SMOM - $\overline{\text{MS}}$ Matching for B_K at Two-loop Order

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Lattice 2018, July 26

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- The Kaon bag parameter is given by

$$\mathrm{B_K} = \frac{\langle \mathrm{K}^0 | \mathcal{O}_{\mathrm{VV+AA}} | \bar{\mathrm{K}}^0 \rangle}{\frac{8}{3} \mathrm{f_K^2 M_K^2}},$$

where $\mathrm{M_K}$ is the kaon mass and $\mathrm{f_K}$ is the leptonic decay constant.

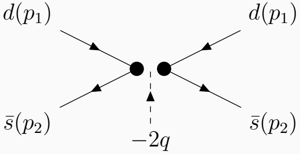
- $\mathrm{B_K}$ parameterizes the QCD hadronic matrix element of the effective weak $\Delta \mathrm{S} = 2$ four quark operator

$$\mathcal{O}_{\mathrm{VV+AA}} = (\bar{\mathrm{s}}\gamma_\mu \mathrm{d})(\bar{\mathrm{s}}\gamma_\mu \mathrm{d}) + (\bar{\mathrm{s}}\gamma_5\gamma_\mu \mathrm{d})(\bar{\mathrm{s}}\gamma_5\gamma_\mu \mathrm{d})$$

that enters a dominant contribution to the indirect CP violation ϵ_K in the kaon sector and can only be computed non-pertubatively on the lattice.

- Perturbative calculations for Wilson coefficients and anomalous dimensions are usually done in the $\overline{\mathrm{MS}}$ NDR scheme. However, dimensional regularization can not be employed on the lattice. Instead, one possibility is a momentum space subtraction (SMOM) scheme. Hence, matching has to be performed.

SMOM schemes are defined with non-exceptional kinematics



where $\mathrm{p}_1^2 = \mathrm{p}_2^2 = (\mathrm{p}_1 - \mathrm{p}_2)^2 = \mathrm{p}^2$ and $\mathrm{q} = \mathrm{p}_1 - \mathrm{p}_2$.

The SMOM renormalization condition is

$$\mathrm{Tr}(\mathrm{P}\Lambda^{\mathrm{tree}}) = 1,$$

where Λ^{tree} is the tree-level Greens function and the projection operators are given by

$$\mathrm{P}^{\mathrm{ij,kl}}_{(1),\alpha\beta,\gamma\delta} = \frac{1}{256\mathrm{N_c}(\mathrm{N_c}+1)}[(\gamma^\nu)_{\beta\alpha}(\gamma_\nu)_{\delta\gamma} + (\gamma^\nu\gamma^5)_{\beta\alpha}(\gamma_\nu\gamma^5)_{\delta\gamma}]\delta_{\mathrm{ij}}\delta_{\mathrm{kl}},$$

$$\mathrm{P}^{\mathrm{ij,kl}}_{(2),\alpha\beta,\gamma\delta} = \frac{1}{64\mathrm{q}^2\mathrm{N_c}(\mathrm{N_c}+1)}[(\not{q})_{\beta\alpha}(\not{q})_{\delta\gamma} + (\not{q}\gamma^5)_{\beta\alpha}(\not{q}\gamma^5)_{\delta\gamma}]\delta_{\mathrm{ij}}\delta_{\mathrm{kl}},$$

where $\mathrm{N_c}$ is the number of colors, $\mathrm{i,j,k,l}$ color and $\alpha,\beta,\gamma,\delta$ spinor indices.

Conversion factors

- Conversion factors are defined as

$$\mathcal{O}^{\mathrm{NDR}}_{\mathrm{VV+AA}}(\mu) = \mathrm{C}^{\mathrm{SMOM}}_{\mathrm{B_K}}(\mathrm{p}^2/\mu^2)\mathcal{O}^{\mathrm{SMOM}}_{\mathrm{VV+AA}}(\mathrm{p}),$$

  where p is the renormalization scale of the SMOM scheme and μ is renormalization scale of the NDR scheme.

- Conversion factors can be calculated using

$$\mathrm{C}^{(\mathrm{X,Y})}_{\mathrm{B_K}} = (\mathrm{C}^{(\mathrm{Y})}_{\mathrm{q}})^2 \mathrm{P}^{\mathrm{ij,kl}}_{(\mathrm{X})\alpha\beta,\gamma\delta} \Lambda^{\mathrm{ij,kl}}_{\alpha\beta,\gamma\delta},$$

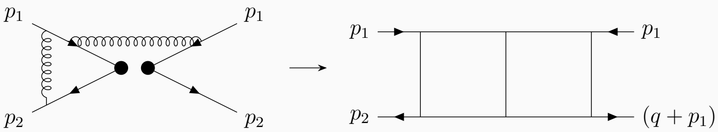
  where $\mathrm{C_q}$ is the conversion factor for the wave-function field renormalization, $\Lambda^{\mathrm{ij,kl}}_{\alpha\beta,\gamma\delta}$ is the amputated four-point Greens function computed in the $\overline{\mathrm{MS}}$-NDR renormalization and at the SMOM point. (X,Y) correspond to different SMOM schemes.

- The calculation has been done at one-loop by Y.Aoki et al. (2010).

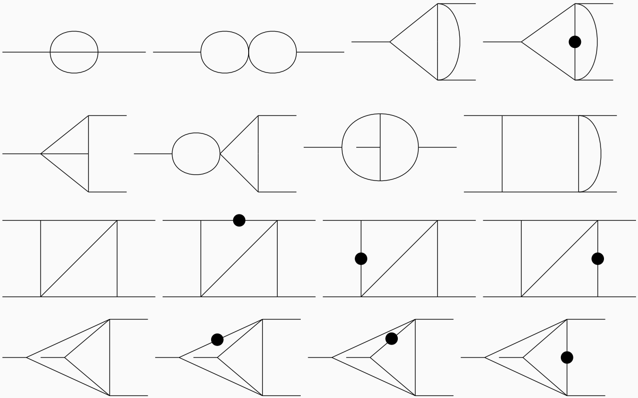
Two-loop Calculation Outline

- At two-loop order there are 28 independent diagrams.
- Dirac algebra results in a large number of two-loop tensor integrals.
- In order to avoid doing tensor integration we are going to contract the diagrams with projectors first. This gives us scalar products in the numerator which can be expressed as inverse powers of propagators but introduces subtleties related to γ_5 in D dimensions.
- We can use the integration by parts (IBP) identities to express the diagrams in terms of a small set of master integrals.
- Calculating fewer master integrals numerically reduces inaccuracies.

IBP's and Auxiliary Topologies



- IBP's might give linear relationships between integrals with "extra" propagators.
- We need to define auxiliary topologies that have a complete set of linearly independent propagators in order to use Reduze2 for the IBP reduction.
- For two loops and two independent external momenta we can make 7 scalar products with loop momenta.
- We can add extra propagators to existing diagrams to form auxiliary topologies.
- Some integrals have linearly dependent propagators.



- All bubble and triangle diagrams have been calculated analyticaly by N. I. Ussyukina and A. I. Davydychev (1994).
- Box diagrams are obtained via sector decomposition method using pySecDec.

In D dimensions we have more operators than in 4 dimensions.

If $\Delta S = 2$ effective four-quark operator is defined as

$$Q = (\bar{s}^i \gamma^\mu P_L d^j)(\bar{s}^k \gamma_\mu P_L d^l),$$

where $P_L = (1 - \gamma_5)/2$, then the evanescent operators can be chosen to be

$$\begin{aligned}
E_F &= (\bar{s}^k \gamma^\mu P_L d^j)(\bar{s}^i \gamma_\mu P_L d^l) - Q,\\
E_1^{(1)} &= (\bar{s}^i \gamma^{\mu_1\mu_2\mu_3} P_L d^j)(\bar{s}^k \gamma_{\mu_1\mu_2\mu_3} P_L d^l) - (16 - 4\epsilon - 4\epsilon^2) Q,\\
E_2^{(1)} &= (\bar{s}^k \gamma^{\mu_1\mu_2\mu_3} P_L d^j)(\bar{s}^i \gamma_{\mu_1\mu_2\mu_3} P_L d^l) - (16 - 4\epsilon - 4\epsilon^2)(Q + E_F),\\
E_1^{(2)} &= (\bar{s}^i \gamma^{\mu_1\mu_2\mu_3\mu_4\mu_5} P_L d^j)(\bar{s}^k \gamma_{\mu_1\mu_2\mu_3\mu_4\mu_5} P_L d^l) - (256 - 224\epsilon - 144\epsilon^2) Q,\\
E_2^{(2)} &= (\bar{s}^k \gamma^{\mu_1\mu_2\mu_3\mu_4\mu_5} P_L d^j)(\bar{s}^i \gamma_{\mu_1\mu_2\mu_3\mu_4\mu_5} P_L d^l) - (256 - 224\epsilon - 144\epsilon^2)(Q + E_F),
\end{aligned}$$

where $\gamma^{\mu_1\mu_2\mu_3}$ denotes the product of gamma matrices $\gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3}$.

All operators but Q vanish in the limit $D \to 4$.

Our choice of evanescent operators allows us to use "Greek projections" of the form $\mathrm{P}^{\alpha\beta,\gamma\delta}\Gamma^1_{\beta\gamma}\Gamma^2_{\delta\alpha} = \mathrm{Tr}(\Gamma^1\gamma^\tau\Gamma^2\gamma_\tau)$ to project out the evanescent part of the result (A. J. Buras and P. H. Weisz (1989), N. Tracas and N. Vlachos (1982)).

This allows us to deal with γ_5 in D dimensions in the following way:

- The total renormalized amplitude is finite.
- Our diagrams when contracted with projectors result in traces such as $\mathrm{Tr}(\Gamma^1 \mathrm{X} \Gamma^2 \mathrm{Y})$.
- Even though these traces contain γ_5 the longest irreducible Dirac structure that remains is $\mathrm{Tr}(\not{p}_1 \not{p}_2 \gamma_5)$ which is consistently equal to zero in 4 and in D dimensions.
- Hence, we can anticommute all γ_5 to the right and drop the terms containing it without introducing ambiguities.

We also avoid having to evaluate tensor integrals.

Scheme change

- Our choice of evanescent operators differ slightly from the ones used by J. Brod and M. Gorbahn (2010) for the Wilson coefficients.
- The resulting change of scheme can be obtained in terms of $1/\epsilon^2$ parts of renormalisation constants.
- The end result is the conversion factor from SMOM to NDR-$\overline{\text{MS}}$ a la Brod-Gorbahn for which the NNLO Wilson coefficients and anomalous dimensions are known.

Results

$$
\begin{aligned}
\mathrm{C}_{\mathrm{B_K}}^{(\gamma_\mu,\not{q})} &= 1 + \frac{\alpha_\mathrm{s}}{4\pi}(-2.45482) + \frac{\alpha_\mathrm{s}^2}{(4\pi)^2}(???) + \mathcal{O}(\alpha_\mathrm{s}^3),\\
\mathrm{C}_{\mathrm{B_K}}^{(\gamma_\mu,\gamma_\mu)} &= 1 + \frac{\alpha_\mathrm{s}}{4\pi}(0.21184) + \frac{\alpha_\mathrm{s}^2}{(4\pi)^2}(???) + \mathcal{O}(\alpha_\mathrm{s}^3),\\
\mathrm{C}_{\mathrm{B_K}}^{(\not{q},\not{q})} &= 1 + \frac{\alpha_\mathrm{s}}{4\pi}(-0.45482) + \frac{\alpha_\mathrm{s}^2}{(4\pi)^2}(???) + \mathcal{O}(\alpha_\mathrm{s}^3),\\
\mathrm{C}_{\mathrm{B_K}}^{(\not{q},\gamma_\mu)} &= 1 + \frac{\alpha_\mathrm{s}}{4\pi}(2.21184) + \frac{\alpha_\mathrm{s}^2}{(4\pi)^2}(???) + \mathcal{O}(\alpha_\mathrm{s}^3).
\end{aligned}
$$

Conclusion

- All of the two-loop integrals are expressed in terms of a small set of master integrals and can be evaluated analytically or numerically.
- The two-loop amplitude is evaluated term by term by contracting with the "Greek projector" and thus removing all contributions from the evanescent operators.
- We can avoid doing tensor integrals.
- Traces involving γ_5 can be consistently set to zero and do not produce ambiguities in D dimensions.
- The conversion factor from SMOM to $\overline{\mathrm{MS}}$ is obtained in Brod-Gorbahn operator scheme for which the NNLO Wilson coefficients and anomalous dimensions are known.
- The new result will reduce the theory uncertainty on ϵ_{K} and increase sensitivity to NP effects.