

Scattering phase shift determinations from a two-scalar field theory and resonance parameters from QCD scattering

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Motivation

- Improve on 1995 model of Rummukainen and Gottlieb to study scattering resonances [5]
 - Not $\lambda \rightarrow \infty$ Ising limit
 - Symanzik-improved so “lattice dispersion” not needed
- Investigate partial wave mixing using box matrix formalism developed by Morningstar et al. [4]

Action

- In Minkowski spacetime:

$$\mathcal{L} = -\frac{1}{2}\phi(\partial^2 + m_\phi^2)\phi - \frac{1}{2}\rho(\partial^2 + m_\rho^2)\rho - \frac{\lambda_\phi}{4!}\phi^4 - \frac{\lambda_\rho}{4!}\rho^4 - \frac{1}{2}g\phi^2\rho$$

- ϕ, ρ real scalar fields
- ρ tuned to appear as resonance
- Quartic terms \rightarrow action bounded from below

Choice of parameters

- Lattices ranging from $n_s = 16$ to $n_s = 53$

Want:

- Renormalized $m_\phi L > 4$, but renormalized mass not known *a priori*
 - Choose $a_t m_\phi = 0.15$
- $m_\rho > 2m_\phi$, but m_ρ suitably less than 3-, 4-particle thresholds.
 - Choose $m_\rho \approx 2.67m_\phi$

Choice of Parameters

- Simple theory \Rightarrow simple vacuum, no S.S.B.
- Minimize $\lambda_\phi, \lambda_\rho$ to simplify model and to minimize mass renormalizations
- Large enough g to yield significant interaction energies

Studying an action with uniform fields yields good candidates:

- $\lambda_\phi = \frac{g^2}{4m_\phi^2}$
- $\lambda_\rho = \frac{g^2}{m_\phi^2}$

Test monte-carlo runs determine $\frac{g}{m_\phi} = 1$ to yield significant interaction energies.

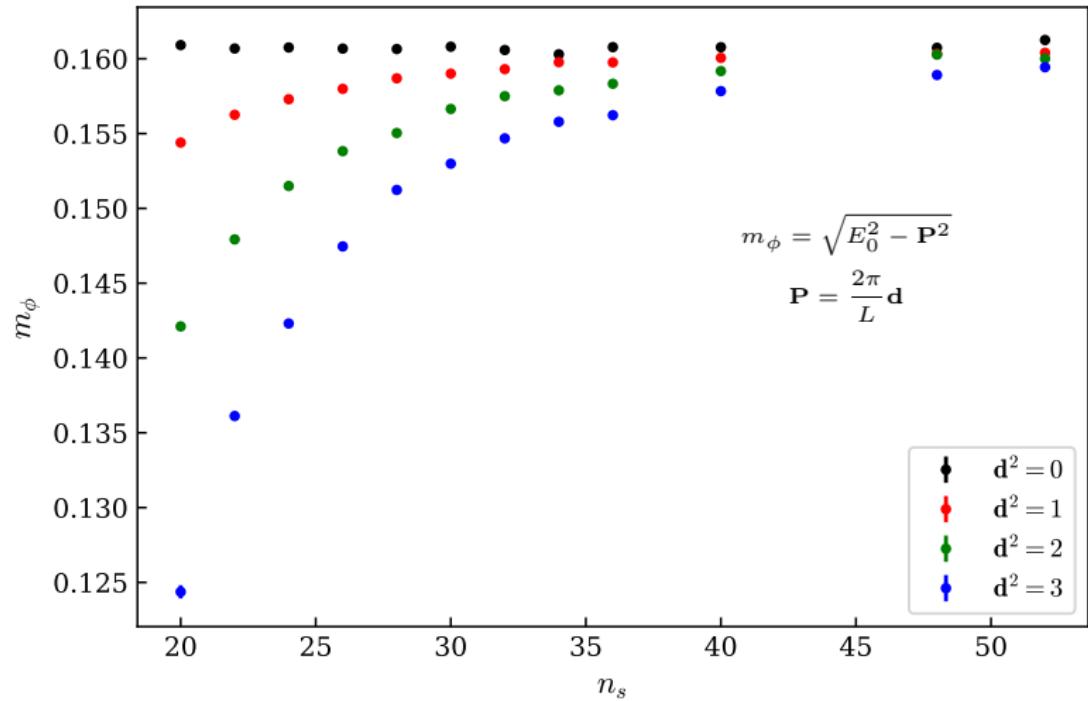
Discretization and the Need for an Improved Action

A naive discretization scheme:

$$S_\phi = a_s^{D-1} a_t \sum_x \sum_\mu \left(\frac{(\phi(x + a_\mu \hat{\mu}) - \phi(x))^2}{2a_\mu} + \frac{1}{2} m_\phi^2 \phi(x)^2 + \frac{\lambda_\phi}{4!} \phi(x)^4 \right)$$

- S_ϕ is portion of action containing ϕ -only terms
- **Large discretization errors!**

Discretization and the Need for an Improved Action



Symanzik Improvement (Improved Finite Difference)

Rewrite kinetic term in action:

$$\int d^D x \partial_\mu \phi \partial_\mu \phi = - \int d^D x \phi \partial_\mu^2 \phi$$

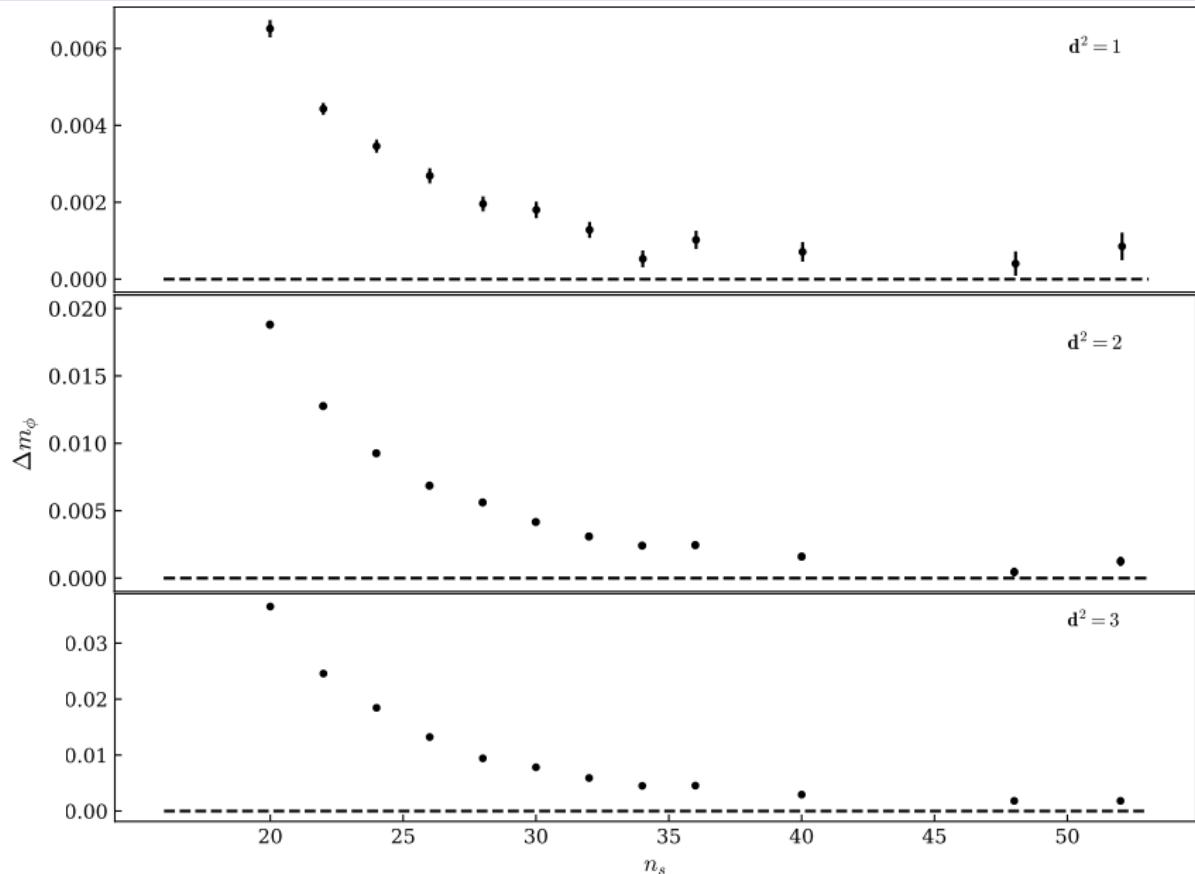
Naive finite-difference:

$$a^{-2}(f(x+a) - f(x-a) - 2f(x)) = f''(x) + \boxed{\mathcal{O}(a^2)}$$

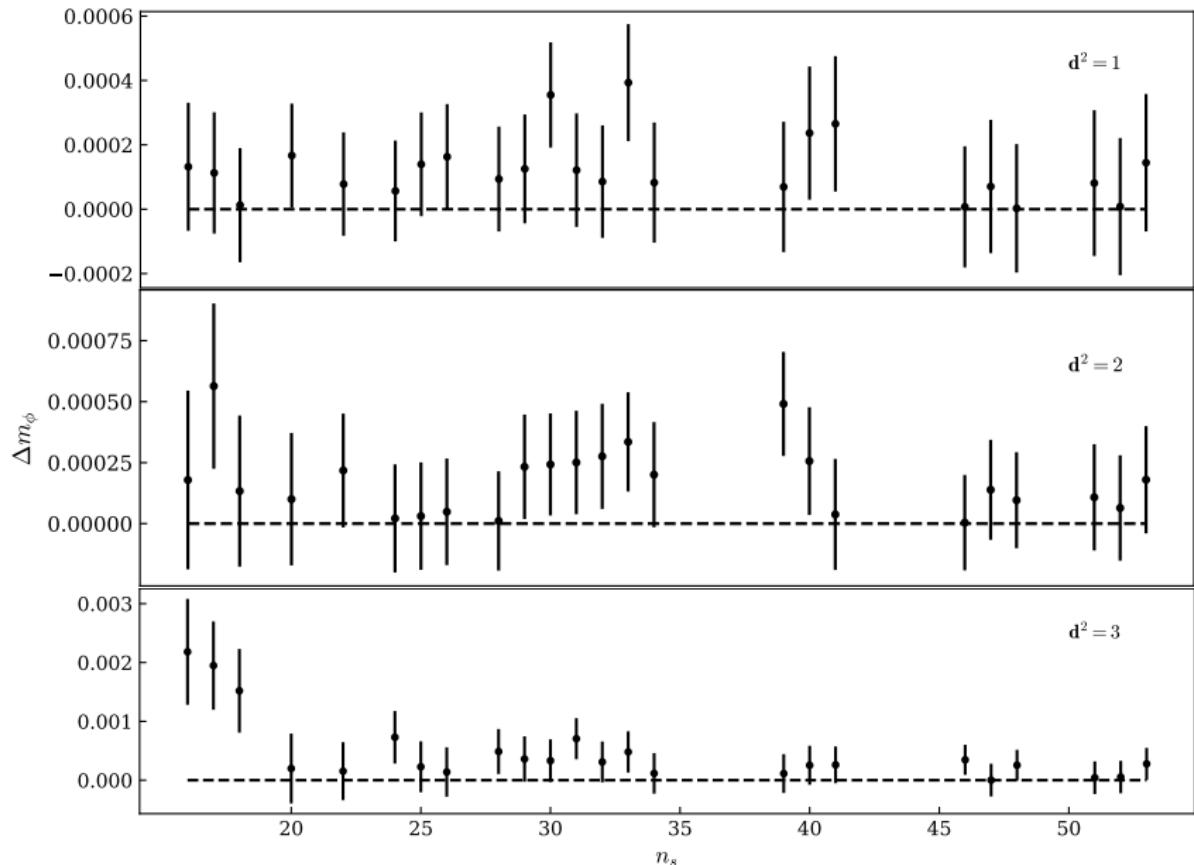
Improved finite difference:

$$\begin{aligned} a^{-2} \left(\frac{4}{3}(f(x+a) + f(x-a)) - \frac{1}{12}(f(x+2a) + f(x-2a)) - \frac{5}{2}f(x) \right) \\ = f''(x) + \boxed{\mathcal{O}(a^4)} \end{aligned}$$

Unimproved Action



Improved action



Updating scheme: Sequential Metropolis + *Microcanonical*

Sequential Metropolis-Hastings

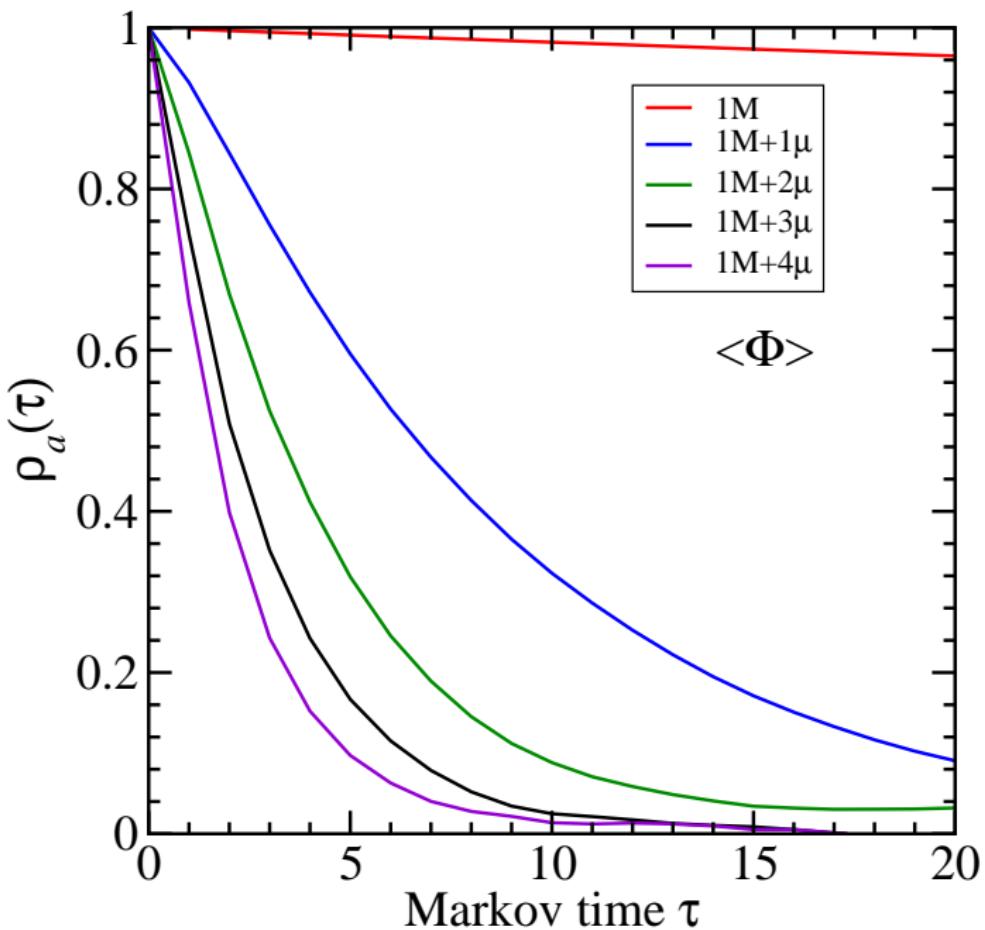
- Not effective at reducing autocorrelations

Microcanonical updating

- Greatly reduces autocorrelations
- Makes *large* but *action-preserving* changes at each site

Measurement parameters

- Each compound sweep: 1 Metropolis + 3 microcanonical
- 5 compound sweeps between measurements
- Number of measurements: 560,000 for larger lattices to 11,000,000 for smaller lattices



Operators: $\mathbf{d}^2 = 0$

Operators, $O_k^{(0)}$, used to extract spectrum in the $[\mathbf{P} = 0, A_{1g}]$ sector.

Name	Operator details
$O_0^{(0)}(t)$	$\Phi(0, 0, 0, t)$
$O_1^{(0)}(t)$	$ \Phi(0, 0, 0, t) ^2$
$O_2^{(0)}(t)$	$ \Phi(1, 0, 0, t) ^2 + \Phi(0, 1, 0, t) ^2 + \Phi(0, 0, 1, t) ^2$
$O_3^{(0)}(t)$	$ \Phi(1, 1, 0, t) ^2 + \Phi(0, 1, 1, t) ^2 + \Phi(1, 0, 1, t) ^2$ $+ \Phi(1, -1, 0, t) ^2 + \Phi(0, 1, -1, t) ^2 + \Phi(1, 0, -1, t) ^2$
$O_4^{(0)}(t)$	$ \Phi(1, 1, 1, t) ^2 + \Phi(-1, 1, 1, t) ^2 + \Phi(1, -1, 1, t) ^2$ $ \Phi(1, 1, -1, t) ^2$
$O_5^{(0)}(t)$	$ \Phi(2, 0, 0, t) ^2 + \Phi(0, 2, 0, t) ^2 + \Phi(0, 0, 2, t) ^2$
$O_6^{(0)}(t)$	$\Xi(0, 0, 0, t)$

Operators: $\mathbf{d}^2 = 1$

Operators, $O_k^{(1)}$, used to extract the spectrum in the $[\mathbf{P} = (2\pi/L)(0, 0, 1), A_1]$ sector.

Name	Operator details
$O_0^{(001)}(t)$	$\Phi(0, 0, 1, t)$
$O_1^{(001)}(t)$	$\Phi(0, 0, 1, t)\Phi(0, 0, 0, t)$
$O_2^{(001)}(t)$	$\Phi(1, 0, 1, t)\Phi(-1, 0, 0, t) + \Phi(-1, 0, 1, t)\Phi(1, 0, 0, t)$ $+ \Phi(0, 1, 1, t)\Phi(0, -1, 0, t) + \Phi(0, -1, 1, t)\Phi(0, 1, 0, t)$
$O_3^{(001)}(t)$	$\Phi(1, 1, 1, t)\Phi(-1, -1, 0, t) + \Phi(-1, -1, 1, t)\Phi(1, 1, 0, t)$ $+ \Phi(1, -1, 1, t)\Phi(-1, 1, 0, t) + \Phi(-1, 1, 1, t)\Phi(1, -1, 0, t)$
$O_4^{(001)}(t)$	$\Phi(0, 0, 2, t)\Phi(0, 0, -1, t)$
$O_5^{(001)}(t)$	$\Xi(0, 0, 1, t)$

Operators: $\mathbf{d}^2 = 2$

Operators, $O_k^{(2)}$, used to extract the spectrum in the $[\mathbf{P} = (2\pi/L)(0, 1, 1), A_1]$ sector.

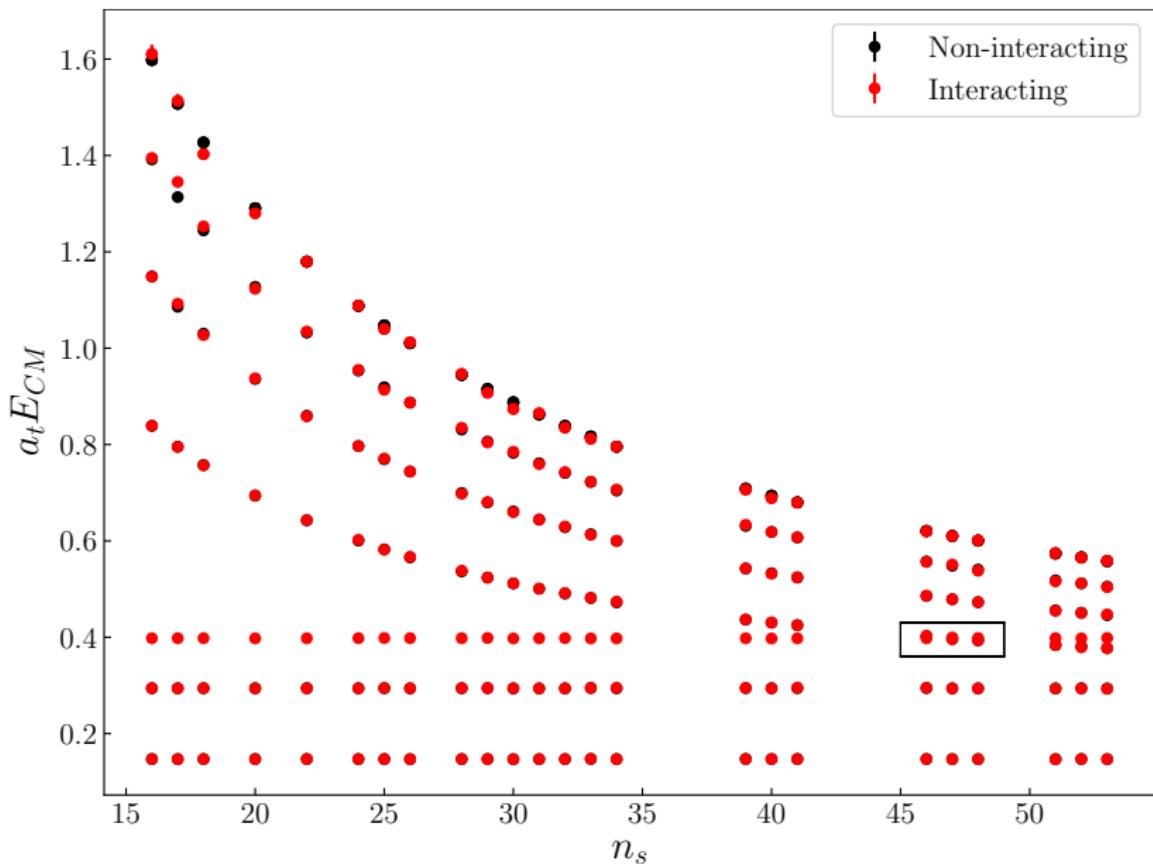
Name	Operator details
$O_0^{(011)}(t)$	$\Phi(0, 1, 1, t)$
$O_1^{(011)}(t)$	$\Phi(0, 0, 0, t)\Phi(0, 1, 1, t)$
$O_2^{(011)}(t)$	$\Phi(0, 0, 1, t)\Phi(0, 1, 0, t)$
$O_3^{(011)}(t)$	$\Phi(-1, 0, 0, t)\Phi(1, 1, 1, t) + \Phi(-1, 1, 1, t)\Phi(1, 0, 0, t)$
$O_4^{(011)}(t)$	$\Phi(0, -1, 0, t)\Phi(0, 2, 1, t) + \Phi(0, 0, -1, t)\Phi(0, 1, 2, t)$
$O_5^{(011)}(t)$	$\Phi(-1, 0, 1, t)\Phi(1, 1, 0, t) + \Phi(-1, 1, 0, t)\Phi(1, 0, 1, t)$
$O_6^{(011)}(t)$	$\Phi(0, -1, 1, t)\Phi(0, 2, 0, t) + \Phi(0, 0, 2, t)\Phi(0, 1, -1, t)$
$O_7^{(011)}(t)$	$\Xi(0, 1, 1, t)$

Operators: $\mathbf{d}^2 = 3$

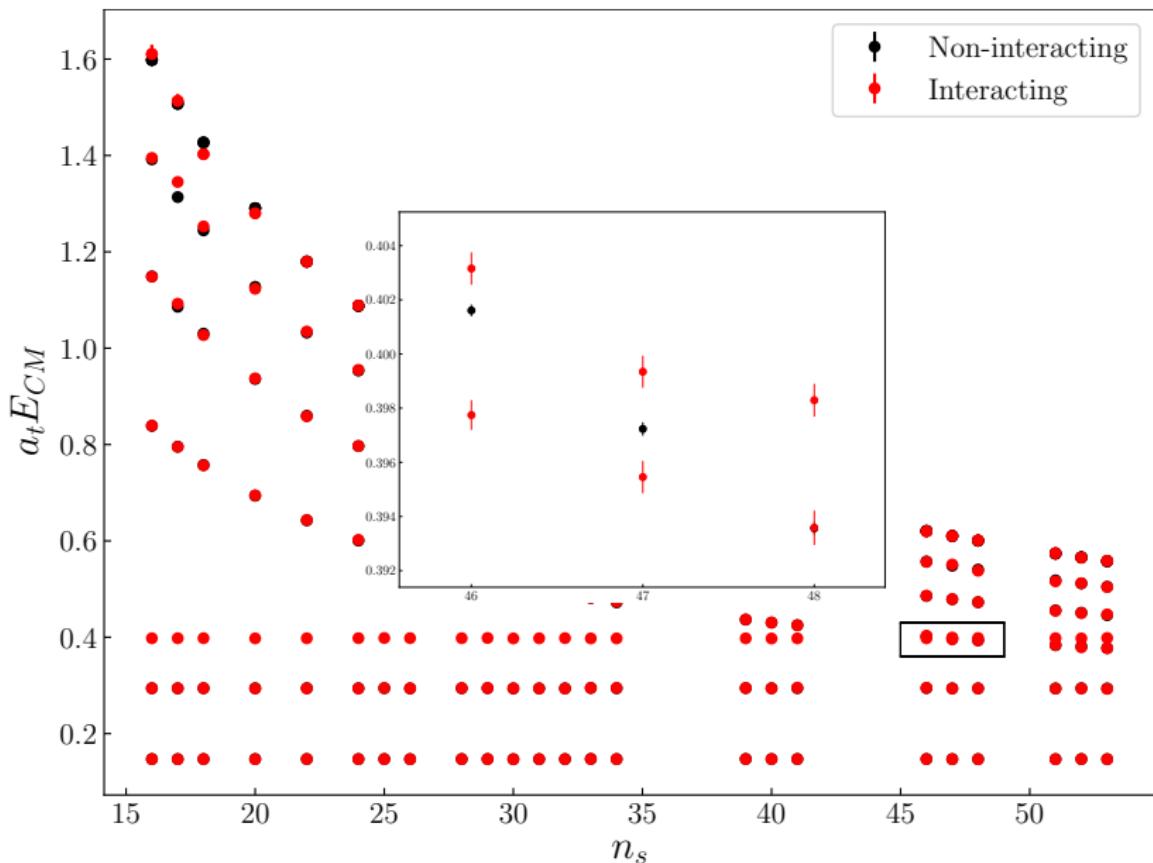
Operators, $O_k^{(3)}$, used to extract the spectrum in the $[\mathbf{P} = (2\pi/L)(1, 1, 1), A_1]$ sector.

Name	Operator details
$O_0^{(111)}(t)$	$\Phi(1, 1, 1, t)$
$O_1^{(111)}(t)$	$\Phi(0, 0, 0, t)\Phi(1, 1, 1, t)$
$O_2^{(111)}(t)$	$\Phi(0, 0, 1, t)\Phi(1, 1, 0, t) + \Phi(0, 1, 0, t)\Phi(1, 0, 1, t)$ $+ \Phi(0, 1, 1, t)\Phi(1, 0, 0, t)$
$O_3^{(111)}(t)$	$\Phi(-1, 0, 0, t)\Phi(2, 1, 1, t) + \Phi(0, -1, 0, t)\Phi(1, 2, 1, t)$ $+ \Phi(0, 0, -1, t)\Phi(1, 1, 2, t)$
$O_4^{(111)}(t)$	$\Phi(-1, 0, 1, t)\Phi(2, 1, 0, t) + \Phi(-1, 1, 0, t)\Phi(2, 0, 1, t)$ $+ \Phi(0, -1, 1, t)\Phi(1, 2, 0, t) + \Phi(0, 1, -1, t)\Phi(1, 0, 2, t)$ $+ \Phi(0, 1, 2, t)\Phi(1, 0, -1, t) + \Phi(0, 2, 1, t)\Phi(1, -1, 0, t)$
$O_5^{(111)}(t)$	$\Phi(-1, 1, 1, t)\Phi(2, 0, 0, t) + \Phi(0, 0, 2, t)\Phi(1, 1, -1, t)$ $+ \Phi(0, 2, 0, t)\Phi(1, -1, 1, t)$
$O_6^{(111)}(t)$	$\Xi(1, 1, 1, t)$

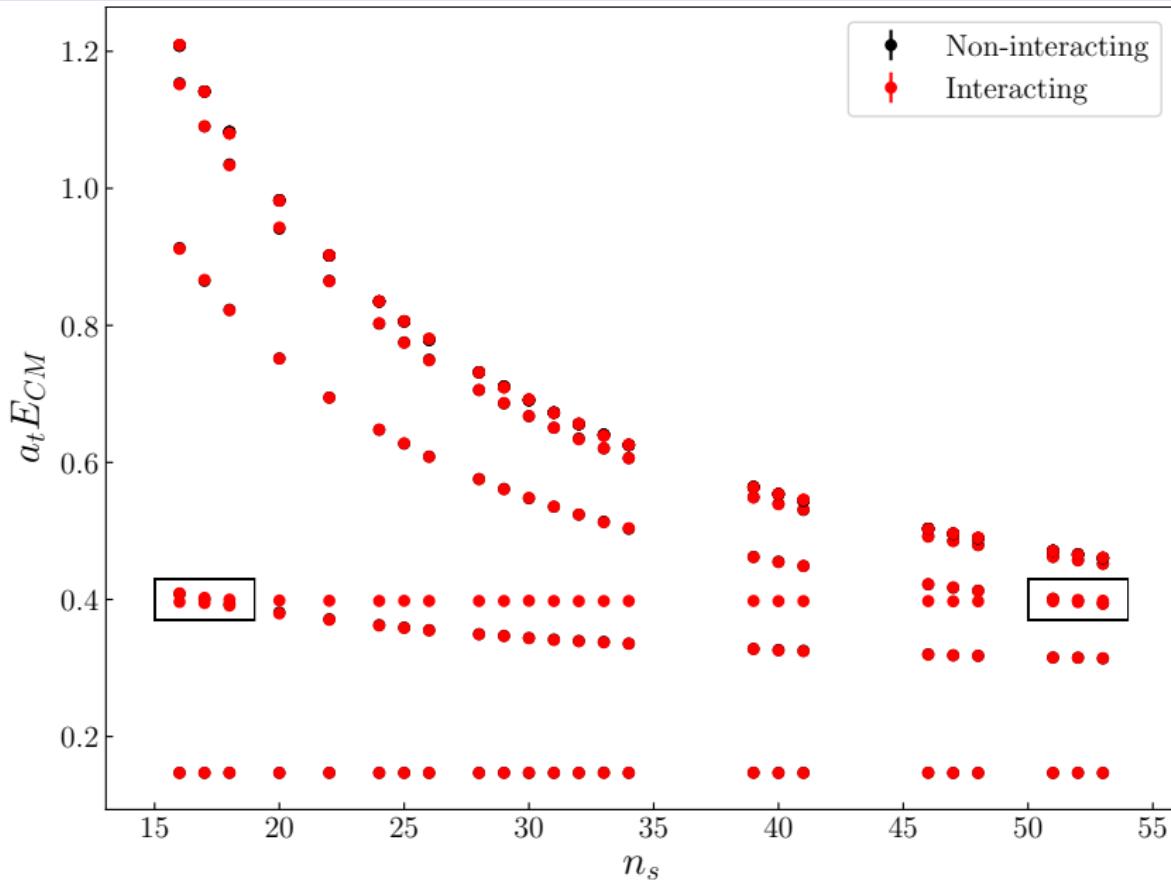
Energy Determinations: $\mathbf{d}^2 = 0$



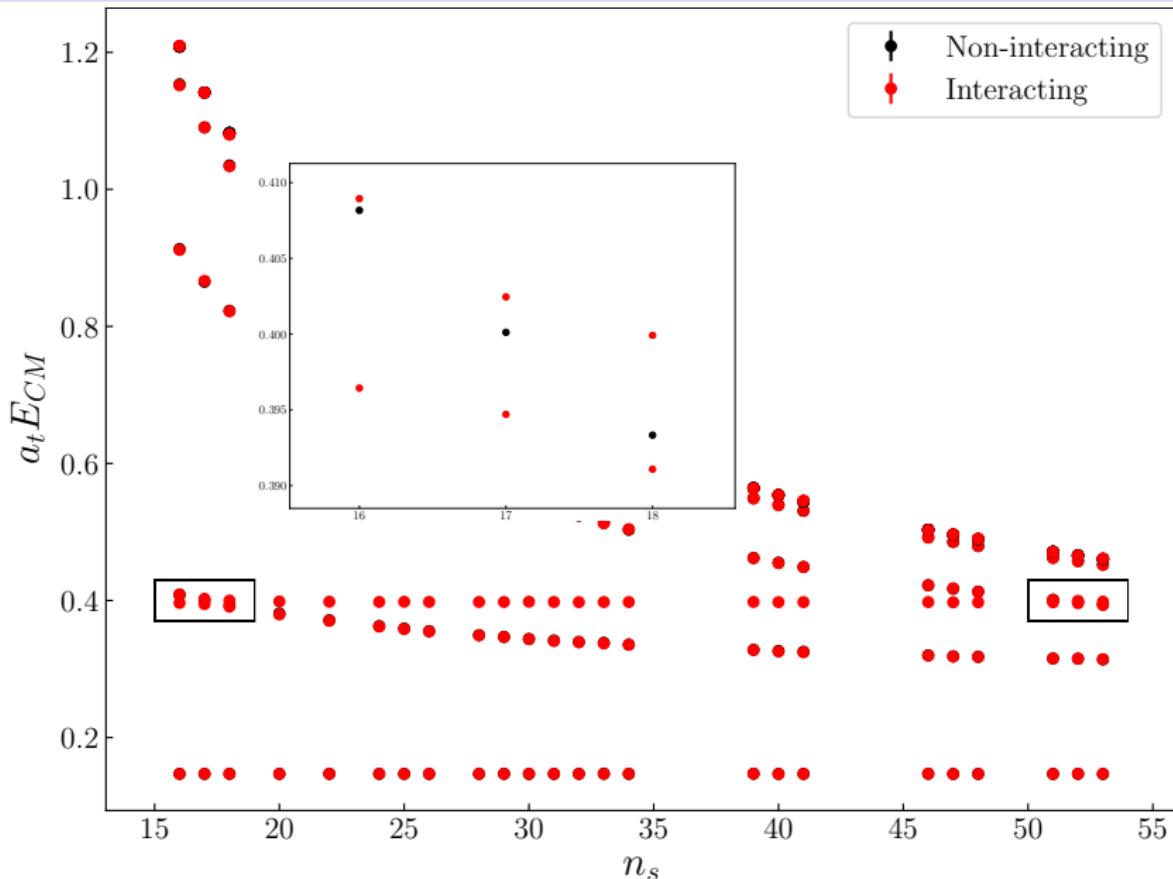
Energy Determinations: $\mathbf{d}^2 = 0$



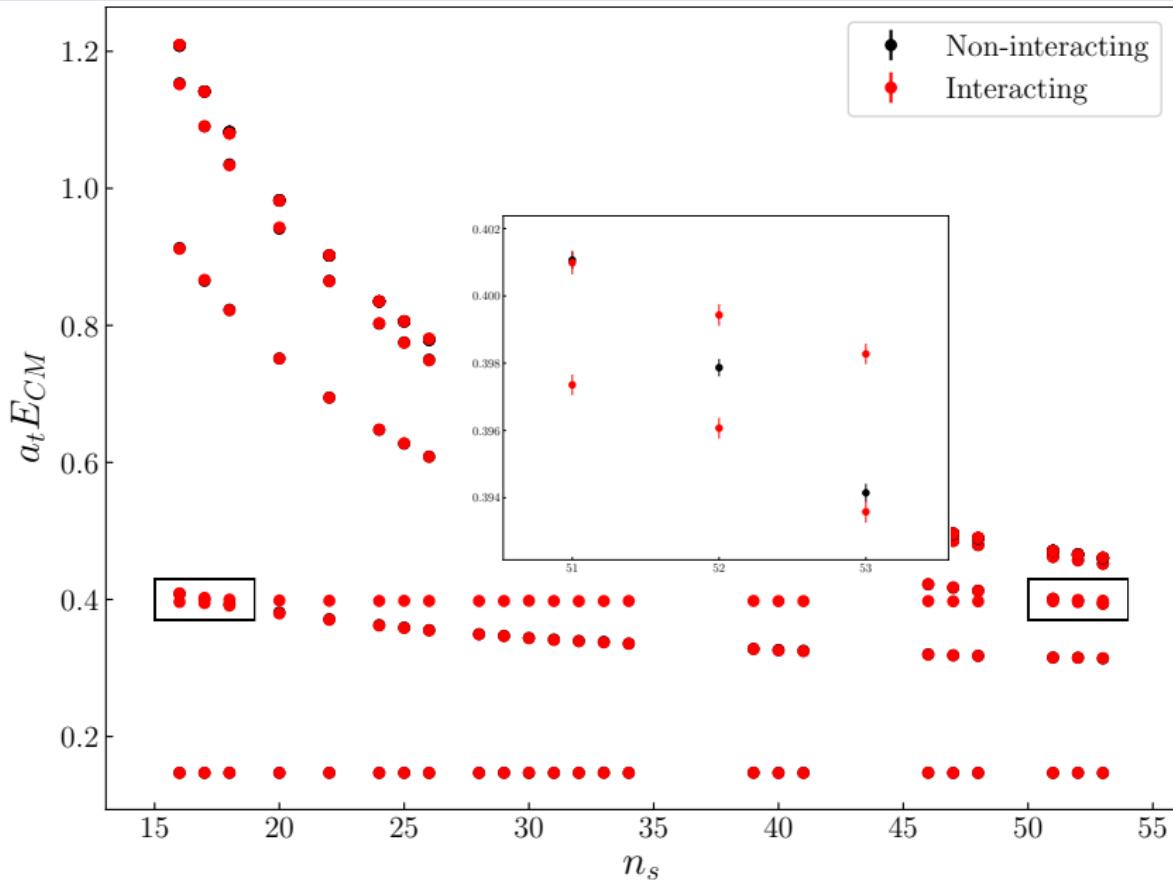
Energy Determinations: $\mathbf{d}^2 = 1$



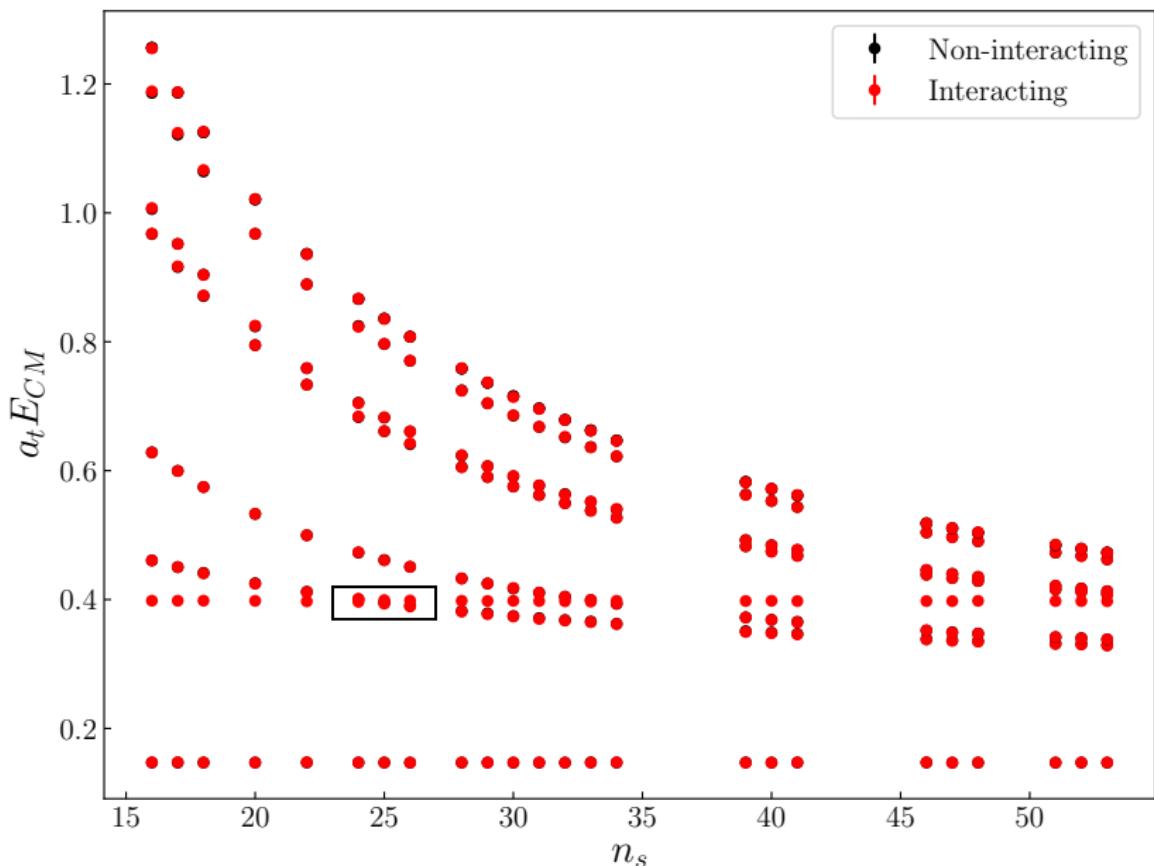
Energy Determinations: $\mathbf{d}^2 = 1$



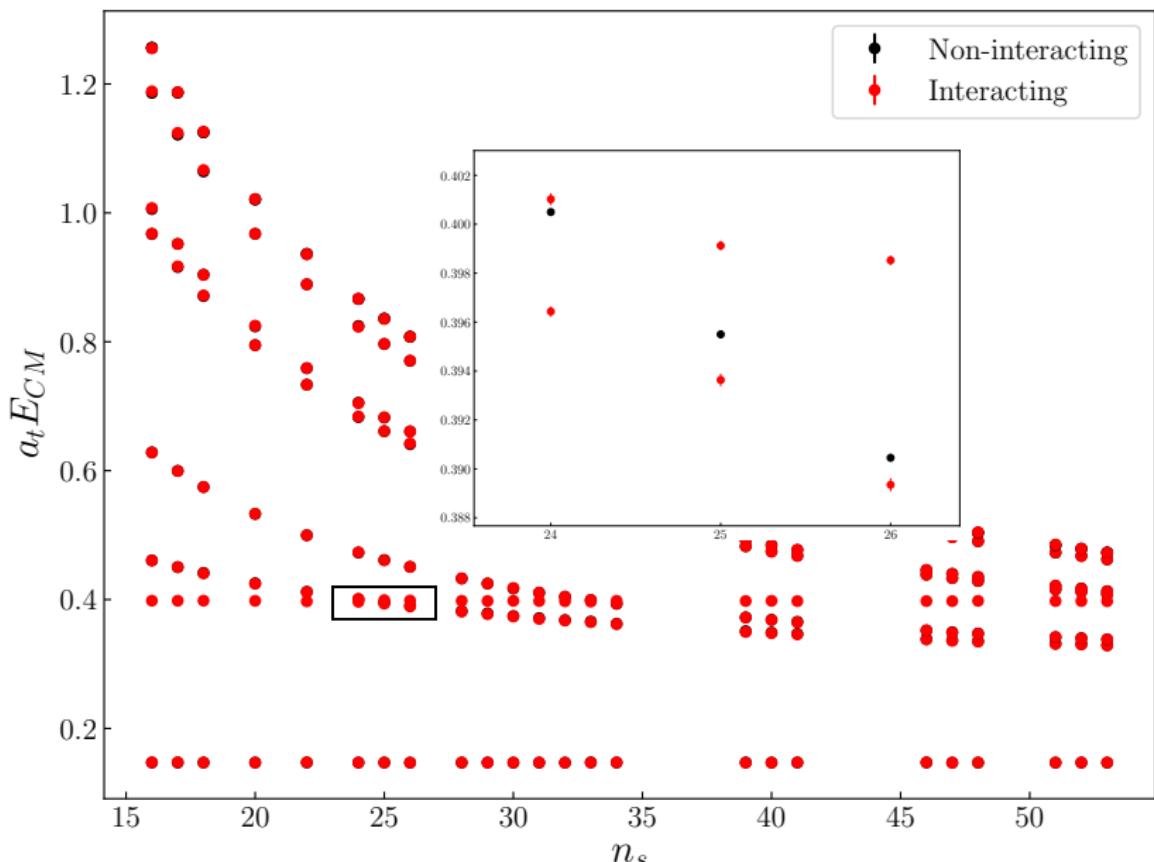
Energy Determinations: $\mathbf{d}^2 = 1$



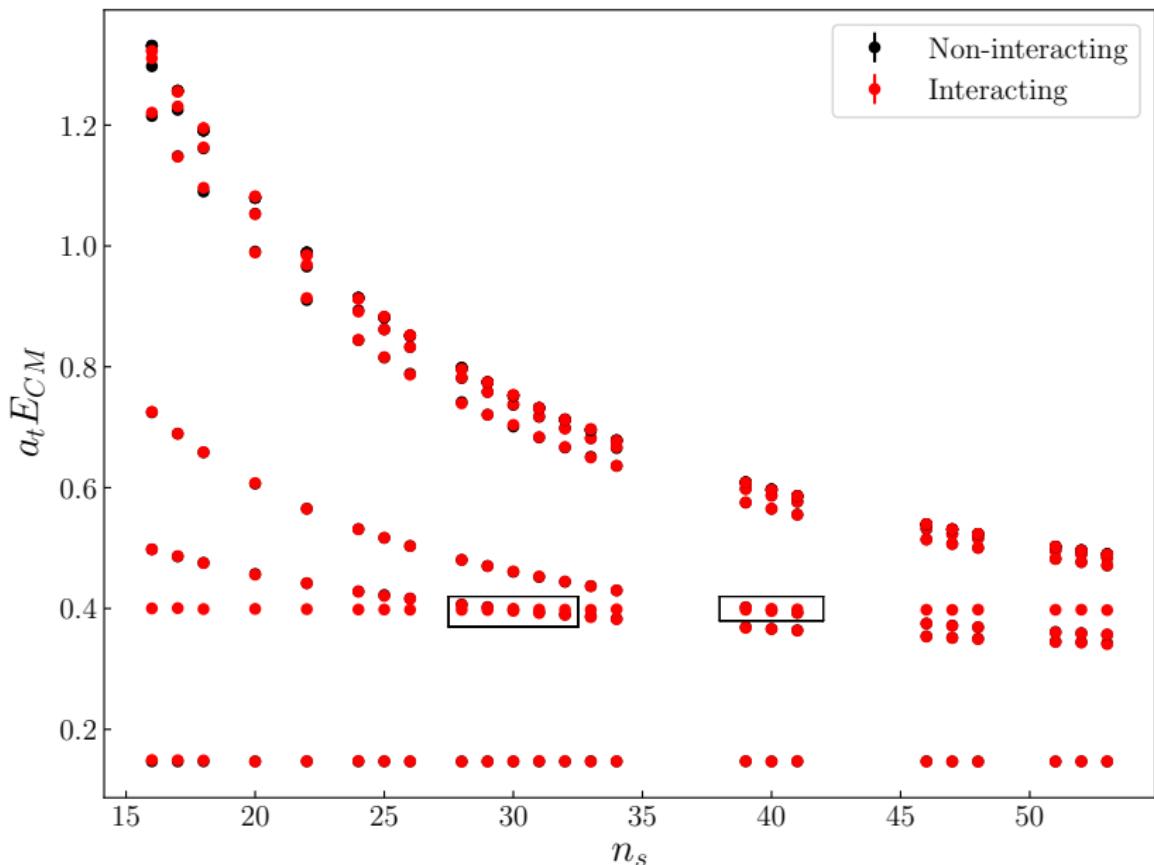
Energy Determinations: $\mathbf{d}^2 = 2$



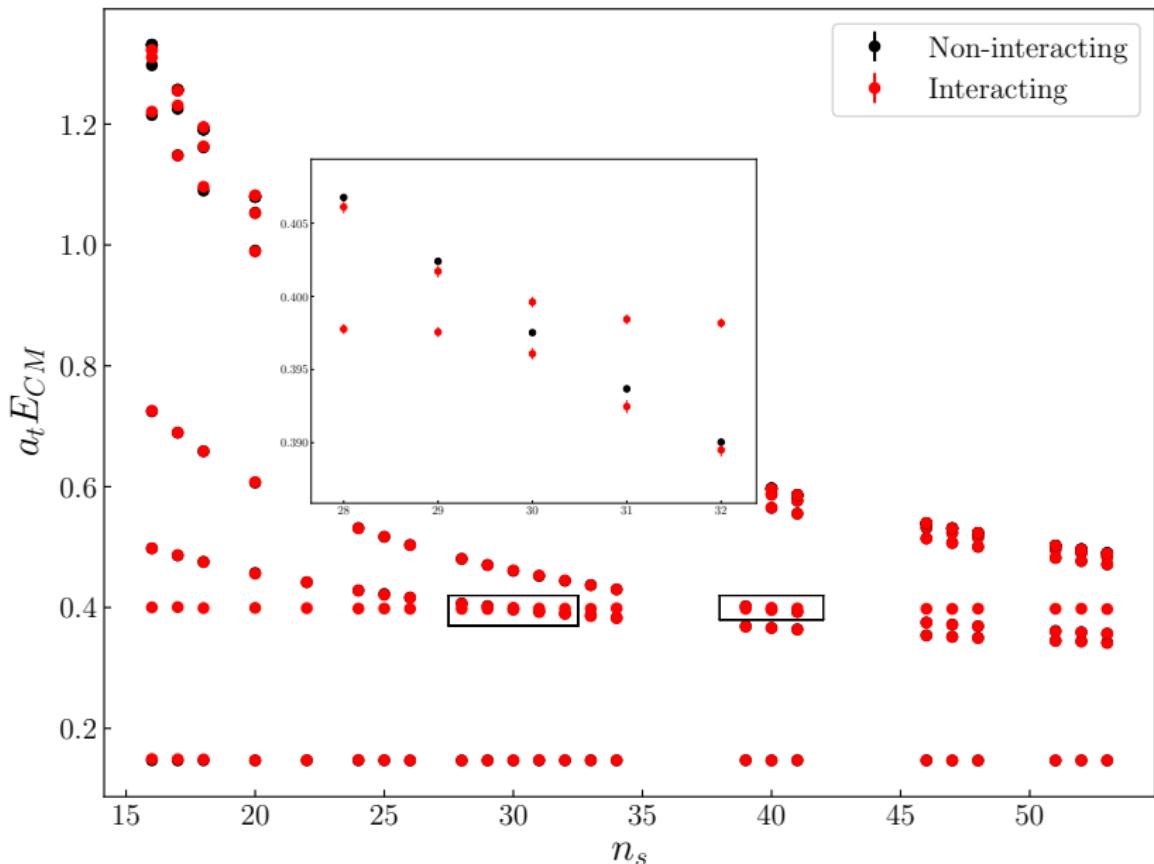
Energy Determinations: $\mathbf{d}^2 = 2$



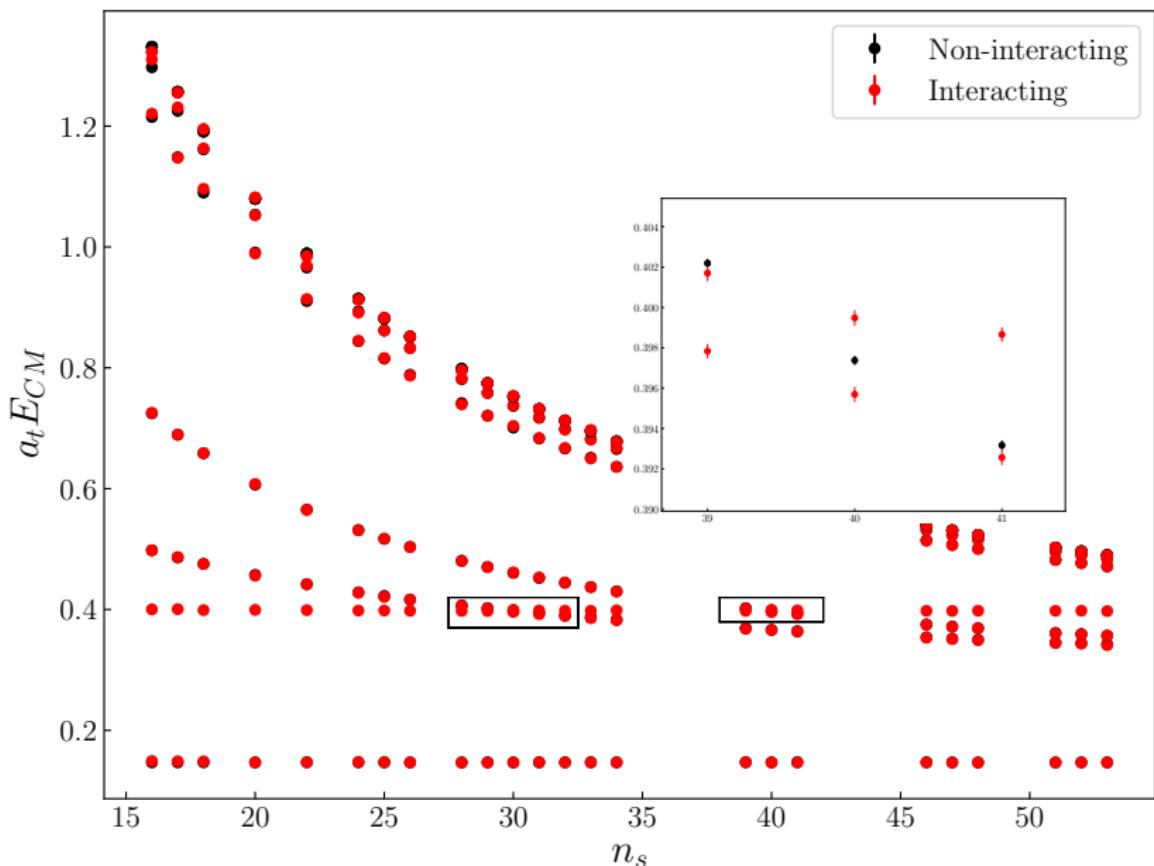
Energy Determinations: $\mathbf{d}^3 = 3$



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Energy Determinations: $\mathbf{d}^3 = 3$



Phase Shift Determination

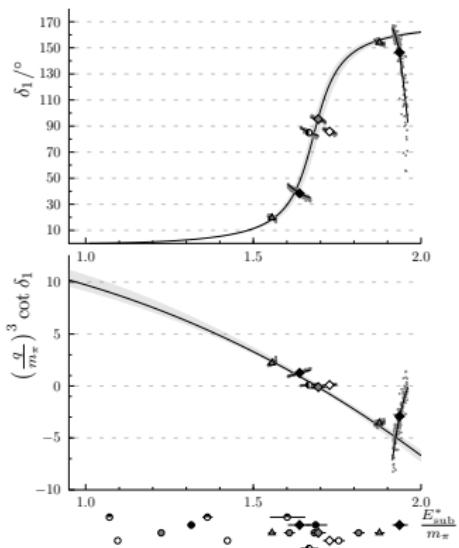
- Lüscher: Finite-volume spectrum \rightarrow infinite-volume scattering [3]
- Morningstar et al.: “box matrix” parametrization of quantization condition [4]:

$$\det \left[\tilde{K}^{-1} - B \right] = 0$$

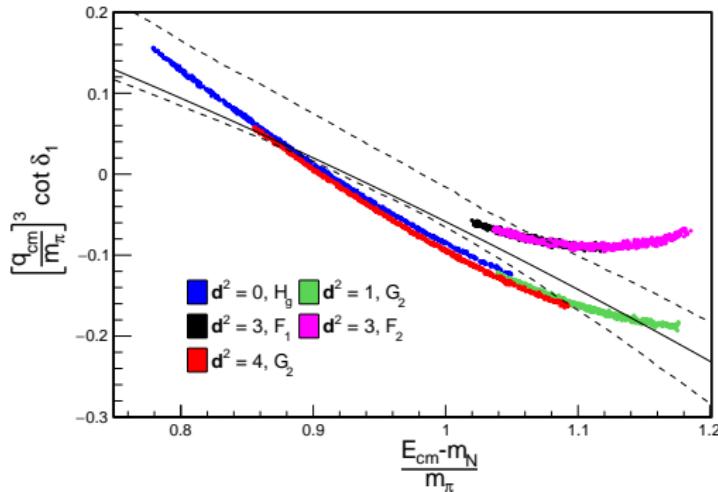
$$\tilde{K}_l^{-1} = \left(\frac{q_{CM}}{m_\phi} \right)^{2l+1} \cot \delta_l \quad B: \text{Known function of energies}$$

Resonance Parameters Determined using Box Matrix Parametrization in QCD

$K^*(892)$ resonance [2]:



$\Delta(1232)$ resonance [1]:



Phase Shift Determinations in ϕ/ρ model

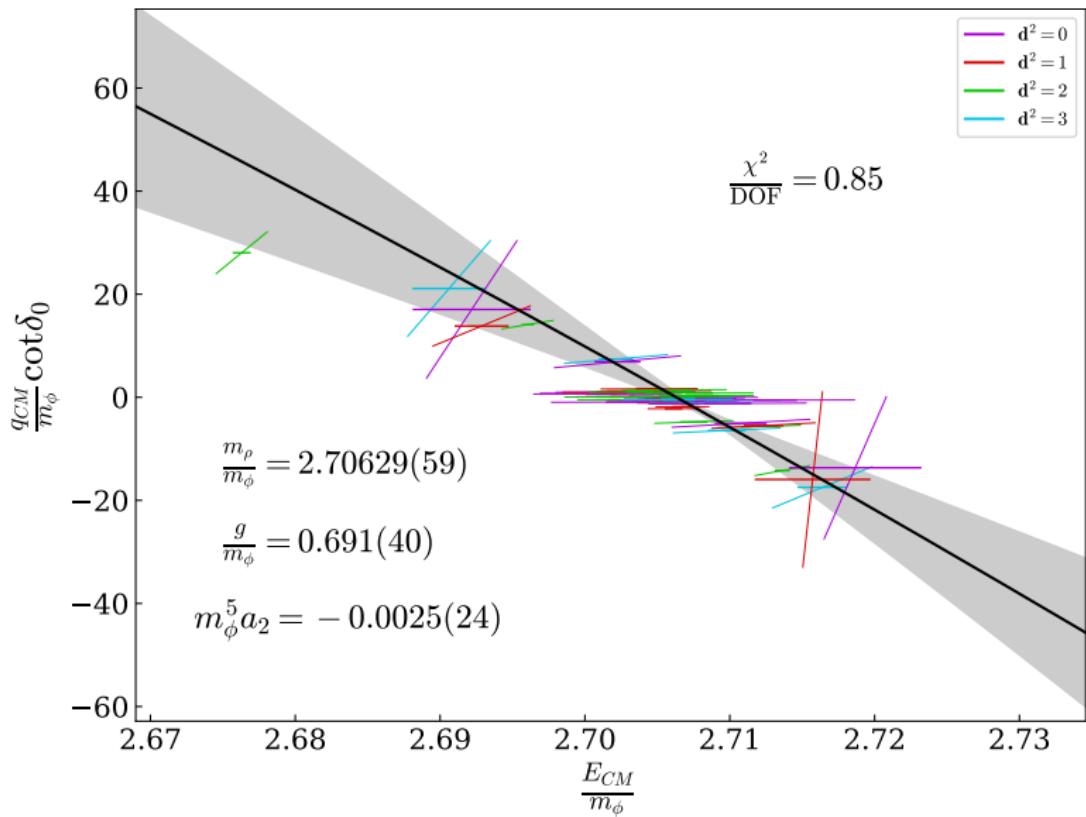
$$\tilde{K}_0^{-1} = \left(\frac{q_{CM}}{m_\phi} \right) \cot \delta_0 = \left(\frac{q_{CM}}{m_\phi} \right) \frac{m_\rho^2 - E_{CM}^2}{m_\rho \Gamma_\rho}$$

$$\tilde{K}_2^{-1} = -\frac{1}{m_\phi^5 a_2}$$

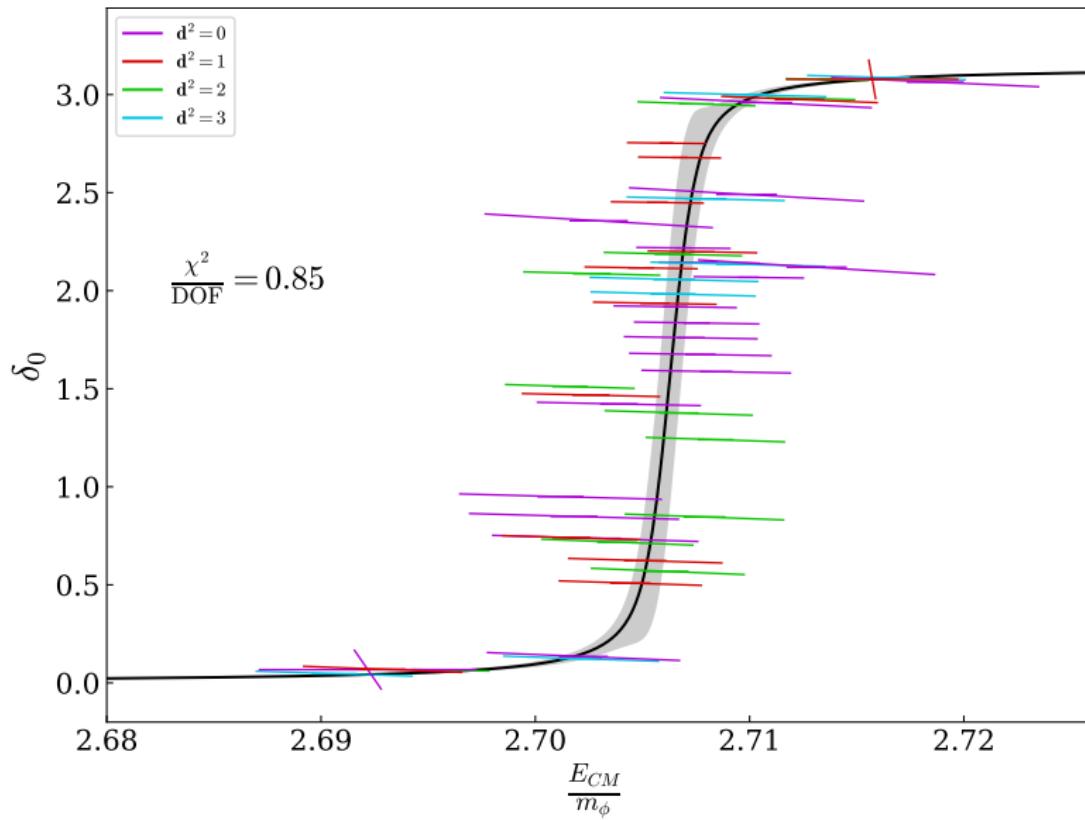
$$\Gamma_\rho = \frac{g^2}{32\pi m_\rho^2} \sqrt{m_\rho^2 - 4m_\phi^2} \quad q_{CM} = \sqrt{\left(\frac{E_{CM}}{2} \right)^2 - m_\phi^2}$$

- m_ρ , m_ϕ , g are renormalized

Phase Shift Determination



Phase Shift Determination



References I



C. W. Andersen, J. Bulava, B. Hörz, and C. Morningstar.

Elastic $I = 3/2$ p -wave nucleon-pion scattering amplitude and the $\Delta(1232)$ resonance from $N_f=2+1$ lattice QCD.

Phys. Rev., D97(1):014506, 2018.



R. Brett, J. Bulava, J. Fallica, A. Hanlon, B. Hörz, and C. Morningstar.

Determination of s-and p-wave $i=1/2$ $k\pi$ scattering amplitudes in $nf=2+1$ lattice qcd.

Nuclear Physics B, 932:29–51, 2018.



M. Lüscher.

Two-particle states on a torus and their relation to the scattering matrix.

Nuclear Physics B, 354(2-3):531–578, 1991.



C. Morningstar, J. Bulava, B. Singha, R. Brett, J. Fallica, A. Hanlon, and B. Hörz.

Estimating the two-particle k-matrix for multiple partial waves and decay channels from finite-volume energies.

Nuclear Physics B, 924:477–507, 2017.

References II



K. Rummukainen and S. Gottlieb.

Resonance scattering phase shifts on a non-rest-frame lattice.

Nuclear Physics B, 450(1-2):397–436, 1995.

Supplementary slides

Symanzik Improvement (Improved Finite Difference)

$$S_{\phi}^I = a_s^{D-1} a_t \sum_x \left\{ \frac{1}{2a_{\mu}^2} \sum_{\mu} \left(-\frac{4}{3}\phi(x + a_{\mu})\phi(x) - \frac{4}{3}\phi(x - a_{\mu})\phi(x) \right. \right. \\ \left. \left. + \frac{1}{12}\phi(x + 2a_{\mu})\phi(x) + \frac{1}{12}\phi(x - 2a_{\mu})\phi(x) + \frac{5}{2}\phi(x)^2 \right) \right. \\ \left. + \frac{1}{2}m_{\phi}^2\phi(x)^2 + \frac{\lambda_{\phi}}{4!}\phi(x)^4 \right\}$$

Updating scheme: Sequential Metropolis + *Microcanonical*

Microcanonical update procedure:

1. Propose a new field with some probability. If decision is to propose new field, continue, otherwise retain current value.
2. For initial value of dimensionless field φ at site x , solve $\delta_\phi S(\varphi) = 0$. 1-3 real solutions.
3. Choose one solution randomly with equal probability as proposed new field value, $\tilde{\varphi}$.
4. Accept this value with probability

$$P_{\text{acc}}(\tilde{\varphi} \leftarrow \varphi) = \min \left(1, \left| \frac{S'(\varphi)}{S'(\tilde{\varphi})} \right| \right)$$