Status of $\bar{B} \to D^* \ell \bar{\nu}$ semileptonic decay and $|V_{cb}|$

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Outline

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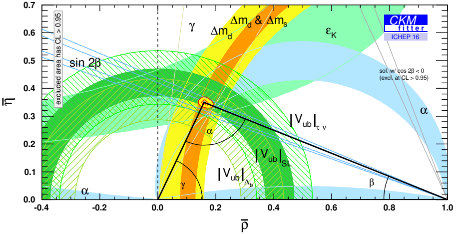
  - The weak decay $\bar{B} \to D^* \ell \bar{\nu}$
  - Available data and simulations
- Status of the analysis
  - Two-point function measurements
  - Three-point function measurements
  - Chiral-continuum limit
  - z-Expansion and parametrizations
- Summary

Introduction: The $|V_{cb}|$ CKM matrix element

- Precision test of the standard model, looking into new physics
- CKM matrix

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



Determination	$\lvert V_{cb}\rvert\ (\cdot 10^{-3})$
Exclusive	39.2 ± 0.7
Inclusive	42.5 ± 0.9

FLAG '17, HFAG '17

- Aparent 2σ tension between inclusive and exclusive determinations
- Forthcoming experiments (LHCb, Belle-II) aim to reduce the uncertainty in the determination of the CKM matrix elements

Introduction: The $|V_{cb}|$ CKM matrix element

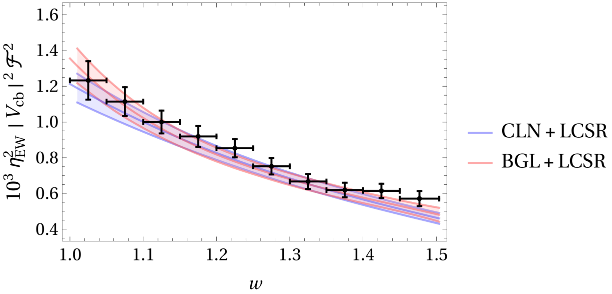
$$\frac{d\Gamma}{dw}\left(\bar{B}\to D^*\ell\bar{\nu}_\ell\right)=\frac{G_F^2 m_B^5}{48\pi^2}\left|V_{cb}\right|^2\left(w^2-1\right)^{\frac{1}{2}}P(w)\left|\eta_{ew}\mathcal{F}(w)\right|^2$$

- Experiments measure the decay rate as a function of $w = v_{D^*}\cdot v_B$
- Reduction in the phase space $(w^2-1)^{\frac{1}{2}}$ limits experimental measurements
- Lattice calculations measure the form factors and reconstruct the whole $\mathcal{F}$ function
  - $\lim_{m_Q\to\infty}\mathcal{F}(w)=\xi(w)$, which is the Isgur-Wise function
  - At large (but finite) mass $\mathcal{F}(w)$ receives corrections $O\left(\alpha_s, \frac{\Lambda_{QCD}}{m_Q}\right)$
- A fit of the form factor to a theory-motivated function (parametrization) allows one to extract V_{cb} from experimental data
- Caprini-Lellouch-Neubert (CLN) *Nucl. Phys.* **B**530 (1998) 153-181

$$F(w)=F(1)-\rho^2 z+cz^2,\quad \text{with } c=f(\rho),\quad z=\frac{\sqrt{w+1}-\sqrt{2}}{\sqrt{w+1}+\sqrt{2}}$$

Introduction: The $|V_{cb}|$ CKM matrix element

- Relies on some strong assumptions
- Tightly constrains $F(w)$: only one independent parameter



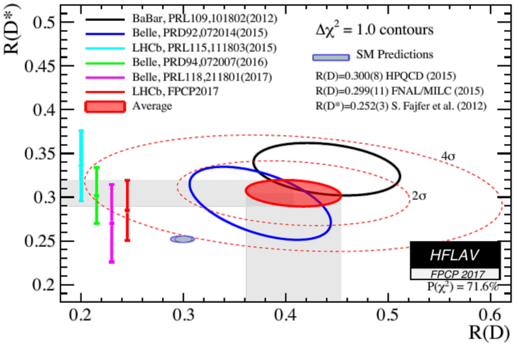
From *Phys. Lett.* **B** *769 (2017) 441-445* using Belle data at non-zero recoil and lattice data at zero recoil

- Our current understanding is that CLN might underestimate the slope at low recoil
- Current discrepancy might be an artifact
- An urgent lattice QCD calculation at $w \gtrsim 1$ is necessary to settle the issue

Introduction: The $|V_{cb}|$ CKM matrix element

Tensions in lepton universality

$$R\left(D^{(*)}\right) = \frac{\mathcal{B}\left(B \to D^{(*)}\tau\nu_\tau\right)}{\mathcal{B}\left(B \to D^{(*)}\ell\nu_\ell\right)}$$



- Current 4σ tension with the SM
- Only one calculation exists for $R(D^*)$

Introduction: The weak decay $\bar{B} \to D^* \ell \bar{\nu}$

- Form factors

$$\frac{\langle D^*(p_{D^*}, \epsilon^\nu)|\, \mathcal{V}^\mu \,|\bar{B}(p_B)\rangle}{2\sqrt{m_B\, m_{D^*}}} = \frac{1}{2}\epsilon^{\nu *} \varepsilon^{\mu\nu}_{\rho\sigma} v_B^\rho v_{D^*}^\sigma h_V(w)$$

$$\frac{\langle D^*(p_{D^*}, \epsilon^\nu)|\, \mathcal{A}^\mu \,|\bar{B}(p_B)\rangle}{2\sqrt{m_B\, m_{D^*}}} = \frac{i}{2}\epsilon^{\nu *} \left[g^{\mu\nu}\,(1+w)\, h_{A_1}(w) - v_B^\nu \left(v_B^\mu h_{A_2}(w) + v_{D^*}^\mu h_{A_3}(w)\right)\right]$$

- Playing with the polarization/momentum of the D^* we can calculate the different h_X form factors
- From the differential decay rate and the form factors (encoded in $\mathcal{F}(w)$) we can extract V_{cb}

$$\frac{d\Gamma}{dw} = \frac{G_F^2 M_B^5}{4\pi^3} r^3 (1-r^2)(w^2-1)^{\frac{1}{2}} \left|\eta_{EW}\right|^2 \left|V_{cb}\right|^2 \chi(w) \left|\mathcal{F}(w)\right|^2$$

Introduction: The weak decay $\bar{B} \to D^* \ell \bar{\nu}$

- Helicity amplitudes

$$H_\pm = \sqrt{m_B\, m_{D^*}}(w+1)\left(h_{A_1}(w) \mp \sqrt{\frac{w-1}{w+1}} h_V(w)\right)$$

$$H_0 = \sqrt{m_B\, m_{D^*}}(w{+}1) m_B \left[(w-r)h_{A_1}(w) + (w-1)\left(r\, h_{A_2}(w) + h_{A_3}(w)\right)\right] / \sqrt{q^2}$$

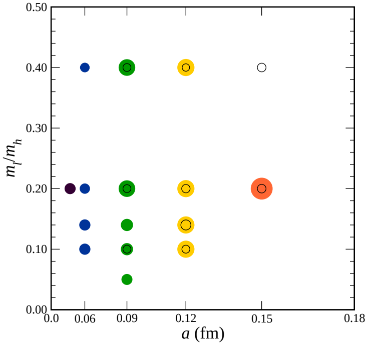
$$H_S = \sqrt{\frac{w^2-1}{r(1+r^2-2wr)}}\left[(1+w)h_{A_1}(w) + (wr-1)h_{A_2}(w) + (r-w)h_{A_3}(w)\right]$$

- Form factor in terms of the helicity amplitudes

$$\chi(w)\,|\mathcal{F}|^2 = \frac{1-2wr+r^2}{12 m_B\, m_{D^*}\,(1-r)^2}\left(H_0^2(w) + H_+^2(w) + H_-^2(w)\right)$$

Introduction: Available data and simulations

- Using 15 $N_f = 2 + 1$ MILC ensembles of sea asqtad quarks
- The heavy quarks are treated using the Fermilab action



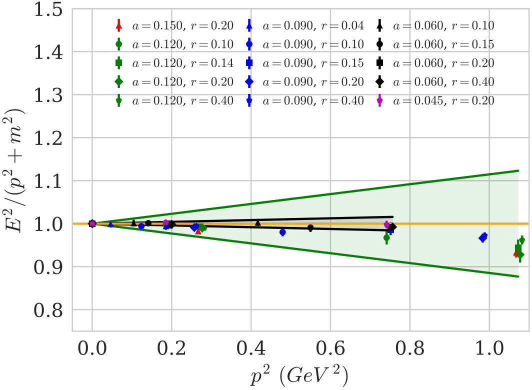
Analysis: Two-point functions

- Used three different smearings: point-point (d, d), smeared-smeared $(1S, 1S)$ and the symmetric average $(d, 1S)$ and $(1S, d)$.
  - The point sources help with the excited states, whereas the smeared sources increase the accuracy of the ground state
- t_{Min} in physical units is common to all the ensembles, t_{Max} is chosen when the points reach 20%-30% error
- Two sets of different data
  - D^* momenta (1,0,0) and (2,0,0) in lattice units, distinguish parallel from perpendicular momenta ($\perp$,$\|$ to the polarization or the current), six correlators per ensemble and momentum
    - We distinguish $Z_{\|}$ and $Z_{\perp}$, as it will be required for the 3pt functions
  - Zero momentum for both mesons and 8 additional momenta for D^* use an average momentum, three correlators per ensemble and momentum
- Done 2 oscillating $+$ 2 non-oscillating and $3 + 3$ fits to ensure stability of the results

Analysis: Two-point functions

- Ansatz for a $N+N$ fit:

$$C_{2pt}(t) = \sum_{i=0,2,4\ldots}^{2N-1} \left[\underbrace{Z_i \left(e^{-E_i t} + e^{-E_i(T-t)}\right)}_{\text{Non-oscillating}} + \underbrace{(-1)^t Z_{i+1} \left(e^{-E_i t} + e^{-E_i(T-t)}\right)}_{\text{Oscillating}} \right]$$



Analysis: Three-point functions

- Used two (three) different smearings
- Fit ratios of three-point functions $R(t,T) = \langle \ldots \rangle / \langle \ldots \rangle$ that cancel some normalization factors and leading exponentials
- The oscillating states are suppressed through a clever weighted average

$$\bar{R}(t,T) = \frac{1}{2}R(t,T) + \frac{1}{4}R(t,T+1) + \frac{1}{4}R(t+1,T+1)$$

- The fit range in physical units is common to all the ensembles per observable
- General ansatz:

$$\bar{R}(t,T) = R\left(1 + Ae^{-\Delta E_X t} + Be^{-\Delta E_Y (T-t)}\right)$$

Analysis: Three-point functions

Calculated three-point functions

$$\frac{\langle D^*(p)|\,\mathbf{V}\,|D^*(0)\rangle}{\langle D^*(p)|\,V_4\,|D^*(0)\rangle} \to x_f, \qquad w = \frac{1+x_f^2}{1-x_f^2}$$

$$\frac{\left\langle D^*(p_\perp,\varepsilon_\parallel)\right|\mathbf{A}\left|\bar{B}(0)\right\rangle \left\langle \bar{B}(0)\right|\mathbf{A}\left|D^*(p_\perp,\varepsilon_\parallel)\right\rangle^*}{\langle D^*(0)|\,V_4\,|D^*(0)\rangle \left\langle \bar{B}(0)\right|V_4\left|\bar{B}(0)\right\rangle} \to R_{A_1}, \qquad h_{A_1} = \left(1-x_f^2\right) R_{A_1}^{\frac{1}{2}}$$

$$\frac{\left\langle D^*(p_\perp,\varepsilon_\perp)\right|\mathbf{V}\left|\bar{B}(0)\right\rangle}{\left\langle D^*(p_\perp,\varepsilon_\parallel)\right|\mathbf{A}\left|\bar{B}(0)\right\rangle} \to X_V, \qquad h_V = \frac{2}{\sqrt{w^2-1}} R_{A_1} X_V$$

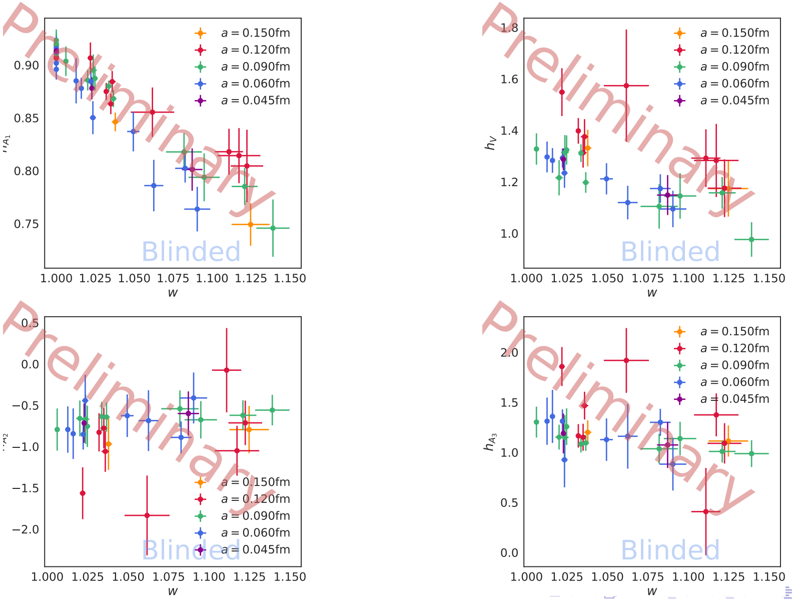
$$\frac{\left\langle D^*(p_\parallel,\varepsilon_\parallel)\right|\mathbf{A}\left|\bar{B}(0)\right\rangle}{\left\langle D^*(p_\perp,\varepsilon_\parallel)\right|\mathbf{A}\left|\bar{B}(0)\right\rangle} \to R_1, \qquad h_{A_3} = \frac{2}{w^2-1} R_{A_1}\left(w-R_1\right)$$

$$\frac{\left\langle D^*(p_\perp,\varepsilon_\parallel)\right|A_4\left|\bar{B}(0)\right\rangle}{\left\langle D^*(p_\perp,\varepsilon_\parallel)\right|\mathbf{A}\left|\bar{B}(0)\right\rangle} \to R_0,$$

$$h_{A_2} = \frac{2}{w^2-1} R_{A_1}\left(wR_1 - \sqrt{w^2-1}R_0 - 1\right)$$

* Phys.Rev. **D**66, 01503 (2002)

Analysis: Uncorrected form factors



Analysis: Heavy quark mistuning corrections

- The simulations are run at approximate physical values of m_c, m_b
- After the runs the results are corrected for the differences between the calculated and the physical masses

Correction process

1. For a particular ensemble correlators are computed at different m_c, m_b
2. All the ratios are calculated for the new values of the heavy quark masses, and the form factors are extracted
3. The derivative of combinations of the form factors with respect to the heavy quark masses is fitted to a suitable function
4. All the form factors are corrected using these results

- Shifts are small, but add a small correlation among all data points
- Corrections in m_c are noticeable, corrections in m_b are much smaller than statistical errors

Phys.Rev. D92, 034506 (2015)

Analysis: The chiral-continuum limit

- Extrapolation to the physical pion mass described by EFTs
- Functional form explicitly known

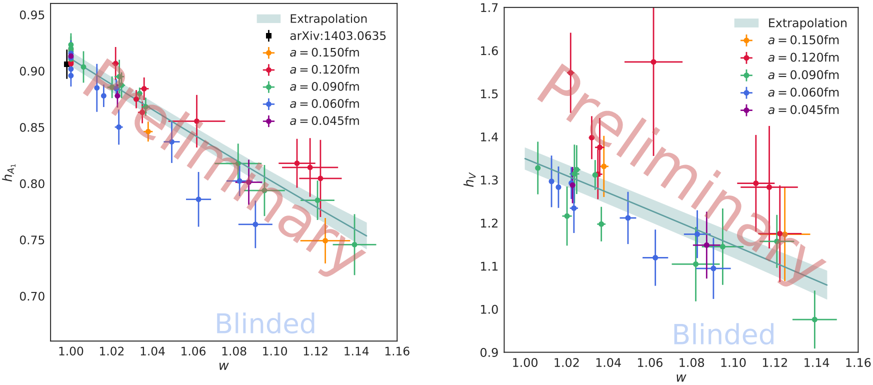
$$h_{A_1}(w) = \underbrace{1 + \frac{X_{A_1}(\Lambda_\chi)}{m_c^2} + \frac{g^2_{D^*-D\pi}}{48\pi^2 f_\pi^2 r_1^2}\mathrm{logs}_{\mathrm{SU3}}(a, m_l, m_s, \Lambda_{QCD})}_{\text{NLO}\,\chi\text{PT}\,+\,\text{HQET}} -$$

$$\underbrace{\rho^2(w-1) + k(w-1)^2}_{w\ \text{dependence}} \underbrace{+c_1 x_l + c_2 x_l^2 + c_{a1} x_{a^2} + c_{a2} x_{a^2}^2 + c_{a,m} x_l x_{a^2}}_{\text{NNLO}\,\chi\text{PT}}$$

with

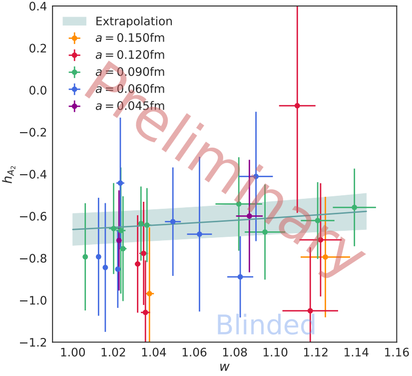
$$x_l = B_0 \frac{m_l}{(2\pi f_\pi)^2}, \qquad x_{a^2} = \left(\frac{a}{4\pi f_\pi r_1^2}\right)^2$$

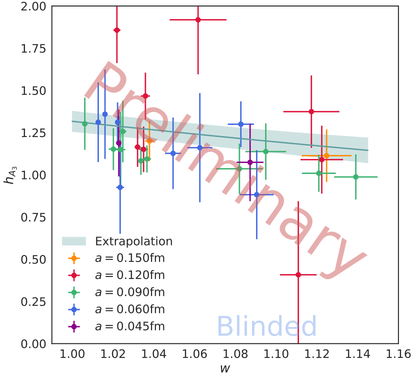
Analysis: Chiral-continuum fits



- Preliminary results, the (blinded) renormalization factors are included

Analysis: Chiral-continuum fits





- Preliminary results, the (blinded) renormalization factors are included

Analysis: z-Expansion

- Conformal transformation

$$z = \frac{\sqrt{w+1} - \sqrt{2}}{\sqrt{w+1} + \sqrt{2}}$$

- Kinematic range $w_{\text{Min}} = 1 \rightarrow z_{\text{Min}} = 0$, $w_{\text{Max}} = \frac{1+r^2}{2r} \rightarrow z_{\text{Max}} = \left(\frac{\sqrt{r}-1}{\sqrt{r}+1}\right)^2$
- Use BGL expansion (less constrained than CLN)

$$f_X(z) = \frac{1}{\phi_{f_X} B_{f_X}} \sum_j k_j z^j$$

- B_{f_X} Blaschke factors, includes contributions from the poles in the kinematic range
- ϕ_{f_X} is called *outer function* and must be computed for each form factor

Analysis: z-Expansion

- The expansion is performed on different (more convenient) form factors

$$
\begin{aligned}
g &= \frac{h_V(w)}{\sqrt{m_B\, m_{D^*}}} &&= \frac{1}{\phi_g(z) B_g(z)} \sum_j a_j z^j \\
f &= \sqrt{m_B\, m_{D^*}}(1+w) h_{A_1}(w) &&= \frac{1}{\phi_f(z) B_f(z)} \sum_j b_j z^j \\
\mathcal{F}_1 &= \sqrt{q^2} H_0 &&= \frac{1}{\phi_{\mathcal{F}_1}(z) B_{\mathcal{F}_1}(z)} \sum_j c_j z^j \\
\mathcal{F}_2 &= \frac{\sqrt{q^2}}{m_{D^*}\sqrt{w^2-1}} H_S &&= \frac{1}{\phi_{\mathcal{F}_2}(z) B_{\mathcal{F}_2}(z)} \sum_j d_j z^j
\end{aligned}
$$

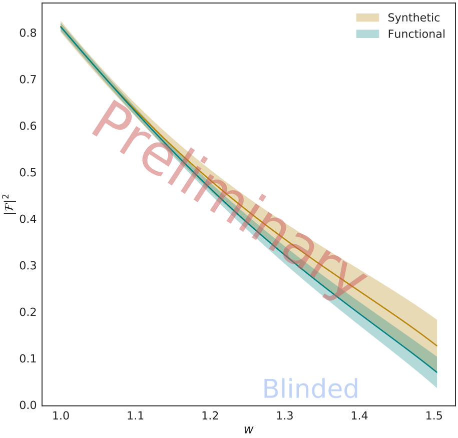
- Constraint $\mathcal{F}_1(z=0) = (m_B - m_{D^*}) f(z=0)$
- BGL unitarity constraints

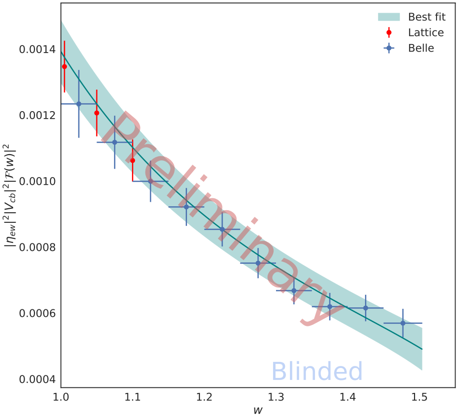
$$
\sum_j a_j^2 \leq 1, \qquad \sum_j b_j^2 + c_j^2 \leq 1
$$

Phys.Lett. **B**769, 441 (2017)

Phys.Lett. **B**771, 359 (2017)

Analysis: Lattice result and joint fit





Summary

- Blinded calculation almost completed
- Potential to improve errors and quality of fits
- Complete error budget is WIK
- Can potentially solve the inclusive-exclusive tension
- Next steps:
  - Calculation of $R(D^*)$
  - Use different actions to improve precision (HISQ + Fermilab, HISQ on HISQ...)