Exclusive Channel Study of the Muon HVP

Aaron S. Meyer (ameyer@quark.phy.bnl.gov)

in collaboration with:
Mattia Bruno, Taku Izubuchi, Christoph Lehner

for the RBC/UKQCD Collaboration

Brookhaven National Laboratory

July 27, 2018

36th International Symposium on Lattice Field Theory

The RBC & UKQCD collaborations

BNL and BNL/RBRC

Yasumichi Aoki (KEK)
Mattia Bruno
Taku Izubuchi
Yong-Chull Jang
Chulwoo Jung
Christoph Lehner
Meifeng Lin
Aaron Meyer
Hiroshi Ohki
Shigemi Ohta (KEK)
Amarjit Soni

UC Boulder

Oliver Witzel

Columbia University

Ziyuan Bai
Norman Christ
Duo Guo
Christopher Kelly
Bob Mawhinney
Masaaki Tomii
Jiqun Tu
Bigeng Wang
Tianle Wang
Evan Wickenden
Yidi Zhao

University of Connecticut

Tom Blum
Dan Hoying (BNL)
Luchang Jin (RBRC)
Cheng Tu

Edinburgh University

Peter Boyle
Guido Cossu
Luigi Del Debbio
Tadeusz Janowski
Richard Kenway
Julia Kettle
Fionn O'haigan
Brian Pendleton
Antonin Portelli
Tobias Tsang
Azusa Yamaguchi

KEK

Julien Frison

University of Liverpool

Nicolas Garron

MIT

David Murphy

Peking University

Xu Feng

University of Southampton

Jonathan Flynn
Vera Guelpers
James Harrison
Andreas Juettner
James Richings
Chris Sachrajda

Stony Brook University

Jun-Sik Yoo
Sergey Syritsyn (RBRC)

York University (Toronto)

Renwick Hudspith

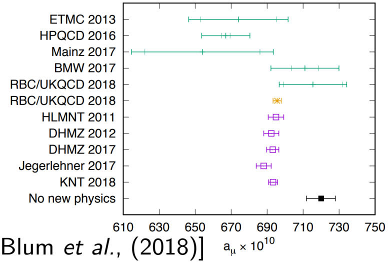
- Introduction
  - Motivation from Experiment
  - Tensions in Experiment
- Computation
  - Lattice Parameters
  - GEVP Study
- Results
  - Correlation Function Reconstruction
  - (Improved) Bounding Method
- Conclusions/Outlook

Introduction

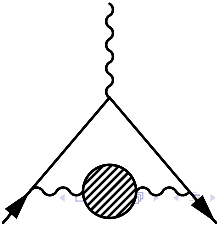
Pieces of Muon $g-2$ Theory Prediction

Contribution	Value $\times 10^{10}$	Uncertainty $\times 10^{10}$
QED	11 658 471.895	0.008
EW	15.4	0.1
HVP LO	692.5	2.7
HVP NLO	-9.84	0.06
HVP NNLO	1.24	0.01
Hadronic light-by-light	10.5	2.6
Total SM prediction	11 659 181.7	3.8
BNL E821 result	11 659 209.1	6.3
Fermilab E989 target		≈ 1.6

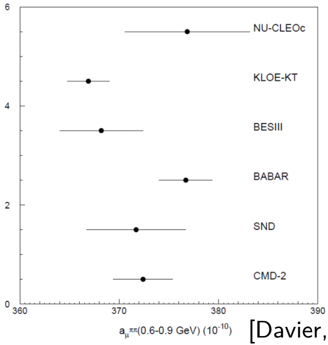
Experiment-Theory difference is 27.4(7.3) $\Longrightarrow$ 3.7σ tension!



[Blum *et al.*, (2018)]



Tensions in Experiment

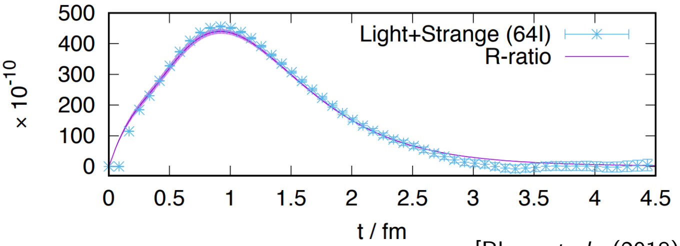


[Davier, KEK (2018)]

R-ratio data for $ee \to \pi\pi$ exclusive channel, $\sqrt{s} = 0.6 - 0.9$ GeV region
Tension between most precise measurements
Other measurements not precise enough to favor one over the other

Avoid tension by computing precise lattice-only estimate of a_μ^{HVP}
Use lattice QCD to inform experiment, resolve discrepancy

Interplay between R-ratio, Lattice



[Blum *et al.*, (2018)]

$$a_\mu^{\text{cont.}} = \int_0^\infty dt\, K(t) C^{\text{cont.}}(t) \qquad a_\mu^{\text{latt.}} = \sum_t w_t C^{\text{latt.}}(t)$$

$$C^{\text{cont.}}(t) = \int_0^\infty d\sqrt{s}\, s\, R(s)\, e^{-\sqrt{s}t} \qquad C^{\text{latt.}}(t) = \sum_n |\langle\Omega|\,\mathcal{O}\,|n\rangle|^2 e^{-E_n t}$$

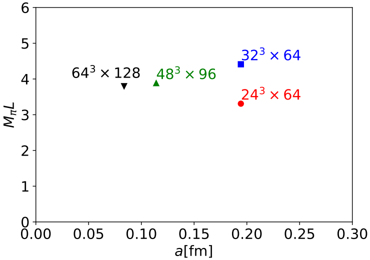
w_t from Bernecker, H. Meyer: 1107.4388 [hep-lat]

R-Ratio, Lattice precise in complimentary regions
Lattice uncertainty dominated by long-distance region
$\Longrightarrow$ need to address long-distance region to reduce lattice uncertainty

Precisely determine E_n and $\langle\Omega|\, V_\mu\, |n\rangle$ from exclusive $\pi\pi$ study
Use those to approximate $C^{\text{latt.}}(t)$ for large t

Computation

Computation Details



Computed on $2+1$ flavor Möbius Domain Wall Fermions for valance and sea, M_π at physical value on all ensembles

All results in this talk on one coarse ensemble:

- $a \approx 0.20 \text{ fm} \approx (1.015 \text{ GeV})^{-1}$,
- $24^3 \times 64$ (4.8 fm)

Extending program to three other ensembles:

- 2 ensembles on same volume - volume dependence (see C. Lehner's talk)
- multiple lattice spacings - continuum extrapolation

Distillation

Phys.Rev.D 80, 054506 (0905.2160 [hep-lat])

Eigenvectors of (spin-diagonal) Laplacian operator used to construct projection matrices ($M \to \infty$ gives identity)

$$\mathcal{P}^{ab}_{t;xy} = \sum_{i=0}^{M-1} \langle x | i^a_t \rangle \langle i^b_t | y \rangle$$

Inserting distillation projection matrices smears quarks in bilinear

$$\sum_a \bar{Q}^a(z) \Gamma Q^a(z) \to \sum_{xyacb} \bar{Q}^a(x) \mathcal{P}^{ac}_{t;xz} \Gamma \mathcal{P}^{cb}_{t;zy} Q^b(y)$$

$$= \sum_{xyacb} \bar{Q}^a(x) f^{ac}(x-z) \Gamma f^{cb}(z-y) Q^b(y)$$

Propagators contracted with eigenvectors at source & sink creates "perambulator" objects

$$M^{ji}_{t,\beta\alpha} = \sum_{xy} \sum_{ab} \langle j^b_t | y \rangle \left(D^{ba}_{yx,\beta\alpha} \right)^{-1} \langle x | i^a_0 \rangle$$

Perambulators stitched together to form desired N-point correlation functions
$\implies$ ideal for creating $2\pi \to 2\pi$ correlation functions

Fit Procedure

Operators in $I=1$ P-wave channel

Local vector current operator:

- Local $\mathcal{O}_0 = \sum_x \bar{\psi}(x)\gamma_\mu\psi(x)$, $\mu \in \{1,2,3\}$

Three 2π operators with $\mathcal{O}_{1,2,3}$ given by $\vec{p}_\pi \in \frac{2\pi}{L} \times \{(1,0,0),(1,1,0),(1,1,1)\}$

$$\mathcal{O}_n = \left|\sum_{xyz} \bar{\psi}(x) f(x-z) e^{-i\vec{p}_\pi\cdot\vec{z}} \gamma_5 f(z-y)\psi(y)\right|^2$$

Correlators arranged in a 4×4 symmetric matrix:

$\otimes$	$\mathcal{O}_0$	$\mathcal{O}_1$	$\mathcal{O}_2$	$\mathcal{O}_3$
$\mathcal{O}_0$	$C^{(2)}_{\rho}$	$C^{(3)}_{\rho\to\pi\pi}$	$C^{(3)}_{\rho\to\pi\pi}$	$C^{(3)}_{\rho\to\pi\pi}$
$\mathcal{O}_1$		$C^{(4)}_{\pi\pi\to\pi\pi}$	$C^{(4)}_{\pi\pi\to\pi\pi}$	$C^{(4)}_{\pi\pi\to\pi\pi}$
$\mathcal{O}_2$			$C^{(4)}_{\pi\pi\to\pi\pi}$	$C^{(4)}_{\pi\pi\to\pi\pi}$
$\mathcal{O}_3$				$C^{(4)}_{\pi\pi\to\pi\pi}$

Extra operator with $\vec{p}_\pi = \frac{2\pi}{L} \times (2,0,0)$ to estimate excited state systematics

Generalized EigenValue Problem (GEVP) to estimate overlaps & energies

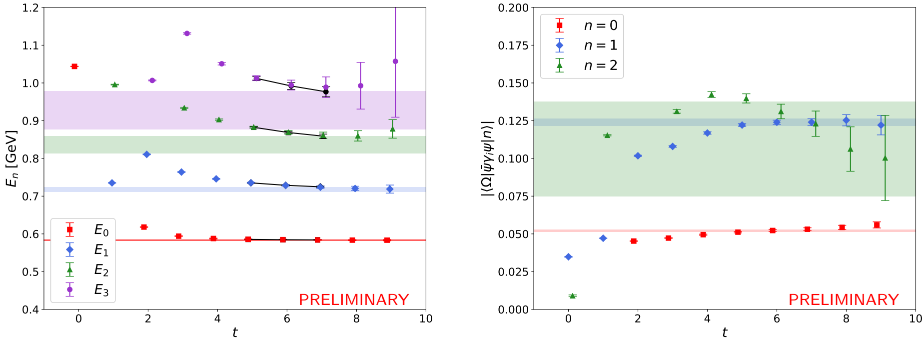
$$C(t)\,V = C(t+\delta t)\,V\,\Lambda(\delta t)\,; \quad \Lambda_{nn}(\delta t) \sim e^{+E_n\delta t}\,,\ V_{im} \propto \langle\Omega|\,\mathcal{O}_i\,|m\rangle$$

Reconstruct exponential dependence of local vector correlation function

$$C^{\text{latt.}}_{ij}(t) = \sum_n^N \langle\Omega|\,\mathcal{O}_i\,|n\rangle\,\langle n|\,\mathcal{O}_j\,|\Omega\rangle\, e^{-E_n t}$$

In practice, only finite N necessary to model correlation function

GEVP Results



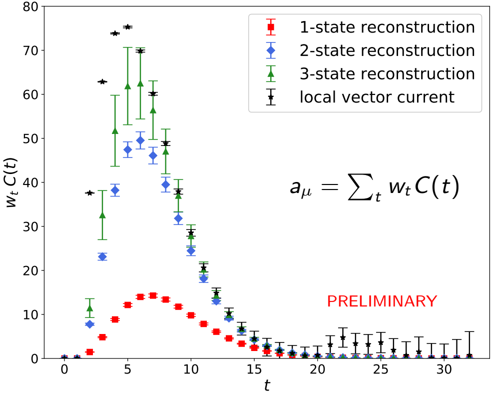
Scatter points from solving GEVP at fixed δt

$$C(t)\,V = C(t+\delta t)\,V\,\Lambda(\delta t)\,, \quad \Lambda_{nn}(\delta t) \sim e^{+E_n \delta t}$$

Black lines are from fit ansatz: $f_i(t) = E_i + \alpha e^{-(E_N - E_i)t}$

Overlaps picked to have approximately same contamination from excited states
Bands are extracted spectrum/overlaps ($= E_i$), with excited state systematics
Systematics estimated from difference between 4- and 5-operator GEVP basis

Correlation Function Reconstruction



GEVP results to reconstruct long-distance behavior of local vector correlation function needed to compute connected HVP

Explicit reconstruction good estimate of correlation function at long-distance, missing excited states at short-distance

More states $\implies$ better reconstruction, can replace $C(t)$ at shorter distances

Improved Bounding Method

Use known results in spectrum to make a precise estimate of
upper & lower bound on a_μ^{HVP}

$$\widetilde{C}(t; t_{\max}, E) = \begin{cases} C(t) & t < t_{\max} \\ C(t_{\max}) e^{-E(t-t_{\max})} & t \geq t_{\max} \end{cases}$$

Upper bound: $E = E_0$, lowest state in spectrum

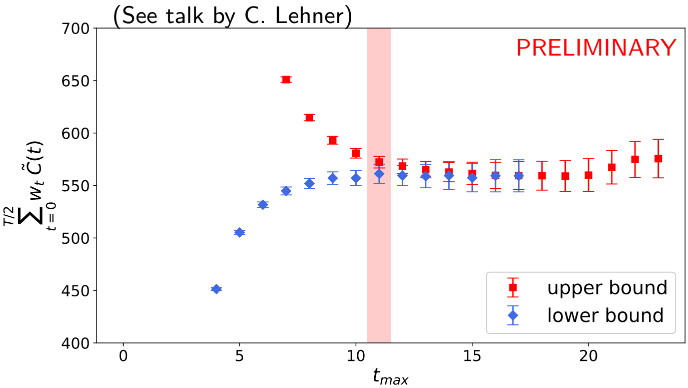
Lower bound: $E = \log[\frac{C(t_{\max})}{C(t_{\max}+1)}]$

Good control over lower states in spectrum with exclusive reconstruction:

Replace $C(t) \to C(t) - \sum_n^N |c_n|^2 e^{-E_n t}$
$\Longrightarrow$ Long distance convergence now $\propto e^{-E_{N+1} t}$
$\Longrightarrow$ Smaller overall contribution from neglected states

Add back contribution from reconstruction after bounding correlator

Bounding Method



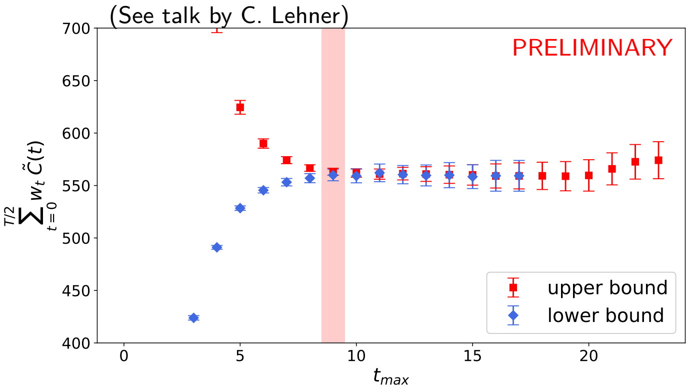
No bounding method: $a_\mu^{HVP} = 577(31)$

Bounding method $t_{\max} = 2.1\ \mathrm{fm}$, no reconstruction: $a_\mu^{HVP} = 566.8(9.0)$

Very large lattice spacing: $a^{-1} = 1.015\ \mathrm{GeV}$, finite volume effects

Could expect $10 - 20\%$ systematic errors

Bounding Method



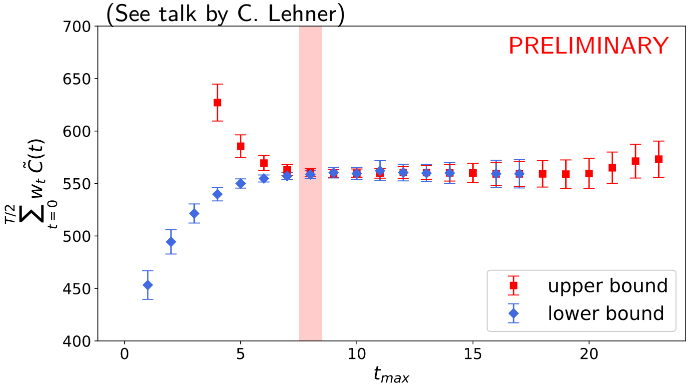
No bounding method: $a_\mu^{HVP} = 577(31)$

Bounding method $t_{\rm max} = 2.1$ fm, no reconstruction: $a_\mu^{HVP} = 566.8(9.0)$

Bounding method $t_{\rm max} = 1.7$ fm, 1 state reconstruction: $a_\mu^{HVP} = 561.5(4.5)$

Very large lattice spacing: $a^{-1} = 1.015$ GeV, finite volume effects
Could expect $10 - 20\%$ systematic errors

Bounding Method



No bounding method:	$a_\mu^{HVP} = 577(31)$
Bounding method $t_{\max} = 2.1$ fm, no reconstruction:	$a_\mu^{HVP} = 566.8(9.0)$
Bounding method $t_{\max} = 1.7$ fm, 1 state reconstruction:	$a_\mu^{HVP} = 561.5(4.5)$
Bounding method $t_{\max} = 1.6$ fm, 2 state reconstruction:	$a_\mu^{HVP} = 559.5(3.8)$

Very large lattice spacing: $a^{-1} = 1.015$ GeV, finite volume effects
Could expect $10 - 20\%$ systematic errors

Outlook and Conclusions

Summary

- $g-2$ is an interesting and exciting topic to work on!
- Tensions in experimental $ee \to \pi\pi$ data make independent study of exclusive channels valuable
- Lattice QCD is a first principles method capable of accessing necessary matrix elements
- Additional studies using correlated fits, additional ensembles in progress
- Study of exclusive channels able to significantly reduce statistical uncertainty on an all-lattice computation of muon HVP $\implies$ expect lattice-only calculation with precision comparable to R-ratio by 2020
- Part of ongoing lattice study to address all lattice systematics in HVP computation

Thanks

Computing time support from many sources:

- ANL
- BNL
- Oak Forest
- Hokusai
- USQCD
- XSEDE

Lots of data to analyze, lots of work ahead of us!

Thank you for your attention!

Backup

Error Budget

$a_\mu^{\text{ud, conn, isospin}}$	$202.9(1.4)_\mathrm{S}(0.2)_\mathrm{C}(0.1)_\mathrm{V}(0.2)_\mathrm{A}(0.2)_\mathrm{Z}$	$649.7(14.2)_\mathrm{S}(2.8)_\mathrm{C}(3.7)_\mathrm{V}(1.5)_\mathrm{A}(0.4)_\mathrm{Z}(0.1)_\mathrm{E48}(0.1)_\mathrm{E64}$
$a_\mu^{\text{s, conn, isospin}}$	$27.0(0.2)_\mathrm{S}(0.0)_\mathrm{C}(0.1)_\mathrm{A}(0.0)_\mathrm{Z}$	$53.2(0.4)_\mathrm{S}(0.0)_\mathrm{C}(0.3)_\mathrm{A}(0.0)_\mathrm{Z}$
$a_\mu^{\text{c, conn, isospin}}$	$3.0(0.0)_\mathrm{S}(0.1)_\mathrm{C}(0.0)_\mathrm{Z}(0.0)_\mathrm{M}$	$14.3(0.0)_\mathrm{S}(0.7)_\mathrm{C}(0.1)_\mathrm{Z}(0.0)_\mathrm{M}$
$a_\mu^{\text{uds, disc, isospin}}$	$-1.0(0.1)_\mathrm{S}(0.0)_\mathrm{C}(0.0)_\mathrm{V}(0.0)_\mathrm{A}(0.0)_\mathrm{Z}$	$-11.2(3.3)_\mathrm{S}(0.4)_\mathrm{V}(2.3)_\mathrm{L}$
$a_\mu^{\text{QED, conn}}$	$0.2(0.2)_\mathrm{S}(0.0)_\mathrm{C}(0.0)_\mathrm{V}(0.0)_\mathrm{A}(0.0)_\mathrm{Z}(0.0)_\mathrm{E}$	$5.9(5.7)_\mathrm{S}(0.3)_\mathrm{C}(1.2)_\mathrm{V}(0.0)_\mathrm{A}(0.0)_\mathrm{Z}(1.1)_\mathrm{E}$
$a_\mu^{\text{QED, disc}}$	$-0.2(0.1)_\mathrm{S}(0.0)_\mathrm{C}(0.0)_\mathrm{V}(0.0)_\mathrm{A}(0.0)_\mathrm{Z}(0.0)_\mathrm{E}$	$-6.9(2.1)_\mathrm{S}(0.4)_\mathrm{C}(1.4)_\mathrm{V}(0.0)_\mathrm{A}(0.0)_\mathrm{Z}(1.3)_\mathrm{E}$
a_μ^{SIB}	$0.1(0.2)_\mathrm{S}(0.0)_\mathrm{C}(0.2)_\mathrm{V}(0.0)_\mathrm{A}(0.0)_\mathrm{Z}(0.0)_\mathrm{E48}$	$10.6(4.3)_\mathrm{S}(0.6)_\mathrm{C}(6.6)_\mathrm{V}(0.1)_\mathrm{A}(0.0)_\mathrm{Z}(1.3)_\mathrm{E48}$
$a_\mu^{\text{udsc, isospin}}$	$231.9(1.4)_\mathrm{S}(0.2)_\mathrm{C}(0.1)_\mathrm{V}(0.3)_\mathrm{A}(0.2)_\mathrm{Z}(0.0)_\mathrm{M}$	$705.9(14.6)_\mathrm{S}(2.9)_\mathrm{C}(3.7)_\mathrm{V}(1.8)_\mathrm{A}(0.4)_\mathrm{Z}(2.3)_\mathrm{L}(0.1)_\mathrm{E48}(0.1)_\mathrm{E64}(0.0)_\mathrm{M}$
$a_\mu^{\text{QED, SIB}}$	$0.1(0.3)_\mathrm{S}(0.0)_\mathrm{C}(0.2)_\mathrm{V}(0.0)_\mathrm{A}(0.0)_\mathrm{Z}(0.0)_\mathrm{E}(0.0)_\mathrm{E48}$	$9.5(7.4)_\mathrm{S}(0.7)_\mathrm{C}(6.9)_\mathrm{V}(0.1)_\mathrm{A}(0.0)_\mathrm{Z}(1.7)_\mathrm{E}(1.3)_\mathrm{E48}$
$a_\mu^{\text{R-ratio}}$	$460.4(0.7)_\mathrm{RST}(2.1)_\mathrm{RSY}$	
a_μ	$692.5(1.4)_\mathrm{S}(0.2)_\mathrm{C}(0.2)_\mathrm{V}(0.3)_\mathrm{A}(0.2)_\mathrm{Z}(0.0)_\mathrm{E}(0.0)_\mathrm{E48}(0.0)_\mathrm{b}(0.1)_\mathrm{c}(0.0)_{\overline{\mathrm{S}}}(0.0)_{\overline{\mathrm{Q}}}(0.0)_\mathrm{M}(0.7)_\mathrm{RST}(2.1)_\mathrm{RSY}$	$715.4(16.3)_\mathrm{S}(3.0)_\mathrm{C}(7.8)_\mathrm{V}(1.9)_\mathrm{A}(0.4)_\mathrm{Z}(1.7)_\mathrm{E}(2.3)_\mathrm{L}(1.5)_\mathrm{E48}(0.1)_\mathrm{E64}(0.3)_\mathrm{b}(0.2)_\mathrm{c}(1.1)_{\overline{\mathrm{S}}}(0.3)_{\overline{\mathrm{Q}}}(0.0)_\mathrm{M}$

TABLE I. Individual and summed contributions to a_μ multiplied by 10^{10}. The left column lists results for the window method with $t_0 = 0.4$ fm and $t_1 = 1$ fm. The right column shows results for the pure first-principles lattice calculation. The respective uncertainties are defined in the main text.

[Blum *et al.*, (2018)]

Full program of computations to reduce uncertainties:

Reduce statistical uncertainties on light connected contribution

Compute QED contribution

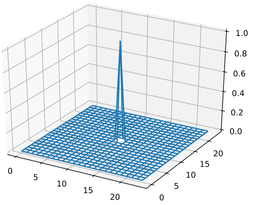
Improve lattice spacing determination

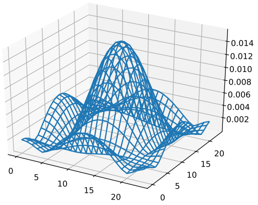
Finite volume and continuum extrapolation study

Distillation Smearing Visualization

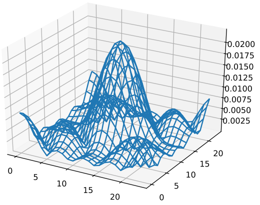
Free-field Laplacian in 2-dimensions, 24^2 volume

More evecs, better ability to localize

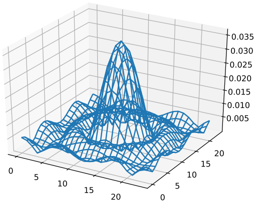




9 evecs (57 equiv), $\sum_i p_i^2 \leq 2$



13 evecs (99 equiv), $\sum_i p_i^2 \leq 4$



21 evecs (171 equiv), $\sum_i p_i^2 \leq 5$