

$\mathcal{N} = 1$ Supersymmetric $SU(3)$ Gauge Theory - Pure Gauge sector with a twist

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Standard model of particle physics

- describes successfully what we observe at $\lesssim \mathcal{O}(10 \text{ TeV})$

Open questions

- Higgs mass
- dark matter
- unification of forces
- ...

⇒ not complete

⇒ more fundamental theory?

Possible solution

introduce supersymmetry

Table of content

1 $\mathcal{N}=1$ Super-Yang-Mills theory (recap)

2 SYM on the Lattice

- Twist term
- Results

3 DD α AMG

$\overline{\mathcal{N}}=1$ Super-Yang-Mills theory

Fields

- Gauge boson (gluon) $A_\mu(x)$ in the adjoint representation
- Super partner (gluino) $\lambda(x)$ is Majorana fermion in the adjoint representation

On-shell Lagrange density

$$\mathcal{L}_{\text{SYM}} = \text{tr} \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{i}{2} \bar{\lambda} \not{D} \lambda \right)$$

Supersymmetry: Relation between fermionic matter particles and bosonic force particles

$$\delta_\epsilon A_\mu = i \bar{\epsilon} \gamma_\mu \lambda, \quad \delta_\epsilon \lambda = i \Sigma_{\mu\nu} F^{\mu\nu} \epsilon$$

$\overline{\mathcal{N}}=1$ Super-Yang-Mills theory

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On-shell Lagrange density

$$\mathcal{L}_{\text{SYM}} = \text{tr} \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{i}{2} \bar{\lambda} \not{D} \lambda - \frac{m_g}{2} \bar{\lambda} \lambda \right)$$

Supersymmetry: Relation between fermionic matter particles and bosonic force particles

$$\delta_\epsilon A_\mu = i \bar{\epsilon} \gamma_\mu \lambda, \quad \delta_\epsilon \lambda = i \Sigma_{\mu\nu} F^{\mu\nu} \epsilon$$

Softly broken by gluino mass term

Symmetries

Chiral (R) symmetry breaking in $SU(3)$ SYM theory

- Global chiral $U(1)_A$ symmetry: $\lambda \mapsto e^{i\alpha\gamma_5} \lambda$
- Due to anomaly only \mathbb{Z}_6 remnant symmetry

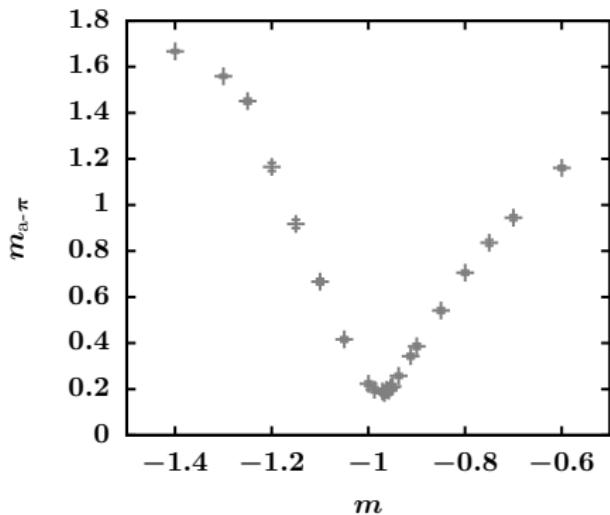
$$\lambda \mapsto e^{i\frac{2\pi n}{6}\gamma_5} \lambda \quad \text{with } n \in \{1, \dots, 6\}$$

- Spontaneously broken to \mathbb{Z}_2 symmetry in consequence of gluino condensate $\langle \bar{\lambda} \lambda \rangle \neq 0 \rightarrow 3$ different vacua

The chiral limit

$$D_W(x, y) = (4 + m) \delta_{x,y} - \frac{1}{2} \sum_{\mu=\pm 1}^{\pm 4} (\mathbb{1} - \gamma_\mu) \mathcal{V}_\mu(x) \delta_{x+\hat{\mu},y}$$

with adjoint representation $[\mathcal{V}_\mu(x)]_{ab} = 2 \text{tr} [\mathcal{U}_\mu^\dagger(x) T_a \mathcal{U}_\mu(x) T_b]$



New approach: Twist Term

$$D_W(x, y) = (4 + m) \delta_{x,y} - \frac{1}{2} \sum_{\mu=\pm 1}^{\pm 4} (\mathbb{1} - \gamma_\mu) \mathcal{V}_\mu(x) \delta_{x+\hat{\mu},y}$$

Feature of SYM

- Particular directions of \mathbb{Z}_6 symmetry are favored by gluino condensate

New approach: Twist Term

$$D_W^{\text{tw}}(x, y) = (4 + m + i\mu\gamma_5)\delta_{x,y} - \frac{1}{2} \sum_{\mu=\pm 1}^{\pm 4} (\mathbb{1} - \gamma_\mu)\mathcal{V}_\mu(x)\delta_{x+\hat{\mu},y}$$

Feature of SYM

- Particular directions of \mathbb{Z}_6 symmetry are favored by gluino condensate
- Deform lattice action by adding parity-breaking mass μ resembling a twisted mass
- m breaks chiral symmetry explicitly and generates a condensate $\sim \langle \bar{\lambda}\lambda \rangle$
- μ leads to a condensate $\sim \langle \bar{\lambda}\gamma_5\lambda \rangle$

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⇒ Possibility to get closer to chiral symmetry and supersymmetry at finite lattice spacing?

Mesonic Supermultiplet

a- η'



a- f_0



Gluino-Glue gg



Mesonic Supermultiplet

a- η'



a- f_0



Mesonic Supermultiplet

a- π

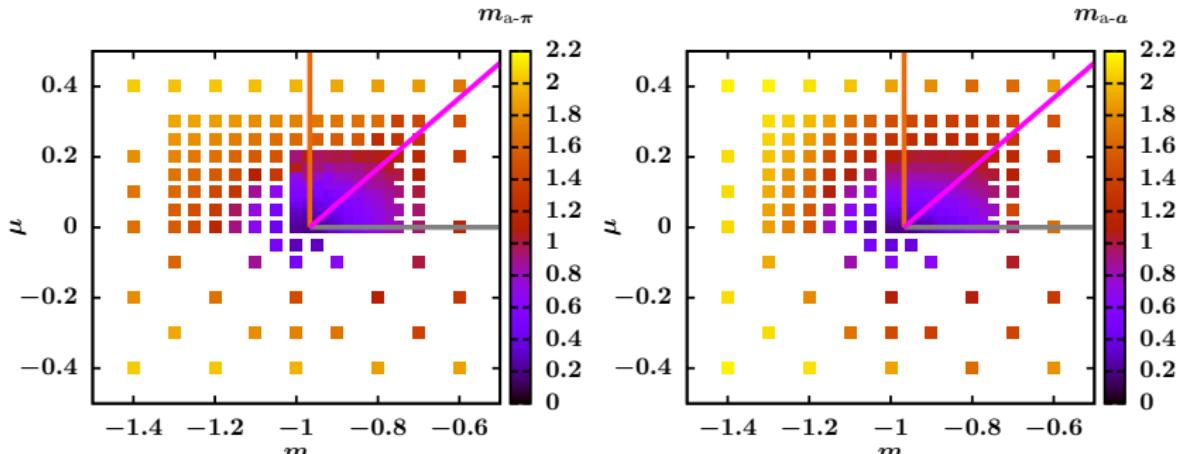


a-a



Connected Correlators

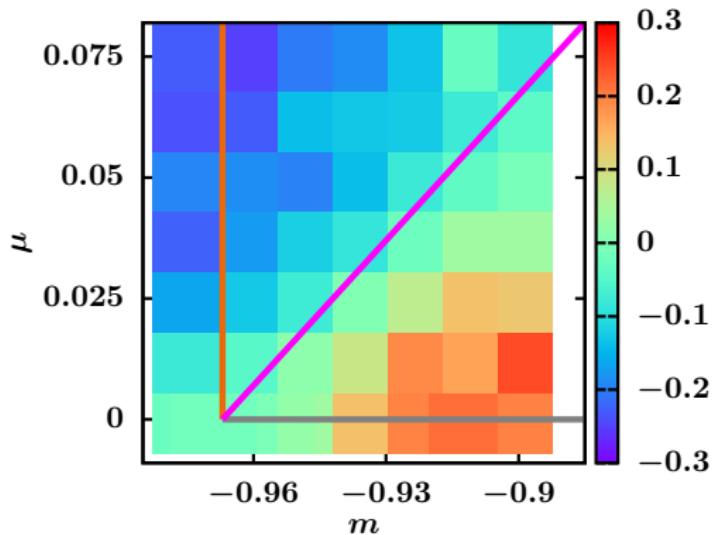
$$D_W^{\text{tw}}(x, y) = (4 + m + i\mu\gamma_5)\delta_{x,y} - \frac{1}{2} \sum_{\mu=\pm 1}^{\pm 4} (\mathbb{1} - \gamma_\mu)\mathcal{V}_\mu(x)\delta_{x+\mu,y}$$



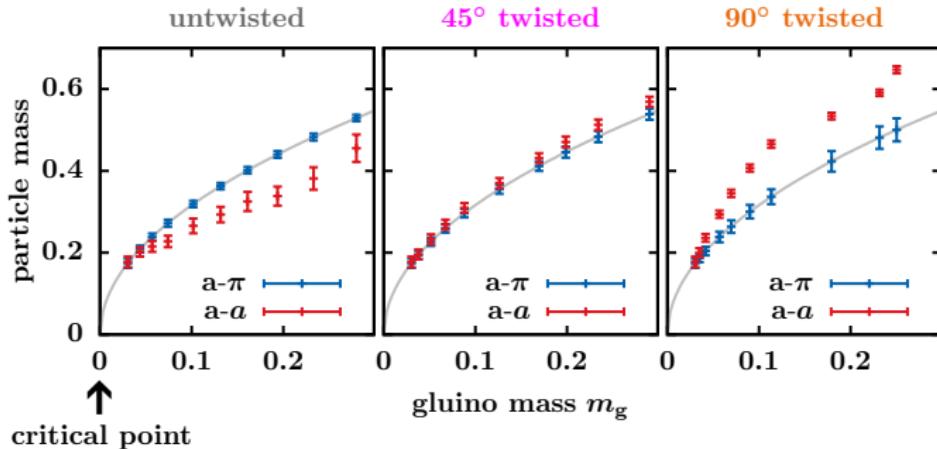
$8^3 \times 16$

Connected Correlators

$$D_W^{\text{tw}}(x, y) = (4 + m + i\mu\gamma_5)\delta_{x,y} - \frac{1}{2} \sum_{\mu=\pm 1}^{\pm 4} (\mathbb{1} - \gamma_\mu)\mathcal{V}_\mu(x) \delta_{x+\mu,y}$$
$$m_{a-\pi}/m_{a-a} - 1$$



Connected Correlators



⇒ improvement of the chiral symmetry & supersymmetry at finite lattice spacing may be possible

Mesonic Supermultiplet

a- η'

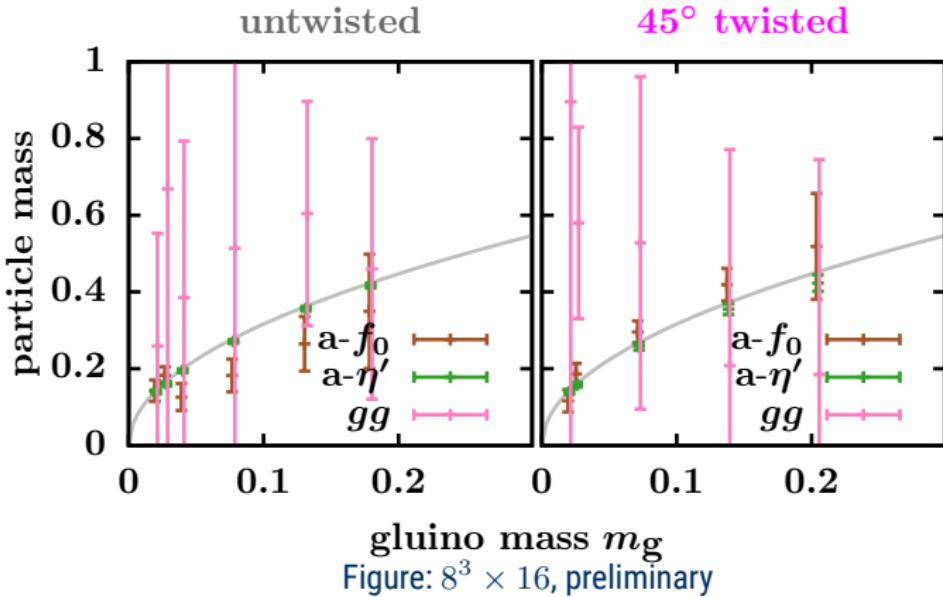


a- f_0



Gluino-Glue gg

Mesonic States



Mesonic States

45° twisted

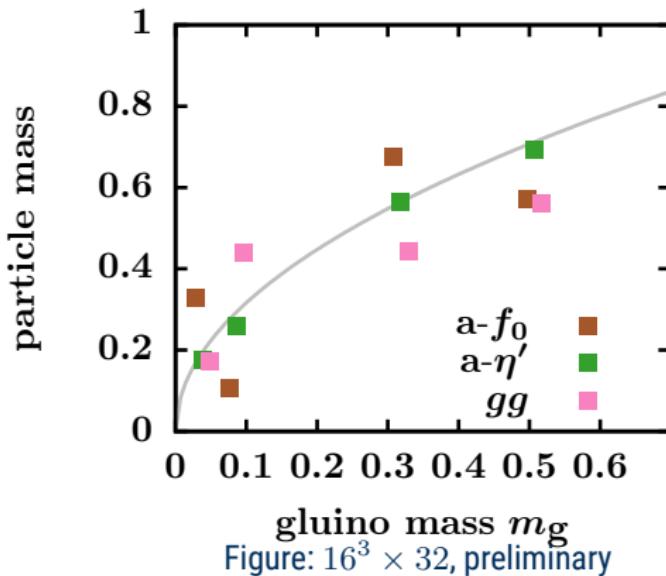


Figure: $16^3 \times 32$, preliminary

Sign of the Pfaffian

Wilson-Dirac operator $D_W = D_W^{\text{tw}}(\mu=0)$

- is γ_5 -Hermitian: $(\gamma_5 D_W)^\dagger = \gamma_5 D_W$
- is \mathcal{C} -Antisymmetric: $(\mathcal{C} D_W)^\top = -\mathcal{C} D_W$
- $\det(D_W) \in \mathbb{R}^+$
- $\text{Pf}(D_W) \in \mathbb{R}$

Sign of the Pfaffian

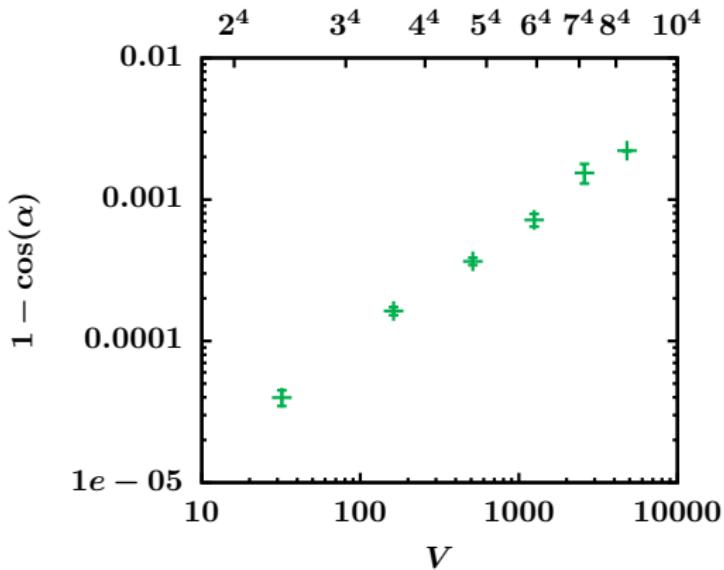
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Twisted Wilson-Dirac operator $D_W^{\text{tw}}(\mu \neq 0)$

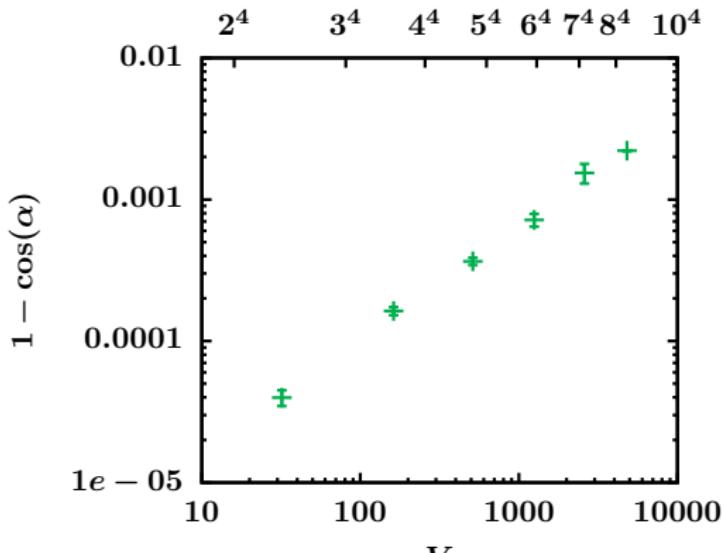
- in general $\text{Pf}(D_W^{\text{tw}}) \in \mathbb{C}$
- in continuum theory $m \rightarrow m_{\text{crit}}, \mu \rightarrow 0, a \rightarrow 0$: $\text{Pf}(D_W^{\text{tw}}) \in \mathbb{R}$
- at finite lattice spacing: phase of $\text{Pf}(D_W^{\text{tw}}) = |\text{Pf}(D_W^{\text{tw}})| \cdot e^{i\alpha}$ negligible

Sign of the Pfaffian



$$m = -0.85, \mu = 0.10, m_{a-\pi} \approx 0.70$$

Sign of the Pfaffian



extrapolated to $16^3 \times 32$:
 $1 - \cos(\alpha) < 0.035$

$$m = -0.85, \mu = 0.10, m_{a-\pi} \approx 0.70$$

$\overline{D}\alpha\text{AMG}$

Why?

- Measurement of $a\cdot\eta'$ & $a\cdot f_0$ correlators requires many stochastic estimators
- Multigrid algorithm accelerates the inversion of $Dx = y$

Code Framework

Code Modifications

Why?

Code Framework

<https://github.com/sbacchio/DDalphaAMG>

- Code designers: Matthias Rottmann, Simone Bacchio, Artur Strelbel, Simon Heybrock, Björn Leder
- Branch of Simone Bacchio includes the twisted mass term
- Hardcoded for gauge group $SU(3)$ in fundamental representation

Code Modifications

Why?

Code Framework

Code Modifications

- Generalization for arbitrary N_c and representation using macros
- Integration into our SYM code framework

Benchmarks

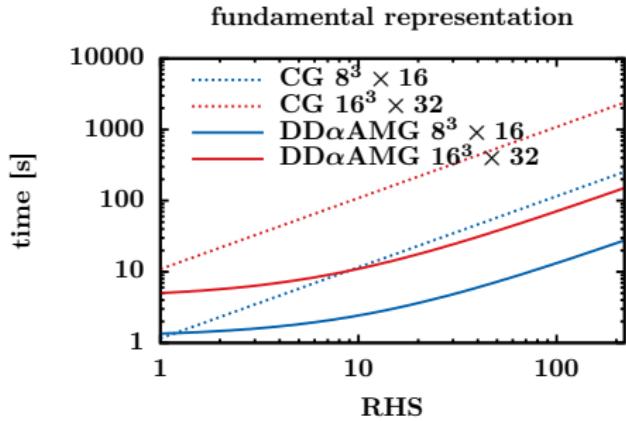
Setting

- 100 stochastic estimators
- 5 point sources
- SU(3) in fundamental & adjoint representation
- DD α AMG parameters
 - 2 levels
 - block size 2^4
 - mixed precision
 - FGMRES + red-black Schwarz

Benchmarks

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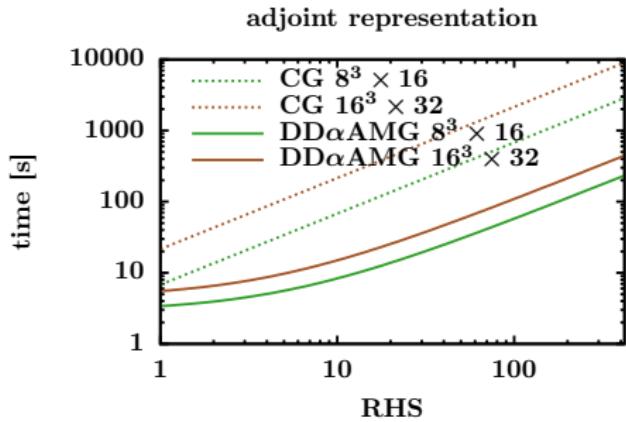


Speedup:
 $8^3 \times 16 : \sim 9$
 $16^3 \times 32 : \sim 16$

Benchmarks

Setting

- 100 stochastic estimators
- 5 point sources
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Speedup:
 $8^3 \times 16 : \sim 12$
 $16^3 \times 32 : \sim 20$

Summary

SYM on the lattice

- Lattice breaks supersymmetry
- Gluino condensate breaks remnant chiral symmetry
- Fine tuning of bare gluino mass m (and μ) necessary
- Chiral symmetry of multiplet improved at finite lattice spacing with 45° twist

Outlook

Summary

SYM on the lattice

Outlook

- More statistics to verify $m_{a-\eta'} \approx m_{a-f_0} \approx m_{gg}$ along 45° twist
⇒ improvement of the susy at finite lattice spacing may be possible
- Spectroscopy at 3 different couplings β for continuum limit
- Test DD α AMG in Monte Carlo updaters