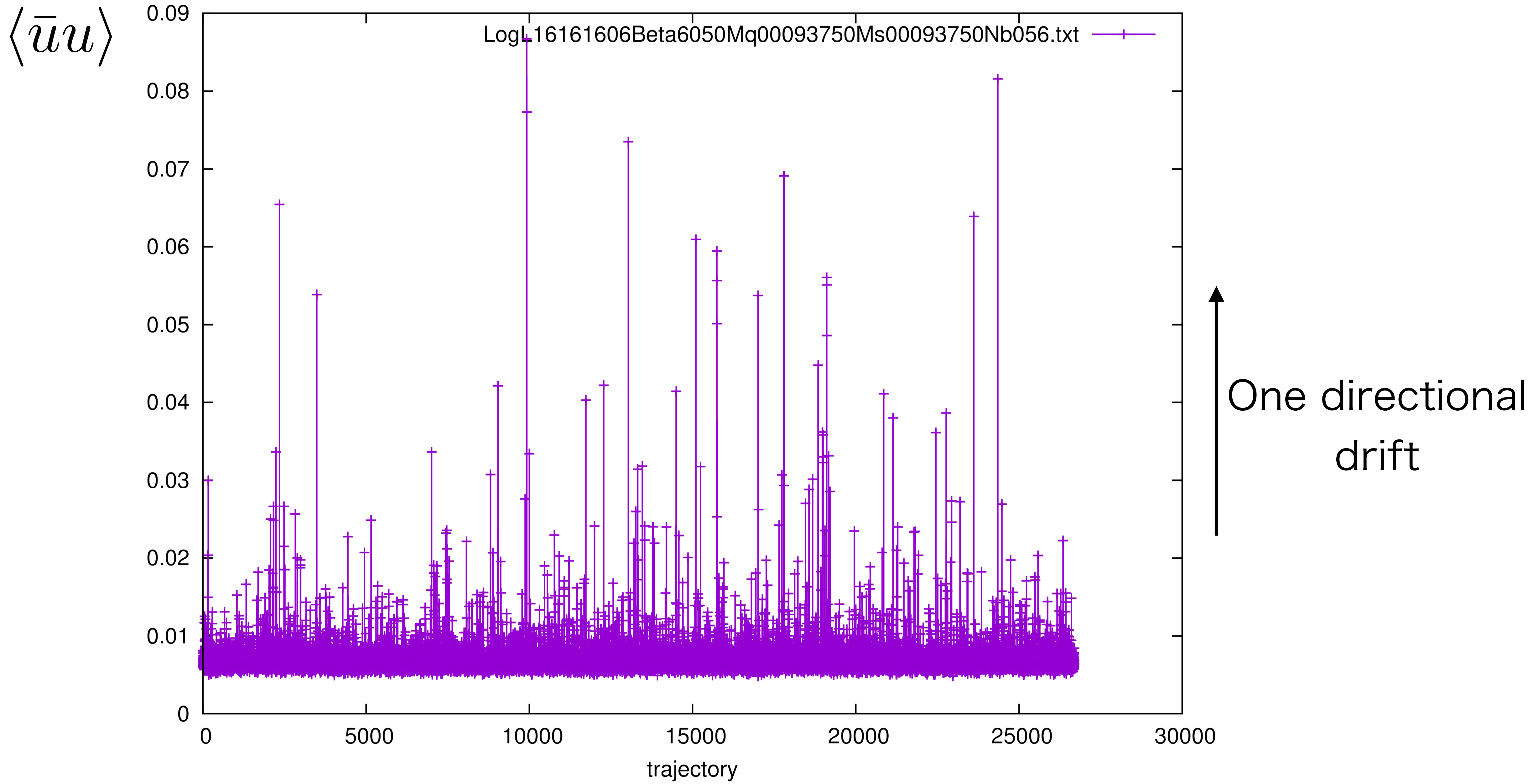


Tc goes up and down along with Nb??

-> Mpi = 80 MeV with Nx=16 is affected by finite size effects.

Results with m_π = 80 MeV

Lighter regime for Nx = 16 is affected by finite volume effects



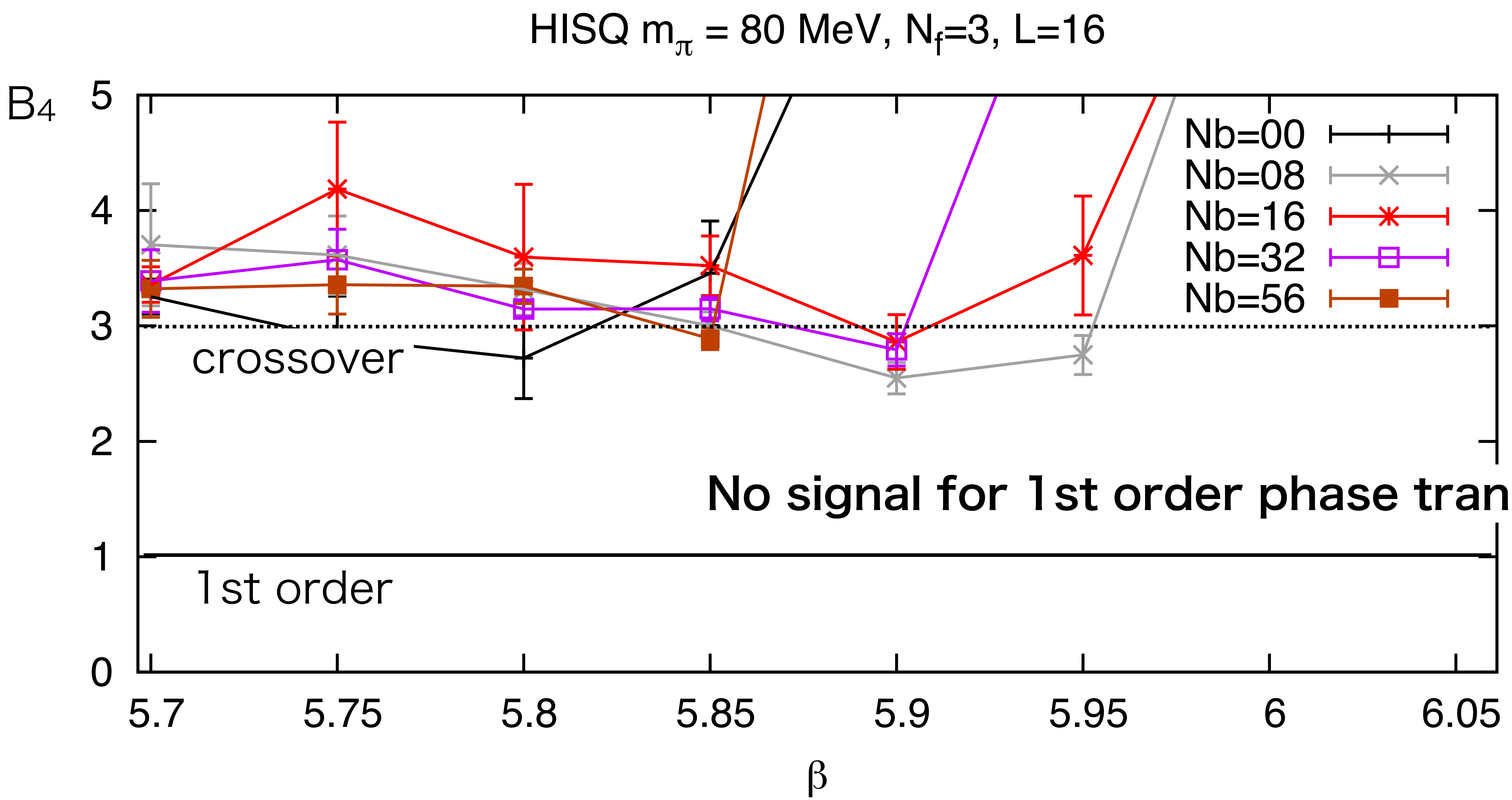
This mass is strongly affected by finite volume effects:

$M_\pi L = 1.7 < 4$

Especially, high temperature and large Nb is serious.

Results with m_π = 80 MeV

Binder cumulant in light pion mass regime (near the SU(3) chiral limit)



No signal for 1st order phase transition.

Summary: QCD phase trs. with magnetic field

A. Tomiya

No 1st order is found for m_π = 320 and 80 MeV ($\sqrt{eB}$ < 870 MeV)

Summary of HISQ with magnetic field

1. m_π = 320 MeV shows crossover, Tc(eB) decreases.
    1. 1st order for standard staggered fermions are probably an artifact
    2. Dirac spectrum for sea part are consistent with inverse magnetic catalysis
    3. Qualitatively similar to the physical point results by stout staggered fermions
2. m_π = 80 MeV shows crossover ($\sqrt{eB}$ < 870 MeV)
    1. Tc goes up and down along with Nb, but Nx=16 lattice affected by finite volume effect, especially high temperature and large Nb. (highly suspected)
    2. Except for Tc(eB) behavior, m_π = 80 MeV system shows qualitatively same behavior to 320 MeV (not showed in this talk)
    3. No indication of the first order
3. In both cases, topological susceptivity behave similar to the chiral condensate. (m_π = 80 MeV is not showed)

Outlook

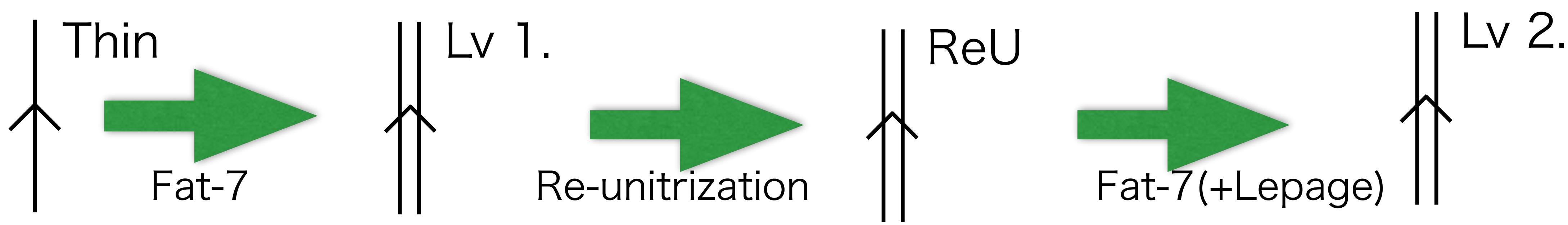
1. To improve the analyses for m_π = 80 MeV, simulations with larger volumes are needed. Analyses on Nx=24 is ongoing.

Backup

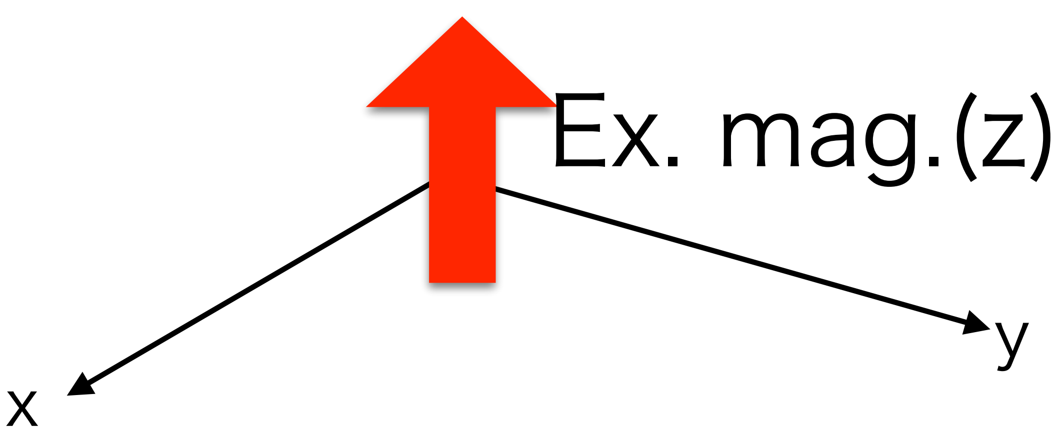
A. Tomiya

HISQ with the magnetic field

HISQ = Highly improved staggered quarks



DHISQ[U] = DKS[Lv 2.] + DNaik[ReU]



$$u_x(n_x, n_y, n_z, n_t) = \begin{cases} \exp[-iqBN_x n_y] & (n_x = N_x - 1) \\ 1 & \text{(Otherwise)} \end{cases}$$

$$u_y(n_x, n_y, n_z, n_t) = \exp[iqBn_x],$$

Multiplying u(1) links **after** the smearing

DKS[u Lv 2.] + DNaik[u ReU]

Topological susceptibility

Previous works by HISQ

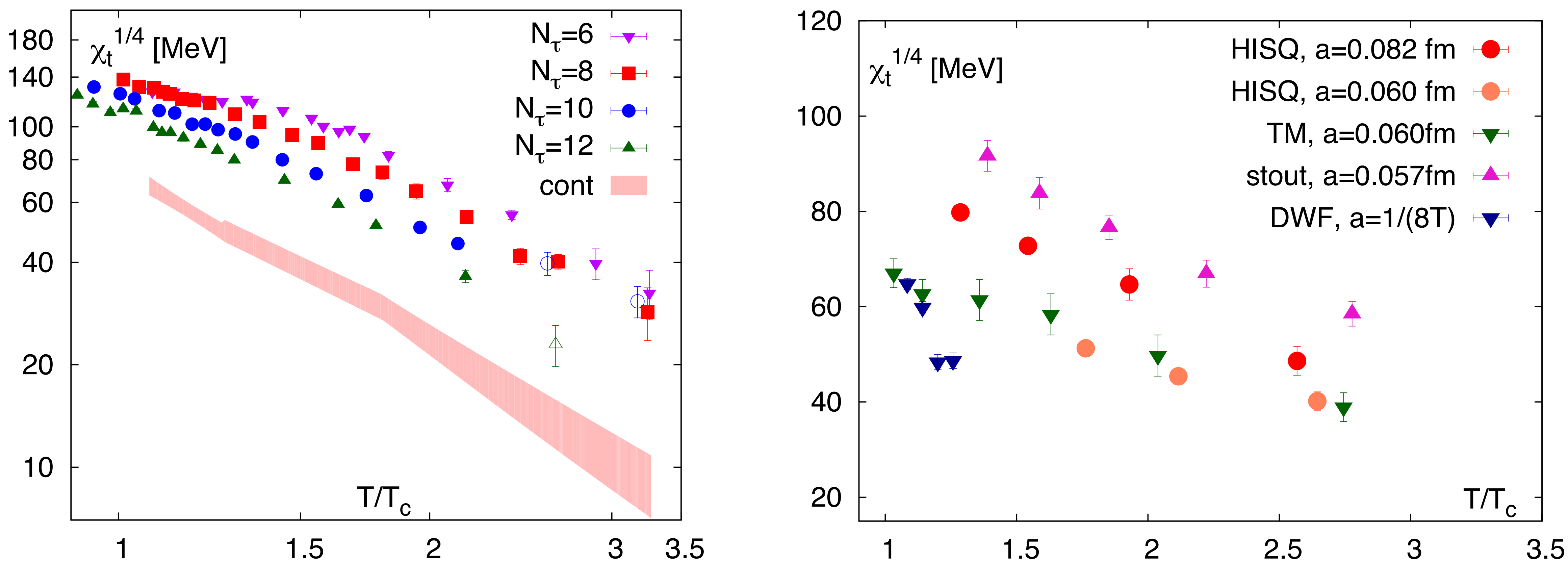


Figure 3: The temperature dependence of χ_t in QCD for HISQ action on lattices with different N_τ (left) and χ_t for HISQ action at two lattice spacings compared with recent results using different fermion actions [19, 36, 49] (right). In the left panel, we also show the continuum result for χ_t disucssed in section 4 and open symbols represent the data points that have not been used in the continuum extrapolation.

Peter Petreczky, Hans-Peter Schadler and Sayantan Sharma

1606.03145

Comparison to conventional method

The projector method gives consistent results to the chiral condensate

We have used stochastic estimator for the Dirac spectrum and calculate the chiral condensate

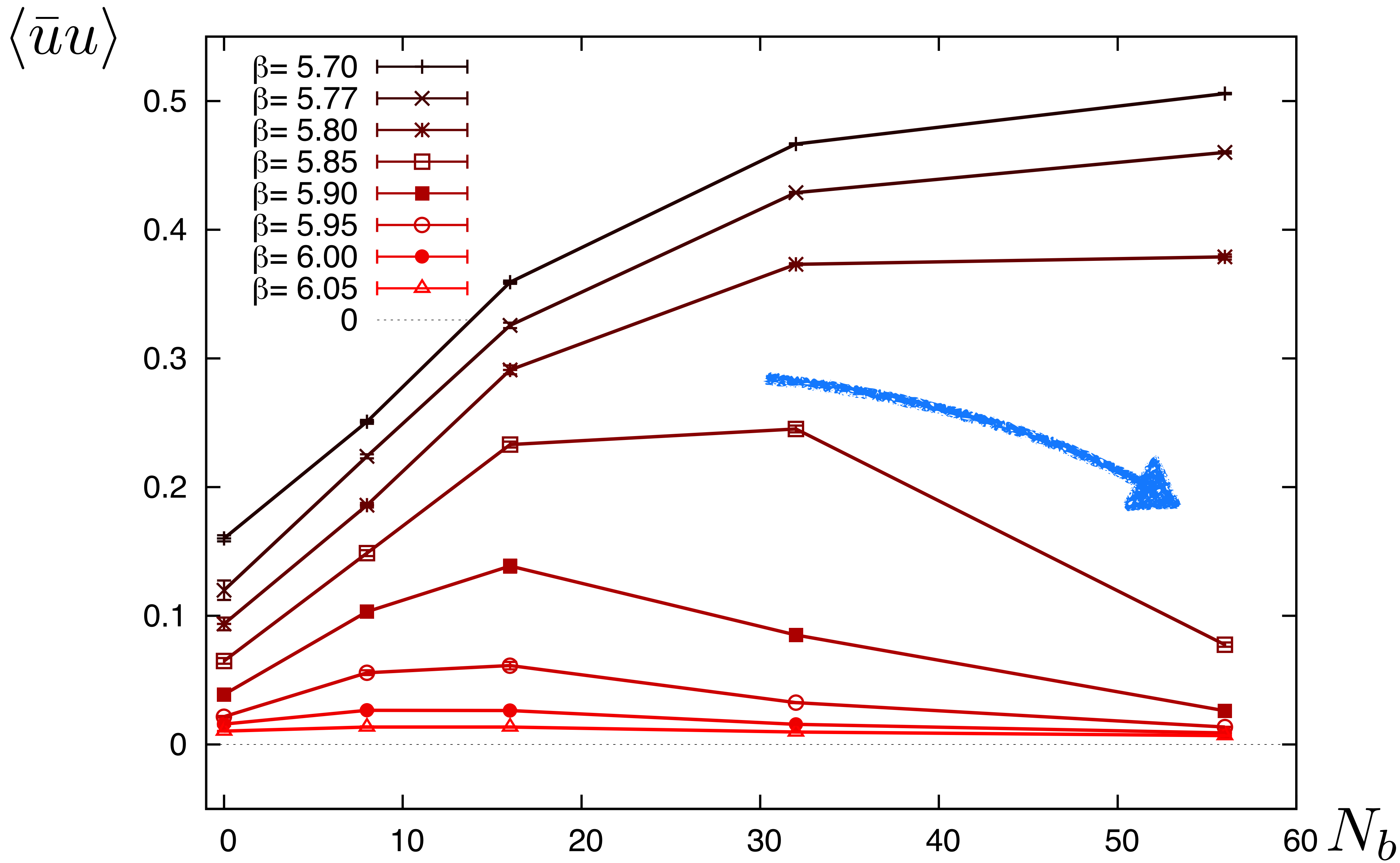
$$|\langle\bar{\psi}\psi\rangle| = \int_0^\infty d\lambda \frac{2m\rho(\lambda)}{\lambda^2+m^2}$$

$$= \frac{1}{V}\left\langle \mathrm{Tr}\left[\frac{1}{D+m}\right]\right\rangle \approx \frac{1}{N_r N_{\mathrm{conf}}}\sum_c^{N_{\mathrm{conf}}}\sum_r^{N_r}\xi_r^\dagger \frac{1}{D[U_c]+m}\xi_r$$

Using stochastic estimator [L. Giusti et al, 2009. G. Cossu et al 2016. P. d. Forcrand et al 2018.]

Results with m_π = 80 MeV

Lighter regime



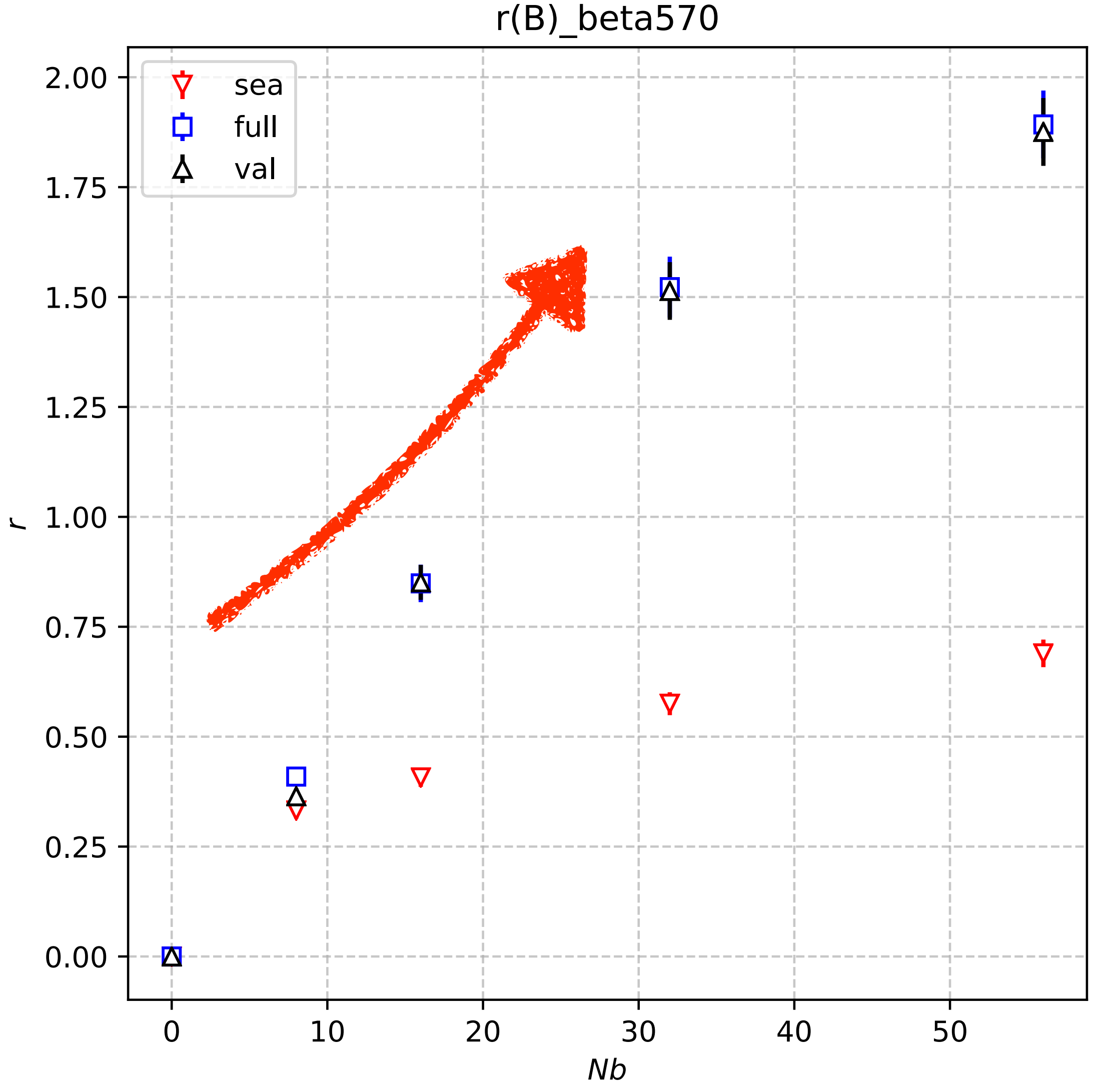
It shows inverse mag. catalysis

Using stochastic estimator [L. Giusti et al, 2009. G. Cossu et al 2016. P. d. Forcrand et al 2018.]

Results with m_π = 80 MeV

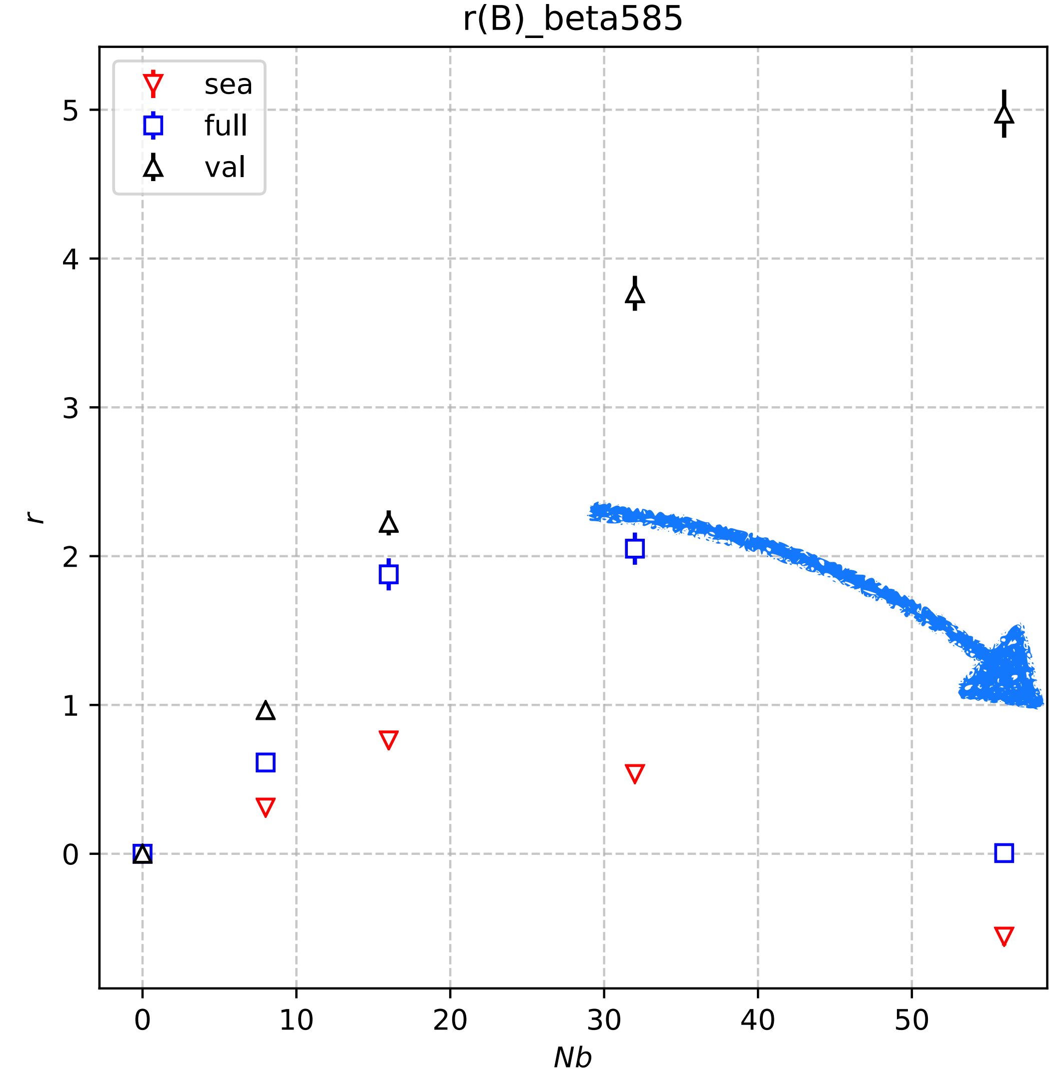
Lighter regime

T < Tc



(Normal) Magnetic Catalysis

T ~ Tc



Inverse Magnetic Catalysis

Sea quarks drive inverse catalysis
Similar to previous results at the physical point

Using stochastic estimator [L. Giusti et al, 2009. G. Cossu et al 2016. P. d. Forcrand et al 2018.]

Results with mpi = 320 MeV

Effects from valence and sea Dirac operators

External magnetic field come into two slots,

$$\langle\bar{\psi}\psi\rangle^{\mathrm{full},f}(B) = \int \mathcal{D}U\, P[U;B]\ \mathrm{Tr}\left[\frac{1}{D^f[U;B]+m}\right],$$

$$\langle\bar{\psi}\psi\rangle^{\mathrm{val},f}(B) = \int \mathcal{D}U\, P[U;0]\ \mathrm{Tr}\left[\frac{1}{D^f[U;B]+m}\right],$$

$$\langle\bar{\psi}\psi\rangle^{\mathrm{sea},f}(B) = \int \mathcal{D}U\, P[U;B]\ \mathrm{Tr}\left[\frac{1}{D^f[U;0]+m}\right],$$

where

$$P[U;B] = \frac{1}{Z(B)} e^{-S_g[U]} \mathrm{Det}\left[D^{\mathrm{up}}[U;B]+m\right]^{1/4} \mathrm{Det}\left[D^{\mathrm{down}}[U;B]+m\right]^{1/2}$$

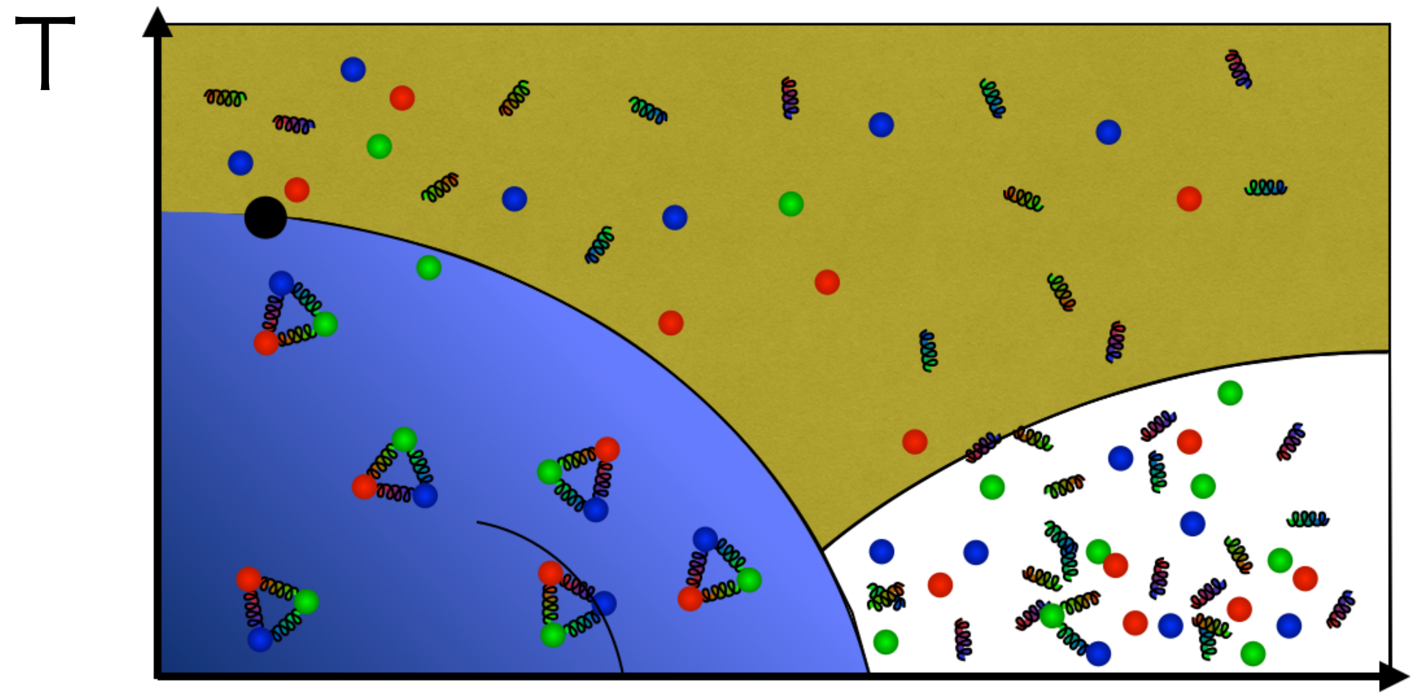
f = up, down

[M. D'Elia *et al* 1103.2080]

QCD phase diagram with magnetic field

Works with external magnetic field (improved KS types)

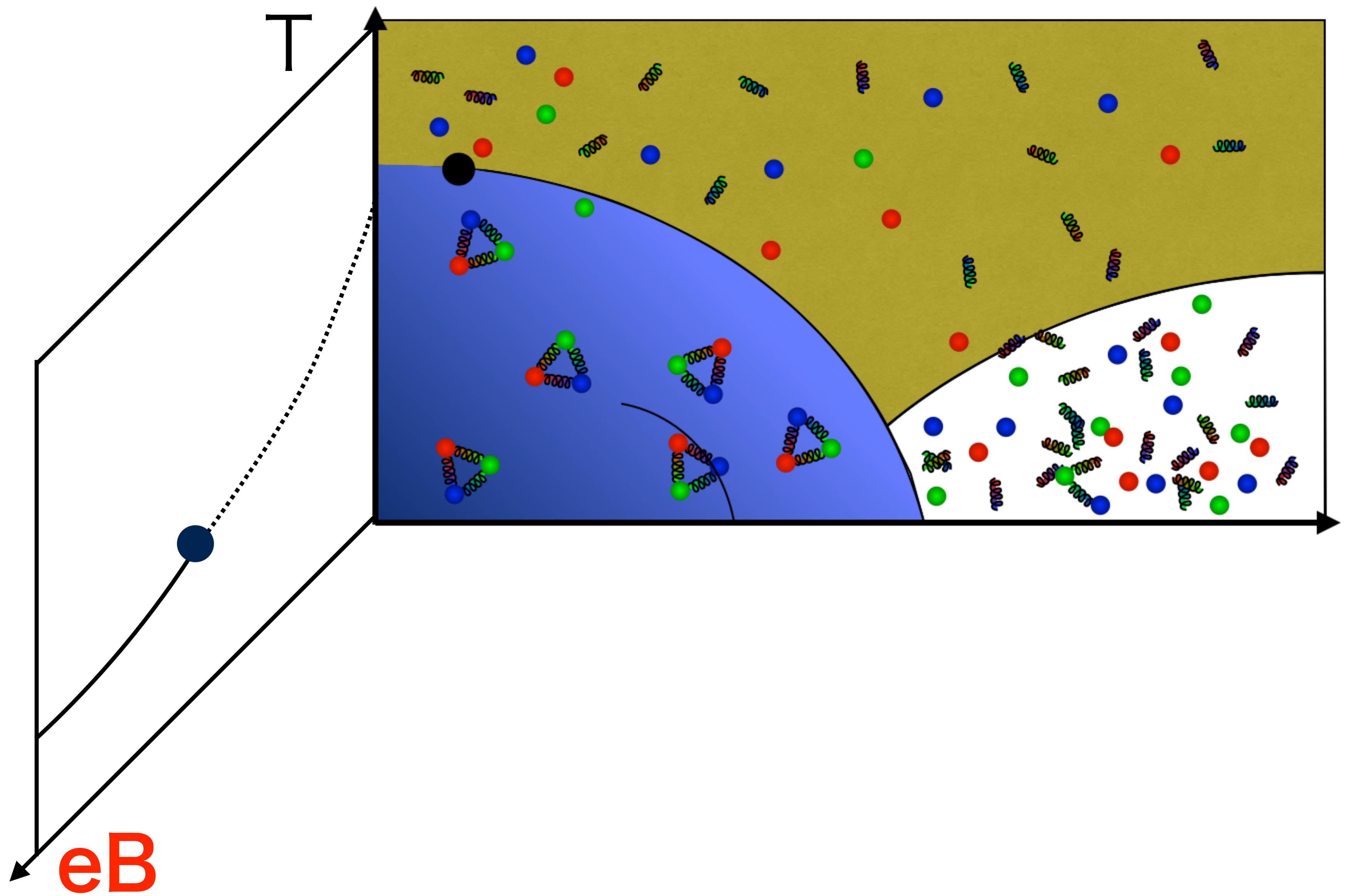
Phase diagram @ m_π= Physical mass



QCD phase diagram with magnetic field

Works with external magnetic field (improved KS types)

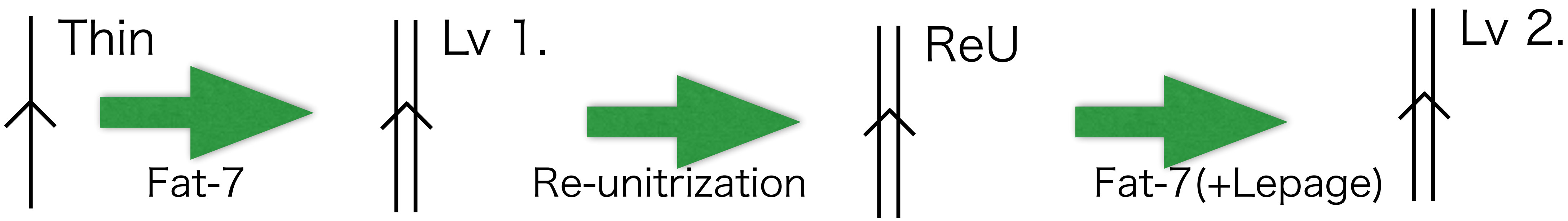
Phase diagram @ m_π= Physical mass



HISQ with the magnetic field

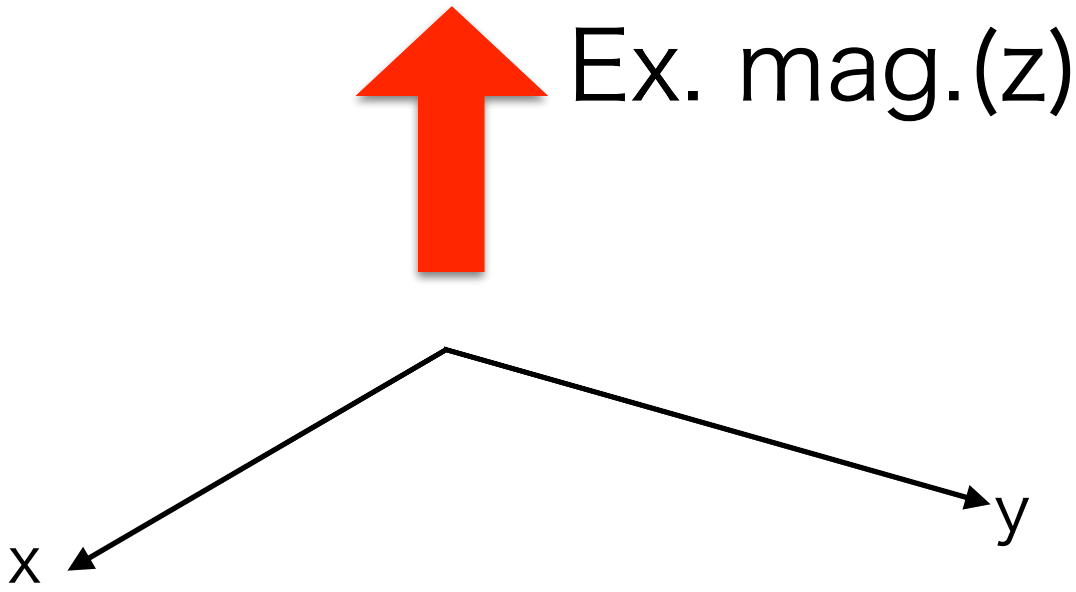
HISQ = Hight improved staggered quarks

HISQ = KS term with 2 level smearing + Naik term with reunitarized links



Quantization of the magnetic field

Magnetic flux and upper limit.



External magnetic field for z-dir.

$$u_x(n_x, n_y, n_z, n_t) = \begin{cases} \exp[-iqBN_x n_y] & (n_x = N_x - 1) \\ 1 & \text{(Otherwise)} \end{cases}$$

$$u_y(n_x, n_y, n_z, n_t) = \exp[iqBn_x],$$

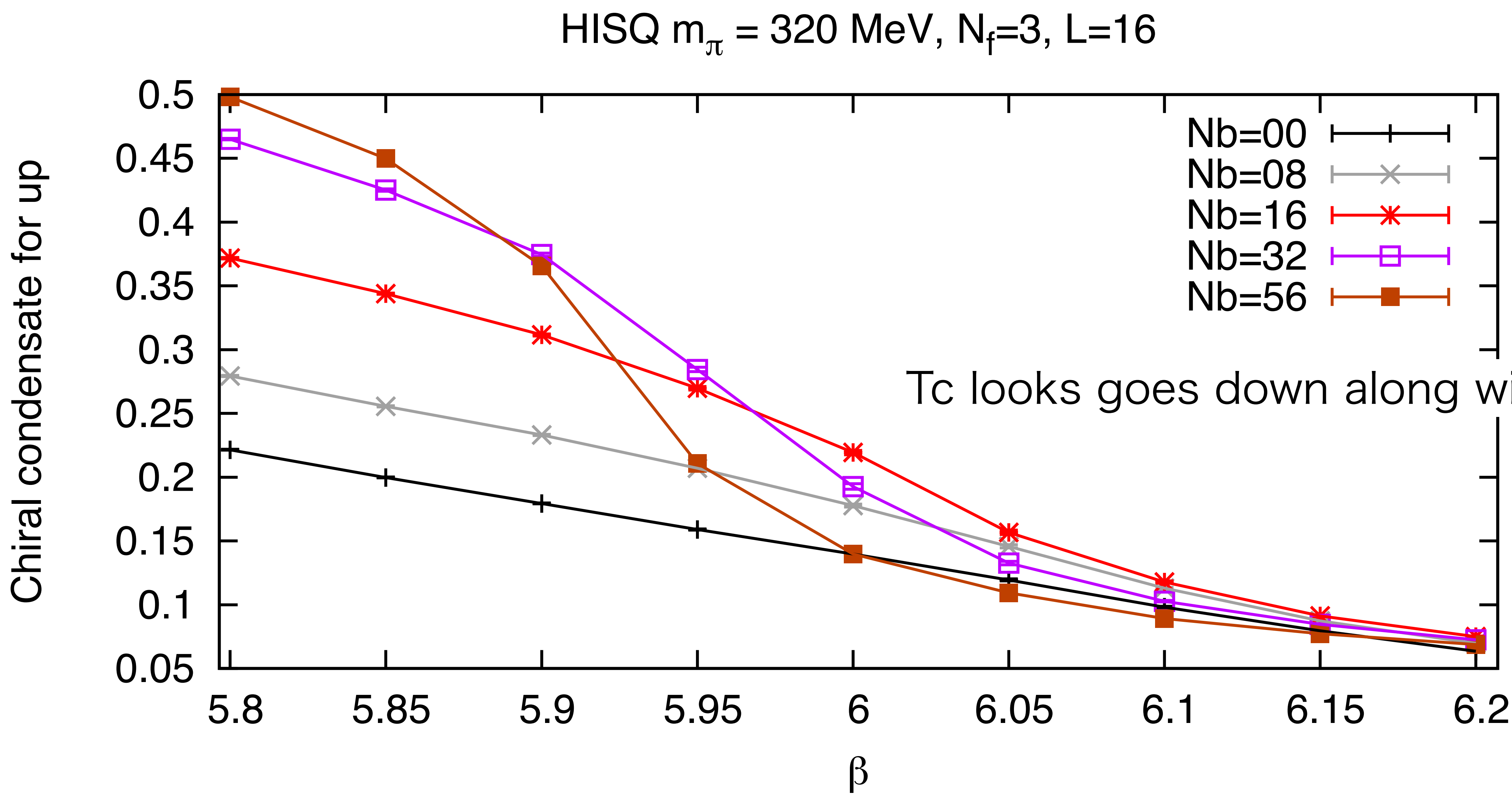
$$eB = \frac{2\pi N_b}{N_x N_y} a^{-2}$$

$$0 \leq N_b < \frac{N_x N_y}{4}$$ (By a periodicity)

Nx = Ny = 16 case, Nb = 64 is the maximum

Results with mpi = 320 MeV

Chiral condensate for heavier regime (~ standard setup)



Tc looks goes down along with Nb

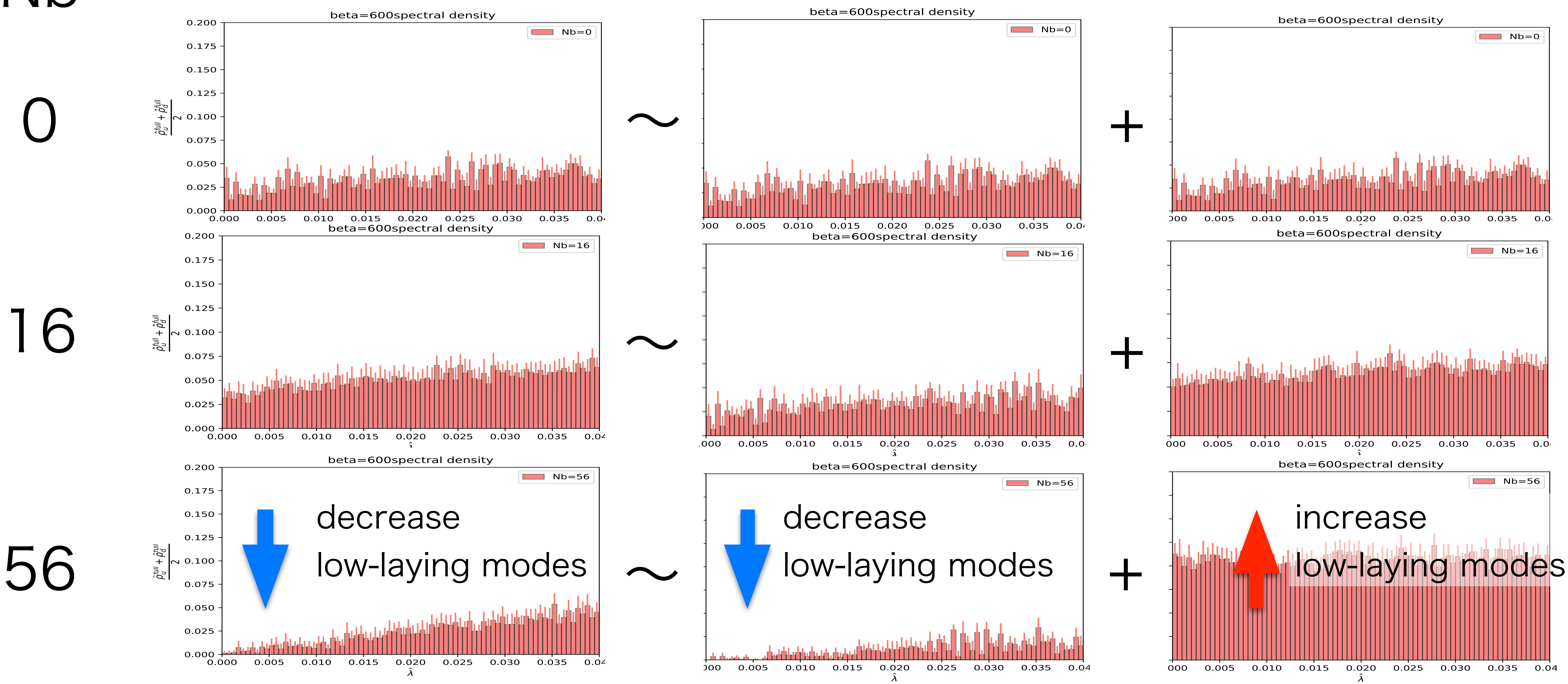
Results with mpi = 320 MeV

Dirac spectrum sea quarks drive inverse catalysis

β=6 (T>Tc)

Nb

Full ~ Sea + Valence

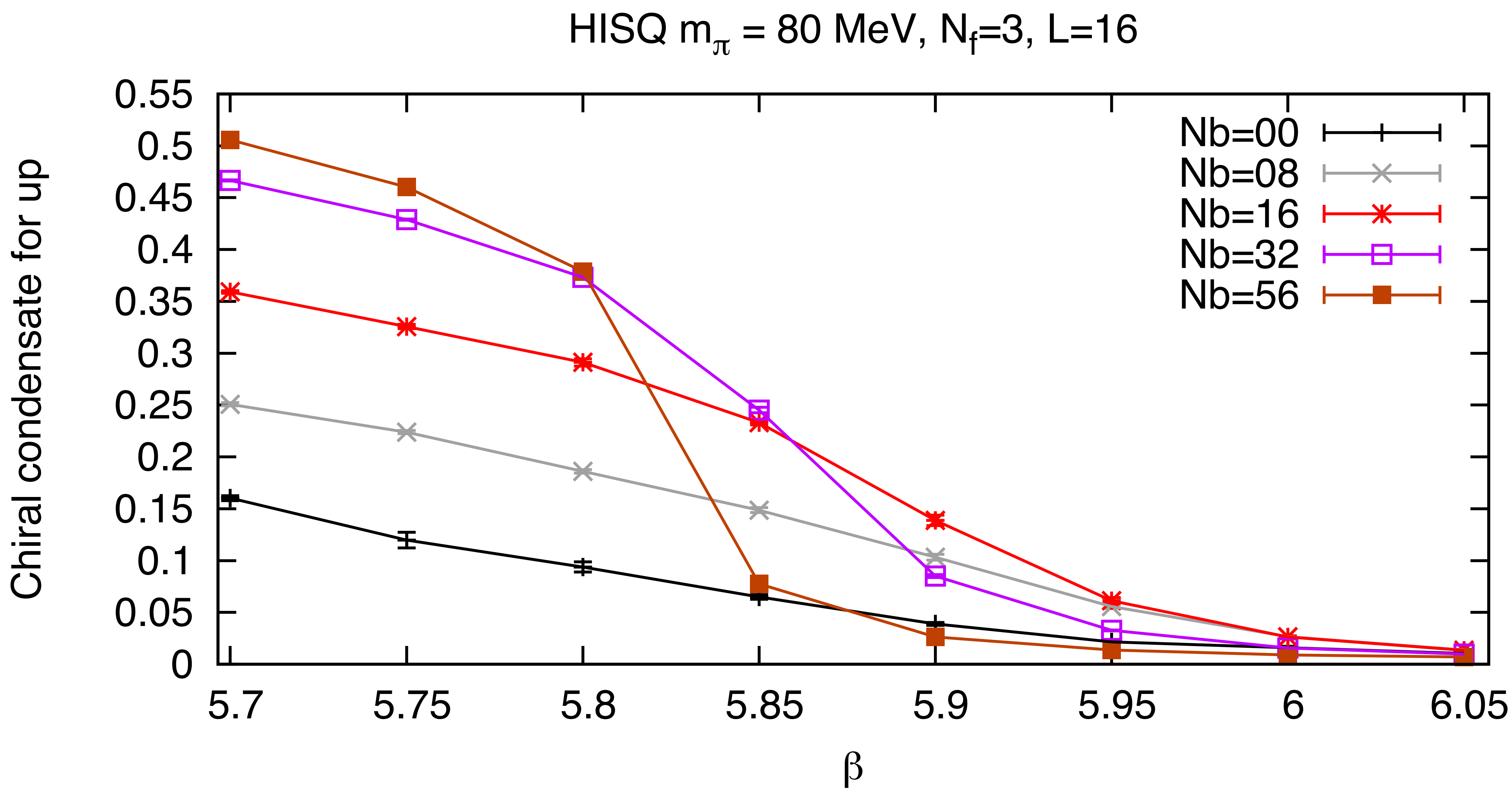


Valence quarks only contribute to normal catalysis

Using stochastic estimator [L. Giusti et al, 2009. G. Cossu et al

Results with mpi = 80 MeV

Chiral cond. in light pion mass regime (near the SU(3) chiral limit)



Results with mpi = 80 MeV

Dirac spectrum for lighter regime

β =X (T>Tc)

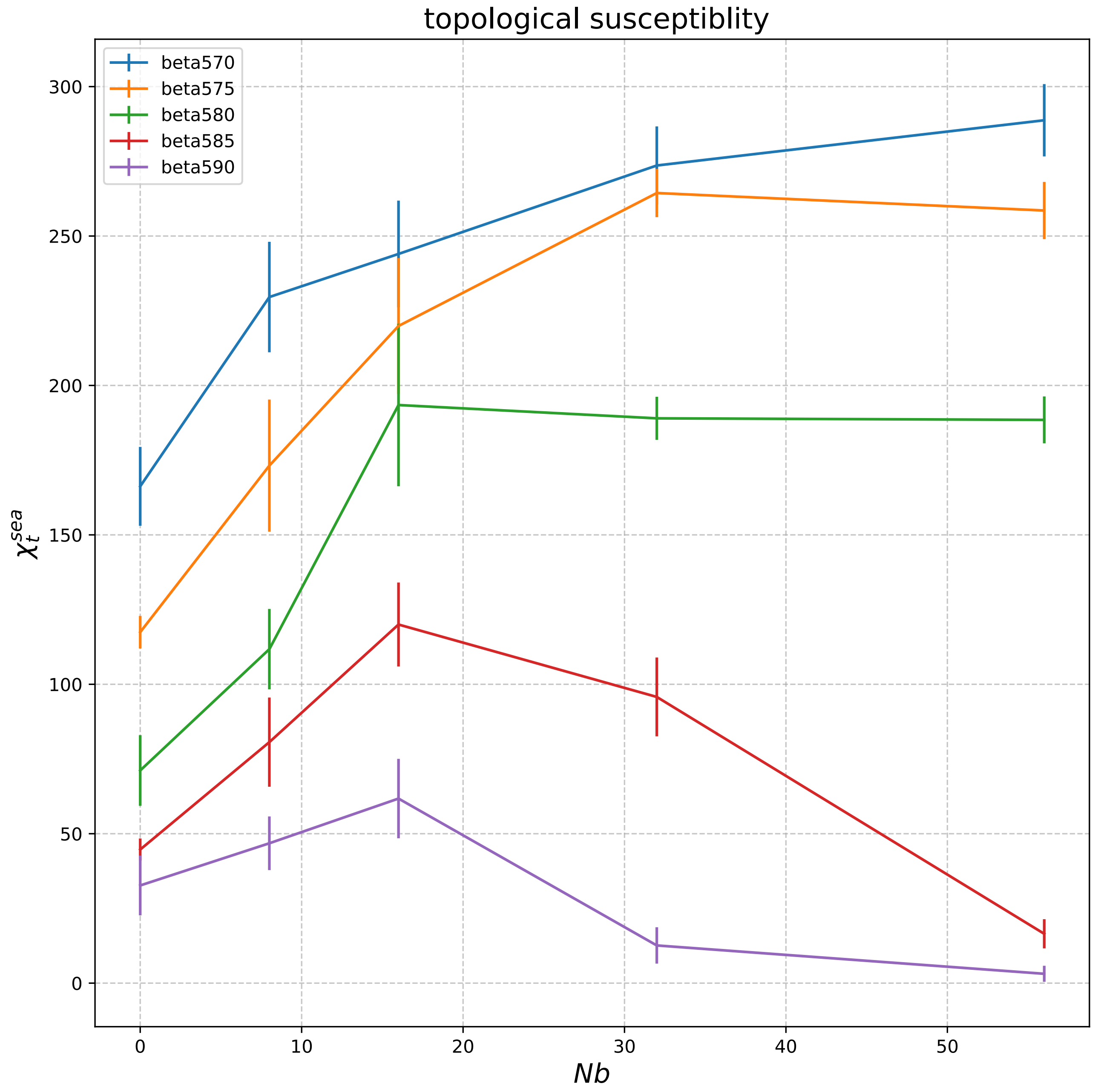
Nb	Full	Sea	Valence
0			
16			
56			

Valence only contribute to normal catalysis

Using stochastic estimator [L. Giusti et al, 2009. G. Cossu et al

Results with mpi = 80 MeV

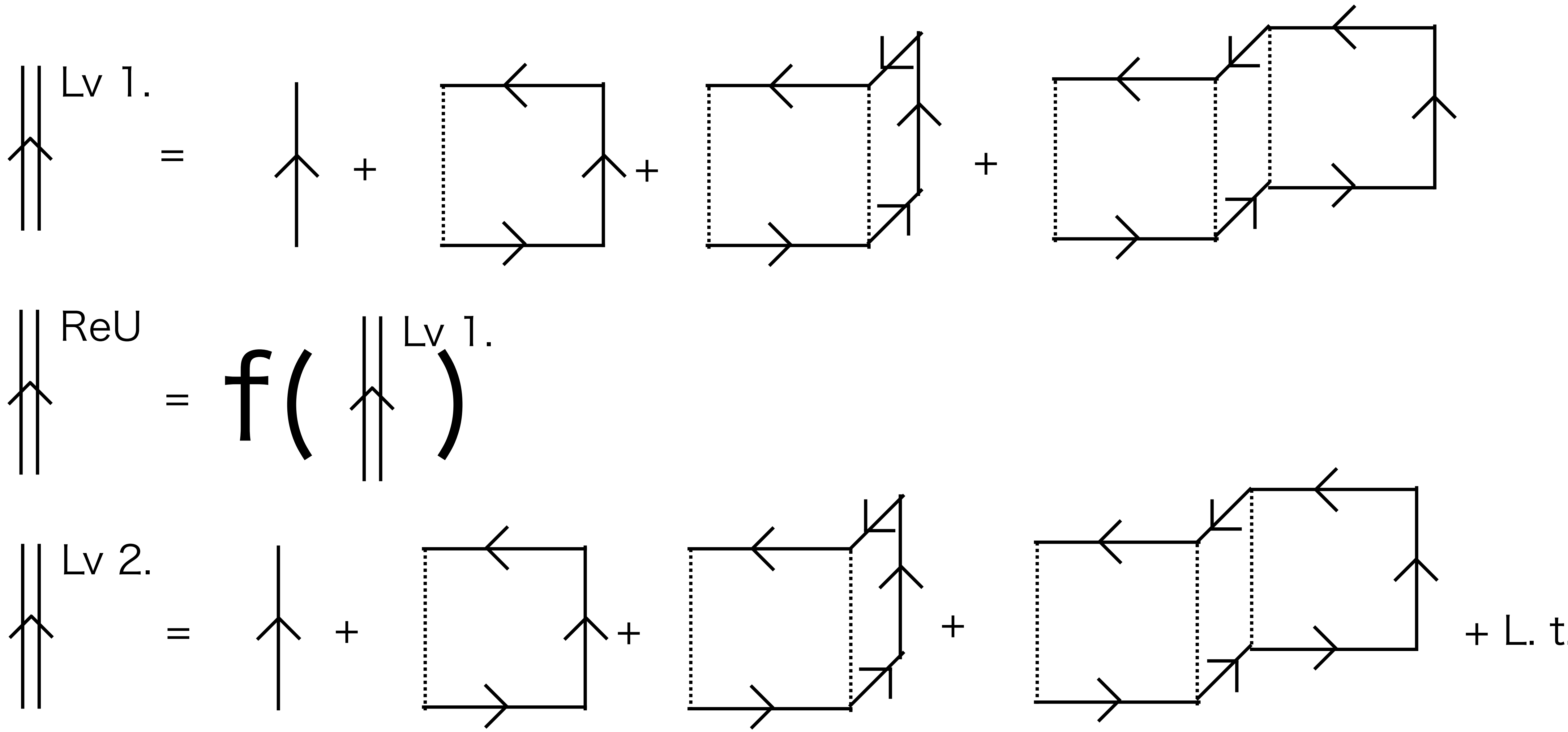
Topological susceptibility for lighter regime

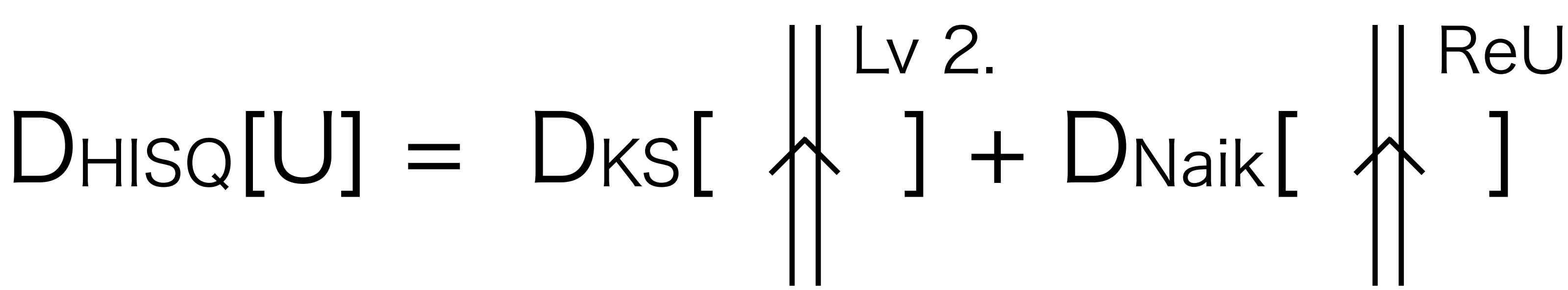


HISQ with the magnetic field

HISQ = Hight improved staggered quarks

HISQ = KS term with 2 level smearing + Naik term with reunitarized links





Related works

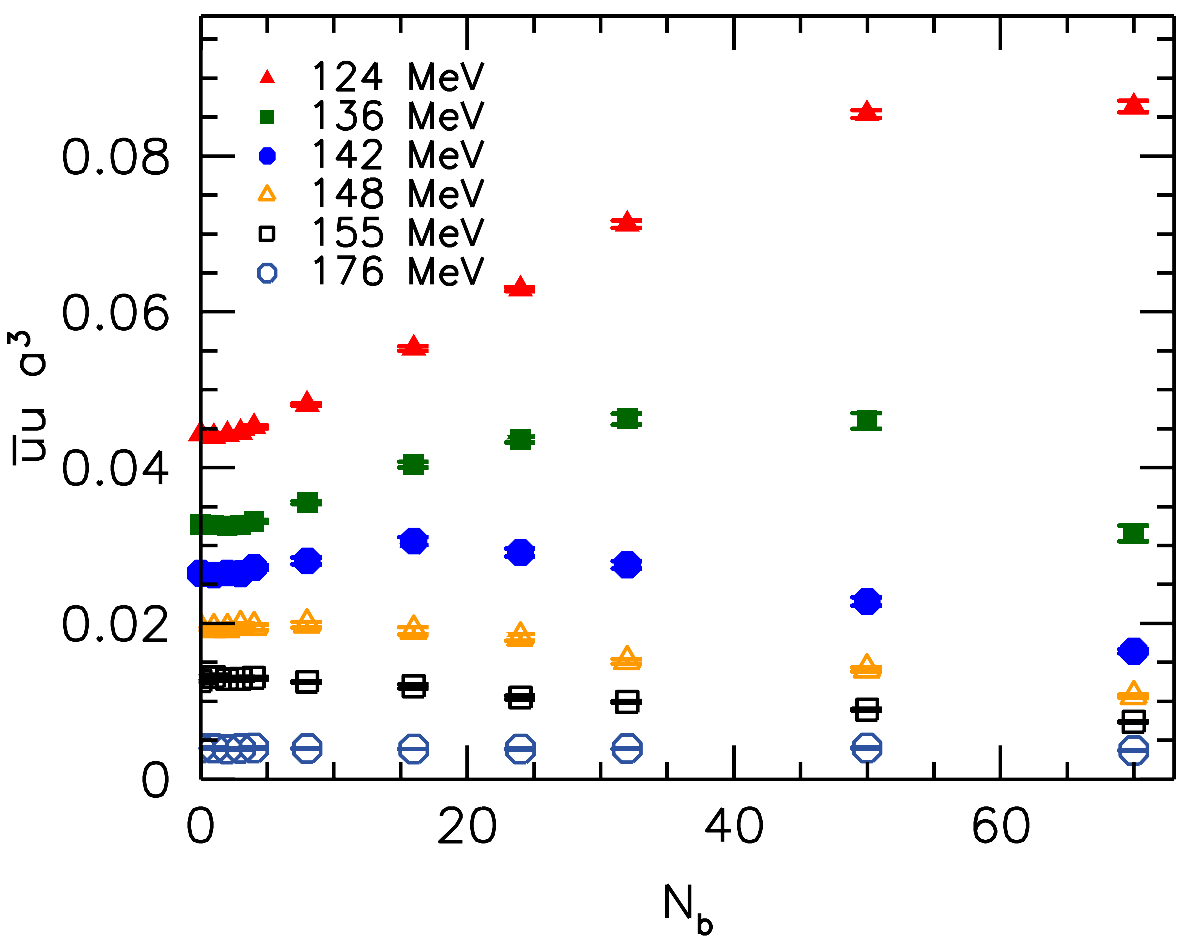
Mpi ~ 300 MeV

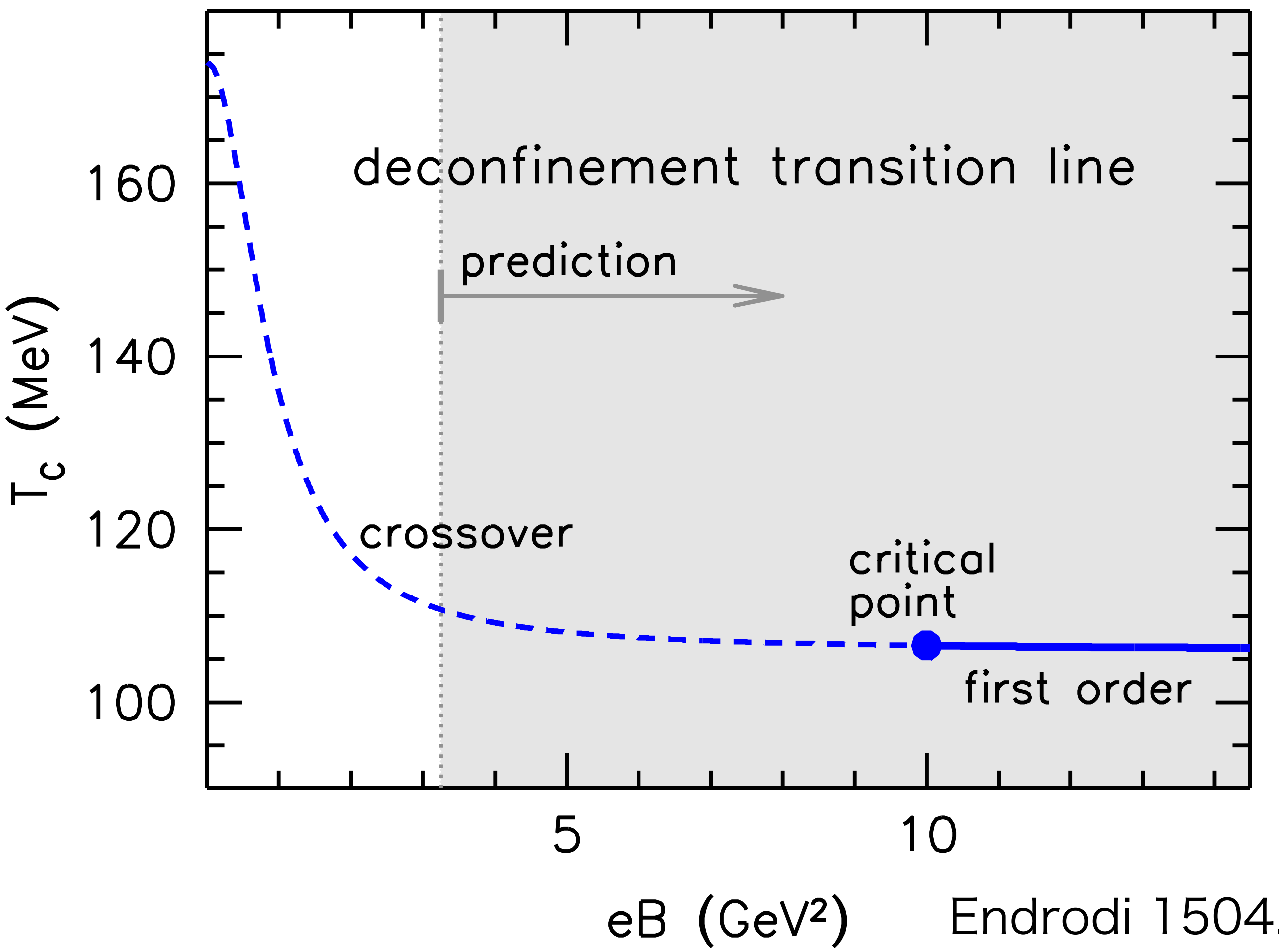
Our last year's one

- Tc goes up
- normal catalysis
- First order??

Related works

Mpi = physical, F. Bruckman et al, arXiv: 1111.4956, 1303.3972



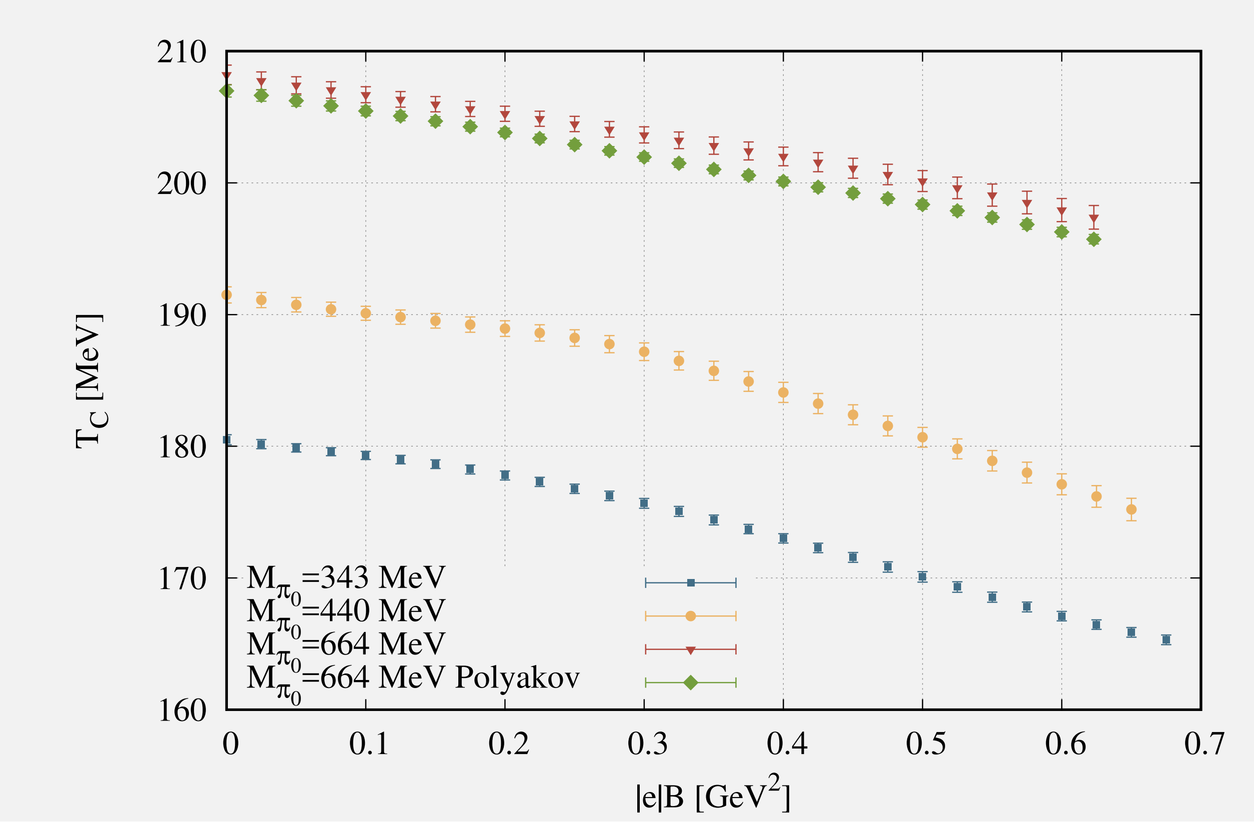


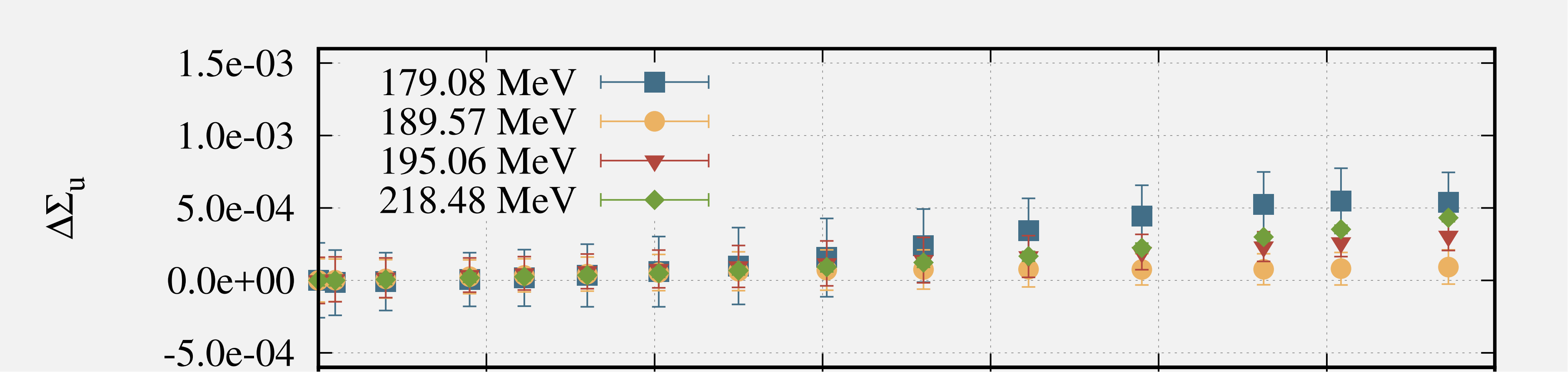
Endrodi 1504.08280

- Tc goes down
- Inverse magnetic Catalysis around Tc
- First order at extremely strong eB (Conjecture)

Related works

Mpi = 600 MeV, Massimo, XQCD2018





- Tc goes down
- No Inverse magnetic Catalysis for the range

Statistics with mpi = 320 MeV

Heavier regime

L	beta	mass	Nb	Trajectories
16	5.80	0.015	00	5500
16	5.80	0.015	08	5000
16	5.80	0.015	16	5000
16	5.80	0.015	32	5000
16	5.80	0.015	56	5000
16	5.85	0.015	00	6500
16	5.85	0.015	08	6500
16	5.85	0.015	16	6500
16	5.85	0.015	32	6500
16	5.85	0.015	56	6500
16	5.90	0.015	00	7500
16	5.90	0.015	08	7500
16	5.90	0.015	16	6500
16	5.90	0.015	32	7500
16	5.90	0.015	56	5466
16	5.95	0.015	00	7500
16	5.95	0.015	08	7500
16	5.95	0.015	16	4500
16	5.95	0.015	32	7500
16	5.95	0.015	56	7500

L	beta	mass	Nb	Trajectories
16	6.00	0.015	00	4500
16	6.00	0.015	08	4500
16	6.00	0.015	16	4500
16	6.00	0.015	32	4500
16	6.00	0.015	56	4500
16	6.05	0.015	00	4000
16	6.05	0.015	08	4000
16	6.05	0.015	16	4000
16	6.05	0.015	32	4000
16	6.05	0.015	56	4000
16	6.10	0.015	00	6500
16	6.10	0.015	08	6500
16	6.10	0.015	16	6500
16	6.10	0.015	32	6500
16	6.10	0.015	56	6500
16	6.15	0.015	00	4170
16	6.15	0.015	08	6500
16	6.15	0.015	16	5500
16	6.15	0.015	32	5500
16	6.15	0.015	56	5500

L	beta	mass	Nb	Trajectories
16	6.20	0.015	00	4485
16	6.20	0.015	08	7500
16	6.20	0.015	16	7500
16	6.20	0.015	32	7500
16	6.20	0.015	56	7500

ma =0.015

Statistics with mpi = 80 MeV

Lighter regime

L	beta	mass	Nb	Trajectories
16	5.700	0.0009375	00	630
16	5.700	0.0009375	08	1637
16	5.700	0.0009375	16	1640
16	5.700	0.0009375	32	10702
16	5.700	0.0009375	56	9638
16	5.750	0.0009375	00	700
16	5.750	0.0009375	08	1652
16	5.750	0.0009375	16	1644
16	5.750	0.0009375	32	11289
16	5.750	0.0009375	56	9363
16	5.800	0.0009375	00	700
16	5.800	0.0009375	08	1721
16	5.800	0.0009375	16	1467
16	5.800	0.0009375	32	12928
16	5.800	0.0009375	56	11486
16	5.850	0.0009375	00	700
16	5.850	0.0009375	08	1734
16	5.850	0.0009375	16	1623
16	5.850	0.0009375	32	11693
16	5.850	0.0009375	56	8512

L	beta	mass	Nb	Trajectories
16	5.900	0.0009375	00	700
16	5.900	0.0009375	08	2329
16	5.900	0.0009375	16	2327
16	5.900	0.0009375	32	8002
16	5.900	0.0009375	56	8276
16	5.950	0.0009375	00	700
16	5.950	0.0009375	08	1840
16	5.950	0.0009375	16	2337
16	5.950	0.0009375	32	13374
16	5.950	0.0009375	56	11990
16	6.000	0.0009375	00	700
16	6.000	0.0009375	08	3603
16	6.000	0.0009375	16	3700
16	6.000	0.0009375	32	13098
16	6.000	0.0009375	56	11690
16	6.050	0.0009375	00	700
16	6.050	0.0009375	08	3700
16	6.050	0.0009375	16	3700
16	6.050	0.0009375	32	14769
16	6.050	0.0009375	56	14010

$ma = 0.0009375$

eB^4 corrections

sea - valence decomposition can be justified below Nb=16

Nx =Ny = 16 case

$$eB = \frac{2\pi N_b}{N_x N_y} a^{-2}$$

$$r^{\text{full}}(B) = r^{\text{sea}}(B) + r^{\text{val}}(B) + O(B^4)$$

$$\det(D+m)(U,B) = \det(D+m) + c(e\hat{B})^2 + O(\hat{B}^4)$$

$$(D+m)^{-1}(U,B) = (D+m)^{-1}(U) + c'(e\hat{B})^2 + O(\hat{B}^4)$$

Nb	8	16	32	56
a^2 eB	0.20	0.39	0.79	1.37
(a^2 eB)^2	0.039	0.154	0.617	1.889
(a^2 eB)^4	0.001	0.024	0.381	3.569
(eB)^4/(eB)^2	4%	15%	62%	189%

↑NNLO contribution is large

eB^4 corrections

sea - valence decomposition can be justified below Nb=16

Nx =Ny = 16 case