

The critical endpoint in the 2D-gauge-Higgs model at topological angle $\theta = \pi$

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Work done in collaboration with
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[arXiv: [1807.07793](https://arxiv.org/abs/1807.07793)]

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Conventional lattice representation with Villain action

Matter fields $\phi_x \in \mathbb{C}$, Gauge angles $A_{x,\mu} \in [-\pi, \pi]$

$$Z = \int D[A] B_G[A] \int D[\phi] e^{-S_\phi[\phi, A]} ,$$

$$B_G[A] = \prod_{x \in \Lambda} \sum_{n_x \in \mathbb{Z}} e^{-\frac{\beta}{2}(F_x + 2\pi n_x)^2 - i \frac{\theta}{2\pi}(F_x + 2\pi n_x)} ,$$

$$F_x = A_{x,1} + A_{x+\hat{1},2} - A_{x+\hat{2},1} - A_{x,2} ,$$

$$S_\phi[\phi, A] = \sum_{x \in \Lambda} \left[(m^2 + 4) |\phi_x|^2 + \lambda |\phi_x|^4 - \sum_{\mu=1}^2 \left(\phi_x^* e^{iA_{x,\mu}} \phi_{x+\hat{\mu}} + c.c. \right) \right] .$$

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Global charge conjugation symmetry \mathcal{C} at $\theta = \pi$:

$$A_{x,\mu} \rightarrow -A_{x,\mu} , \quad \phi_x \rightarrow \phi_x^*$$

- Implemented exactly with Villain action

Worldline representation solves complex action problem

$$Z = \sum_{\{j,p\}} W_H[j] W_G[p] \prod_x \delta(\vec{\nabla} j_x) \delta(j_{x,1} + p_x - p_{x-2}) \delta(j_{x,2} - p_x + p_{x-1})$$

Dual variables:

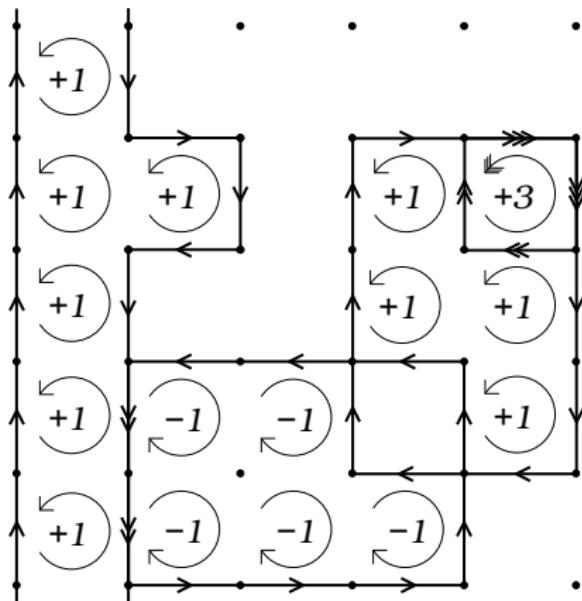
- $p_x \in \mathbb{Z}$
- $j_{x,\mu} \in \mathbb{Z}$

Constraints:

- Vanishing divergence for j -flux at each lattice point
- Combination of j - and p -flux has to cancel at each link

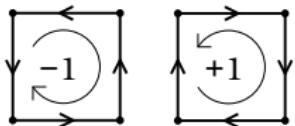
Real and positive weights

$W_H[j]$, $W_G[p]$



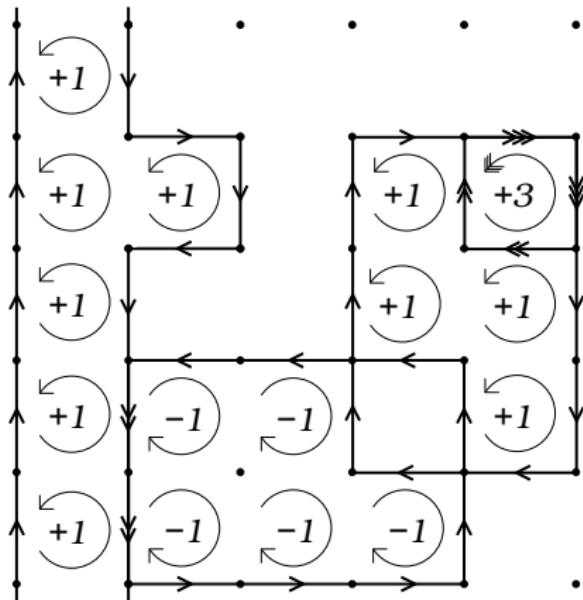
Dual MC-Updates

- ▶ Inserts loop around plaquette in either orientation:



- ▶ Fulfils constraints and ergodicity.

Example configuration

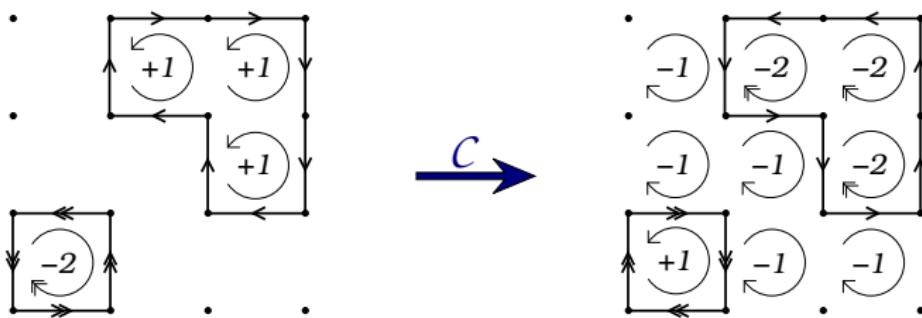


Charge conjugation symmetry at $\theta = \pi$

Also the dual form of the Villain action implements global charge conjugation symmetry at $\theta = \pi$ as an exact Z_2 symmetry!

► Symmetry transformation:

$$p_x \xrightarrow{c} p'_x \equiv -p_x - 1 \quad , \quad j_{x,\mu} \xrightarrow{c} j'_{x,\mu} \equiv -j_{x,\mu} \quad , \quad \forall x, \mu$$

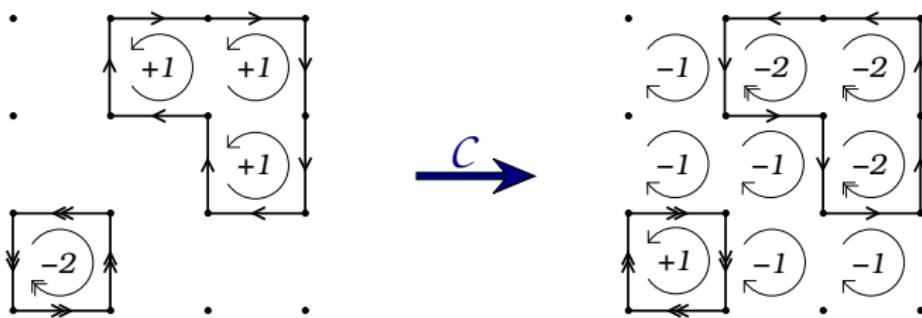


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► Z_2 nature:

Applying transformation twice gives the identity transformation:

$$p_x \xrightarrow{C} -p_x - 1 \xrightarrow{C} -(-p_x - 1) - 1 = p_x$$

$$j_{x,\mu} \xrightarrow{C} -j_{x,\mu} \xrightarrow{C} j_{x,\mu}$$

Observables

Topological charge, topological susceptibility, gauge action density:

$$\langle q \rangle = -\frac{1}{V} \frac{\partial}{\partial \theta} \ln(Z) , \quad \chi_t = \frac{1}{V} \frac{\partial^2}{\partial \theta^2} \ln(Z) , \quad \langle s_G \rangle = -\frac{1}{V} \frac{\partial}{\partial \beta} \ln(Z)$$

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In worldline representation:

$$\langle q \rangle = \frac{1}{V} \left\langle \frac{1}{2\pi\beta} \sum_x \left[p_x + \frac{\theta}{2\pi} \right] \right\rangle ,$$

$$\chi_t = \frac{1}{V} \left[\left\langle \left(\frac{1}{2\pi\beta} \sum_x \left[p_x + \frac{\theta}{2\pi} \right] \right)^2 \right\rangle - \left\langle \frac{1}{2\pi\beta} \sum_x \left[p_x + \frac{\theta}{2\pi} \right] \right\rangle^2 \right] ,$$

$$\langle s_G \rangle = \frac{1}{2\beta V} \left\langle \sum_x \left[1 - \frac{(p_x + \frac{\theta}{2\pi})^2}{\beta} \right] \right\rangle$$

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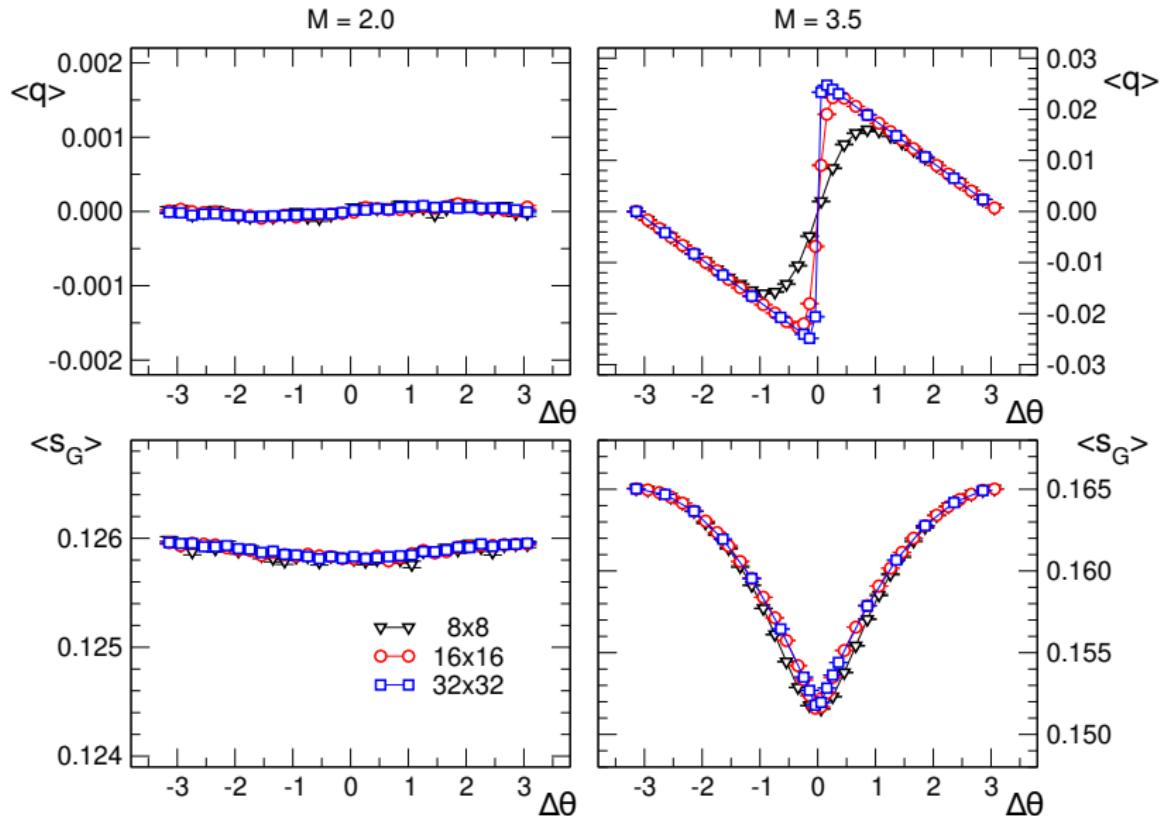
Note: $\langle q \rangle$ is odd under \mathcal{C} transformation at $\theta = \pi$.

$\Rightarrow \langle q \rangle$ is order parameter for breaking of \mathcal{C} symmetry!

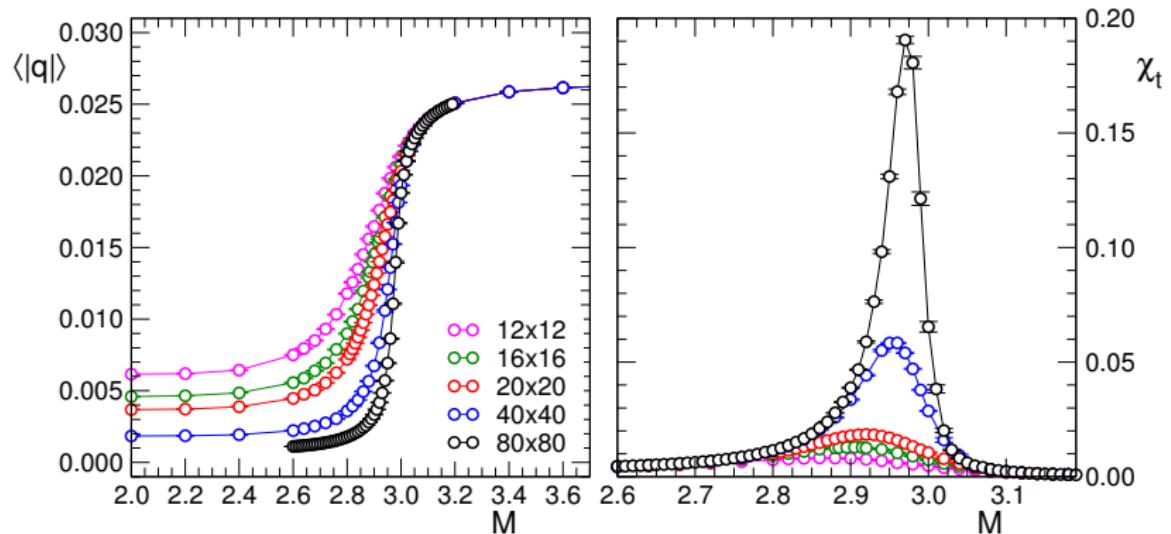
Breaking of \mathcal{C} symmetry

- ▶ Conjectured: \mathcal{C} symmetry is broken at large m^2 and restored at sufficiently negative m^2 . [Komargodski et.al., ArXiv: 1705.04786]
- ▶ 2-d Ising transition between the two regimes?
- ▶ $\langle q \rangle$ corresponds to the Ising magnetization.
- ▶ We cannot observe symmetry breaking on a finite lattice \implies study $\langle |q| \rangle$.
- ▶ $M = 4 + m^2$ drives the system through the phase transition. Corresponds to temperature in Ising model.
- ▶ $\Delta\theta = \theta - \pi$ plays the role of the external magnetic field in Ising model.
 $\Delta\theta = 0$ corresponds to the symmetrical point.

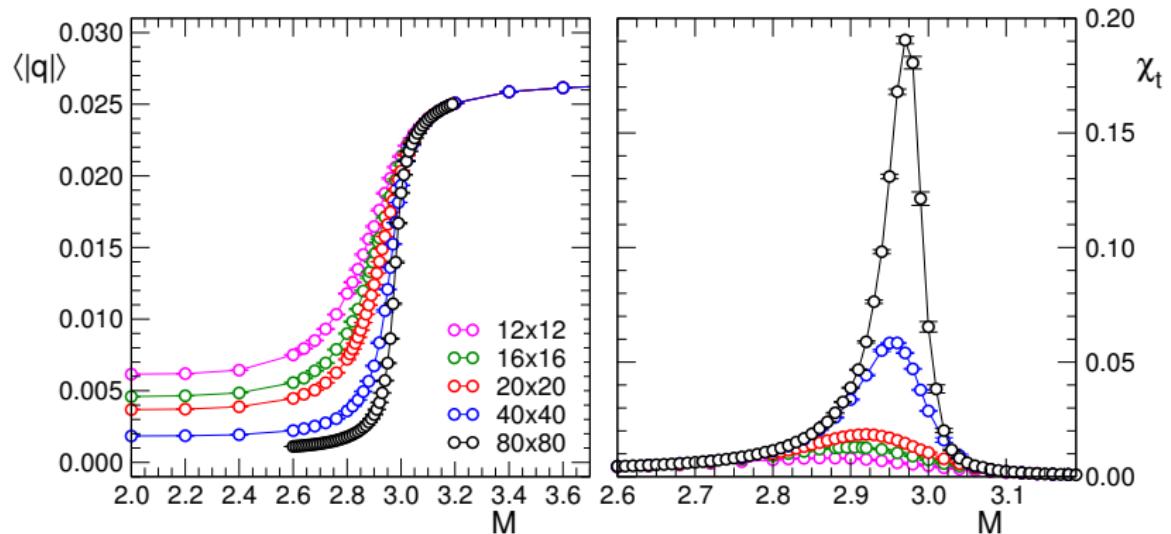
Fist-Order transition as function of θ : ($\lambda = 0.5$, $\beta = 3$), $M = 4 + m^2$



Critical endpoint at $\Delta\theta = 0$: ($\lambda = 0.5$, $\beta = 3$), $M = 4 + m^2$



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- ▶ What is the scaling behavior? Critical exponents?

Finite Size Scaling for determination of critical exponents

We follow the following procedure:

1. Study emerging divergences in observables as we increase the volume.
2. Determine exponent ν from scaling of $\langle |q| \rangle$, $\langle q^2 \rangle$ and Binder cumulant U :

$$\frac{dU}{dM} \Big|_{\max}, \quad \frac{d}{dM} \ln \langle |q| \rangle \Big|_{\max}, \quad \frac{d}{dM} \ln \langle q^2 \rangle \Big|_{\max} \propto L^{\frac{1}{\nu}}$$

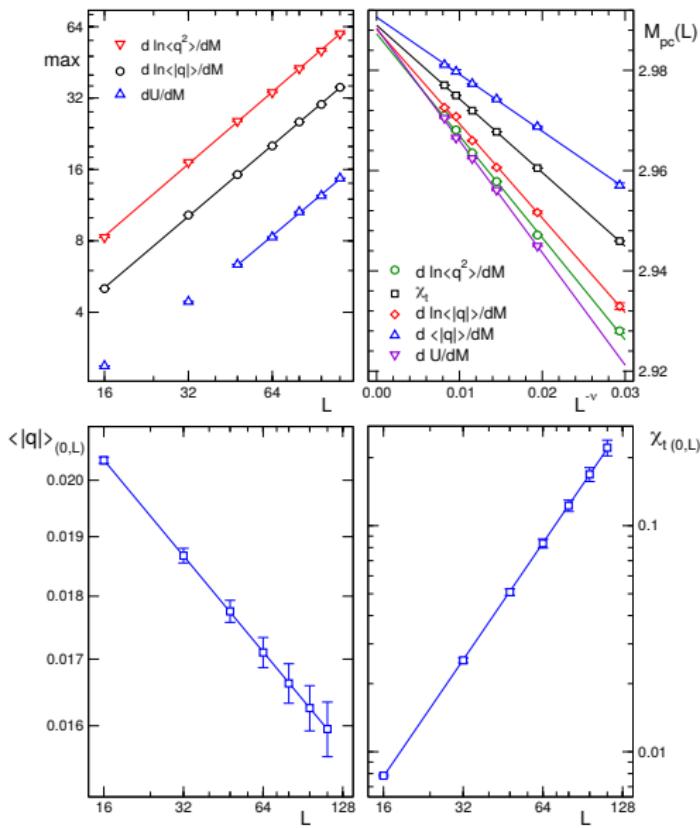
3. Estimate M_C from scaling of pseudo-critical mass defined as position of maximum:

$$M_{pc}(L) = M_C + A L^{-\frac{1}{\nu}}$$

4. Extract critical exponents β and γ from scaling of observables at critical Mass M_C :

$$\langle |q| \rangle_{(M_C, L)} = L^{-\frac{\beta}{\nu}} F_q(x), \quad \chi_t(M_C, L) = L^{\frac{\gamma}{\nu}} F_\chi(x)$$

Critical exponents



► Final results for $U(1)$ gauge-Higgs model:

$$\nu = 1.003(11)$$

$$\beta = 0.126(7)$$

$$\gamma = 1.73(7)$$

► 2-d Ising values:

$$\nu = 1$$

$$\beta = 0.125$$

$$\gamma = 1.75$$

Summary

- ▶ We study the critical endpoint of the U(1) gauge-Higgs model at topological angle $\theta = \pi$.
- ▶ The Villain action implements the charge conjugation symmetry at $\theta = \pi$ as an exact Z_2 symmetry.
- ▶ Complex action problem is solved by simulating in the world line representation.
- ▶ We identify the critical endpoint and determine the critical exponents from a finite size scaling analysis.
- ▶ We show that the critical endpoint is in the 2d Ising universality class:

$$\nu = 1.003(11) \quad , \quad \beta = 0.126(7) \quad , \quad \gamma = 1.73(7)$$