Curvature Correlators in Lattice Quantum Gravity

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Outline

The Einstein-Hilbert Action and Assymptotic Safety

- A quick lightning review for those just arriving!
- The Einstein-Hilbert action for gravity is:
  $S_{EH} = \int d^4x \sqrt{-g} \frac{1}{16\pi G}(R - 2\Lambda)$
- Since G has dimension -2, the theory is not renormalizable.
- Asymptotic Safety: a non-trivial fixed point exists in the UV. Couplings tend to finite, but not small, values.
- Need a non-perturbative approach to gravity; to the lattice!

EDT: Euclidean Dynamical Triangulation

- Break up 4D spacetime into discrete pieces, called 4-simplices glued together along their 3D edges, tetrahedra.
- Curvature is measured at the D-2 objects (triangles) via parallel transport.
- Construct the lattice by starting with the minimal four-sphere and randomly applying the ergodic Pachner moves, allowing us to explore the space of configurations, i.e. Universe geometries.
- The discretized version of the action, called the Einstein-Regge action, is:
  $S_{ER} = -\kappa_2 N_2 + \kappa_4 N_4$,
  where N_i is the number of i-dimensional simplices, and the κ_is are coupling constants
- Use the action as the Metropolis criterion, sampling only the important configurations.
- The lattice itself is the dynamical quantity!

Motivation

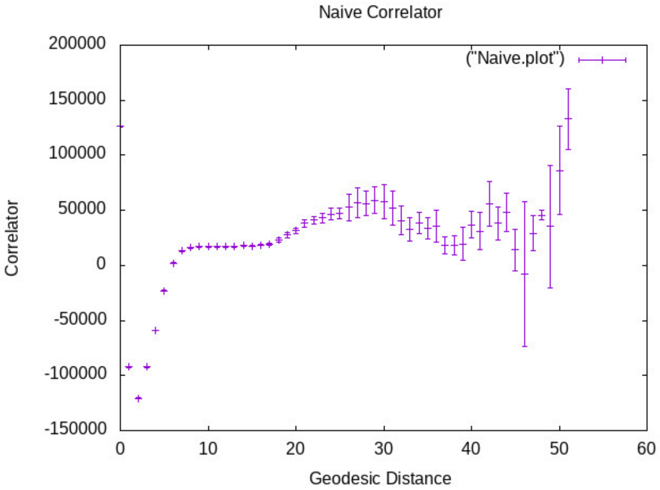
- My goal is to calculate the two-point correlator functions for curvature.
- Difficulties arise in this calculation that do not normally appear for many other two-point correlation functions.
- Hopefully we can match the expectation for gravity.

The Correlator on the Lattice

- The form of the correlator we wish to calculate is $\langle RR \rangle (r) = \frac{1}{N_2} \sum_x \frac{1}{N_{yx}} \sum_y R(x)R(y)\delta_{d(x,y),r} - \bar{R}^2$
- On the lattice, the D-2 objects (triangles) are the sites of curvature. For a given triangle, the curvature is $R_h = \frac{2A_h\delta_h}{V_h} = \frac{2\frac{\sqrt{3}}{4}(2\pi - \Theta(h)acos(1/4))}{\frac{1}{10}\frac{\sqrt{5}}{96}\Theta(h)} = 96\sqrt{15}(\frac{2\pi}{\Theta(h)} - acos(1/4))$ where the triangle h has deficit angle δ_h, area A_h, associated volume V_h, and order $\Theta(h)$.
- Distance can be measured discretely as hops between neighboring triangles. Neighbor can be defined simply as triangles sharing an edge, or those sharing an edge and existing within the same simplex. We use those that share an edge and a common simplex.
- Need to average across configurations within each ensemble.

Procedure

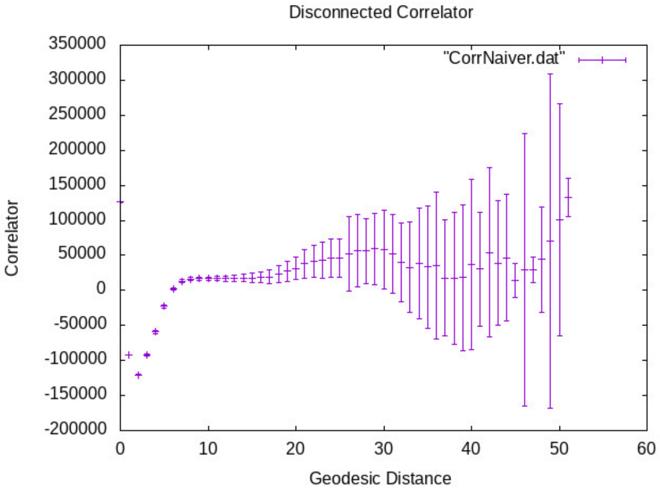
1. Select one of the configurations within the ensemble
2. Randomly select a triangle in the bulk Universe
3. Calculate the correlator treating that triangle as being at the origin, and determining neighbors, neighbors of neighbors, etc.
4. Repeat for many starting triangles.
5. Average to get the correlator for that configuration
6. Repeat for every configuration in the ensemble.
7. Average and get statistics.


Naive Correlator
("Naive.plot")
Correlator
Geodesic Distance
200000
150000
100000
50000
0
-50000
-100000
-150000
0
10
20
30
40
50
60

The Disconnected Piece

- The correlator does not asymptote, let alone to 0.
- The reason: curvature is a geometric dependent object, but so is the distance between two points. These two objects correlate!
- We see this by plotting $\langle 1R \rangle (r)$, that is, how unity at the origin correlates with distance.
- This disconnected piece must be subtracted to get the correct correlation function.

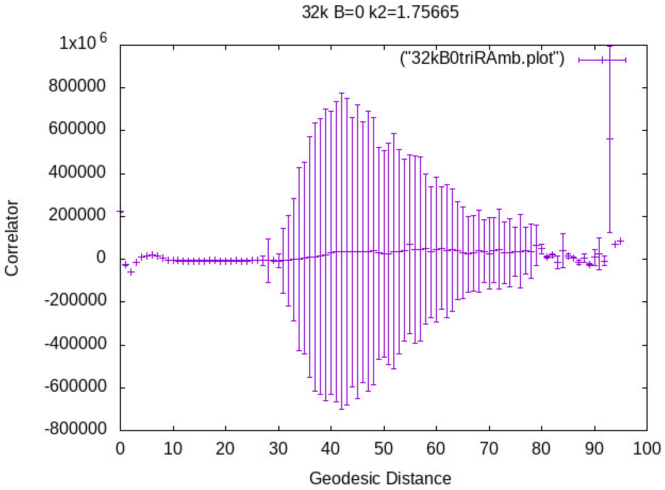
Plot of the Disconnected Piece



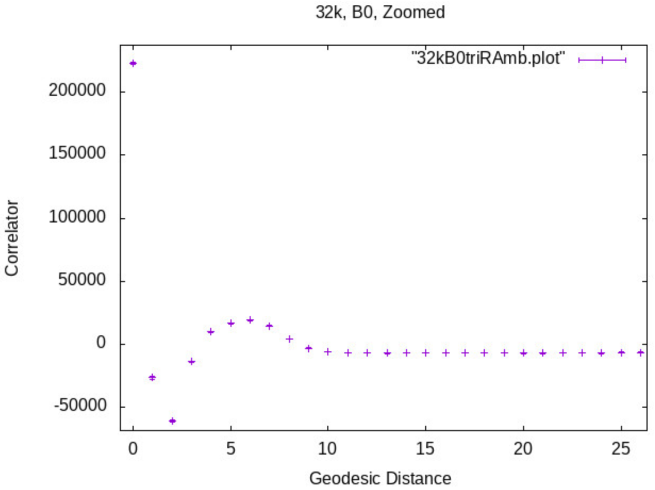
The Connected Correlator

- The simplest correction to this is to subtract off the disconnected piece, $\langle RR\rangle(r) - (\langle 1R\rangle(r))^2$
- Per Ambjorn, the disconnected contribution can be further reduced with a more complicated prescription.
- Define the functions $G^{AB}(r) = \sum_x \sum_y A(x)B(y)\delta_{d(x,y),r}$, where A and B can be R or 1.
- Take as an Ansatz that $G^{1R}(r) = AG^{11}(r+d)$, where A and d are determined by fitting.
- The desired correlator is now $\langle RR\rangle(r) = \frac{G^{RR}(r) - A^2 G^{11}(r+2d)}{G^{11}(r)}$

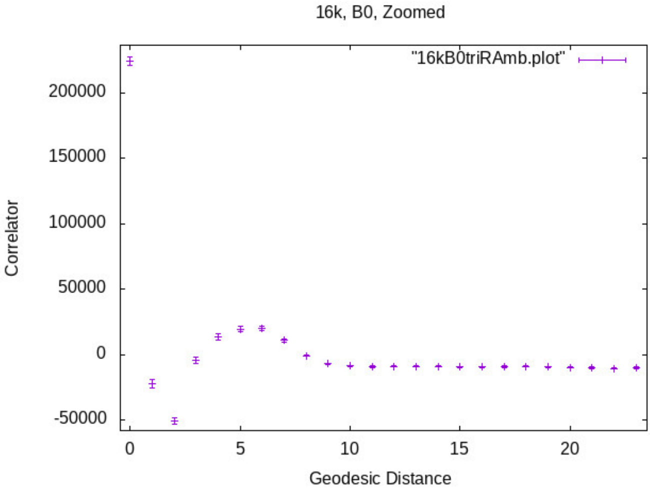
Shape of the Correlator



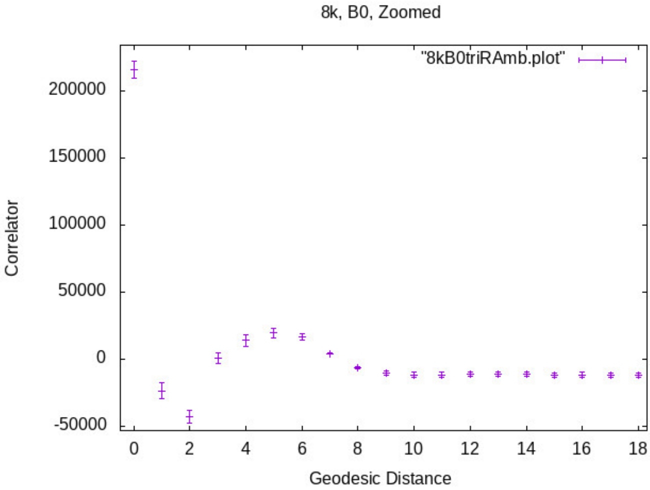
Shape of the Correlator: Zoomed In



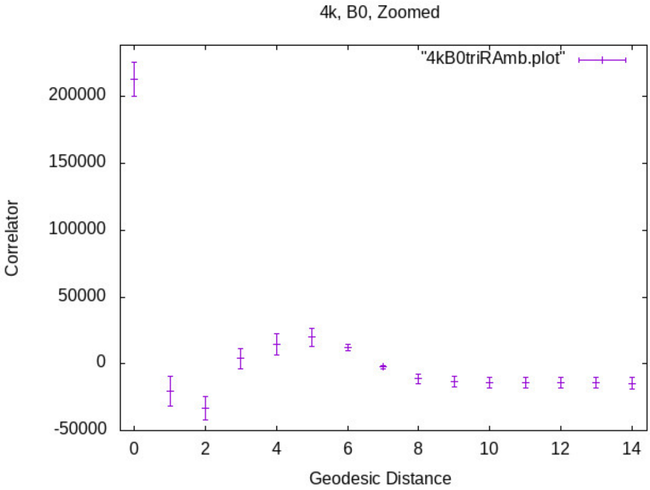
Shape of the Correlator: Zoomed In



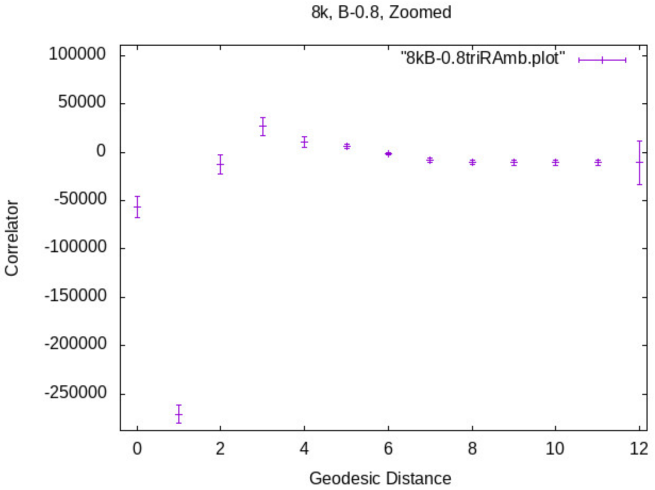
Shape of the Correlator: Zoomed In



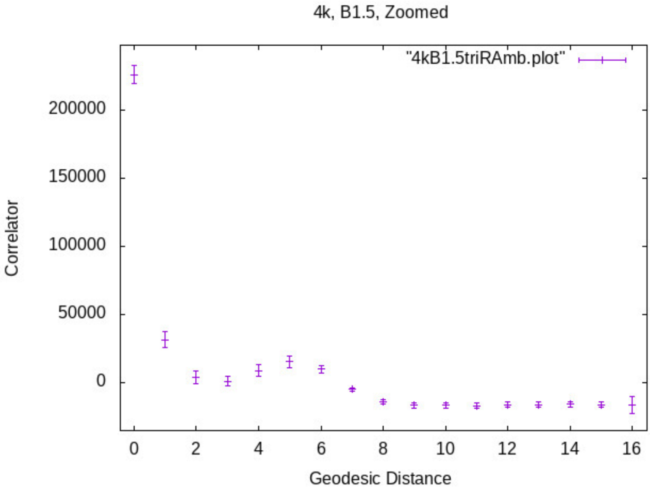
Shape of the Correlator: Zoomed In



Shape of the Correlator: Zoomed In



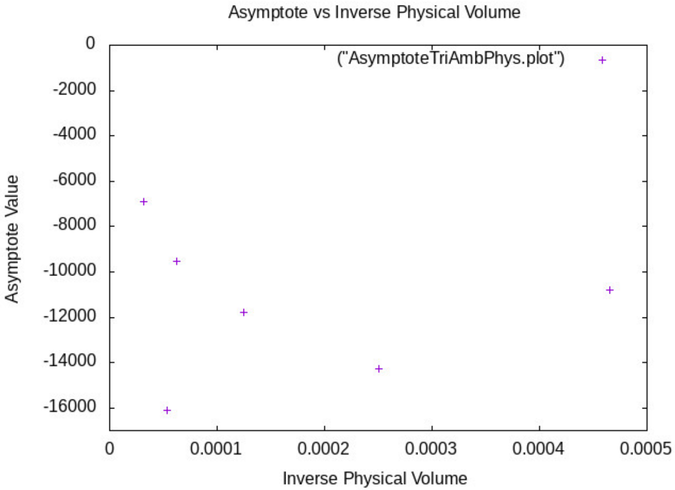
Shape of the Correlator: Zoomed In



The Asymptote

- After a large enough number of lattice spacing, the errors become very large. This is the result of the lattices not all spanning the same number of "hops", and the branched polymer baby Universes.
- For sufficiently large distances, asymptotic behavior does emerge, but the asymptotes do not tend to zero.
- This appears to be a finite size effect. Comparing the 4k, 8k, 16k, and 32k ensembles, the magnitude of the asymptote gets smaller as lattice volume increases
- Plotted against physical volume, the asymptotes appear as though they might tend to 0 in the continuum limit.

The Continuum of the Asymptotes



In Search of the Power Law Decay

- Now that the correlator appears to be well behaved, we can try to find the power law
- The power law should only manifest in the semi-classical region; i.e. large enough distances.
- Each correlator appears to have such a region consistent with a power-law.
- Fit function: $f(x) = ax^p + b$.

How the Fits Look

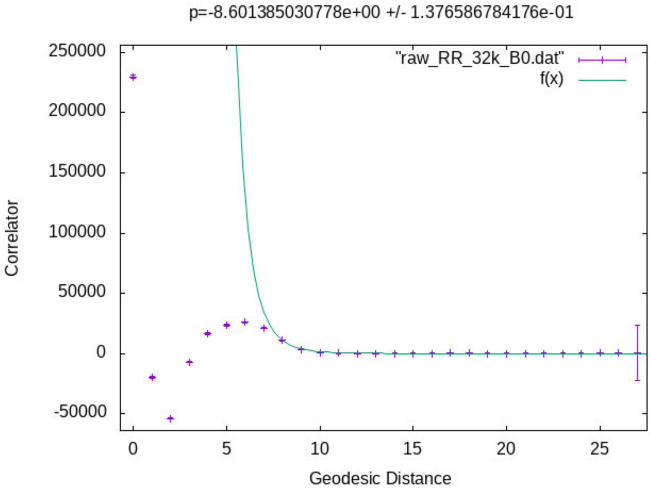


Figure: 32k, β=0, κ_2=1.75665

How the Fits Look

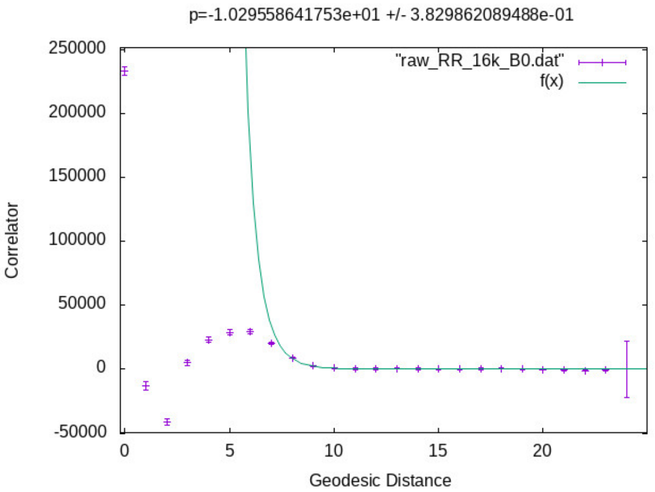


Figure: 16k, β=0, κ_2=1.7325

How the Fits Look

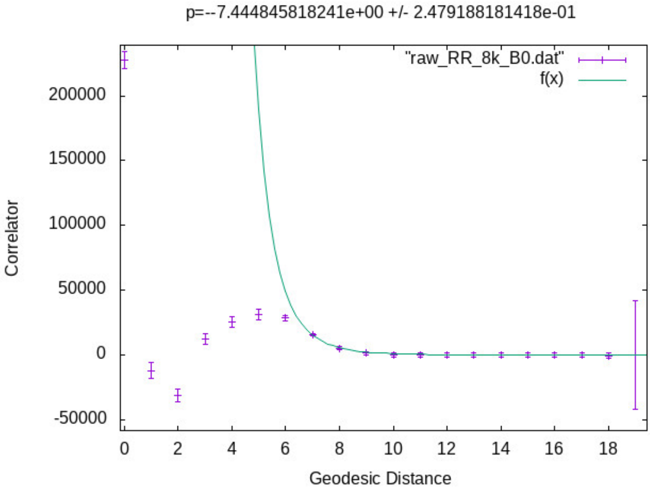


Figure: 8k, β=0, κ_2=1.7024

How the Fits Look

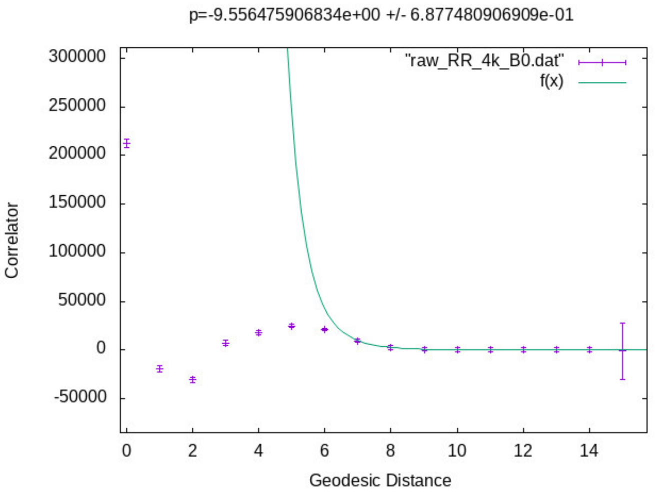


Figure: 4k, β=0, κ_2=1.669

How the Fits Look

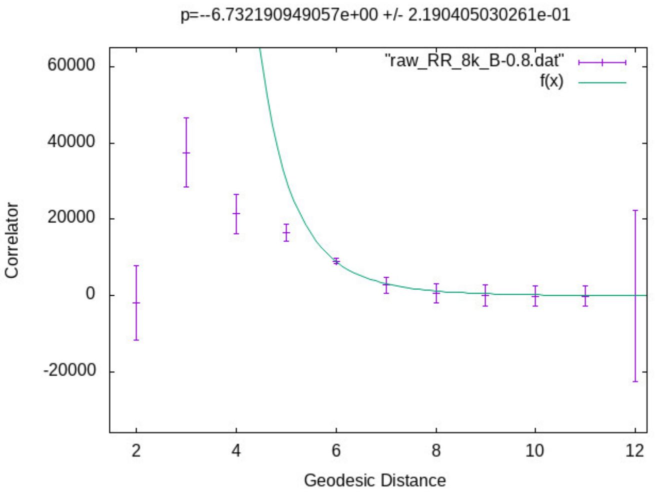


Figure: 8k, β=-0.8, κ_2=3.0

How the Fits Look

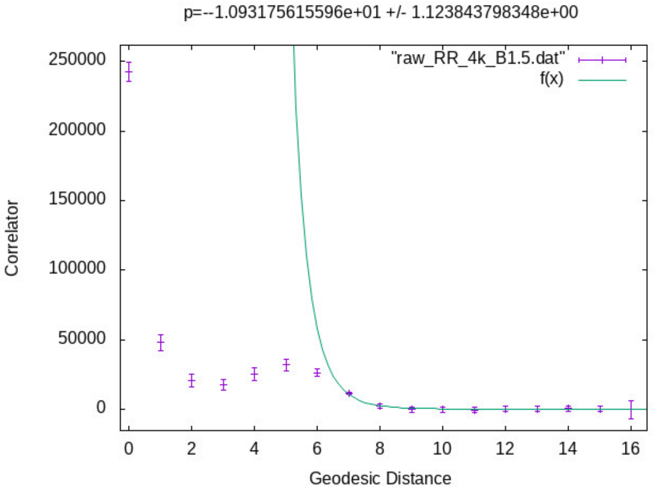
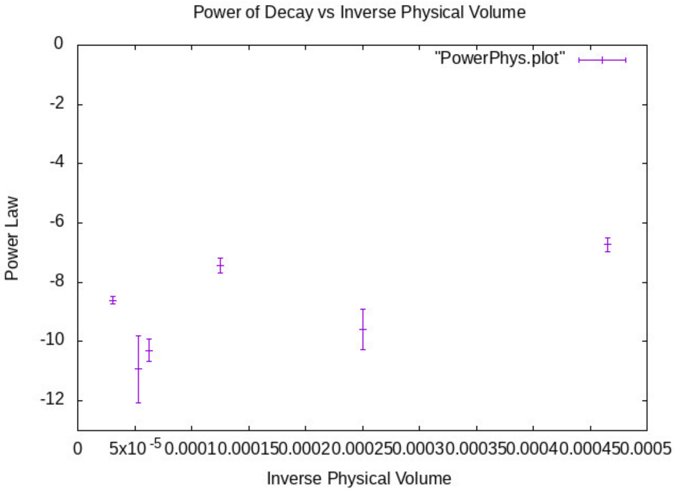


Figure: 4k, β=1.5, κ_2=0.5886

The Fit Results



The Fit Results

- A clear continuum limit behavior doesn't seem to exist in the same way it did for the drop off of the asymptote .
- If errors are being underestimated, these values could all be consistent with one another. Might this quantity be relatively unaffected by finite size effects/discretization errors?
- Semi-classical arguments suggest that the power we would have expected to get would be -8. The values we obtained seem close to this value!

Summary

- A disconnected piece contributes to the two point correlation function for curvature in quantum gravity. To get the correct behavior, this effect must be subtracted out.
- Once the disconnected piece is subtracted out, we obtain a correlation function that asymptotes, and exhibits a power law decay.
- The continuum limit for this power requires refinement, but the initial results may be consistent with semi-classical gravity.

Thank you!