Progress on the nature of the QCD thermal transition as a function of quark flavors and masses

Francesca Cuteri, Owe Philipsen and Alessandro Sciarra

partially based on Phys.Rev. D93 (2016) no.5, 054507

36th Annual International Symposium on Lattice Field Theory @ MSU

July 26th, 2018

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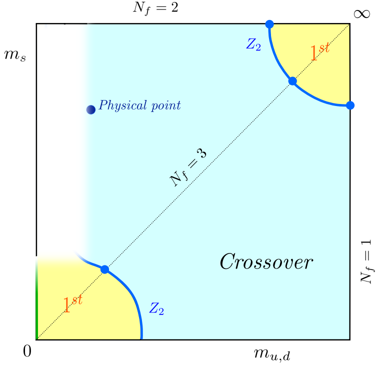
Dependence of the order of the QCD thermal phase transition on the quark masses



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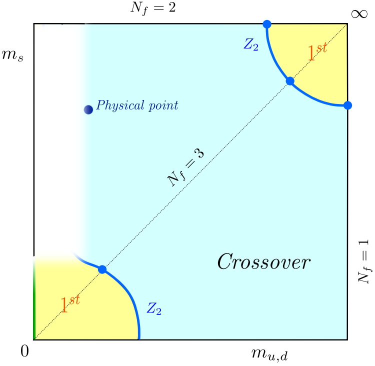

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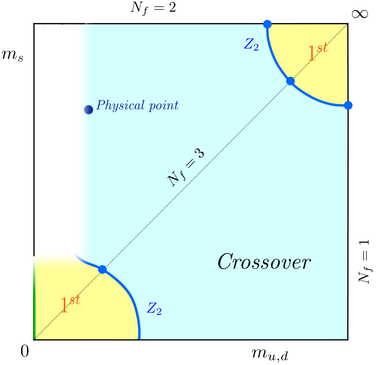

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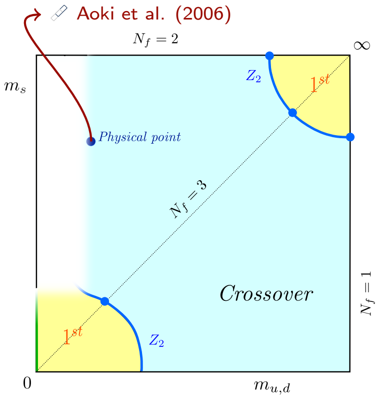

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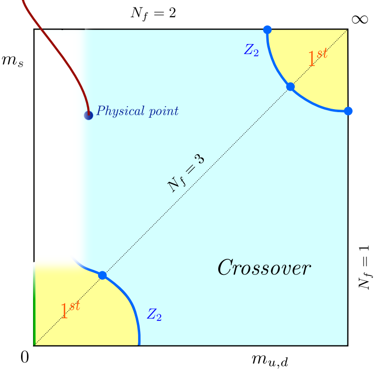

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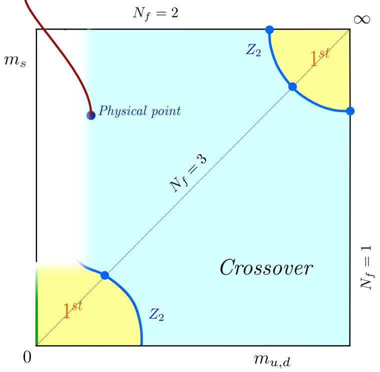

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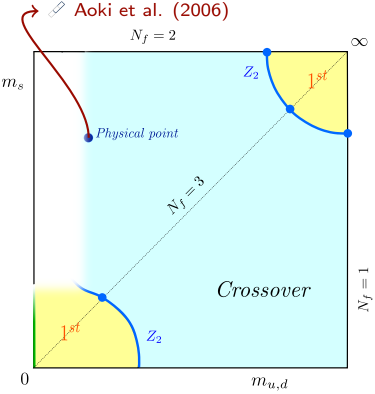

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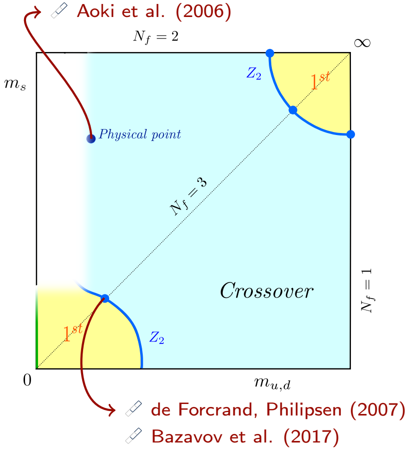

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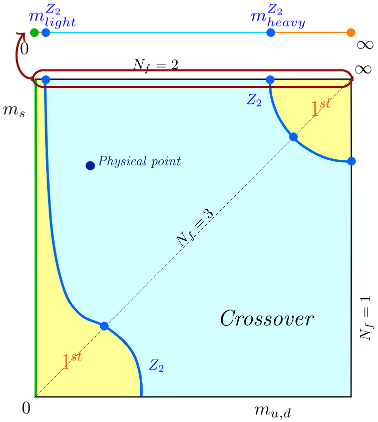
- Chiral 1^{st} order region wider for larger N_f, until $N_f = 4$ de Forcrand, D'Elia (2017)
- Strong cut-off and discretization dependence of chiral Z_2 boundary
  - Critical quark masses unreachably small for highly improved actions

$(m_s, m_{u,d})$ Columbia plot in the continuum

- Columbia plot from the "unimproved viewpoint". Focus on $N_\mathsf{f} = 2$.
  - Qualitative/~~Quantitative~~ agreement between Wilson and staggered studies



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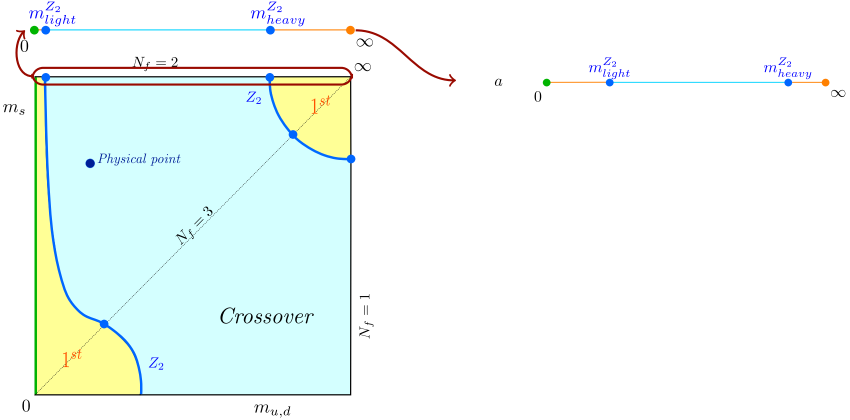

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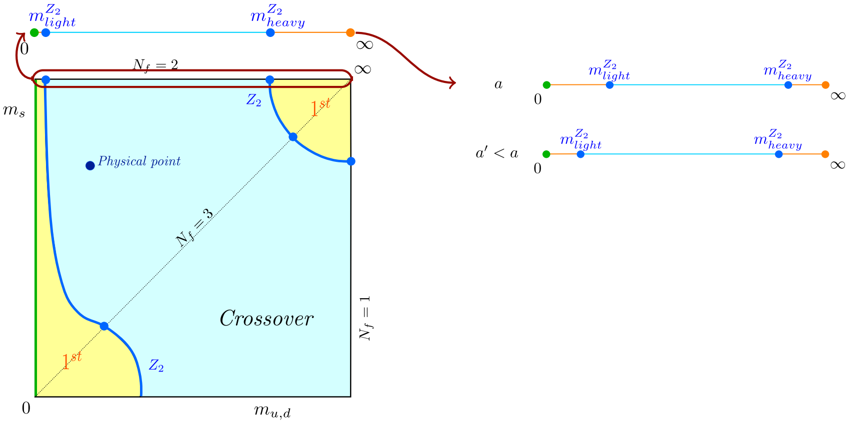

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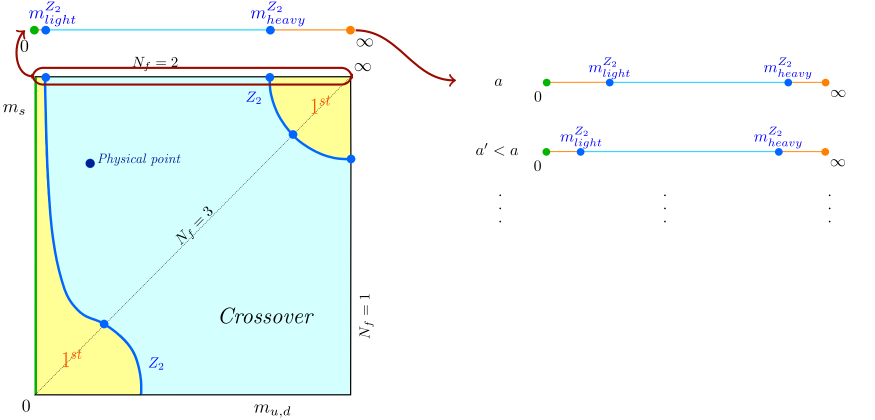

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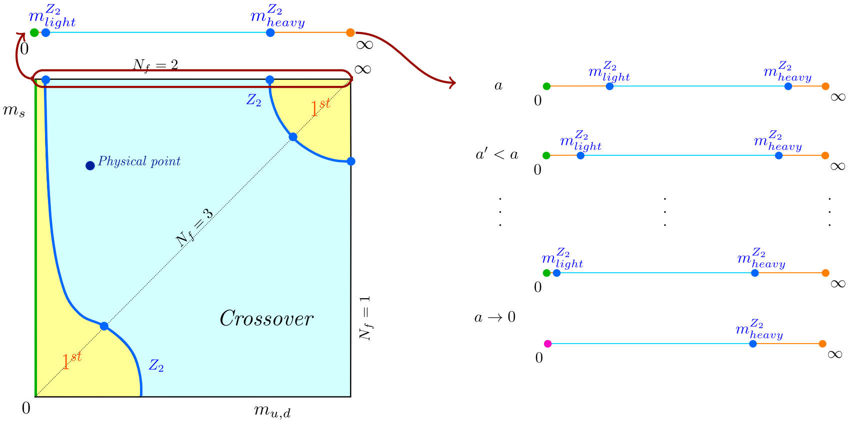

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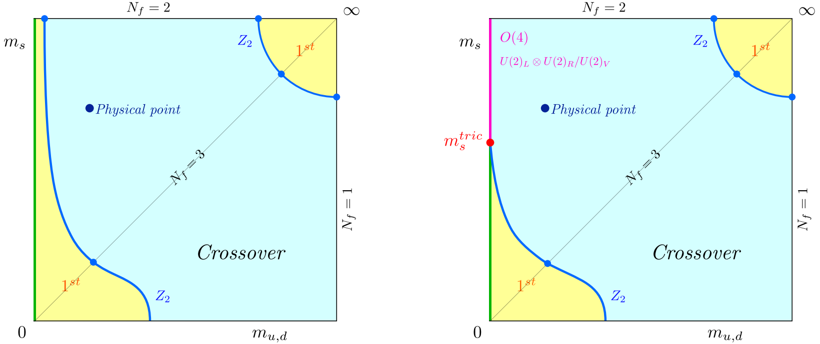
At least two possible scenarios for the nature of $N_f = 2$ chiral transition



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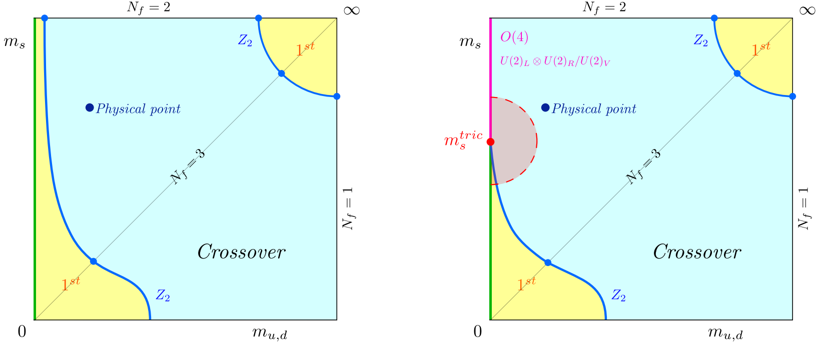

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- "Indirect approaches" exploit tricritical scaling laws for controlled extrapolations to the chiral limit
  - From imaginary chemical potential $\mu = i\mu_i$
  - At $\mu = 0$ and $(2+1)N_f$

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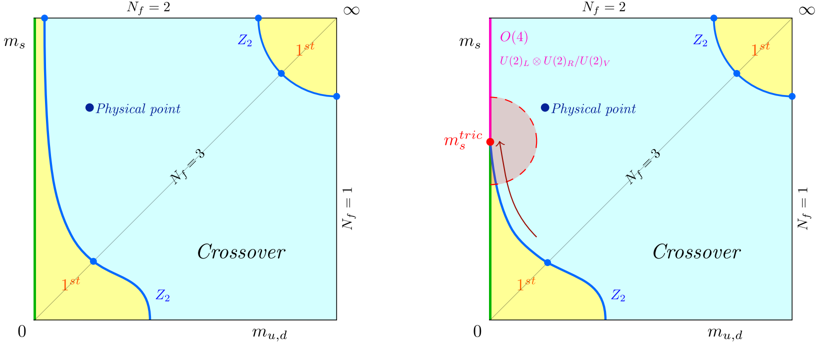

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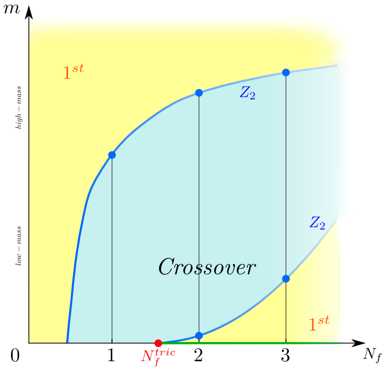
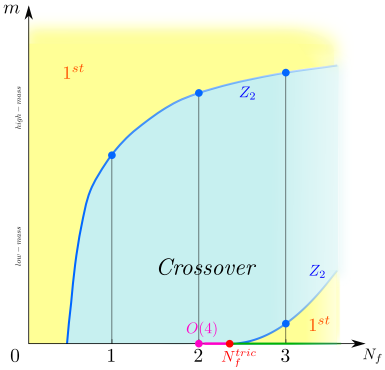
Alternative (m, N_F) Columbia plot

Yet another "indirect approach", promoting N_f to continuous real parameter $m_{Z_2}(N_f)$ according to the two considered scenarios



$N_f^{tric} < 2$



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Continuous N_F in the path integral

- Partition function describing N_f flavors of degenerate mass m

$$Z_{N_f}(m) = \int \mathcal{D}U \left[\det M(U,m)\right]^{N_f} e^{-\mathcal{S}_G}$$

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- ∞ many interpolations such that $\lim_{N_f \to 2,3} \mathcal{Z}(N_f = 2.\#) = \mathcal{Z}(N_f = 2,3)$

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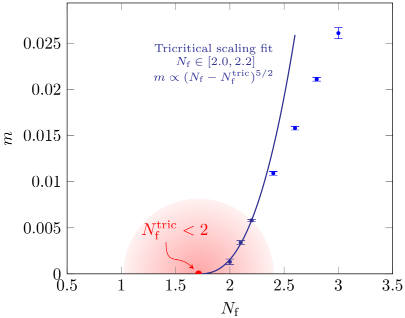
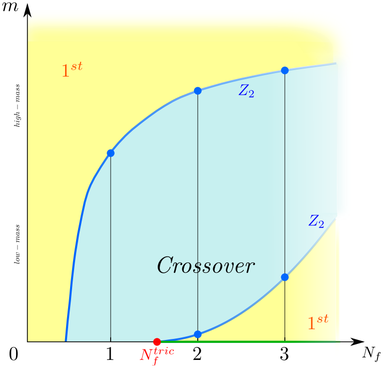
- $Z_{N_f=2.\#}$ partition function of a statistical system that represents one particular interpolation between QCD with integer $N_f^!$
- Relative position of N_f^{tric} with respect to $N_f = 2$ uniquely determined, its precise value has no meaning other than being located between two integer $N_f^!$

Conclusion from the tricritical scaling





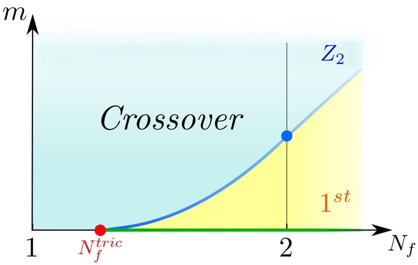
- Coarse $N_\tau = 4$ lattices explored with rooted staggered fermion discretization
- Width of the scaling window in m same as found in the extrapolation from μ_i
- Cheapest extrapolation while changing N_τ?

Outline

1. How could linearity help?
2. Strategy
3. Results

Z_2 BOUNDARY LINEAR IN SOME N_F RANGE?

- If it is reasonable to expect both linearity within some range in N_f and tricritical scaling closer to the chiral limit
  - make use of a linear extrapolation to $m = 0$ to get an upper bound for N_f^{tricr}, without the need to enter the tricritical scaling region
    - lower cost
    - no need for simulations at non integer number of flavors?!



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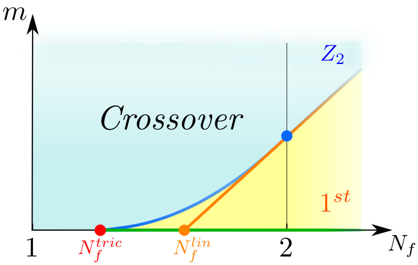

- If $N_f^{\text{lin}} < 2$, while one simulates at larger and larger N_τ towards the continuum limit
  - Transition in the $N_f = 2$ chiral limit keeps being a first order one
- As soon as $N_f^{\text{lin}} \geqslant 2$
  - No conclusion can be drawn

Outline

Code & investigated parameter space

- Simulations employ the OpenCL-based and publicly available CL^{2}QCD code
  Philipsen et al. (2014)
  - Unimproved rooted staggered fermion discretization (RHMC algorithm)

- No. of flavors $\longrightarrow$ $N_f = 3, 4, 5$ on $N_\tau = 4$
  $N_f = 3.6, 4, 4.4$ on $N_\tau = 6$
- Chemical potential $\longrightarrow$ $\mu = 0$
- Scan in mass $\longrightarrow$ $m \in [0.0075, 0.0900]$
- Finite size scaling $\longrightarrow$ $N_\sigma / N_\tau = 2, 3, 4$
- Scan in temperature $\longrightarrow$ $(3-5)$ β values, then reweighting

Strategy to map out the Z_2 boundary

- Sample the (approximate) order parameter $\mathcal{O} = \langle\bar{\psi}\psi\rangle$ and extract central moments of the distribution, which gets shifted and deformed while β is varied

$$B_n(\beta, m, N_\sigma) = \frac{\langle(\mathcal{O} - \langle\mathcal{O}\rangle)^n\rangle}{\left\langle(\mathcal{O} - \langle\mathcal{O}\rangle)^2\right\rangle^{n/2}} .$$

  - On phase boundaries, i.e. at β_c and in the thermodynamic limit

$$B_3(\beta_c, m) = 0; \qquad B_4(\beta_c, m) = \begin{cases} 1, & 1^{st} \text{ order} \\ 1.5, & 1^{st} \text{ order triple} \\ 1.604, & 2^{nd} \text{ order } Z_2 \\ 2, & \text{tricritical} \\ 3, & \text{crossover} \end{cases}$$

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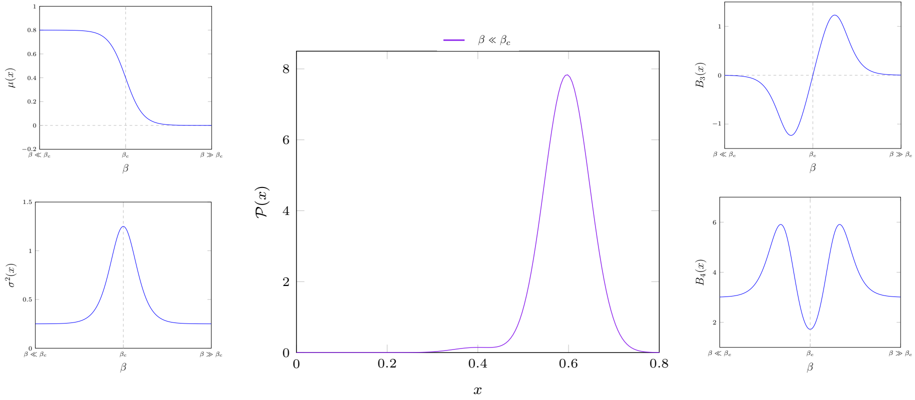
  F. C., O. Philipsen, A. Sciarra (2017)



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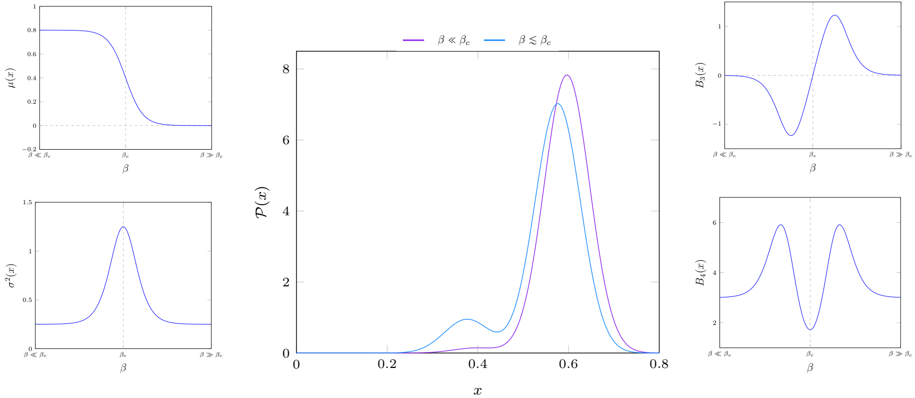

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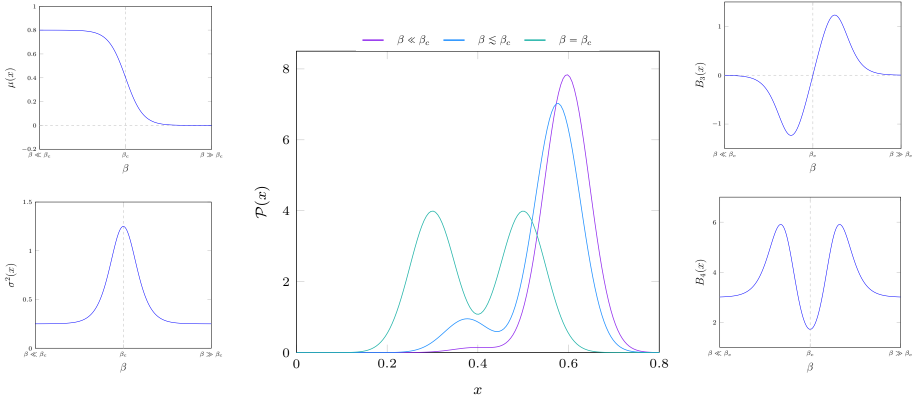

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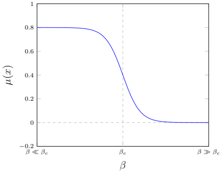

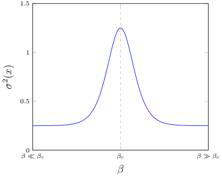

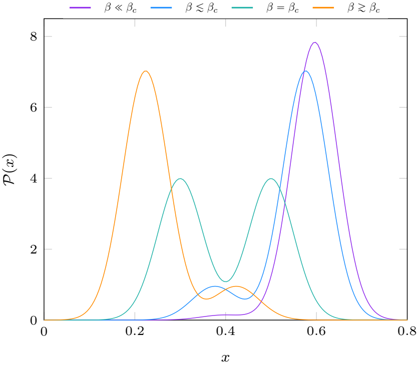

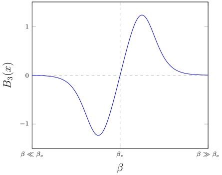

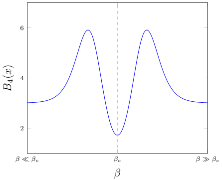

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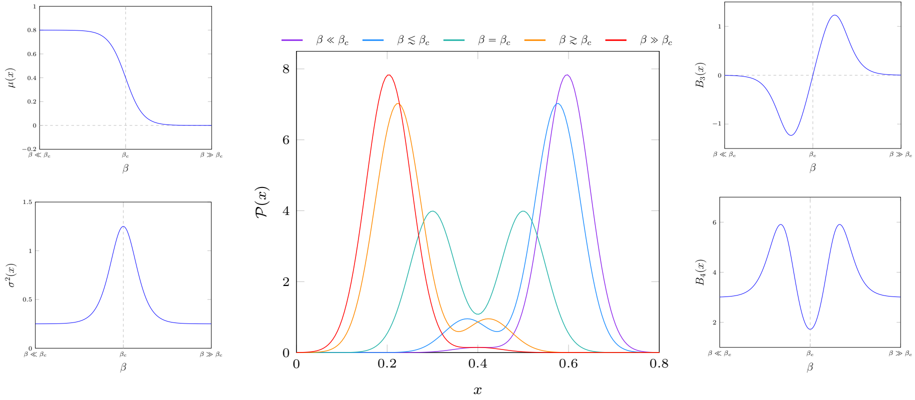
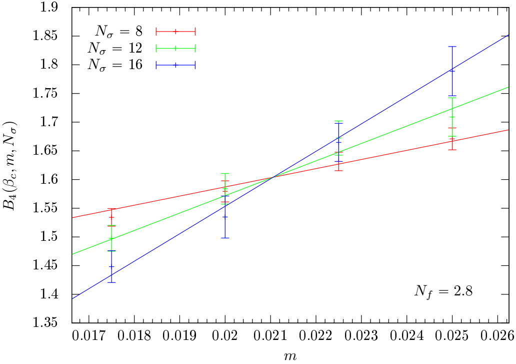


$B_3(\beta_c) = 0$ pinpoints β_c & $B_4(\beta_c)$ reveals the order of the transition

Critical parameters for Z_2 transitions

Fitting $B_4(X, N_\sigma)$ for sets at different spatial extent N_σ, as function of X

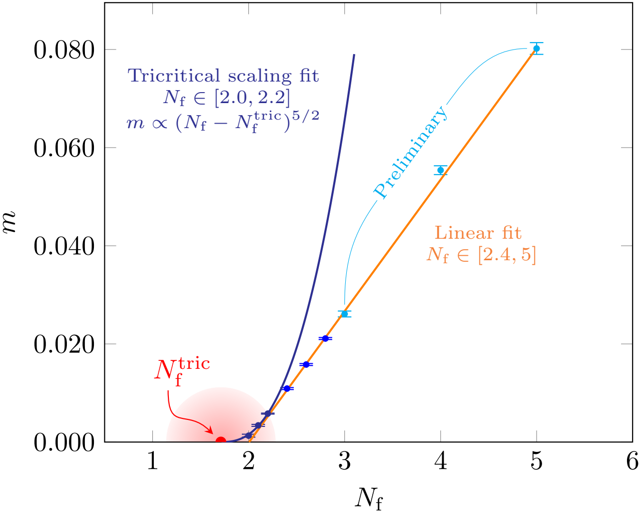


$$B_4(X, N_\sigma) = B_4(X_c, \infty) + a_1\, x + a_2\, x^2 + \mathcal{O}(x^3)$$

$$x \equiv (X - X_c) N_\sigma^{1/\nu}, \qquad X = m$$

Outline

Results, so far...



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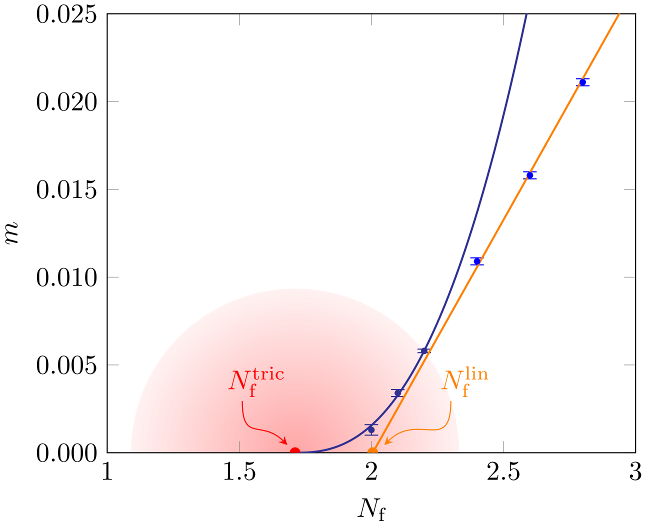
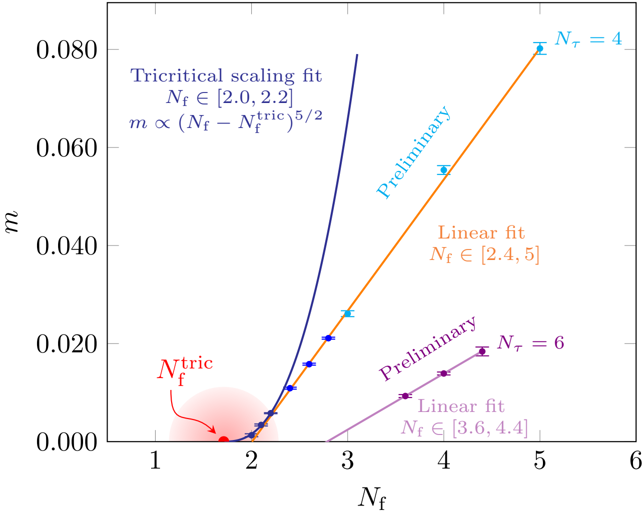
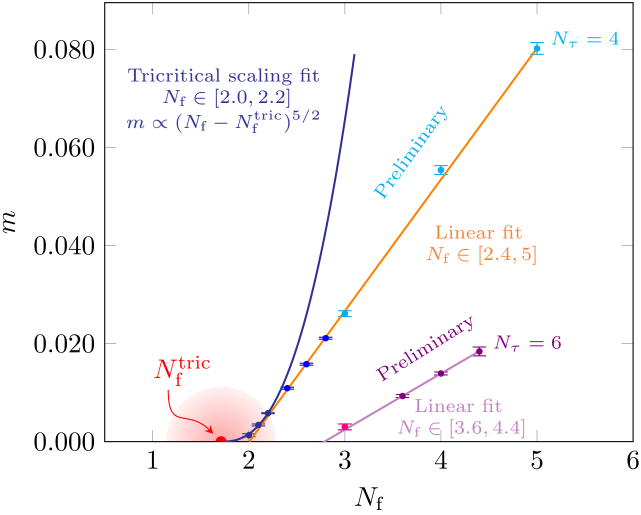


Results, so far...

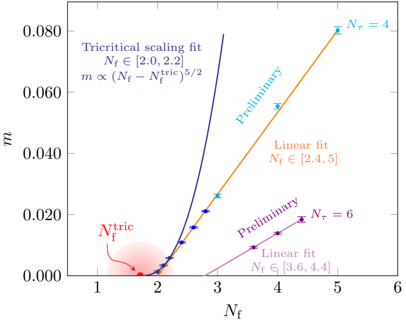


Results, so far...



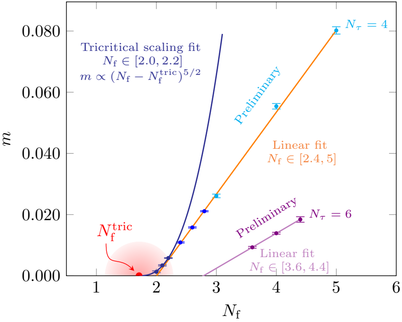
$N_f = 3$, $N_\tau = 6$ from de Forcrand, Kim, Philipsen (2007)

Conclusions, so far...



- Found $N_f^{lin} \geqslant 2$ for $N_\tau = 4, 6$.
- On the one hand
  - Nature of the chiral phase transition at $N_f = 2$ remains elusive to our linear extrapolation already on coarse lattices.
  - While being interesting on its own, linearity does not seem to help in resolving the $N_f = 2$ puzzle.
- On the other hand
  - If we think of the size of the shift in the boundary, the first order scenario would look more and more contrived with larger and larger N_τ values.
  - It would be interesting to obtain results for other fermion discretizations based on this method.

Conclusions, so far...



Thank you for your attention!

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