Nuclear Matrix Elements for Neutrinoless Double Beta Decay from Lattice QCD

David Murphy with Will Detmold for the NPLQCD Collaboration

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The Neutrino Sector

- Neutrino oscillation experiments are becoming increasingly precise:

Δm^2_{12} [eV^2]	$7.5(2) \times 10^{-5}$	2.7%
Δm^2_{13} [eV^2] (NH)	$2.50(3) \times 10^{-3}$	1.2%
Δm^2_{13} [eV^2] (IH)	$2.42(^{+3}_{-4}) \times 10^{-3}$	1.4%
$\sin^2\theta_{12}$	$3.2(2) \times 10^{-1}$	5.5%
$\sin^2\theta_{13}$ (NH)	$2.2(^{+8}_{-7}) \times 10^{-2}$	3.5%
$\sin^2\theta_{13}$ (IH)	$2.2(^{+7}_{-8}) \times 10^{-2}$	
$\sin^2\theta_{23}$ (NH)	$5.5(^{+2}_{-3}) \times 10^{-1}$	4.7%
$\sin^2\theta_{23}$ (IH)		4.4%
δ/π (NH)	1.2(2)	10%
δ/π (IH)	$1.6(^{+1}_{-2})$	9%

Table 1: PMNS [arXiv:1708.01186]

$\sin^2\theta_{12}$	$5.09(4) \times 10^{-2}$	0.8%
$\sin^2\theta_{13}$	$1.2(1) \times 10^{-5}$	8.3%
$\sin^2\theta_{23}$	$1.72(9) \times 10^{-3}$	5.2%
δ/π	0.38(3)	7.9%

Table 2: CKM [PDG]

- Interesting, fundamental questions remain:
  1. Absolute Neutrino masses / mixing angles?
  2. Are neutrinos Dirac or Majorana?
  3. Is lepton number conserved in nature?
- Observing $0\nu\beta\beta$ would tell us a lot!

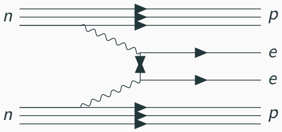


Figure 1: Quark-level $0\nu\beta\beta$ decay via long-distance Majorana exchange

$0\nu\beta\beta$ Searches

- Active experimental hunt for $0\nu\beta\beta$
- Need NME to relate half-life to eff. mass

$$\left(T_{1/2}^{0\nu}\right)^{-1} \propto \left|m_{\beta\beta}\right|^2 G^{0\nu} \left|M^{0\nu}\right|^2$$

- Effective mass: $m_{\beta\beta} = \sum_{i=1}^{3} U_{ei}^2 m_i$

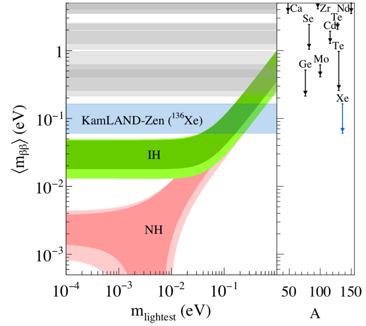


Figure 2: KamLAND-Zen [arXiv:1605.02889]

- **Goal:** address $M^{0\nu}$ in LQCD for light Majorana exchange mechanism
  - ▶ Compute $M^{0\nu}_{nn\rightarrow ppee}$ and use EFT to connect to large nuclei
  - ▶ Directly probe systematics of nuclear models for small systems?
- Complimentary to short-distance LQCD calculations [arXiv:1805.02634]

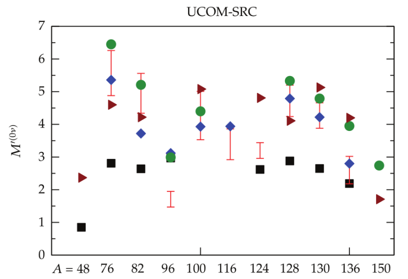


Figure 3: [Giuliani and Poves, Adv. High Energy Phys. 2012 857016]

Neutrinoless Double Beta Decay in the Standard Model

- For lattice scales $a^{-1} \ll m_W$ suffices to work in Fermi effective theory

$$H_W = \frac{G_F}{\sqrt{2}} \Big\{ V_{ud} \overline{u}(x) \gamma_\alpha \left(1-\gamma_5\right) d(x) \otimes \overline{e}(x) \gamma^\alpha \left(1-\gamma_5\right) \nu_e(x) \Big\}$$

- Treat H_W as perturbation to $H_{\rm QCD}$ → $0\nu\beta\beta$ induced at second order
- Majorana condition: $\nu^\top(x) = \overline{\nu}(x) C^\top$
- Matrix element decomposes into **leptonic** and **hadronic** pieces

$$\int d^4x \, d^4y \, \langle fee | T\{H_W(x) H_W(y)\} | i \rangle \propto \int d^4x \, d^4y \left[\overline{u}_e(p_1) \boldsymbol{L_{\alpha\beta}(x,y)} \overline{u}_e^\top(p_2) \right] \boldsymbol{H^{\alpha\beta}(x,y)}$$

$$\boldsymbol{L_{\alpha\beta}(x,y) \equiv \gamma_\alpha \left(1-\gamma_5\right) S_\nu(x,y) C^\top \left(1-\gamma_5\right) \gamma_\beta^\top}$$

$$\boldsymbol{H_{\alpha\beta}(x,y) \equiv \langle f | T \left\{ \overline{u}(x) \gamma_\alpha \left(1-\gamma_5\right) d(x) \overline{u}(y) \gamma_\beta \left(1-\gamma_5\right) d(y) \right\} | i \rangle}$$

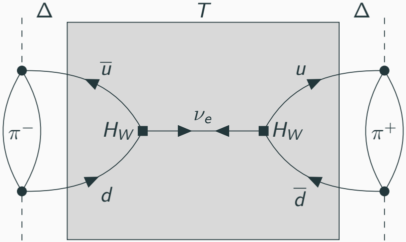
- Develop lattice methods by first computing $\pi^- \to \pi^+ e^- e^-$ amplitude

(a) $nn \to ppee$ **(b)** $\pi^- \to \pi^+ e^- e^-$

Lattice Formalism for $0\nu\beta\beta$

- Adapt Δm_K [arXiv:1406.0916] and rare kaon [arXiv:1701.02858] techniques
  - Compute $\mathcal{M}^{0\nu} = \int d^4x\, d^4y \langle fee | T\{H_W(x) H_W(y)\} | i \rangle$ non-perturbatively
  - Extract $M^{0\nu} = \sum_n \frac{\langle fee|H_W|n\rangle\langle n|H_W|i\rangle}{E_n-(E_i+E_f)/2}$ to compute e.g. decay rate by fitting
- On the lattice, one can show integrated bilocal matrix element is given by

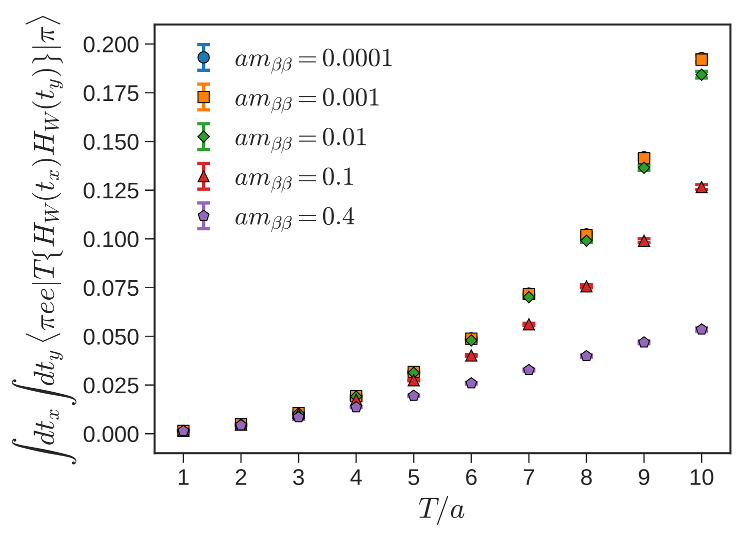
$$\mathcal{M}^{0\nu}(T) = |Z_\pi|^2\, e^{-m_\pi(T+2\Delta)} \sum_n \frac{\langle \pi ee|H_W|n\rangle\langle n|H_W|\pi\rangle}{E_n - m_\pi} \left[T + \frac{e^{-(E_n-m_\pi)T} - 1}{E_n - m_\pi} \right]$$



- Choose Δ to suppress excited states
- To see $\mathcal{M}^{0\nu} \sim T$, must remove:
  - Source/sink dependence
  - $|e\overline{\nu}_e\rangle \propto e^{(m_\pi-(m_{\beta\beta}+m_e))T}$
  - $|\pi^0 e\overline{\nu}_e\rangle \propto \frac{1}{2}T^2$
- **Strategy:** remove $|e\overline{\nu}_e\rangle$ state, then extract **ME** from quadratic fit

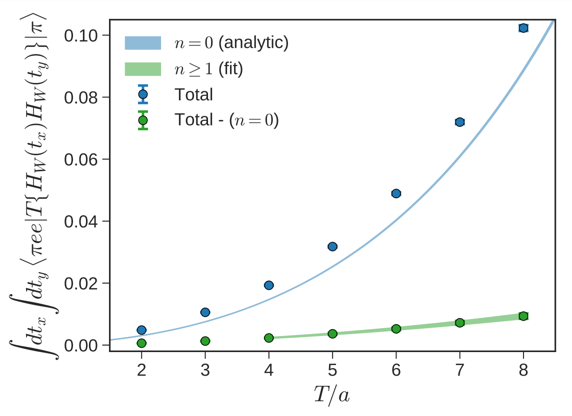
- Pilot study: $16^3 \times 32 \times 16$ Iwasaki+DWF ensemble [arXiv:hep-lat/0701013]
  - $m_\pi = 400$ MeV, $a^{-1} = 1.6$ GeV, $L = 2$ fm
  - (Free) overlap fermion propagator for neutrino
  - Coulomb gauge-fixed wall source propagators for quarks

Preliminary Results for Integrated Bilocal Matrix Element



- Scan over wide range of neutrino masses $am_e/3 \lesssim am_{\beta\beta} \lesssim 2am_\pi$
- ME insensitive to choice of (experimentally relevant) $am_{\beta\beta}$

Preliminary Results for $am_{\beta\beta} = 0.0001$



- Remove $n = 0$ analytically

$$\propto f_\pi^2 e^{(m_\pi - (m_{\beta\beta} + m_e))T}$$

- Fit quadratic to $n \geq 1$
- Reconstruct ME

$$M^{0\nu} = \sum_{n=0}^{\infty} \frac{\langle \pi ee | H_W | n \rangle \langle n | H_W | \pi \rangle}{E_n - m_\pi}$$

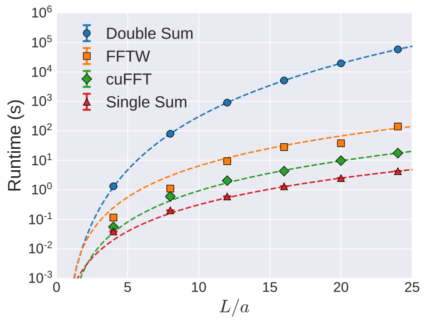
from fit

	$\vert e\overline{\nu}_e\rangle$ $(n=0)$	$\vert \pi^0 e\overline{\nu}_e\rangle$ $(n=1)$	$n \geq 2$
$\left[\frac{\langle \pi ee \vert H_W \vert n\rangle\langle n \vert H_W \vert \pi\rangle}{E_n - m_\pi}\right] / \left[\sum_{n=0}^{\infty} \frac{\langle \pi ee \vert H_W \vert n\rangle\langle n \vert H_W \vert \pi\rangle}{E_n - m_\pi}\right]$	-0.0082(15)	1.0082(13)	0.00009(26)

- Sum dominated by $n = 1$ contribution for $am_{\beta\beta} \ll 1$
- $n = 0$ $(n \geq 2)$ correction is $\mathcal{O}(1\%)$ $(\mathcal{O}(0.01\%))$

Improved Methods: Exact Double Sum via FFTs

- Explicit double sum $\sum_{\vec{x}}\sum_{\vec{y}} H_W(x)H_W(y)$ is expensive
- Standard approach is to fix $H_W(x)$ and sum $\sum_{\vec{y}} H_W(y)$
- Can do explicit double sum in $\mathcal{O}(V \log V)$ via FFTs
  - Continuum: $\int d^3x\, d^3y\, f_\alpha(x) S_\nu^{\alpha\beta}(x-y) g_\beta(y) = \int d^3x\, f_\alpha(x) [\mathscr{F}^{-1}\{\mathscr{F}\{S_\nu^{\alpha\beta}\}\mathscr{F}\{g_\beta\}\}](x)$
  - Exploit block-Toeplitz structure of $S_\nu \to \sum_{\vec{y}}\sum_\beta$ as 1d FFTs of length $4^2(2L-1)^3$ for each spin/color component [Microw. Opt. Technol. Lett. 31, 28-32 (2001)]
  - Further gain by using GPUs to perform FFTs in large, parallel batches



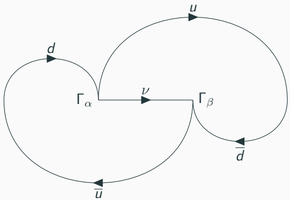


Figure 4: Example diagram non-trivially coupling x and y.

$$\mathrm{Tr}\left[\Gamma_\alpha S_d(t_-;x)S_u^\dagger(t_-;y)\Gamma_\beta S_d(t_+;y)S_u^\dagger(t_+;x)\right] = \left[\Gamma_\alpha S_d(t_-;x)S_u^\dagger(t_+;x)\right]^{ab}\left[\Gamma_\beta S_d(t_+;y)S_u^\dagger(t_-;y)\right]^{ba}$$

Conclusions

- We have explored using lattice QCD techniques for second-order EW matrix elements to compute $M^{0\nu}$
- We have also studied some refinements of these techniques:
  - Explicit FFT double sums
  - Ported back-end inverter to GPUs using QUDA
  - UV-regulated continuuum propagators for the neutrino
- Pilot study of the $\pi^- \to \pi^+ e^- e^-$ decay amplitude suggests calculation is feasible with modest computational resources
- Near-future plans:
  - Repeat for multiple lattice ensembles with improved methods
  - Match m_π dependence to χPT and extract LEC $g_\nu^{\pi\pi}$ [arXiv:1710.01729]
  - Explore mixing with short-distance four-quark operators and renormalization / matching to Standard Model
- Beginning to think about generalization to $nn \to ppee$ and light nuclei
  - Automatic contraction generator?

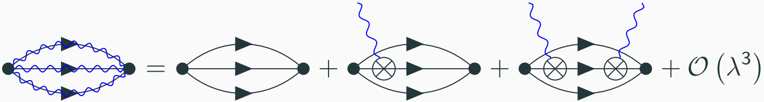
Thank You!

$M^{2\nu}_{GT}$ for $nn \rightarrow ppee\overline{\nu}_e\overline{\nu}_e$ from Lattice QCD [arXiv:1702.02929]

- Calculation on $SU(3)$-symmetric lattice with $m_\pi \approx 806$ MeV
- Compute **compound propagators** in background axial field $\propto \lambda$

$$S_\lambda(x,y) = S(x,y) + \lambda \int d^4z\, S(x,z)A_3(z)S(z,y) + \mathcal{O}(\lambda^2)$$

- $\mathcal{O}(\lambda^n)$ compound correlation function has n axial current insertions:

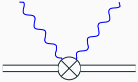


Results for NME:

$$\frac{\Delta}{g_A^2}\frac{M^{2\nu}_{GT}}{6} = -1.04(4)(4)$$

$$\frac{\Delta}{g_A^2}\frac{|\langle pp|A_3^+|d\rangle|^2}{\Delta} = 1.00(3)(1)$$

Matching to pionless EFT:



$$\mathbb{H}_{2,S} = 4.7(1.3)(1.8)\ \mathrm{fm}$$

- New methods required for $0\nu\beta\beta$!