



Quark mass determination with the RI-SMOM intermediate scheme and HISQ action

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Motivations



- Lattice QCD is well suited to high precision determination of SM parameters
- Previous HPQCD result matching time moments of heavyonium correlators to perturbation theory yielded 1% precision [1408.4169]
- Check of previous result using a different method
- Recent result using a different methodology provides further check (Fermilab/MILC/TUMQCD [1802.04248]: J. Komijani's talk)



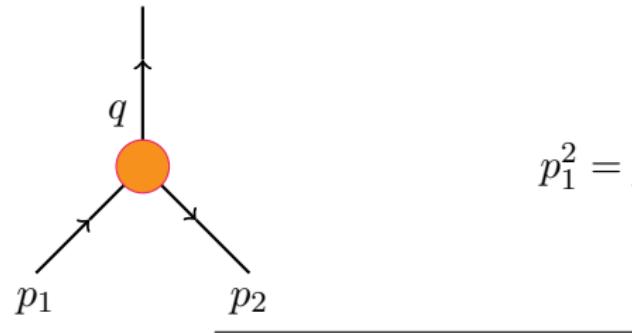


- Fix to Landau gauge
- Z factors defined in terms of momentum space propagators and vertex functions
- Use momentum sources $e^{-ip \cdot x}$ ($DS = e^{-ip \cdot x}$): multiple solves per configuration
- Only 20 configurations required for good precision

Sturm et. al. [0901.2599]



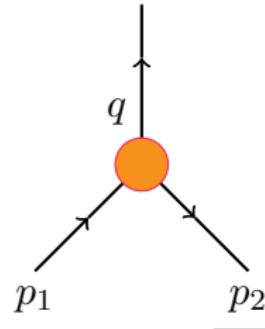
RI-SMOM



$$p_1^2 = p_2^2 = q^2 = \mu^2$$



RI-SMOM



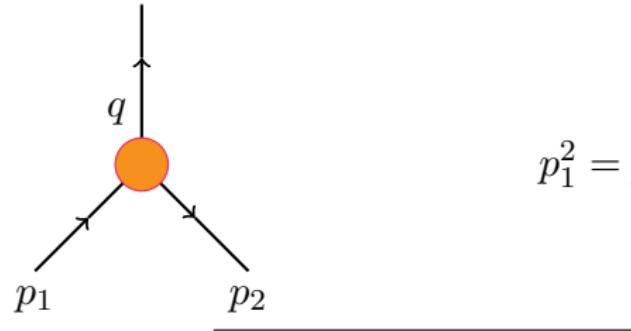
$$p_1^2 = p_2^2 = q^2 = \mu^2$$



$$G_S = \langle \chi(p'_1 + \pi A) \left(\sum_x \bar{\chi}(x) \chi(x) e^{i(p'_1 - p'_2)x} \right) \bar{\chi}(p'_2 + \pi B) \rangle$$



RI-SMOM



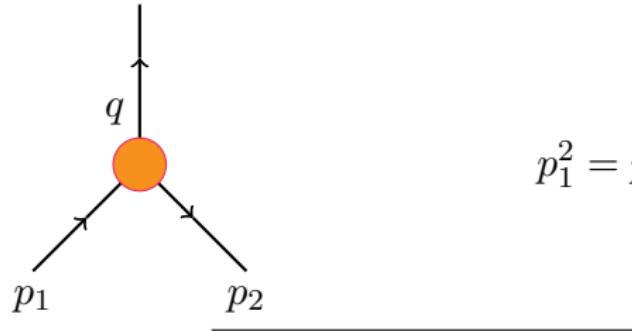
$$p_1^2 = p_2^2 = q^2 = \mu^2$$



$$G_S = \sum_x S(p_1, x) e^{i(p'_1 - p'_2)x} (-1)^x S^\dagger(p_2, x)$$



RI-SMOM



$$p_1^2 = p_2^2 = q^2 = \mu^2$$



$$\textcolor{orange}{G}_S = \sum_x S(p_1, x) e^{i(p'_1 - p'_2)x} (-1)^x S^\dagger(p_2, x)$$

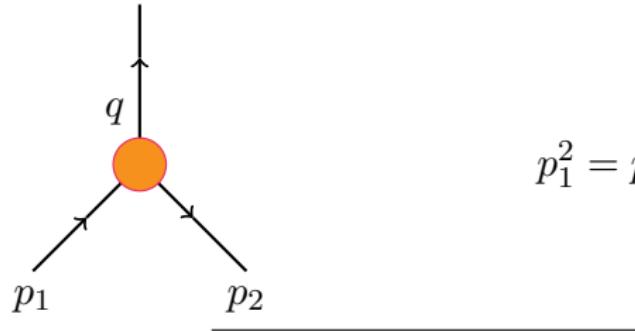
$$\textcolor{teal}{Z}_{\textcolor{blue}{q}} = \tfrac{1}{12p^2} \text{Tr}(S^{-1}(p)\not{p})$$

$$Z_q = -\frac{i}{48} \sum_{\mu} \frac{\hat{p}'_{\mu}}{(\hat{p}')^2} \text{Tr} \left[\overline{(\gamma_{\mu} \otimes I)} S^{-1}(p') \right] \text{ (staggered [1306.3881])}$$

A. Lytle & S. Sharpe



RI-SMOM



$$p_1^2 = p_2^2 = q^2 = \mu^2$$



$$\textcolor{orange}{G}_S = \sum_x S(p_1, x) e^{i(p'_1 - p'_2)x} (-1)^x S^\dagger(p_2, x)$$

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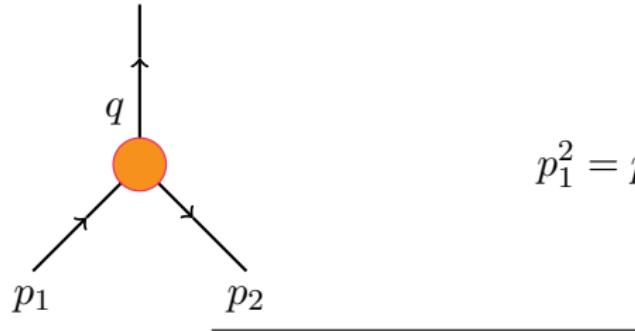
$$Z_q = -\frac{i}{48} \sum_{\mu} \frac{\hat{p}'_\mu}{(\hat{p}')^2} \text{Tr} \left[\overline{(\gamma_\mu \otimes I)} S^{-1}(p') \right] \text{ (staggered [1306.3881])}$$

A. Lytle & S. Sharpe

$$\Lambda_S(p') = S^{-1}(p'_1) \textcolor{orange}{G}_S S^{-1}(p'_2)$$



RI-SMOM



$$p_1^2 = p_2^2 = q^2 = \mu^2$$



$$\textcolor{orange}{G}_S = \sum_x S(p_1, x) e^{i(p'_1 - p'_2)x} (-1)^x S^\dagger(p_2, x)$$

$$\textcolor{teal}{Z}_q = \frac{1}{12p^2} \text{Tr}(S^{-1}(p)\not{p})$$

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A. Lytle & S. Sharpe

$$\Lambda_S(p') = S^{-1}(p'_1) \textcolor{orange}{G}_S S^{-1}(p'_2)$$

$$\textcolor{violet}{Z}_m = \textcolor{violet}{Z}_S = \frac{1}{48\textcolor{teal}{Z}_q} \text{Tr} \Lambda_S(p')$$



Lattices



HISQ 2+1+1 lattices generated by the MILC collaboration ([1004.0342] [1212.4768])

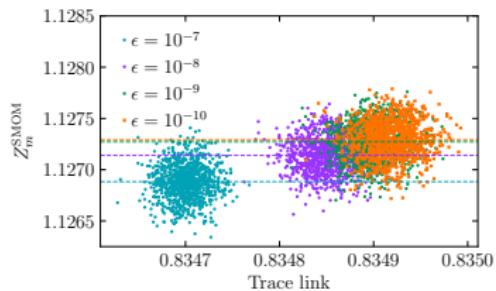
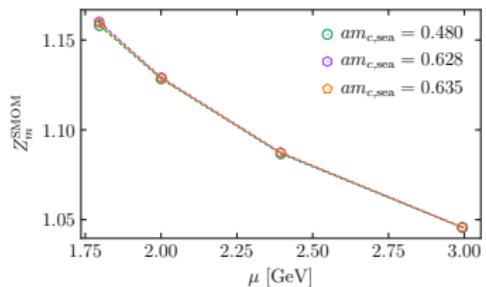
Set	β	L_s	L_t	am_l^{sea}	am_s^{sea}	am_c^{sea}
1	6.0	20	64	0.008	0.040	0.480
2	6.0	24	64	0.0102	0.0509	0.635
3	6.0	24	64	0.00507	0.0507	0.628
4	6.0	32	64	0.00507	0.0507	0.628
5	6.0	40	64	0.00507	0.0507	0.628
6	6.0	32	64	0.00507	0.00507	0.628
7	6.0	32	64	0.00507	0.012675	0.628
8	6.0	32	64	0.00507	0.022815	0.628
9	6.0	48	64	0.00184	0.0507	0.628
10	6.30	48	96	0.00363	0.0363	0.430
11	6.30	64	96	0.00120	0.0363	0.432
12	6.72	48	144	0.0048	0.024	0.286



Checks of systematics



- No observable sea quark mass dependence for strange and charm quarks
- No observable volume dependence
- Gauge fixing tolerance is a larger issue
- Strict gauge fixing tolerance is required to remove the impact on Z_m^{SMOM} values

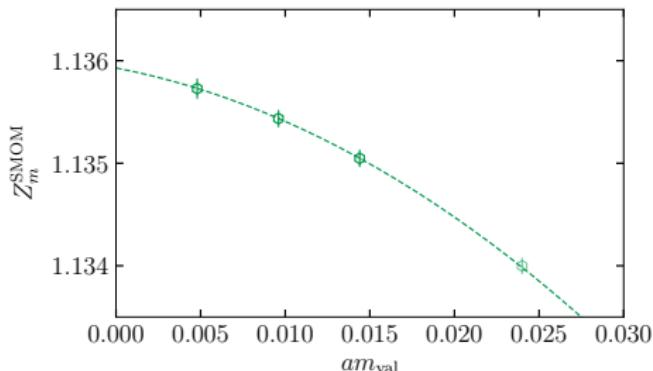


Valence mass extrapolation



- RI-SMOM scheme defined at $am_{\text{val}} \rightarrow 0$
- Extrapolation performed by calculating at 3 am_{val} values
- Fit form $Z_m^{\text{SMOM}}(\mu, am_{\text{val}}) =$

$$Z_m^{\text{SMOM}}(\mu) + d_1(\mu) \frac{am_{\text{val}}}{am_s} + d_2(\mu) \left(\frac{am_{\text{val}}}{am_s} \right)^2 + d_3(\mu) \left(\frac{am_{\text{val}}}{am_s} \right)^3$$



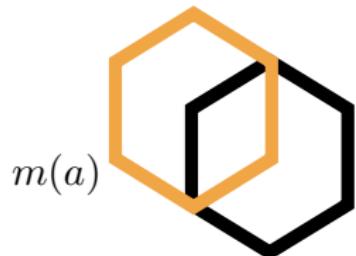
Condensate contributions



- Expect $m\bar{\psi}\psi/\mu^4$ terms in both Z_q and Λ_S
 - Removed by valence mass extrapolation
- Expect gauge-noninvariant condensate as working in Landau gauge
 - $\langle A^2 \rangle / \mu^2$
- $\langle A^2 \rangle$ could be $\sim (1\text{GeV})^2$ so should be accounted for
- Also allow for higher dimension gluon condensate operators



Construction of fitting data



Bare mass determined from η_c or η_s (one for each β at physical sea masses)



Construction of fitting data

$$Z_m^{\text{SMOM}}(\mu, a)m(a)$$



Bare mass determined from η_c or η_s (one for each β at physical sea masses)

Multiply by Z_m^{SMOM}



Construction of fitting data

$$Z_m^{\overline{\text{MS}}/\text{SMOM}}(\alpha_s(\mu)) Z_m^{\text{SMOM}}(\mu, a) m(a)$$



Bare mass determined from η_c or η_s (one for each β at physical sea masses)

Multiply by Z_m^{SMOM}

Convert to $\overline{\text{MS}}$ (we use a conversion factor corrected for non-zero charm mass in the sea)



Construction of fitting data

$$R(3\text{GeV}, \mu) Z_m^{\overline{\text{MS}}/\text{SMOM}}(\alpha_s(\mu)) Z_m^{\text{SMOM}}(\mu, a) m(a)$$



Bare mass determined from η_c or η_s (one for each β at physical sea masses)

Multiply by Z_m^{SMOM}

Convert to $\overline{\text{MS}}$ (we use a conversion factor corrected for non-zero charm mass in the sea)

Multiple μ values (2, 2.5, 3 and 4 GeV)
to control condensate

Run all to 3 GeV (four loop β function)



Continuum extrapolation



$$\begin{aligned} \overline{m}(\mu_{\text{ref}}, \mu, a) &= \overline{m}(\mu_{\text{ref}}) \times \\ &\left[1 + \sum_{n=1}^4 c_{\Lambda^2 a^2}^{(n)} (\Lambda a / \pi)^{2n} \right] \times \\ &\left(1 + \sum_{n=1}^{10} c_{\mu^2 a^2}^{(n)} (\mu a / \pi)^{2n} + c_\alpha \alpha_{\overline{\text{MS}}}^3(\mu) + \right. \\ &h_\ell^{\text{sea}} \frac{\delta_\ell^{\text{sea}}}{m_s} + h_c^{\text{sea}} \frac{\delta_c^{\text{sea}}}{m_c} + \\ &\left. \left[1 + k_\ell^{\text{sea}} \frac{\delta_\ell^{\text{sea}}}{m_s} + k_c^{\text{sea}} \frac{\delta_c^{\text{sea}}}{m_c} \right] \times \right. \\ &\left. \sum_{n=1}^3 c_{\text{cond}}^{(n)} \alpha_{\overline{\text{MS}}}(\mu) \frac{(1 \text{ GeV})^{2n}}{\mu^{2n}} \times \left[1 + c_{\text{cond}, a^2}^{(n)} (\tilde{\Lambda} a / \pi)^2 \right] \right). \end{aligned}$$



Continuum extrapolation



Accounts for a dependence of bare masses

$$\begin{aligned} \overline{m}(\mu_{\text{ref}}, \mu, a) &= \overline{m}(\mu_{\text{ref}}) \times \\ &\left[1 + \sum_{n=1}^4 c_{\Lambda^2 a^2}^{(n)} (\Lambda a / \pi)^{2n} \right] \times \\ &\left(1 + \sum_{n=1}^{10} c_{\mu^2 a^2}^{(n)} (\mu a / \pi)^{2n} + c_\alpha \alpha_{\overline{\text{MS}}}^3(\mu) + \right. \\ &h_\ell^{\text{sea}} \frac{\delta_\ell^{\text{sea}}}{m_s} + h_c^{\text{sea}} \frac{\delta_c^{\text{sea}}}{m_c} + \\ &\left. \left[1 + k_\ell^{\text{sea}} \frac{\delta_\ell^{\text{sea}}}{m_s} + k_c^{\text{sea}} \frac{\delta_c^{\text{sea}}}{m_c} \right] \times \right. \\ &\left. \sum_{n=1}^3 c_{\text{cond}}^{(n)} \alpha_{\overline{\text{MS}}}(\mu) \frac{(1 \text{ GeV})^{2n}}{\mu^{2n}} \times \left[1 + c_{\text{cond}, a^2}^{(n)} (\tilde{\Lambda} a / \pi)^2 \right] \right). \end{aligned}$$



Continuum extrapolation



Accounts for a dependence of Z_m^{SMOM}

$$\begin{aligned} \overline{m}(\mu_{\text{ref}}, \mu, a) &= \overline{m}(\mu_{\text{ref}}) \times \\ &\left[1 + \sum_{n=1}^4 c_{\Lambda^2 a^2}^{(n)} (\Lambda a / \pi)^{2n} \right] \times \\ &\left(1 + \sum_{n=1}^{10} c_{\mu^2 a^2}^{(n)} (\mu a / \pi)^{2n} + c_\alpha \alpha_{\overline{\text{MS}}}^3(\mu) + \right. \\ &h_\ell^{\text{sea}} \frac{\delta_\ell^{\text{sea}}}{m_s} + h_c^{\text{sea}} \frac{\delta_c^{\text{sea}}}{m_c} + \\ &\left. \left[1 + k_\ell^{\text{sea}} \frac{\delta_\ell^{\text{sea}}}{m_s} + k_c^{\text{sea}} \frac{\delta_c^{\text{sea}}}{m_c} \right] \times \right. \\ &\left. \sum_{n=1}^3 c_{\text{cond}}^{(n)} \alpha_{\overline{\text{MS}}}(\mu) \frac{(1 \text{ GeV})^{2n}}{\mu^{2n}} \times \left[1 + c_{\text{cond}, a^2}^{(n)} (\tilde{\Lambda} a / \pi)^2 \right] \right). \end{aligned}$$



Continuum extrapolation



Missing α_s^3 term in $\overline{\text{MS}}$ conversion

$$\begin{aligned} \overline{m}(\mu_{\text{ref}}, \mu, a) &= \overline{m}(\mu_{\text{ref}}) \times \\ &\left[1 + \sum_{n=1}^4 c_{\Lambda^2 a^2}^{(n)} (\Lambda a / \pi)^{2n} \right] \times \\ &\left(1 + \sum_{n=1}^{10} c_{\mu^2 a^2}^{(n)} (\mu a / \pi)^{2n} + c_\alpha \alpha_{\overline{\text{MS}}}^3(\mu) + \right. \\ &h_\ell^{\text{sea}} \frac{\delta_\ell^{\text{sea}}}{m_s} + h_c^{\text{sea}} \frac{\delta_c^{\text{sea}}}{m_c} + \\ &\left. \left[1 + k_\ell^{\text{sea}} \frac{\delta_\ell^{\text{sea}}}{m_s} + k_c^{\text{sea}} \frac{\delta_c^{\text{sea}}}{m_c} \right] \times \right. \\ &\left. \sum_{n=1}^3 c_{\text{cond}}^{(n)} \alpha_{\overline{\text{MS}}}(\mu) \frac{(1 \text{ GeV})^{2n}}{\mu^{2n}} \times \left[1 + c_{\text{cond}, a^2}^{(n)} (\tilde{\Lambda} a / \pi)^2 \right] \right). \end{aligned}$$



Continuum extrapolation



Sea quark mass dependence in Z_m^{SMOM}

$$\begin{aligned} \overline{m}(\mu_{\text{ref}}, \mu, a) &= \overline{m}(\mu_{\text{ref}}) \times \\ &\left[1 + \sum_{n=1}^4 c_{\Lambda^2 a^2}^{(n)} (\Lambda a / \pi)^{2n} \right] \times \\ &\left(1 + \sum_{n=1}^{10} c_{\mu^2 a^2}^{(n)} (\mu a / \pi)^{2n} + c_\alpha \alpha_{\overline{\text{MS}}}^3(\mu) + \right. \\ &h_\ell^{\text{sea}} \frac{\delta_\ell^{\text{sea}}}{m_s} + h_c^{\text{sea}} \frac{\delta_c^{\text{sea}}}{m_c} + \\ &\left. \left[1 + k_\ell^{\text{sea}} \frac{\delta_\ell^{\text{sea}}}{m_s} + k_c^{\text{sea}} \frac{\delta_c^{\text{sea}}}{m_c} \right] \times \right. \\ &\left. \sum_{n=1}^3 c_{\text{cond}}^{(n)} \alpha_{\overline{\text{MS}}}(\mu) \frac{(1 \text{ GeV})^{2n}}{\mu^{2n}} \times \left[1 + c_{\text{cond}, a^2}^{(n)} (\tilde{\Lambda} a / \pi)^2 \right] \right). \end{aligned}$$



Continuum extrapolation



Condensate terms

$$\begin{aligned} \overline{m}(\mu_{\text{ref}}, \mu, a) &= \overline{m}(\mu_{\text{ref}}) \times \\ &\left[1 + \sum_{n=1}^4 c_{\Lambda^2 a^2}^{(n)} (\Lambda a / \pi)^{2n} \right] \times \\ &\left(1 + \sum_{n=1}^{10} c_{\mu^2 a^2}^{(n)} (\mu a / \pi)^{2n} + c_\alpha \alpha_{\overline{\text{MS}}}^3(\mu) + \right. \\ &h_\ell^{\text{sea}} \frac{\delta_\ell^{\text{sea}}}{m_s} + h_c^{\text{sea}} \frac{\delta_c^{\text{sea}}}{m_c} + \\ &\left. \left[1 + k_\ell^{\text{sea}} \frac{\delta_\ell^{\text{sea}}}{m_s} + k_c^{\text{sea}} \frac{\delta_c^{\text{sea}}}{m_c} \right] \times \right. \\ &\left. \sum_{n=1}^3 c_{\text{cond}}^{(n)} \alpha_{\overline{\text{MS}}}(\mu) \frac{(1 \text{ GeV})^{2n}}{\mu^{2n}} \times \left[1 + c_{\text{cond}, a^2}^{(n)} (\tilde{\Lambda} a / \pi)^2 \right] \right). \end{aligned}$$



Continuum extrapolation



Possible a dependence of condensates

$$\begin{aligned} \overline{m}(\mu_{\text{ref}}, \mu, a) &= \overline{m}(\mu_{\text{ref}}) \times \\ &\left[1 + \sum_{n=1}^4 c_{\Lambda^2 a^2}^{(n)} (\Lambda a / \pi)^{2n} \right] \times \\ &\left(1 + \sum_{n=1}^{10} c_{\mu^2 a^2}^{(n)} (\mu a / \pi)^{2n} + c_\alpha \alpha_{\overline{\text{MS}}}^3(\mu) + \right. \\ &h_\ell^{\text{sea}} \frac{\delta_\ell^{\text{sea}}}{m_s} + h_c^{\text{sea}} \frac{\delta_c^{\text{sea}}}{m_c} + \\ &\left. \left[1 + k_\ell^{\text{sea}} \frac{\delta_\ell^{\text{sea}}}{m_s} + k_c^{\text{sea}} \frac{\delta_c^{\text{sea}}}{m_c} \right] \times \right. \\ &\left. \sum_{n=1}^3 c_{\text{cond}}^{(n)} \alpha_{\overline{\text{MS}}}(\mu) \frac{(1 \text{ GeV})^{2n}}{\mu^{2n}} \times \left[1 + c_{\text{cond}, a^2}^{(n)} (\tilde{\Lambda} a / \pi)^2 \right] \right). \end{aligned}$$



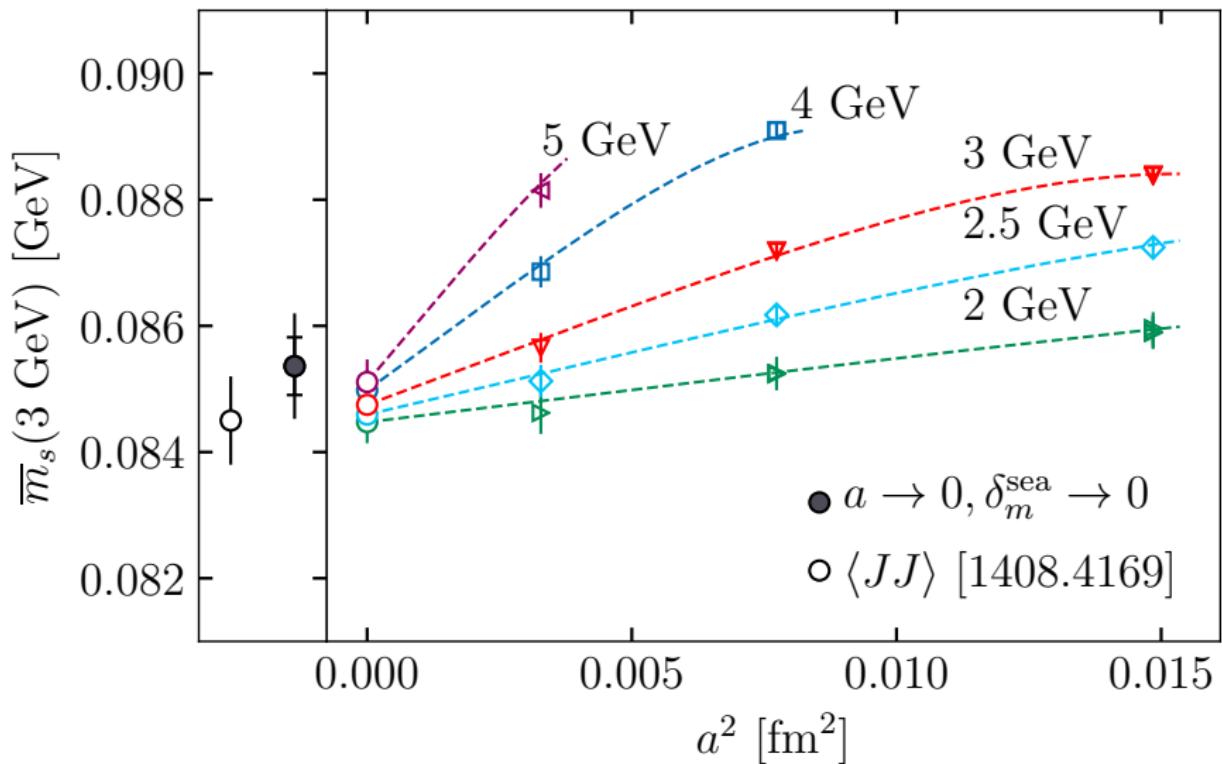
Continuum extrapolation

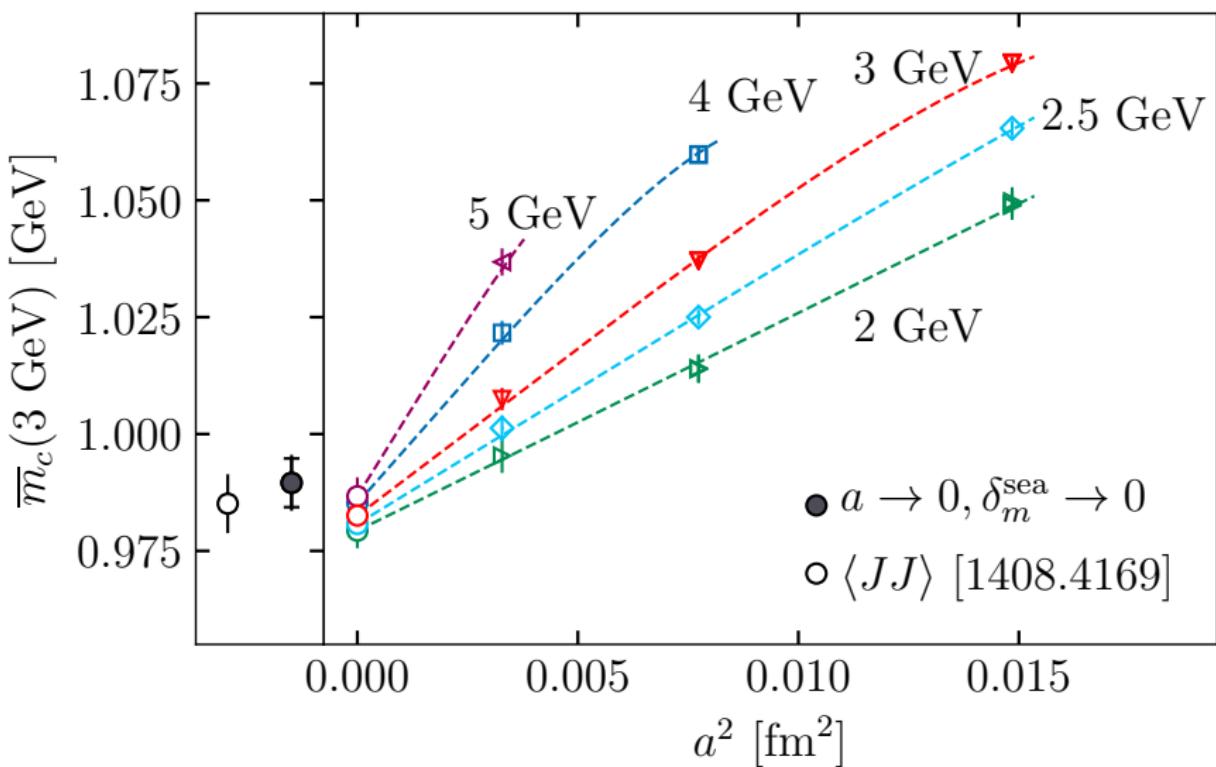


Possible sea quark mass dependence of condensates

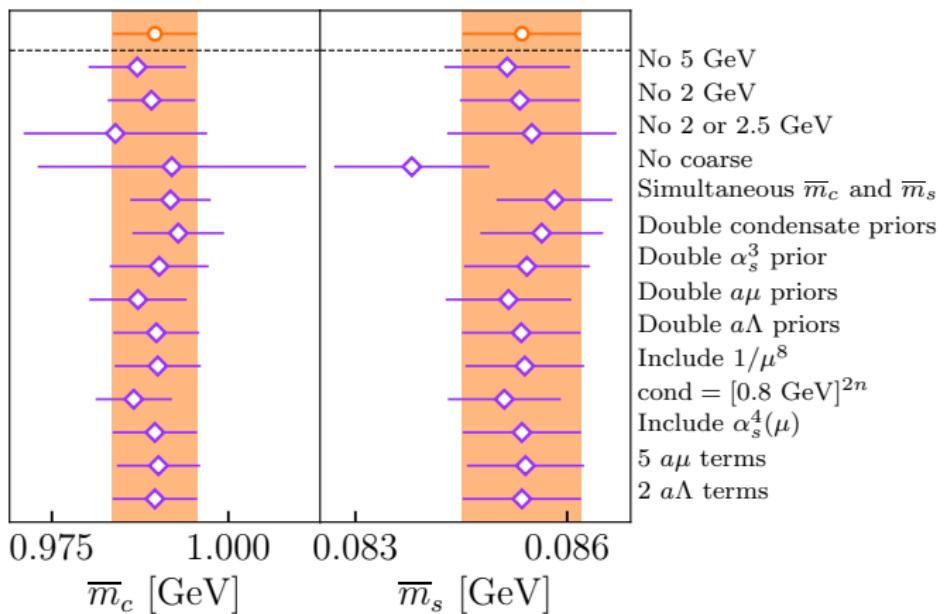
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Fit tests



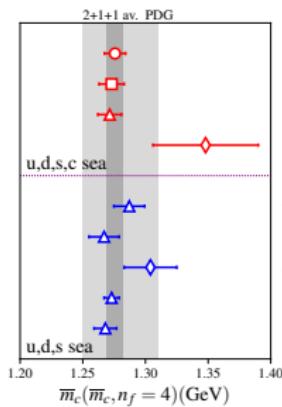
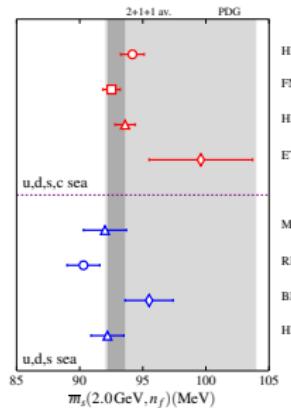
Error budget



	$\overline{m}_c(3 \text{ GeV})$	$\overline{m}_s(3 \text{ GeV})$
$a^2 \rightarrow 0$	0.28	0.28
Missing α_s^3 term	0.22	0.22
Condensate	0.23	0.23
m_{sea} effects	0.00	0.00
$Z_m^{\overline{\text{MS}}/\text{SMOM}}$ and R	0.04	0.04
Z_m^{SMOM}	0.13	0.13
Uncorrelated m^{tuned}	0.20	0.23
Correlated m^{tuned}	0.30	0.82
Gauge fixing	0.11	0.11
μ error from w_0	0.12	0.12
Total:	0.62%	0.99%



Summary



2+1+1 averages

$$\overline{m}_s(2 \text{ GeV}, n_f = 4) = 0.09291(78) \text{ GeV}$$

$$\overline{m}_c(\overline{m}_c, n_f = 4) = 1.2753(65) \text{ GeV}$$

