Meson electromagnetic form factors from lattice QCD

C. T. H. Davies, J. Koponen, G. P. Lepage,
A. T. Lytle, A. C. Zimermmane-Santos

Abstract

Lattice QCD can provide a direct determination of meson electromagnetic form factors as a function of momentum-transfer from the photon, making predictions for upcoming experiments at Jefferson Lab. The form factors are a reflection of the bound-state nature of the meson and so these calculations give information about how confinement by QCD affects meson internal structure.

The region of high squared (space-like) momentum-transfer, Q^2, is of particular interest because perturbative QCD predictions take a simple form in that limit that depends on the meson decay constant (see below). We previously showed in [1] that, up to Q^2 of 6 GeV^2, the form factor for a 'pseudo-pion' made of s quarks was significantly larger than the asymptotic perturbative QCD result and showed no sign of heading towards that value at higher Q^2.

Here we give predictions for real mesons, the K^+ and K^0, in anticipation of JLAB results for the K^+ in the next few years. These results show a number of interesting qualitative features discussed below as well as a mismatch, up to Q^2 of 4 GeV^2, with asymptotic perturbative QCD expectations.

We also give results for a heavier meson, the η_c, up to Q^2 of 20 GeV^2 for a comparison to perturbative QCD in a higher Q^2 regime.

Lattice QCD calculation

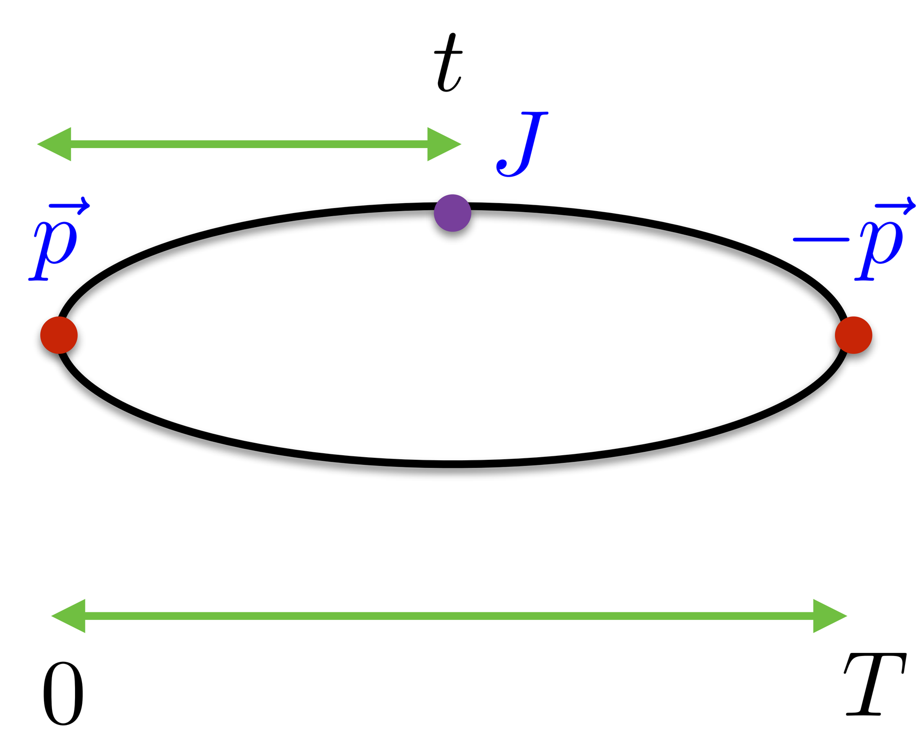
We use the Highly Improved Staggered Quark (HISQ) action on high-statistics ensembles of gluon field configurations that include 2+1+1 flavours of HISQ quarks in the sea, generated by the MILC collaboration. For the K meson results we use ensembles at 3 different values of the lattice spacing (0.15 fm, 0.12 fm and 0.09 fm approximately). The lattice spacing is determined by w_0, with the physical value of w_0 fixed from the pion decay constant [2]. We have well-tuned valence s quarks on each ensemble, and use valence light quarks with the same mass as the sea light quarks. The ensembles have sea light quarks with masses from 0.2 x that of the s quark down to the physical point. We compare two different spatial volumes to test for finite-volume effects.

For the η_c results we use well-tuned valence c quarks on ensembles with lattice spacing values of 0.09 fm and 0.06 fm. These have sea light quarks with masses 0.2 x that of s quarks only.

We calculate 2-point and 3-point correlation functions from quark propagators with zero and non-zero momentum. We use multiple time sources for increased statistics. We insert momentum using twisted boundary conditions. For 3-point correlators (figure above right) we use the Breit frame in which the initial and final states have equal and opposite spatial momentum. This maximises Q^2 for a given pa value.

The reach in Q^2 that is possible is limited more by the statistical errors that grow with pa than systematic errors at large pa [1]. Access to higher Q^2 is possible on finer lattices.

For the electromagnetic current, J, we use a one-link temporal current between 'Goldstone' pseudoscalar mesons. For the K meson we must calculate results for both light-quark and s-quark currents.



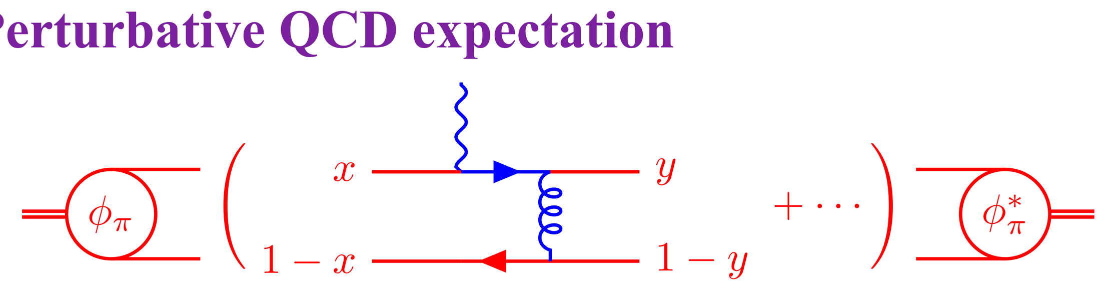
Simultaneous fits to 2-point and 3-point correlators as a function of t, for multiple T, at multiple momenta yield results for the matrix element

$$\langle P(\vec{p})|J|P(-\vec{p})\rangle = 2EF_P(Q^2)$$

with $Q^2 = |2\vec{p}|^2$

We normalise the current by dividing the form factor $F(Q^2)$ by its value at Q^2=0.

Perturbative QCD expectation

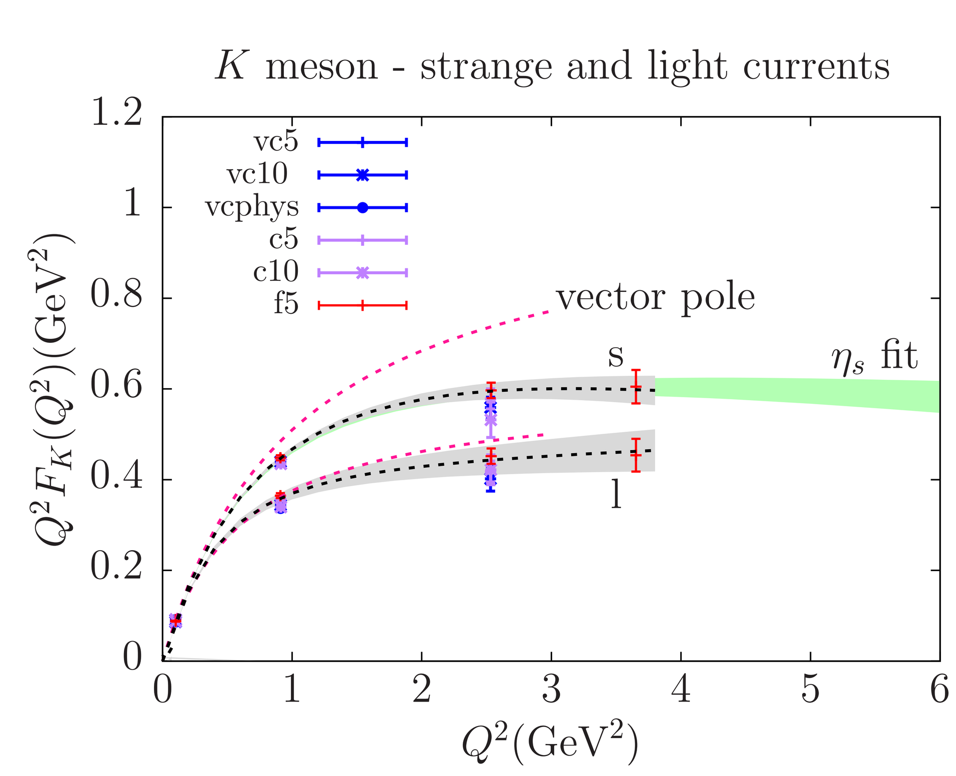


The central hard-photon scattering factorises from the 'distribution amplitudes' , φ, that describe the internal structure of the meson [3]. Redistribution of the photon momentum by gluons means that $F(Q^2)$ starts at $O(\alpha_s)$. Normalisation of φ gives, at very high Q^2 , :

$$Q^2F_P(Q^2) = 8\pi\alpha_s f_P^2$$

with decay constant, f_P. The key question is: at what Q^2 does perturbative physics start to be relevant?

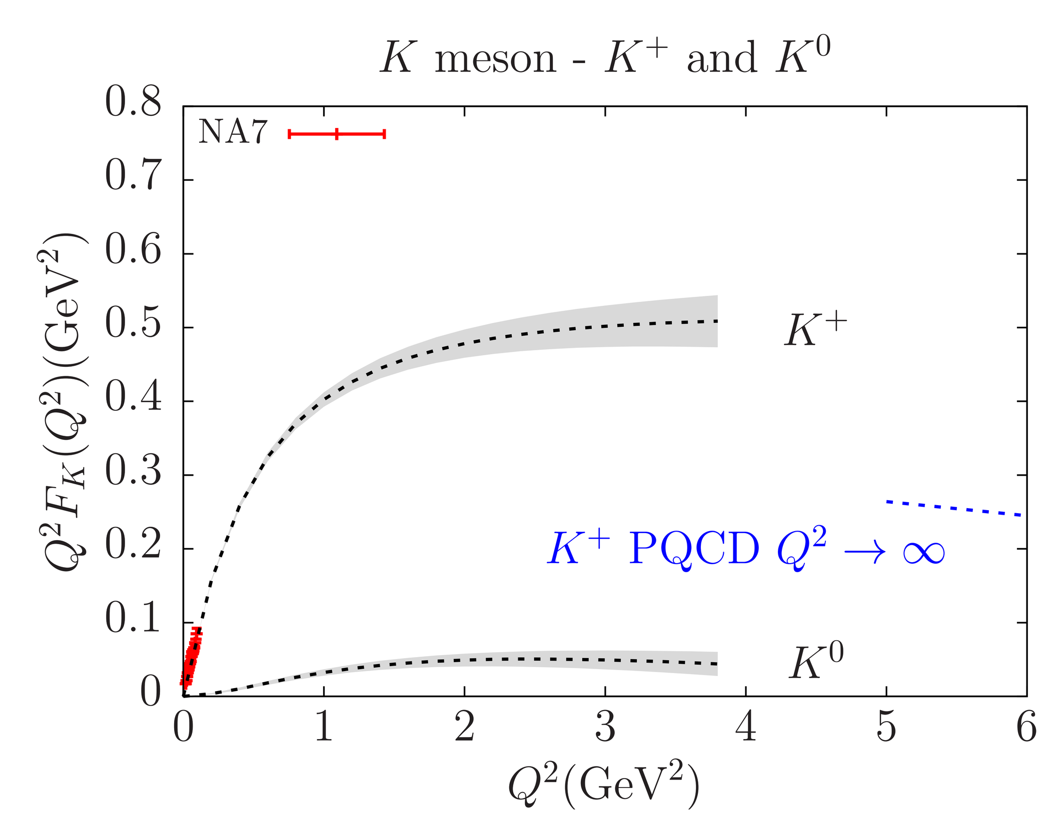
Results - K



To interpolate in Q^2 and allow extrapolation to physical light quark masses and a=0, we transform from Q^2 to z-space [1] and fit PF to a power-series in z, with coefficients that allow for a- and quark mass dependence. Here P is $(1+Q^2/M_V^2)$ where M_v is the appropriate vector mass expected from pole-dominance of the form factor at low Q^2 (i.e. ρ for the light-quark current and φ for the s-quark current).

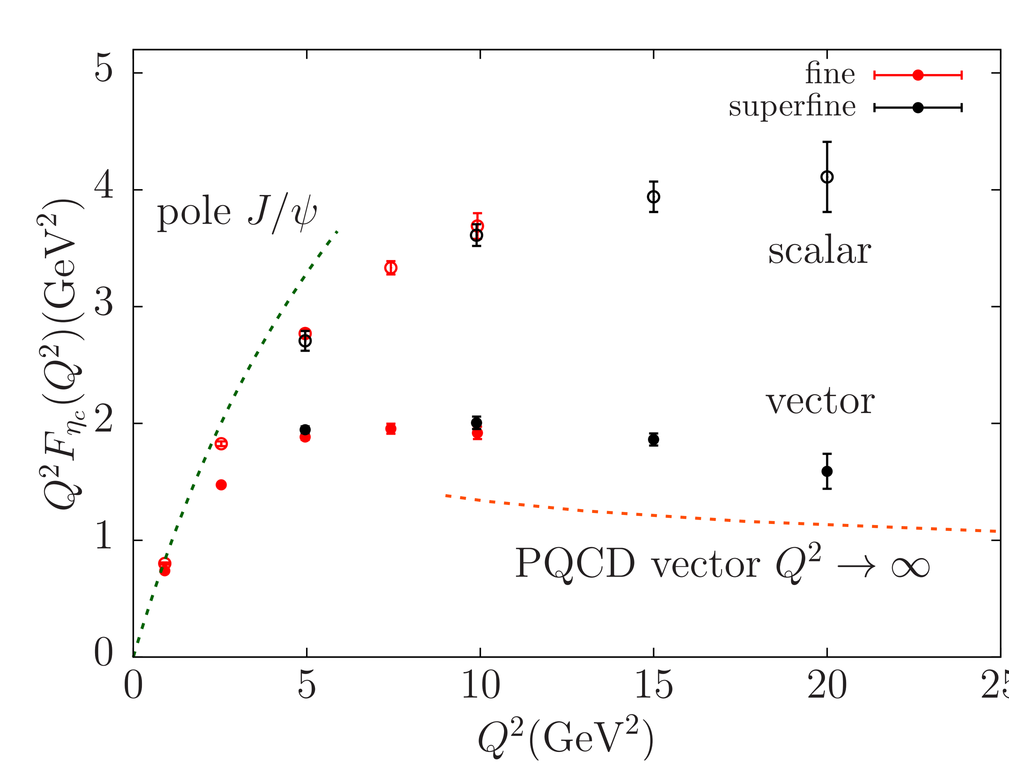
The plot above shows K form-factor results separately for light- and s-quark currents (with electric charge set to 1 in both cases). The grey band gives the continuum and chiral fit. Q^2F is flat for both currents above 2 GeV^2. Note how the s-current result agrees with our earlier η_s results [1], even though the 'spectator' quark is now a light one.

Combining form factors with appropriate electric charge weights allows us to obtain form factors for K^+ and K^0 - see plot below.



Note good agreement with NA7 results [4] at low Q^2, but poor agreement with asymptotic perturbative QCD. There is also no sign of a trend downwards towards the perturbative result, as might be obtained from corrections to the asymptotic distribution amplitude. These will be Q^2-dependent and are being calculated in lattice QCD by several groups.

Results - η_c



For valence c-quarks we are able to push to higher Q^2 values with good statistical precision. The plot above shows both the vector and scalar form factors as a function of Q^2. Both form factors have been normalised by their value at Q^2=0.

Once again the vector form factor disagrees with the asymptotic perturbative QCD result, now up to Q^2 of 20 GeV^2. Also note the shape of the scalar form factor. Perturbative QCD predicts that this should fall faster than $1/Q^2$ from helicity arguments [3].

Conclusion: By direct calculation in lattice QCD we can predict the electromagnetic form factor for the K for JLAB. Our results show that the approach of the form factor to the perturbative QCD regime requires much higher Q^2 than is often assumed.

[1] J. Koponen et al, HPQCD, 1701.04250.
[2] R. Dowdall et al, HPQCD, 1303.1670.
[3] G. P. Lepage and S. J. Brodsky, Phys Rev. D22, 2157 (1980).
[4] Amendolia et al, NA7, Phys. LettB 178 (1986).
Our calculations used Darwin@Cambridge, part of the UK STFC's DiRAC facility.