Semileptonic decays of $B_{(s)}$ mesons to light pseudoscalar mesons on MILC ensembles

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Fermilab Lattice and MILC Collaborations

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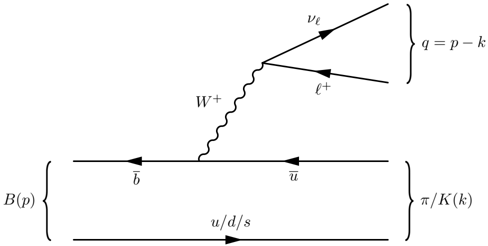
Overview

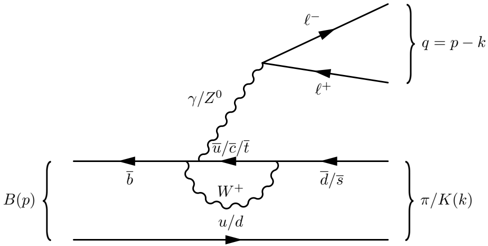
Part I — Introduction

$B_{(s)} \to \pi(K)\ell\nu$: charged currents



$$\frac{d\Gamma}{dq^2} = \text{(known factors)} \times |V_{ub}|^2 \times \{|f_+(q^2)|^2, |f_0(q^2)|^2\}$$

$B \to \pi(K)\ell^+\ell^-$: flavor-changing neutral currents



$$\frac{d\Gamma}{dq^2} = \text{(known factors)} \times |V_{tb}V_{tf}^*|^2 \times \{|f_+(q^2)|^2, |f_0(q^2)|^2, |f_T(q^2)|^2\}$$

Form factors I

These transitions can be mediated by vector, scalar, and tensor currents.

Taking Lorentz and discrete symmetries into account:

$$\langle P(k)|\mathcal{V}^\mu|B(p)\rangle = f_+(q^2)\left(p^\mu + k^\mu - \frac{M_B^2 - M_P^2}{q^2}q^\mu\right) + f_0(q^2)\frac{M_B^2 - M_P^2}{q^2}q^\mu$$

$$\langle P(k)|\mathcal{S}|B(p)\rangle = f_0(q^2)\frac{M_B^2 - M_P^2}{m_b - m_q}$$

$$\langle P(k)|\mathcal{T}^{\mu\nu}|B(p)\rangle = f_T(q^2)\frac{2}{M_B + M_P}\left(p^\mu k^\nu - p^\nu k^\mu\right)$$

PCVC ensures that the vector and scalar currents lead to the same f_0.

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$$= \sqrt{2M_B}\left[k_\perp^\mu f_\perp(E_P) + v^\mu f_\parallel(E_P)\right], \quad v = p/M_B$$

$$\langle P(k)|\mathcal{S}|B(p)\rangle = f_0(q^2)\frac{M_B^2 - M_P^2}{m_b - m_q}$$

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Form factors II

It is straightforward to extract the matrix elements

$$f_\perp(E_P) = \frac{\langle P|\mathcal{V}^i|B\rangle}{\sqrt{2M_B}}\frac{1}{k^i}$$

$$f_\parallel(E_P) = \frac{\langle P|\mathcal{V}^0|B\rangle}{\sqrt{2M_B}}$$

$$f_T(E_P) = \frac{M_B + M_P}{\sqrt{2M_B}}\frac{\langle P|\mathcal{T}^{0i}|B\rangle}{\sqrt{2M_B}}\frac{1}{k^i}$$

from three-point correlation functions.

Then f_+ and f_0 are linear combinations of $f_\perp$ and $f_\parallel$.

Correlation functions

Energies of pseudoscalar mesons are extracted from two-point correlators:

$$C_2(t;\boldsymbol{k}) = \sum_{\boldsymbol{x}} e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \left\langle \mathcal{O}_P(0,\mathbf{0})\, \mathcal{O}_P^\dagger(t,\boldsymbol{x}) \right\rangle$$

$$\Rightarrow \sum_m \frac{\left|\left\langle 0|\mathcal{O}_P|P^{(m)}\right\rangle\right|^2}{2E_P^{(m)}} e^{-E_P^{(m)}t}$$

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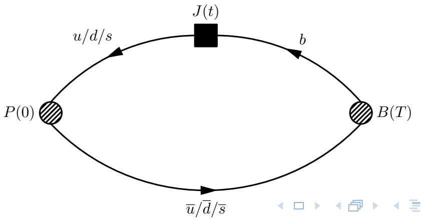
Form factors are extracted from three-point correlators:

$$C_3^{\mu(\nu)}(t,T;\boldsymbol{k}) = \sum_{\boldsymbol{x},\boldsymbol{y}} e^{i\boldsymbol{k}\cdot\boldsymbol{y}} \left\langle \mathcal{O}_P(0,\boldsymbol{0})\, J^{\mu(\nu)}(t,\boldsymbol{y})\, \mathcal{O}_B^\dagger(T,\boldsymbol{x}) \right\rangle$$



Correlation-function fits

We use a mostly nonperturbative matching $Z_J = \rho_J \sqrt{Z_{V^4_{bb}} Z_{V^4_{qq}}}$, along with a blinding procedure, for the form factors:

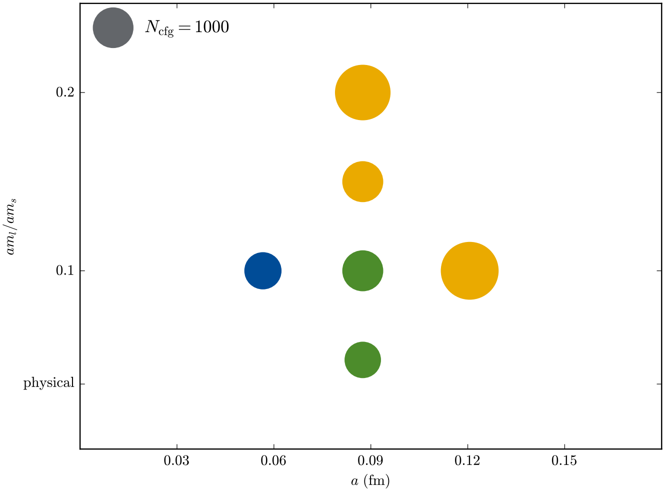
$$
\begin{aligned}
f_\perp(E_P) &= Z_\perp \frac{\widehat{C}_3^i(\boldsymbol{k})}{k^i} \\
f_\parallel(E_P) &= Z_\parallel \widehat{C}_3^4(\boldsymbol{k}) \\
f_T(E_P) &= Z_T \frac{M_B + M_P}{\sqrt{2M_B}} \frac{\widehat{C}_3^{4i}(\boldsymbol{k})}{k^i}
\end{aligned}
$$

Form factors are extrapolated to the chiral-continuum limit using heavy meson rooted staggered chiral perturbation theory. [PRD:73.014515, PRD:76.014002]

Finally, they are extended to the full kinematic range using z-expansion methods.

Part II — (2+1)-flavor asqtad $B_s \to K$

MILC asqtad ensembles



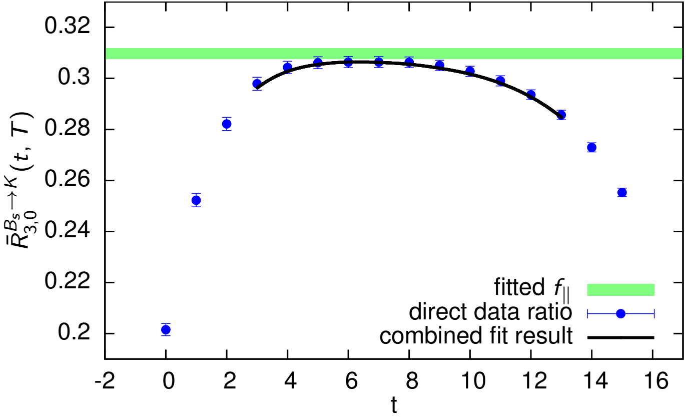
Actions and parameters

- MILC $N_f = 2 + 1$ ensembles
- Lüscher-Weisz gauge action $\to O(\alpha_s a^2)$
- asqtad action for $q_l, s \to O(\alpha_s a^2)$
- Clover action with Fermilab interpretation for $b \to O(\alpha_s a, a^2)\, f\left((m_b a)^2\right)$
- Scale set with r_1, where $r_1^{a=0} = 0.3117(22)$ fm
- Partially quenched: $m_s' \neq m_s$

$\approx a$ (fm)	0.12	0.09	0.09	0.09	0.09	0.06
$N_{\text{cfg}} \times N_{\text{src}}$	2099×4	1931×4	1015×8	1015×8	791×4	827×4
$N_s^3 \times N_4$	$24^3 \times 64$	$28^3 \times 96$	$32^3 \times 96$	$40^3 \times 96$	$64^3 \times 96$	$64^3 \times 144$
am_l'	0.0050	0.0062	0.00465	0.0031	0.00155	0.0018
am_s	0.050	0.031	0.031	0.031	0.031	0.018
κ_b'	0.0901	0.0979	0.0977	0.0976	0.0976	0.1052
$\approx r_1/a$	2.7386	3.7887	3.7716	3.7546	3.7376	5.3073
$\approx \alpha_V(2/a)$	0.3104	0.2608	0.2608	0.2608	0.2608	0.2249

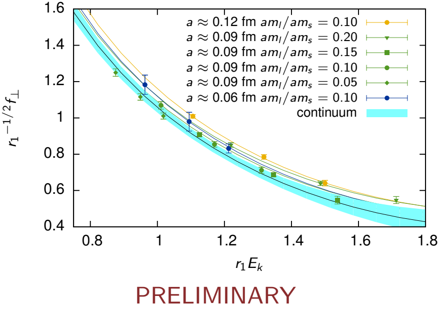
Form factors

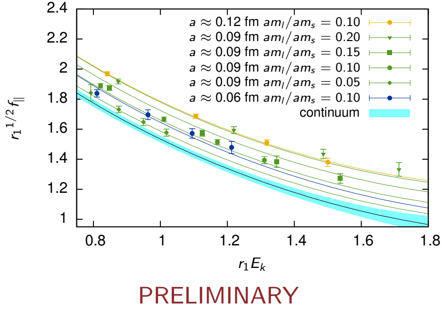
Form factors are obtained from fits to three-point correlators and from fits to ratios of three- to two-point correlators, which provide consistent results.



Plot from Liu et al. in arXiv:1711.08085

Form factors in the chiral continuum

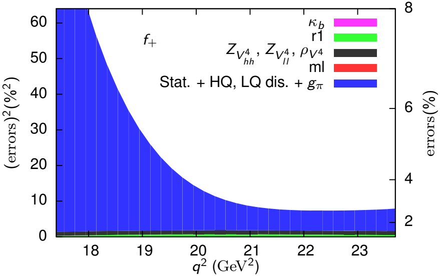




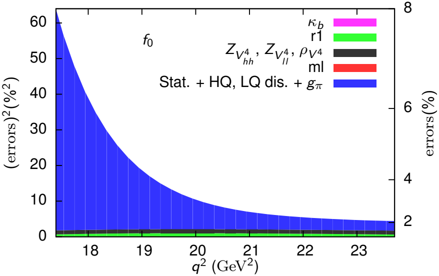
- $f_\perp$ and $f_\parallel$ are fit simultaneously.
- NNLO HMrSχPT is used as the central fit.

arXiv:1711.08085

Error budget



PRELIMINARY



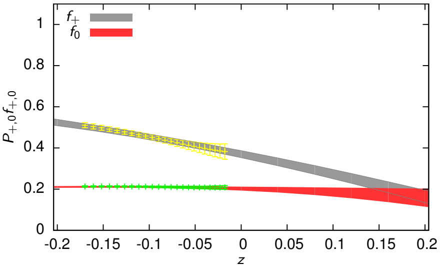
PRELIMINARY

- Errors in the (lattice) low-q^2 region are large and are dominated by those of statistics, discretization, and the chiral-continuum extrapolation.
- z-expansion methods are then used to extrapolate the form factors to $q^2 = 0$.

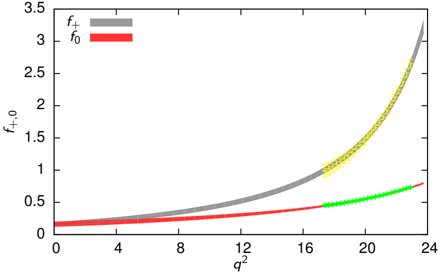
arXiv:1711.08085

z expansion

- Functional method with BCL parameterization is used.
- f_+ and f_0 are fit simultaneously, including terms up to z^3 in the central fit.
- The data satisfy the kinematic constraint $f_+(q^2 = 0) = f_0(q^2 = 0)$.
- The data satisfy the unitarity condition $\sum_{m,n=0}^{N_z} B_{mn} b_m b_n \le 1$.



PRELIMINARY

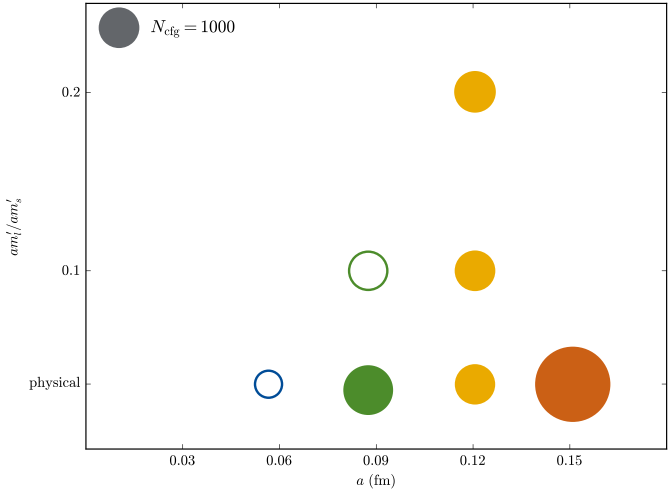


PRELIMINARY

arXiv:1711.08085

Part III — (2+1+1)-flavor HISQ $B_s \to K$, $B \to K$, $B \to \pi$

MILC HISQ ensembles



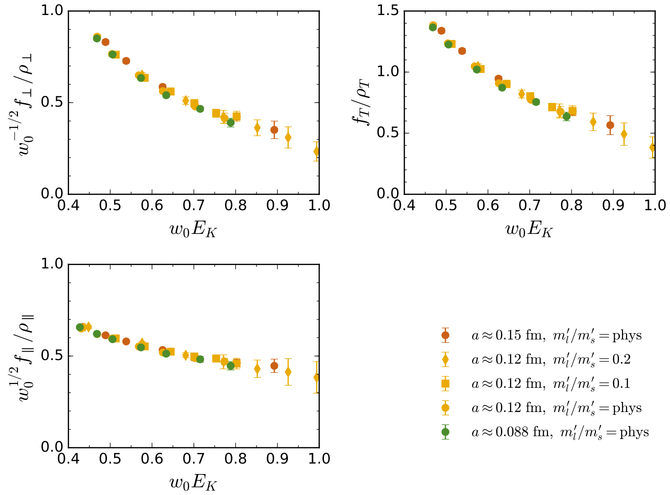
Open circles: datasets that are in progress and not used in this analysis.

Actions and parameters

- MILC $N_f = 2+1+1$ ensembles
- Lüscher-Weisz gauge action $\to O(\alpha_s^2 a^2)$
- HISQ action for $q_l, s, c \to O(\alpha_s a^2)$
- Clover action with Fermilab interpretation for $b \to O(\alpha_s a, a^2)\, f\left((m_b a)^2\right)$
- Scale set with w_0, where $w_0^{a=0} = 0.1714(15)$ fm
- Includes physical quark masses at each lattice spacing

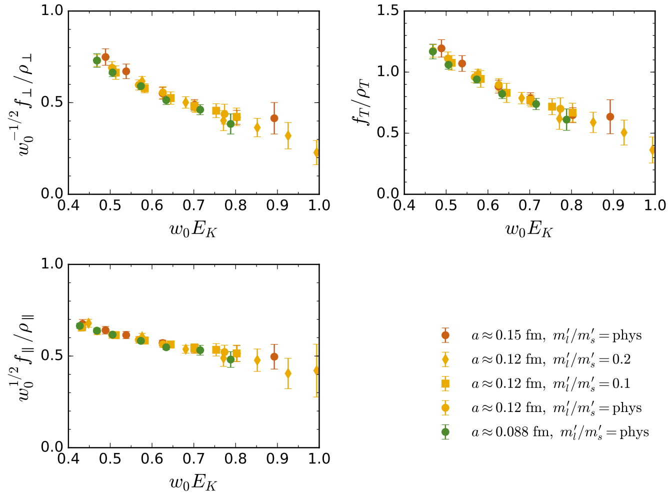
a (fm)	0.1509(14)	0.1206(14)	0.1206(11)	0.1206(11)	0.0875(8)
$N_{\text{cfg}} \times N_{\text{src}}$	3630×8	1053×8	1000×8	986×8	1535×8
$N_s^3 \times N_4$	$32^3 \times 48$	$24^3 \times 64$	$32^3 \times 64$	$48^3 \times 64$	$64^3 \times 96$
am_l'	0.00235	0.0102	0.00507	0.00184	0.0012
am_s'	0.0647	0.0509	0.0507	0.0507	0.0363
am_c'	0.831	0.635	0.628	0.628	0.432
κ_b'	0.07732	0.08574	0.08574	0.08574	0.09569
w_0/a	1.1468(4)	1.3835(10)	1.4047(9)	1.4168(10)	1.9470(13)
$\alpha_V(2/a)$	0.45275	0.38138	0.38138	0.38138	0.31391

Form factors for $B_s \to K$



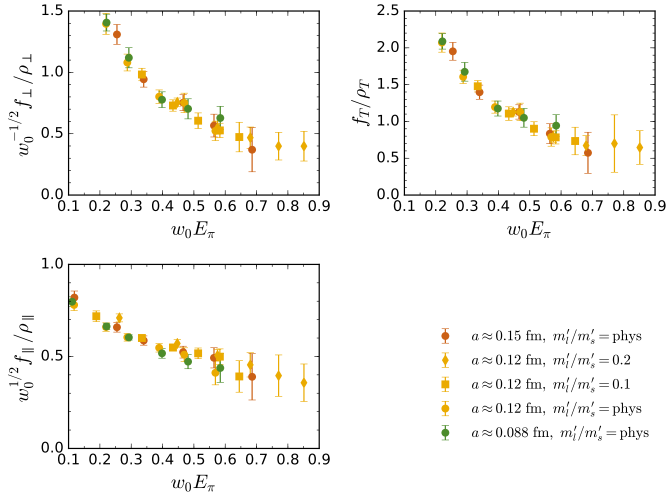
PRELIMINARY

Form factors for $B \to K$



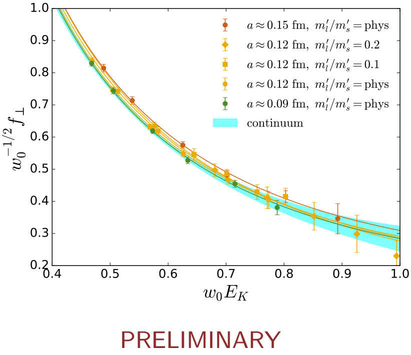
PRELIMINARY

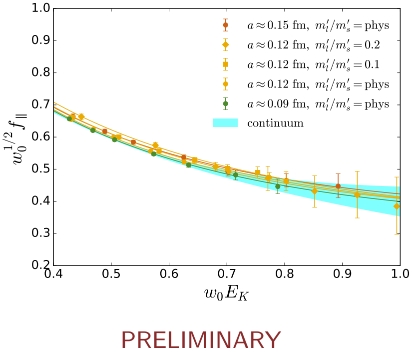
Form factors for $B \to \pi$



PRELIMINARY

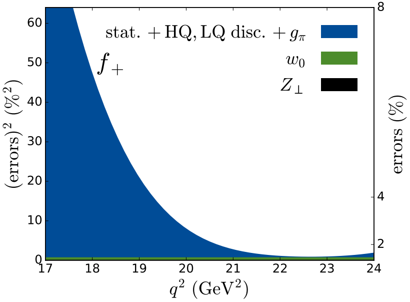
Form factors for $B_s \to K$ in the chiral continuum



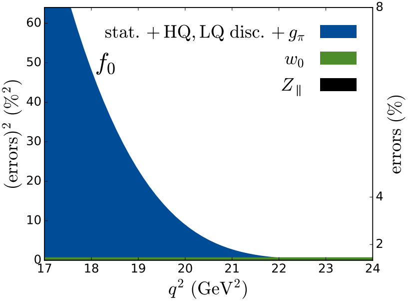


- The asqtad perturbative matching factors ρ_J are used here.
- NNLO HMrSχPT is used as the central fit.

Error budget for $B_s \to K$



PRELIMINARY



PRELIMINARY

- Statistical errors at high q^2 are reduced.
- Errors due to the chiral-continuum fit are removed.

Part IV — Conclusion

1. Introduction
   a. Form factors
   b. Methods of analysis

2. (2+1)-flavor asqtad $B_s \to K$
   - Analysis led by Yuzhi Liu

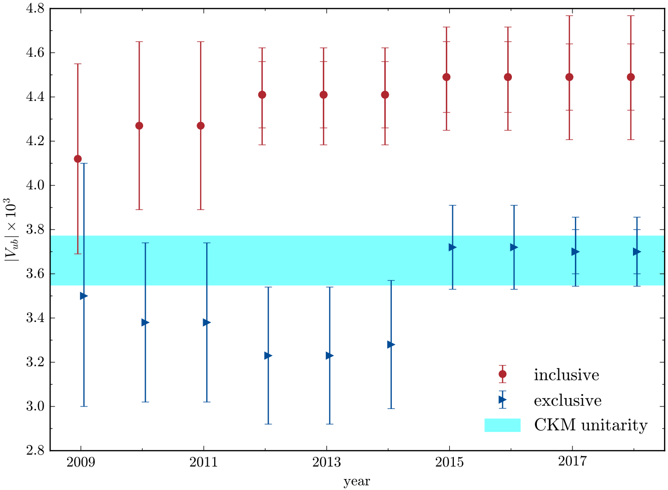
3. (2+1+1)-flavor HISQ $B_s \to K$, $B \to K$, $B \to \pi$
   - Analysis led by Z. Gelzer

4. Conclusion

Outlook

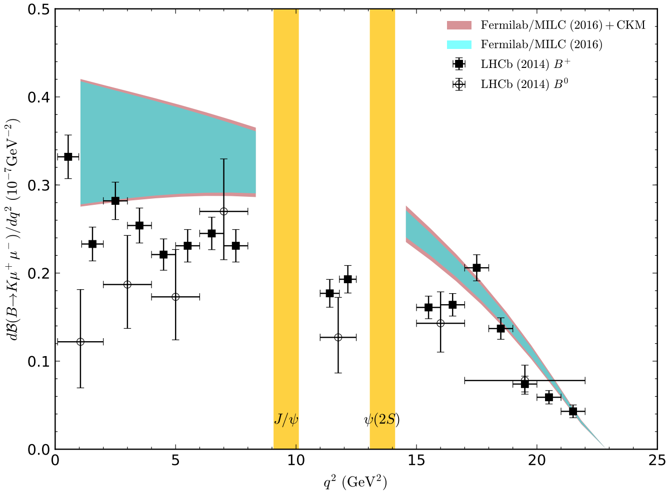
1. Complete error budget
2. Unblind current renormalization factors
3. Confront experiment
   a. Determine $|V_{ub}|$ from charged-current decays
   b. Compare $\mathcal{B}$ observables from neutral-current decays to test for new physics

Status of $|V_{ub}|$



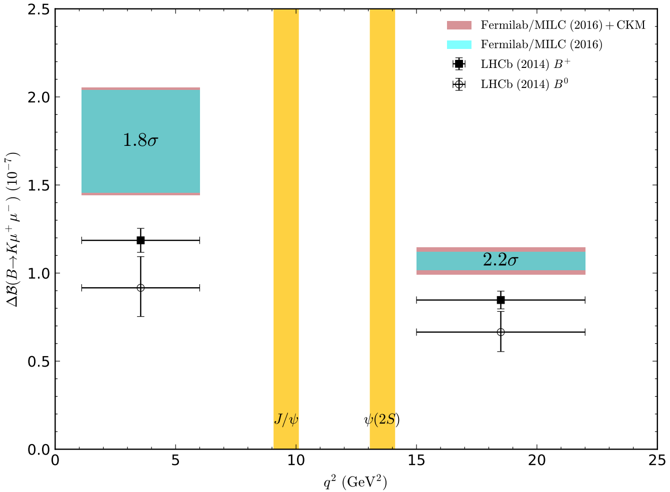
Update of plot in arXiv:1711.08085

Tension in $B \to K\mu^+\mu^-$



Older, less precise experiments omitted; cf. PRD:93.034005 [arXiv:1510.02349]

Tension in $B \to K\mu^+\mu^-$

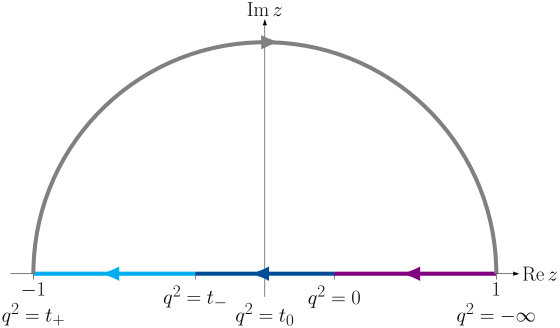


Older, less precise experiments omitted; cf. PRD:93.034005 [arXiv:1510.02349]

Thank you!

z expansion

Conformal mapping $q^2 \mapsto |z| \leq 1$ exploits analytic structure in complex plane to extend chiral-continuum form factors to low q^2.



$$z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

where $t_\pm = (M_B \pm M_P)^2$

$$\Downarrow$$

$$f(q^2) = \frac{1}{1 - \frac{q^2}{M^2}} \sum_n a_n z^n(q^2)$$

- t_0^{opt} minimizes $|z|$ in physical region $\Rightarrow |z| \leq \{0.30, 0.15\}$ for $P = \{\pi, K\}$
- Smallness of $|z|$ controls truncation
- Unitarity guarantees convergence

return to analysis