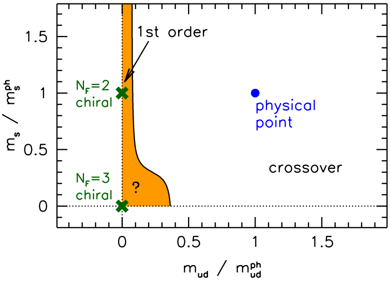
Chiral transition using the Banks-Casher relation

Gergely Endrődi, Lukas Gonglach

Goethe University of Frankfurt

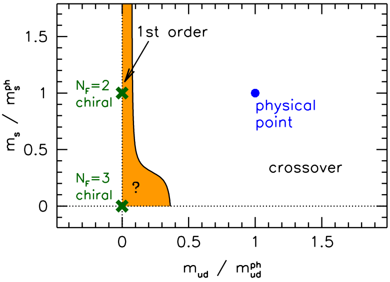
Lattice '18, 26. July 2018

Columbia plot



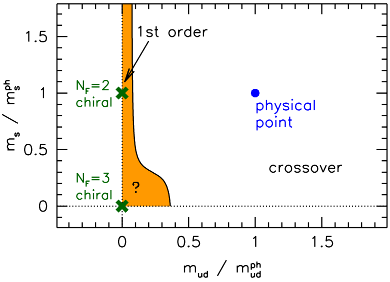
- nature of chiral transition as function of m_{ud} and m_s

Columbia plot



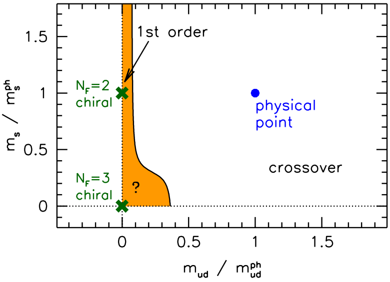
- nature of chiral transition as function of m_{ud} and m_s
- crossover at physical point
  [Aoki et al '06, Bhattacharya et al '14]

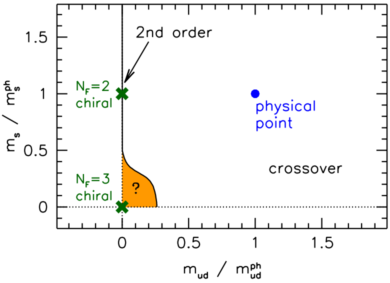
Columbia plot



- nature of chiral transition as function of m_{ud} and m_s
- crossover at physical point
  [Aoki et al '06, Bhattacharya et al '14]
- 1st order region around origin?
  [Pisarski, Wilczek '84]

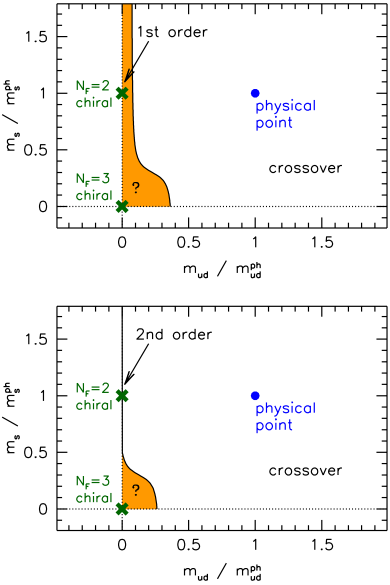
Columbia plot





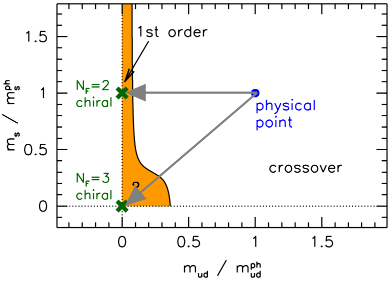
- nature of chiral transition as function of m_{ud} and m_s
- crossover at physical point
  [Aoki et al '06, Bhattacharya et al '14]
- 1st order region around origin?
  [Pisarski, Wilczek '84]
- relevant for: $\mathrm{U}(1)_A$ symmetry, critical endpoint at $\mu_B > 0$

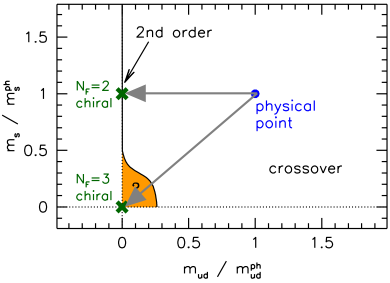
Columbia plot



- nature of chiral transition as function of m_{ud} and m_s
- crossover at physical point
  [Aoki et al '06, Bhattacharya et al '14]
- 1st order region around origin?
  [Pisarski, Wilczek '84]
- relevant for: $\mathrm{U}(1)_A$ symmetry, critical endpoint at $\mu_B > 0$
- $m \to 0$ limit controversial

Columbia plot





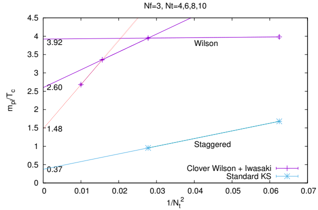
- nature of chiral transition as function of m_{ud} and m_s
- crossover at physical point
  [Aoki et al '06, Bhattacharya et al '14]
- 1st order region around origin?
  [Pisarski, Wilczek '84]
- relevant for: $\mathrm{U}(1)_A$ symmetry, critical endpoint at $\mu_B > 0$
- $m \to 0$ limit controversial
- here: learn about the chiral limit using a novel technique

Outline

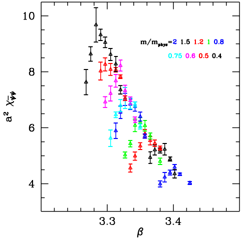
- problems of the chiral limit
- new approach
  - Banks-Casher relation
  - determination of the spectral density
  - chiral extrapolations
- results
- conclusions

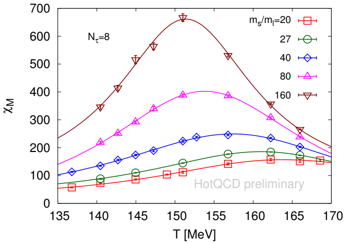
Towards the chiral limit

- with unimproved actions: critical point with huge lattice artefacts [de Forcrand, D‘Elia ’17]



- with improved actions: no critical point only strengthening

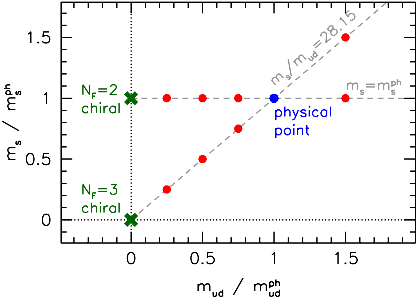




[Endrődi et al ’07] [Ding et al ’18]

Strategy

- attempt an extrapolation to the chiral limit directly



- chiral condensate

$$\left\langle \bar{\psi}\psi(m) \right\rangle_m = \frac{1}{Z_m} \int \mathcal{D}U \, e^{-S_g} \det[\not{D} + m] \, \mathrm{tr}[(\not{D} + m)^{-1}]$$

- $m \to 0$ using Banks-Casher relation [Banks,Casher '80]
- $m \to 0$ using leading-order reweighting

Banks-Casher relation

- in the eigenbasis of $\not{D}$, the condensate $\bar{\psi}\psi \propto \mathrm{tr}(\not{D} + m)^{-1}$

$$\bar{\psi}\psi(m) = \frac{T}{V} \sum_i \frac{m}{\lambda_i^2 + m^2} \xrightarrow{V \to \infty} \int_{-\infty}^{\infty} \mathrm{d}\lambda \, \rho(\lambda) \, \frac{m}{\lambda^2 + m^2} \xrightarrow{m \to 0} \pi \, \rho(0)$$

- the eigenvalues contain much more information than just $\bar{\psi}\psi(m)$, they encode also its dependence on m

Leading-order reweighting

- reweight configurations towards $m = 0$

$$\langle \rho(\lambda) \rangle_0 = \frac{\langle \rho(\lambda)\, W(m) \rangle_m}{\langle W(m) \rangle_m}$$

with

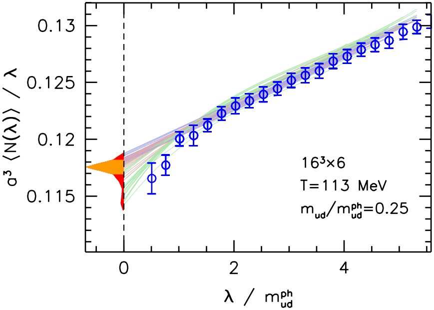
$$W(m) = \frac{\det[\not{D}]}{\det[\not{D} + m]} = \exp\left[-\frac{V}{T} m \cdot \bar{\psi}\psi(m) + \mathcal{O}(m^4) \right]$$

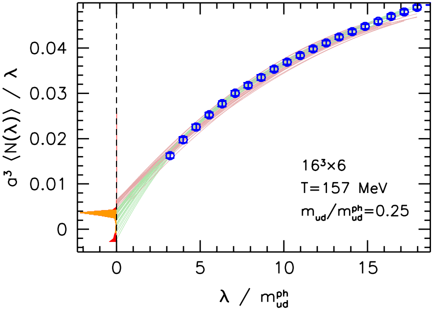
- work with the so reweighted spectral density in the following

Spectral density

- find $\rho(0)$ via extrapolation of integrated spectral density

$$N(\lambda) = \int_0^\lambda \mathrm{d}\lambda' \rho(\lambda') \qquad \rho(0) = \lim_{\lambda \to 0} \frac{N(\lambda)}{\lambda}$$

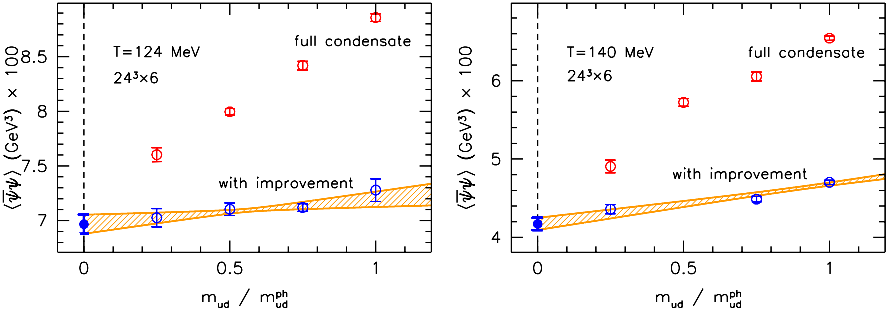




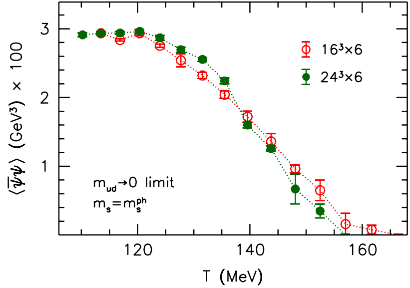
- build histogram of intersects to define mean and systematic error of fit

Extrapolations

- remaining m_{ud}-dependence much smaller than in the full condensate $\left\langle \bar{\psi}\psi(m) \right\rangle_m$

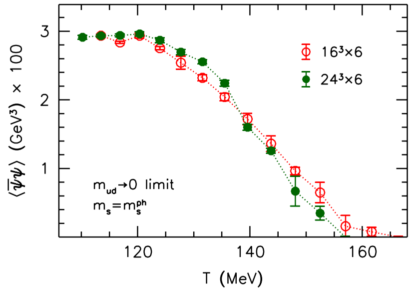


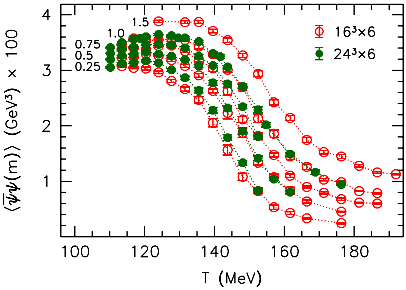
Chiral transition



- sharpening of the order parameter as V grows
  $\rightsquigarrow$ real phase transition?
- chiral transition temperature at crossing point of two volumes:
  $T_c^{N_f=2+1} \approx 140$ MeV

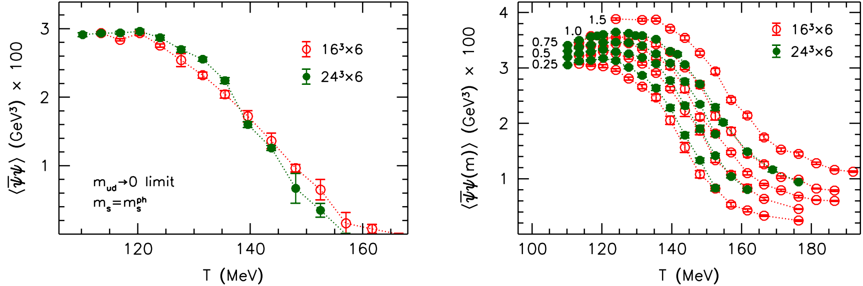
Chiral transition





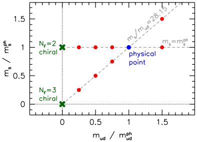
- sharpening of the order parameter as V grows
  $\rightsquigarrow$ real phase transition?
- chiral transition temperature at crossing point of two volumes: $T_c^{N_f=2+1} \approx 140$ MeV
- the same signal is hidden in the full condensate

Chiral transition

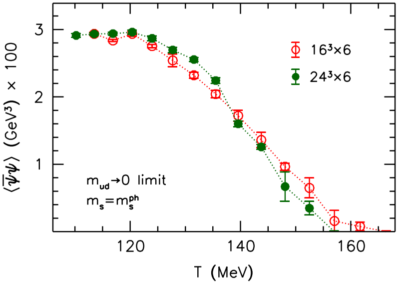


- sharpening of the order parameter as V grows
  $\rightsquigarrow$ real phase transition?
- chiral transition temperature at crossing point of two volumes: $T_c^{N_f=2+1} \approx 140$ MeV
- the same signal is hidden in the full condensate
- for $\left\langle \bar{\psi}\psi(m=0) \right\rangle$, no additive renormalization necessary

Number of massless flavors

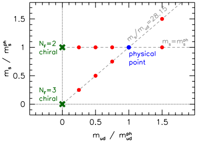


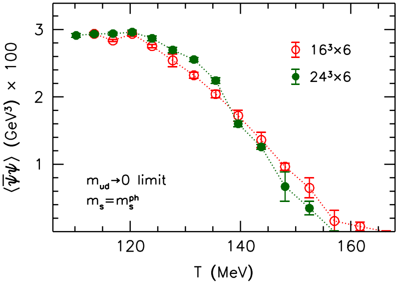
- same analysis along $m_s/m_{ud} = \text{const.}$ line

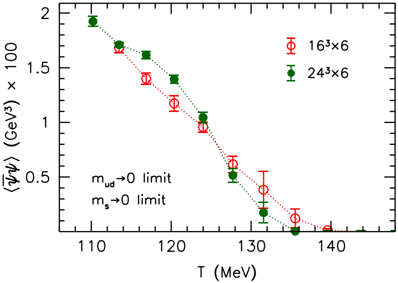


Number of massless flavors

- same analysis along $m_s/m_{ud} = \text{const.}$ line

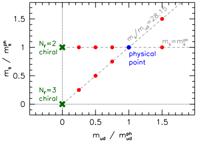


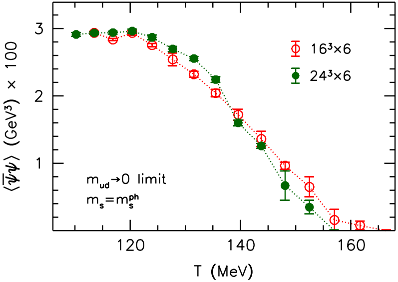


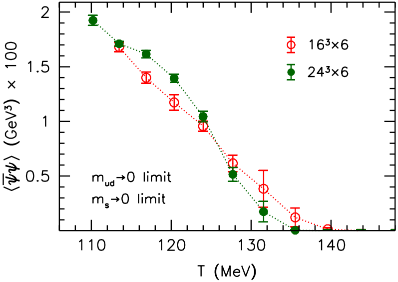


Number of massless flavors

- same analysis along m_s/m_{ud} = const. line



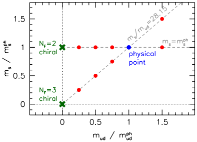


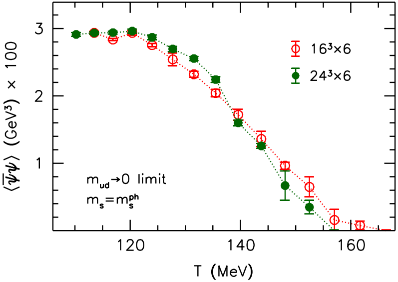


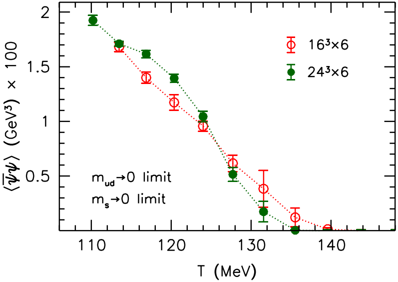
- vacuum condensate reduced
  consistent with χPT [Moussalam '99, Descotes et al '99]

Number of massless flavors

- same analysis along $m_s/m_{ud} = \text{const.}$ line

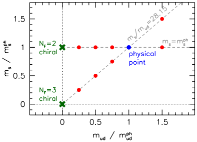




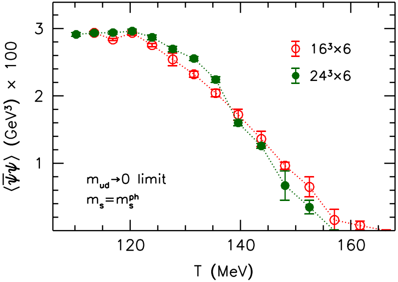


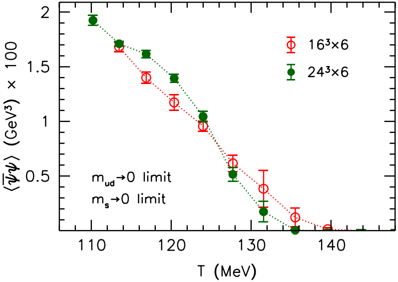
- vacuum condensate reduced
  consistent with χPT [Moussalam '99, Descotes et al '99]
- volume-dependence more pronounced $\rightsquigarrow$ stronger transition?

Number of massless flavors



- same analysis along $m_s/m_{ud} = \text{const.}$ line

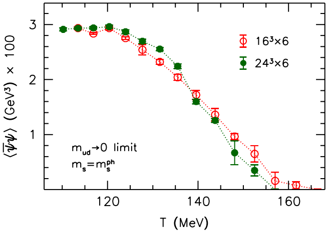




- vacuum condensate reduced
  consistent with χPT [Moussalam '99, Descotes et al '99]
- volume-dependence more pronounced $\rightsquigarrow$ stronger transition?
- chiral transition is reduced to $T_c^{N_f=3} \approx 125$ MeV

Nature of the transition

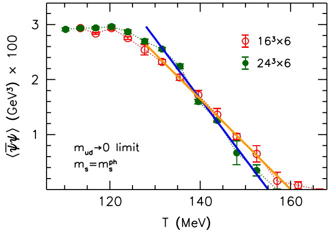
- fit for slope of order parameter



- critical scaling: $\bar{\psi}\psi'_{T=T_c} \xrightarrow{V\to\infty} \infty$

Nature of the transition

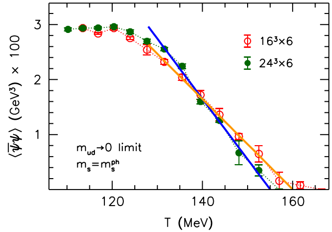
- fit for slope of order parameter



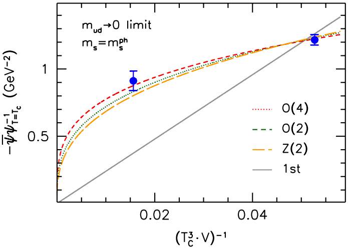
- critical scaling: $\bar{\psi}\psi'_{T=T_c} \xrightarrow{V\to\infty} \infty$

Nature of the transition

- fit for slope of order parameter

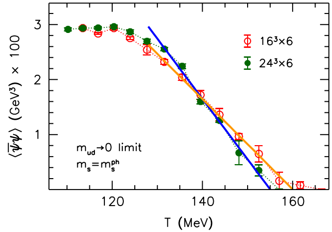


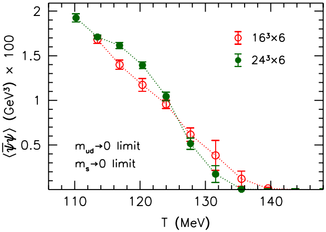
- critical scaling: $\bar\psi\psi'_{T=T_c} \xrightarrow{V\to\infty} \infty$



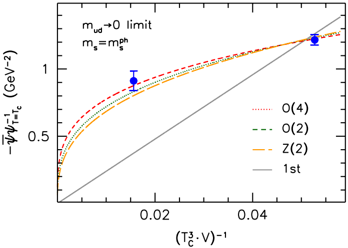
Nature of the transition

- fit for slope of order parameter



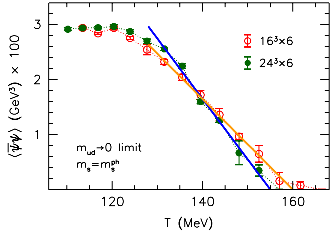


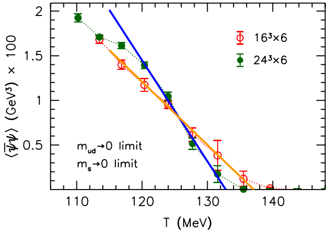
- critical scaling: $\bar\psi\psi'_{T=T_c} \xrightarrow{V\to\infty} \infty$



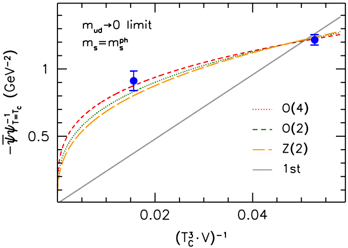
Nature of the transition

- fit for slope of order parameter



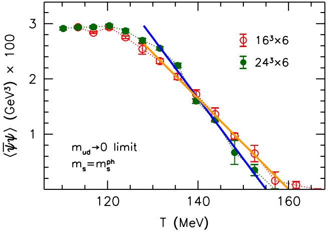


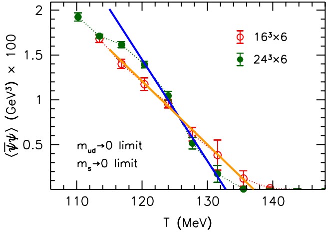
- critical scaling: $\bar\psi\psi'_{T=T_c} \xrightarrow{V\to\infty} \infty$



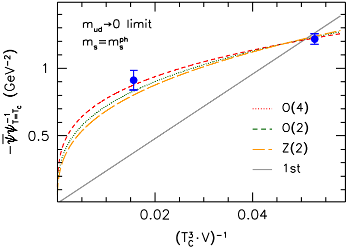
Nature of the transition

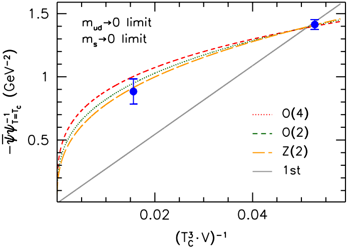
- fit for slope of order parameter





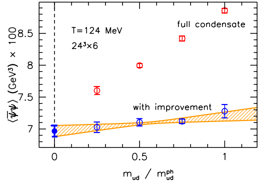
- critical scaling: $\bar{\psi}\psi'_{T=T_c} \xrightarrow{V\to\infty} \infty$



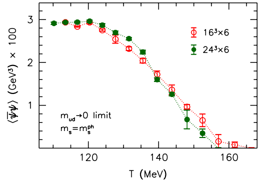


Summary

- extract chiral condensate via Banks-Casher relation
  $\rightsquigarrow$ flat extrapolation



- finite volume analysis of chiral condensate
  (no additive renormalization required)



- $N_F = 2+1$ chiral limit
  consistent with $\mathrm{O}(4)$ scenario