

Visualizations of Centre Vortex Structure in Lattice Simulations

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In collaboration with: James Biddle, Waseem Kamleh, Daniel Trewartha



THE UNIVERSITY
of ADELAIDE

Outline

Projected Vortices: What are they and how are they located?

Why do we care?

Recent results on quark and gluon propagators

What do they look like?

3D rendering techniques for 2D surfaces in 4D

How are they related to topological charge?

Examine their location and form

Search for singular points where the surface(s) span all four dimensions.

New insights into vacuum instability under centre-vortex removal

Conclusions and future directions

What are Centre Vortices?

1. Gauge fix gluon configurations to Maximal Centre Gauge
 - o Bring the links $U_\mu(x)$ close to the centre elements of $SU(3)$

$$\exp\left(2\pi i \frac{m}{3}\right) \mathbf{I}, \text{ with } m = -1, 0, 1.$$

- o On the lattice, search for the gauge transformation Ω

$$\sum_{x,\mu} \left| \text{tr } U_\mu \Omega(x) \right|^2 \xrightarrow{\Omega} \max$$

What are Centre Vortices?

1. Gauge fix gluon configurations to Maximal Centre Gauge
2. Project the gluon field to the centre phase

$$U_\mu(x) \rightarrow Z_\mu(x) \text{ where } Z_\mu(x) = \exp\left(2\pi i \frac{m_\mu(x)}{3}\right),$$

with

$$m_\mu(x) = -1, 0, 1.$$

- o Implemented by

$$\frac{1}{3} \text{tr } U_\mu^{\Omega}(x) = \underbrace{r_\mu(x)}_{\text{real}} \underbrace{\exp(i\varphi_\mu(x))}_{\text{phase}},$$

$$\underbrace{\cos\left(\varphi_\mu(x) - \frac{2\pi}{3} m_\mu(x)\right)}_{\text{close to zero}} \xrightarrow{m_\mu} \max.$$

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3. Projected Vortices (P vortices) are identified through the centre charge

$$z(x) = \prod_{\square} Z_\mu(x) = \exp\left(2\pi i \frac{n}{3}\right)$$

- If $\text{mod}(n, 3) = 0$ no vortex pierces the plaquette
- If $\text{mod}(n, 3) = -1$, or 1 a vortex with charge z pierces the plaquette

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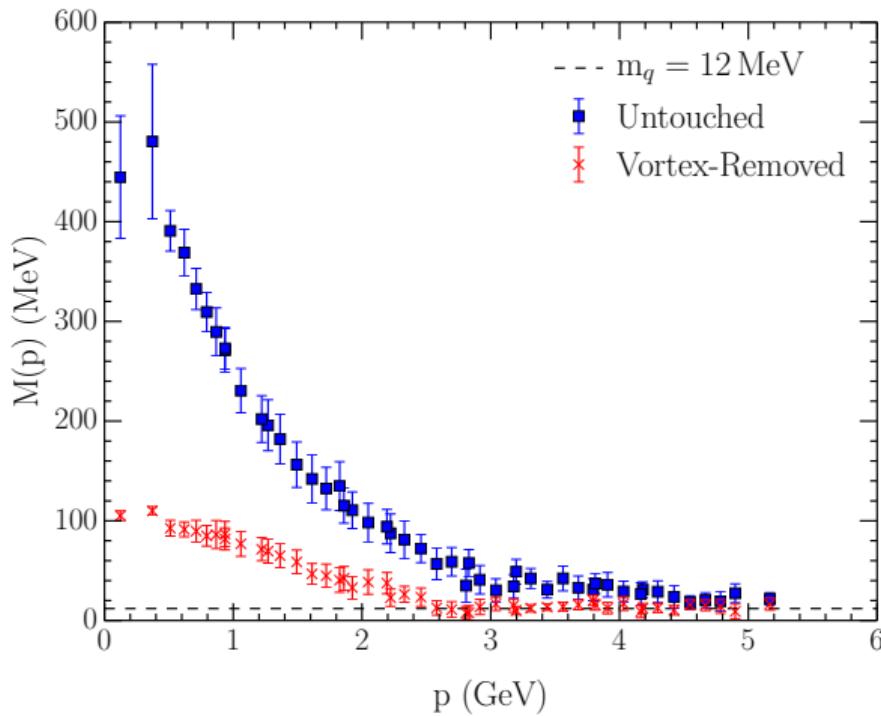
4. Vortices are removed by removing the centre phase

$$U_\mu(x) \rightarrow U'_\mu(x) = Z_\mu^*(x) \cdot U_\mu(x),$$

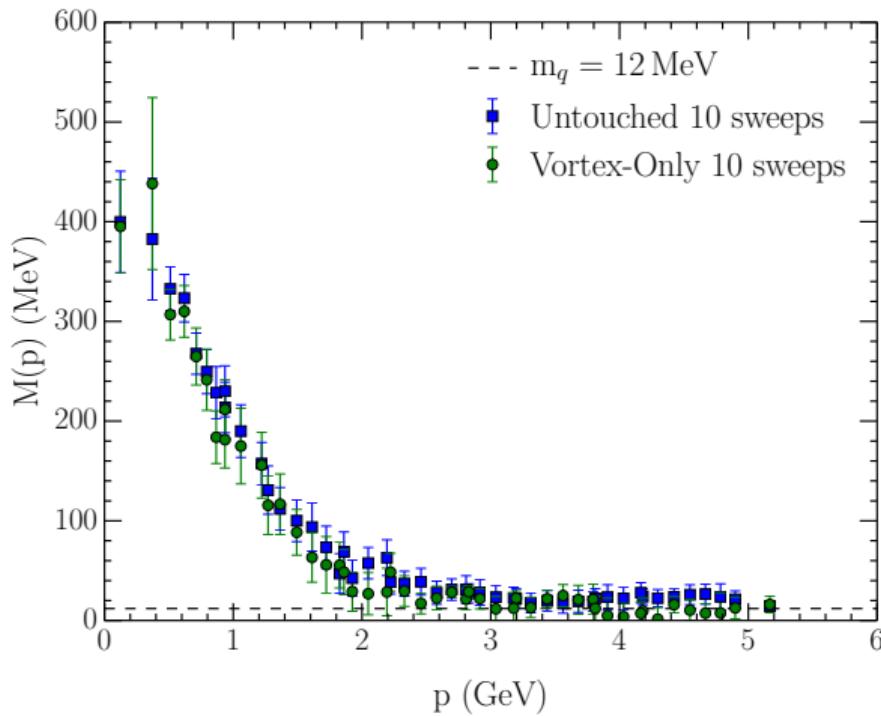
Simulation Details

- $SU(3)$ gauge-field configurations are considered.
 - Lattice size of $20^3 \times 40$, with $a = 0.125$ fm.
 - Lušcher-Weisz mean-field improved action.
- Cooling
 - Minimise the local action.
 - Successive sweeps remove short-range noise.
 - Performed using an $\mathcal{O}(a^4)$ -three-loop improved action.
- Topological charge density
 - Calculated using an $\mathcal{O}(a^4)$ -five-loop improved $F_{\mu\nu}$.
 - 96% of contributions are from 1×1 and 1×2 clover terms.

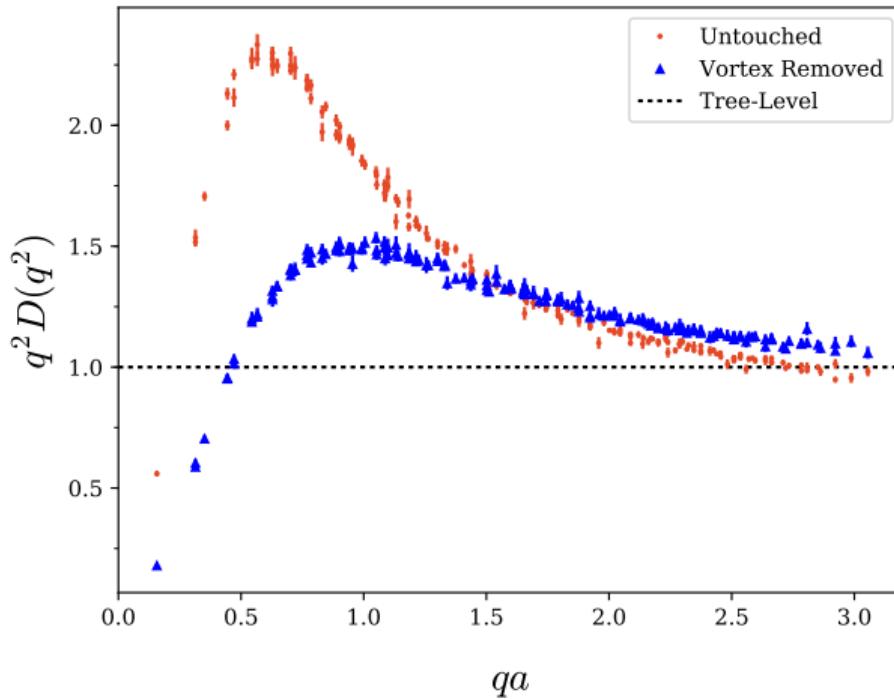
Vortex-Removed Mass Function from Overlap Fermions



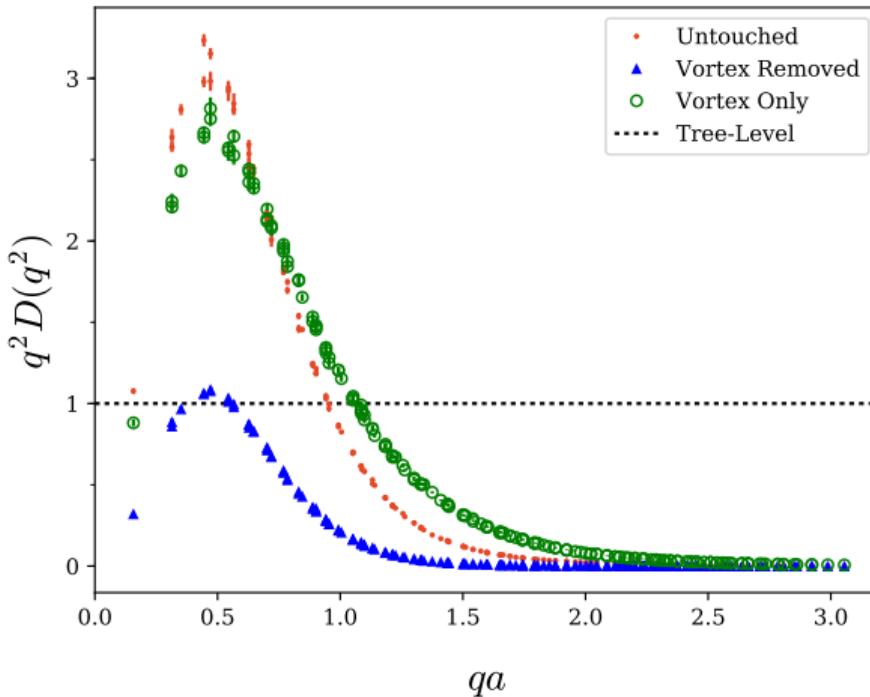
Vortex-Only Mass Function (with $\mathcal{O}(a^4)$ -improved cooling)



Vortex-Removed Gluon Propagator



Vortex-Only Gluon Propagator with $\mathcal{O}(a^4)$ -improved cooling



Projected Centre Vortices

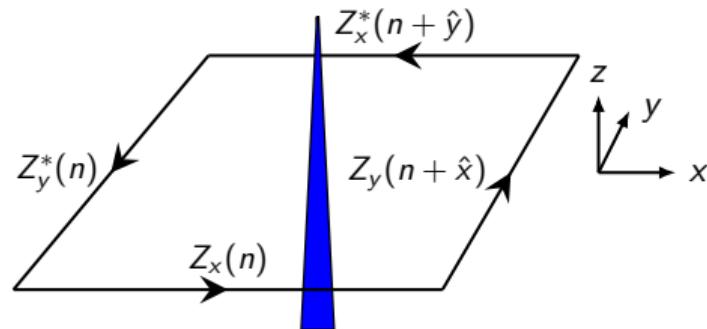
- The hadron spectrum on vortex-removed fields shows
 - A theory of weakly-interacting constituent quarks at heavy quark masses,
 - Chiral symmetry restoration at light values of the bare quark mass.
- D. Trewartha, W. Kamleh and DBL, J. Phys. G **44** (2017) no.12, 125002

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D. Trewartha, W. Kamleh and DBL, J. Phys. G **44** (2017) no.12, 125002
- Projected centre vortices are the seeds of dynamical chiral symmetry breaking.

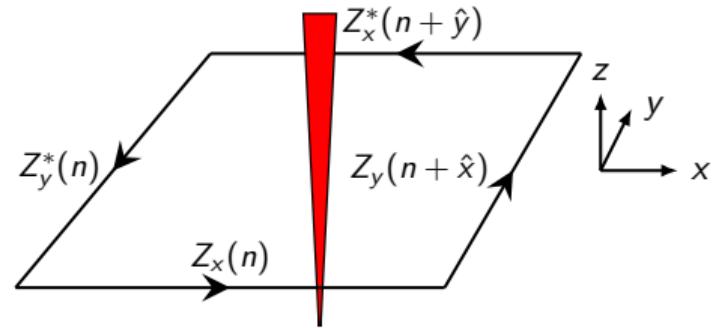
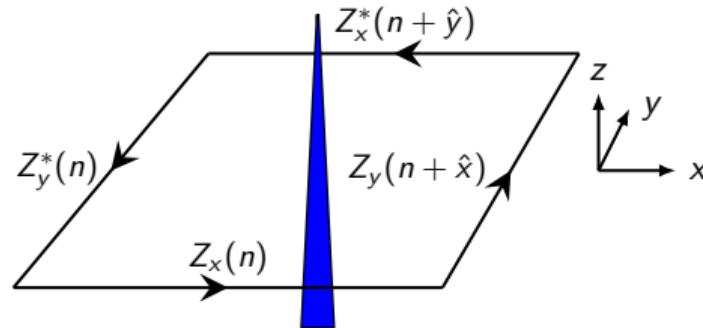
Rendering Projected Vortices

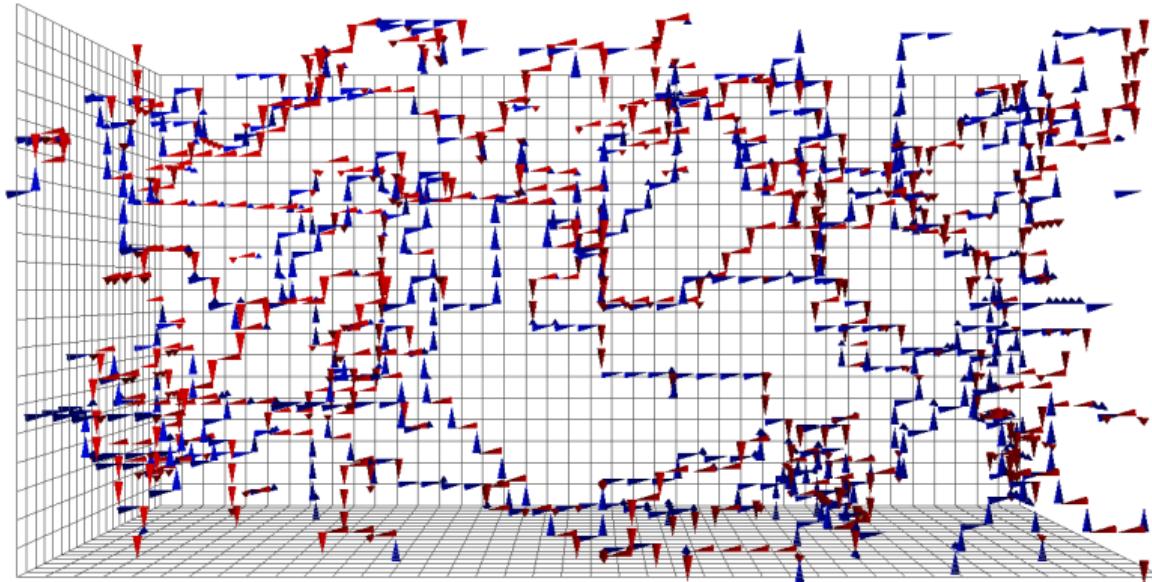
- Vortex directions are indicated using a right-handed coordinate system.
- For example,
 - An $m = +1$ vortex in the x - y plane is plotted in the $+\hat{z}$ direction as a blue jet.

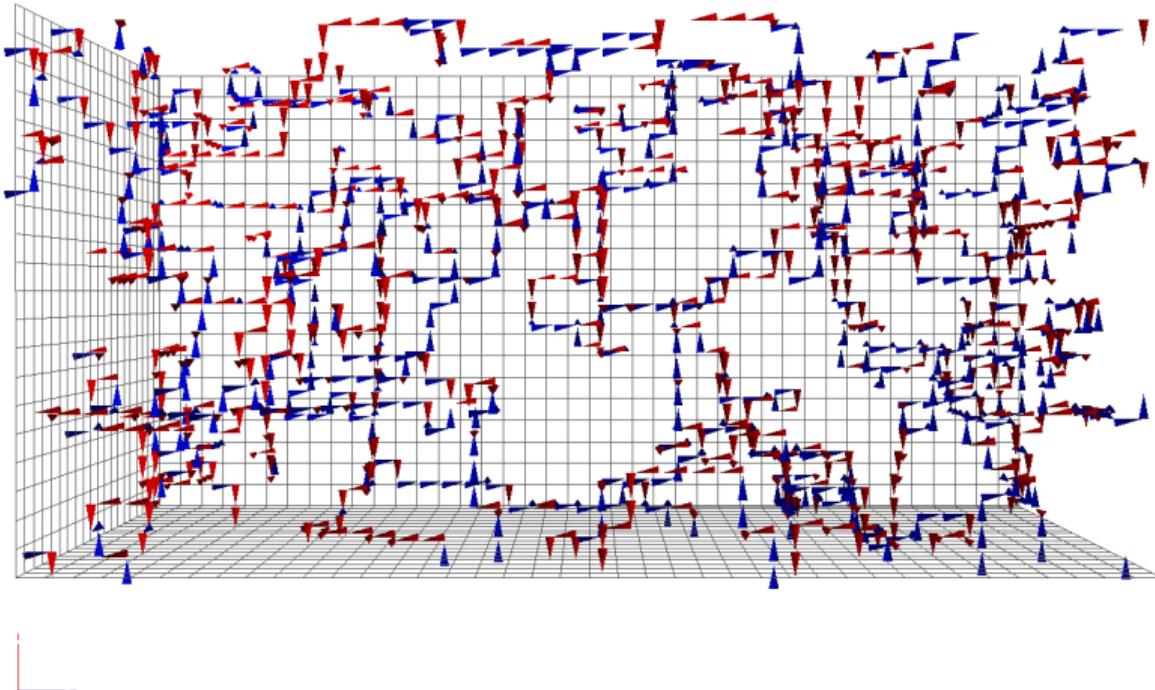


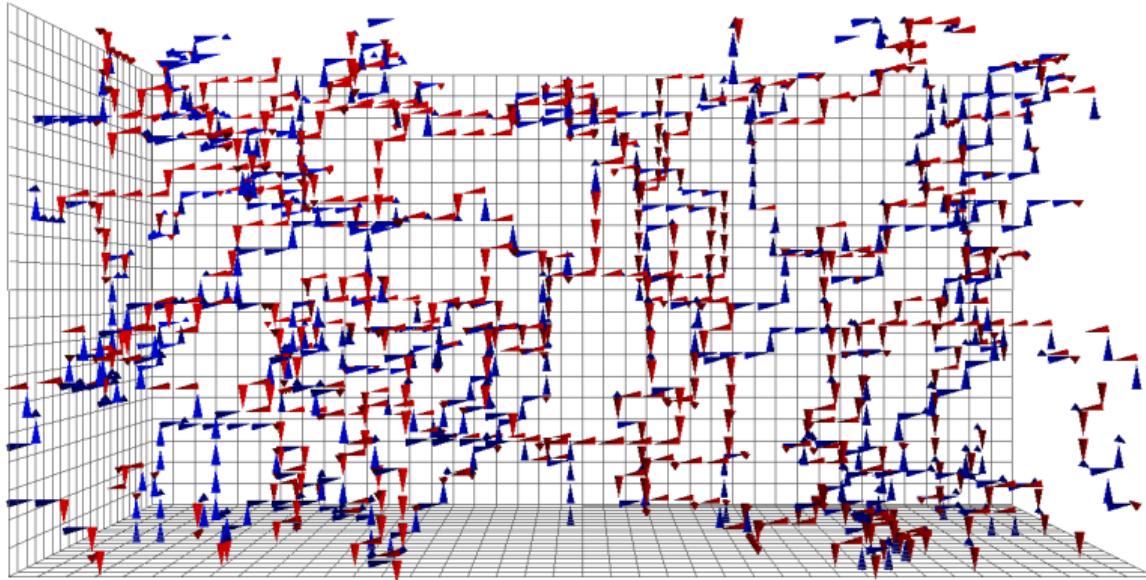
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- For example,
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 - An $m = -1$ vortex in the $x-y$ plane is plotted in the $-\hat{z}$ direction as a **red** jet.









Rendering Space-Time Oriented Projected Vortices

- Every link in the spatial volume has a forward and backward time-oriented plaquette associated with it.

Rendering Space-Time Oriented Projected Vortices

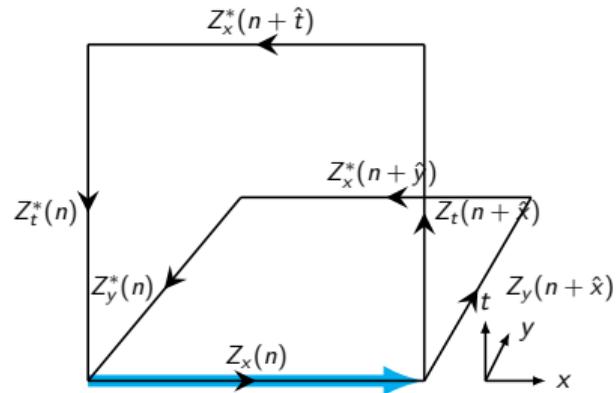
- Every link in the spatial volume has a forward and backward time-oriented plaquette associated with it.
- The three jets associated with the spatial $x-y$, $y-z$ and $z-x$ plaquettes, are complemented by
 - Jets in the three forward time $x-t$, $y-t$ and $z-t$ plaquettes, and
 - Jets in the three backward time $x-t$, $y-t$ and $z-t$ plaquettes.

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 - Jets in the three backward time $x-t$, $y-t$ and $z-t$ plaquettes.
- Space-time oriented P vortices are illustrated in the spatial three-volume by rendering the link associated with the space-time P vortex.

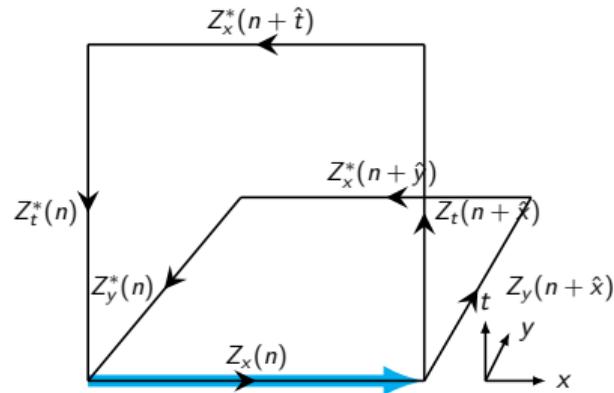
Rendering Space-Time Oriented Projected Vortices

- If a spatial link belongs to a P vortex in a space-time plaquette then:
 - The link is rendered in **cyan** for an $m = +1$ vortex.



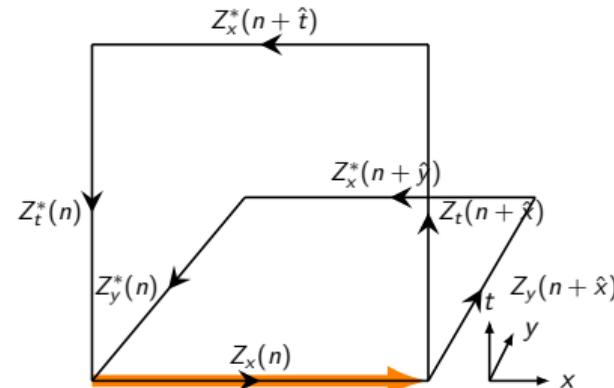
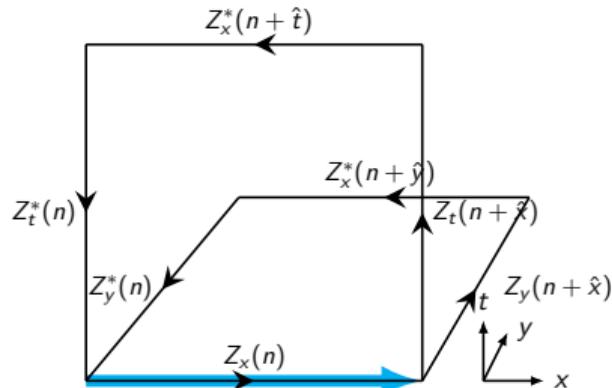
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- If a spatial link belongs to a P vortex in a space-time plaquette then:
 - The link is rendered in **cyan** for an $m = +1$ vortex.
 - The link is rendered as a positively-directed arrow for forward space-time plaquettes.



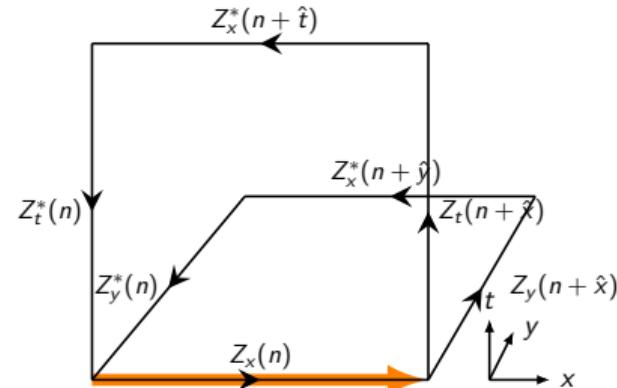
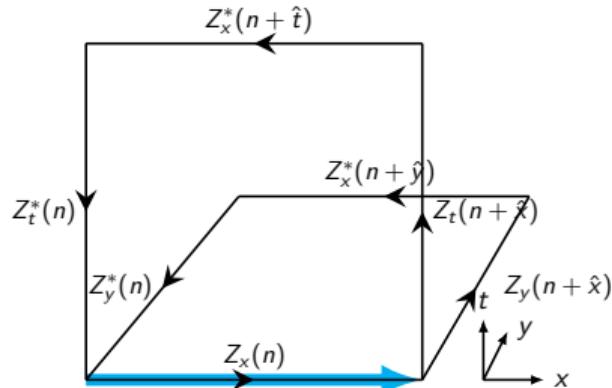
Rendering Space-Time Oriented Projected Vortices

- If a spatial link belongs to a P vortex in a space-time plaquette then:
 - The link is rendered in **cyan** for an $m = +1$ vortex, and in **orange** for $m = -1$.
 - The link is rendered as a positively-directed arrow for forward space-time plaquettes.



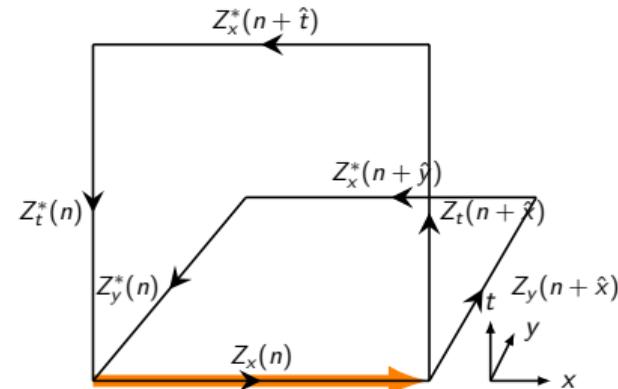
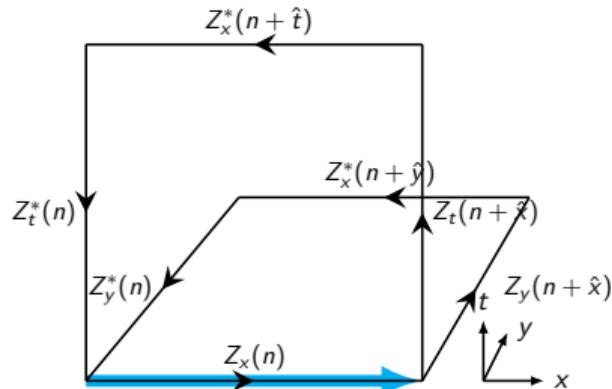
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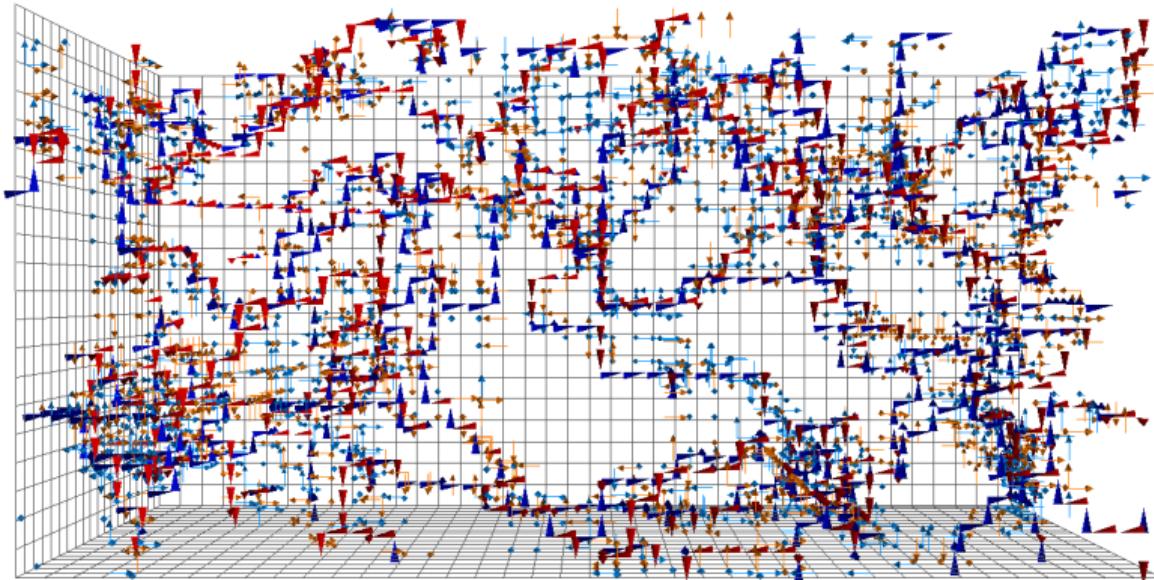
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 - The link is rendered as a positively-directed arrow for forward space-time plaquettes.
 - The link is rendered as a negatively-directed arrow for backward space-time plaquettes.

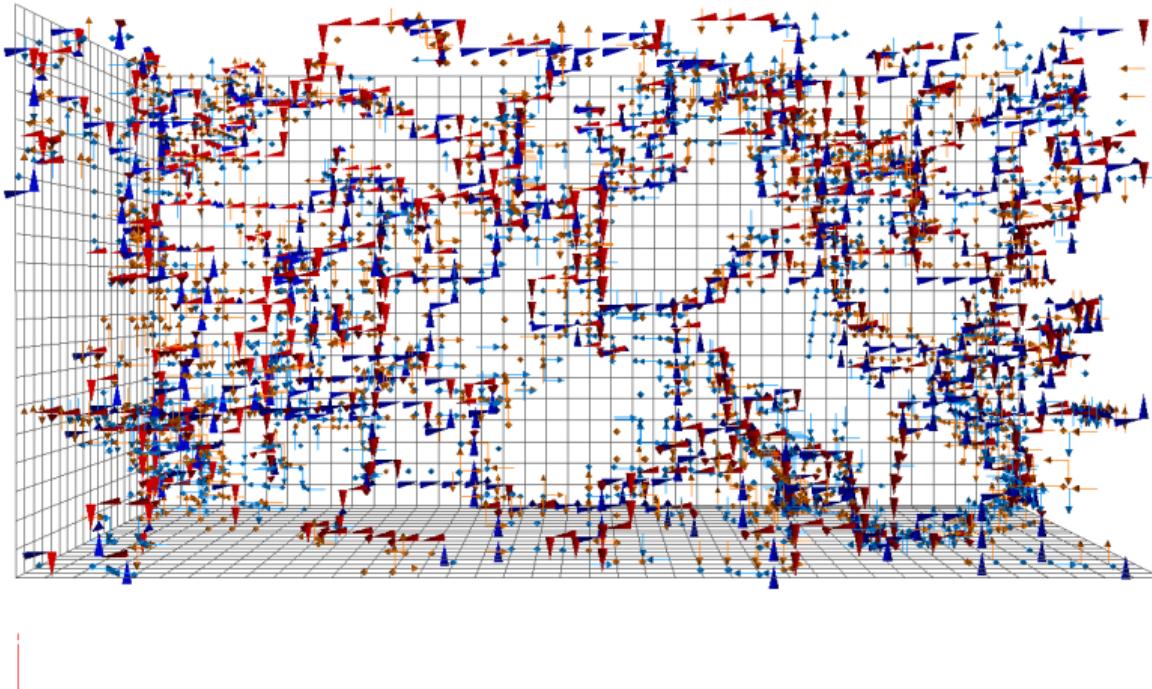


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 - The link is rendered in **cyan** for an $m = +1$ vortex
 - The link is rendered as a positively-directed arrow for forward space-time plaquettes.
 - The link is rendered as a negatively-directed arrow for backward space-time plaquettes.
- As one steps forwards in time, positively-directed links become negatively-directed.

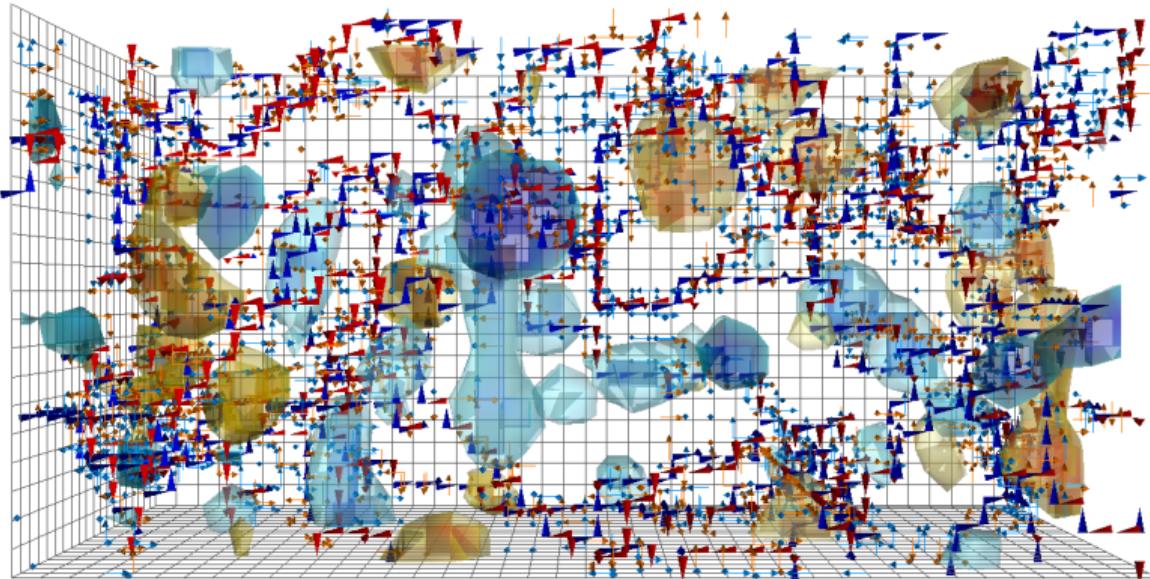






Relationship to Topological Charge Density

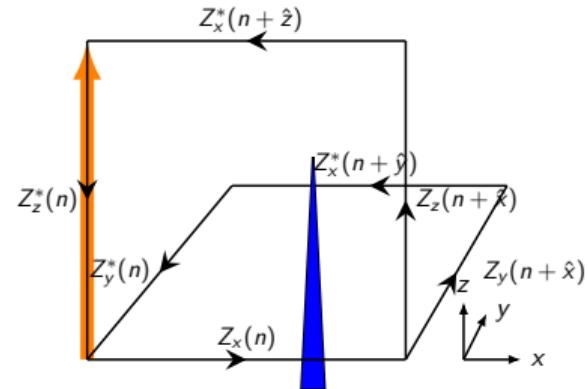
- $q(x)$ is calculated on the original configurations.
- Three-loop $\mathcal{O}(a^4)$ -improved cooling is used to remove ultra-violet fluctuations.
- Eight sweeps is sufficient to render the action and field-strength tensor accurate.
- In rendering the topological charge density
 - Render areas of positive charge density in red through to yellow.
 - Render areas of negative charge density in blue through to cyan.
 - Low charge-density regions are not rendered to allow us to see into the configuration.



Signature of a Singular Point

M. Engelhardt, Nucl. Phys. B 585 (2000) 614

- Singular points are points at which the tangent vectors of the vortex surface(s) span all four dimensions – touching points, intersection points and writhing points.

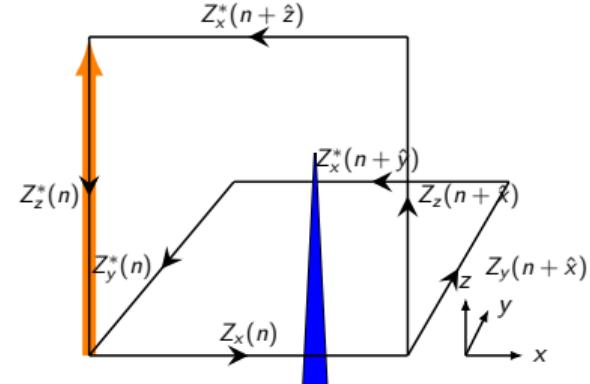


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- Relevant to the creation of non-trivial topological charge density

$$q(x) = \frac{g^2}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} F_{\mu\nu}^{ab}(x) F_{\rho\sigma}^{ba}(x).$$



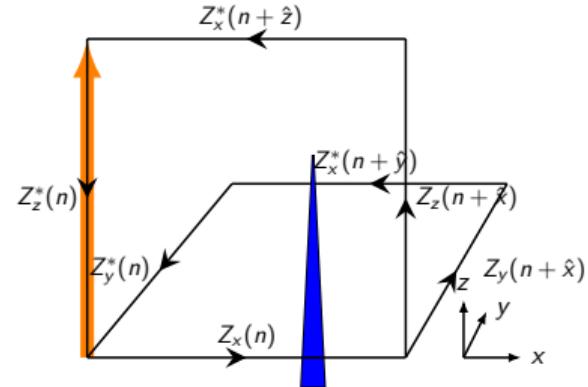
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- Search for a plaquette with a jet, and a parallel link on the corner.
- Any colour combination and link orientation is fine.



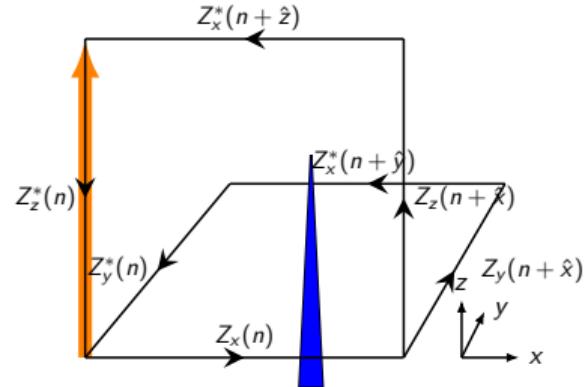
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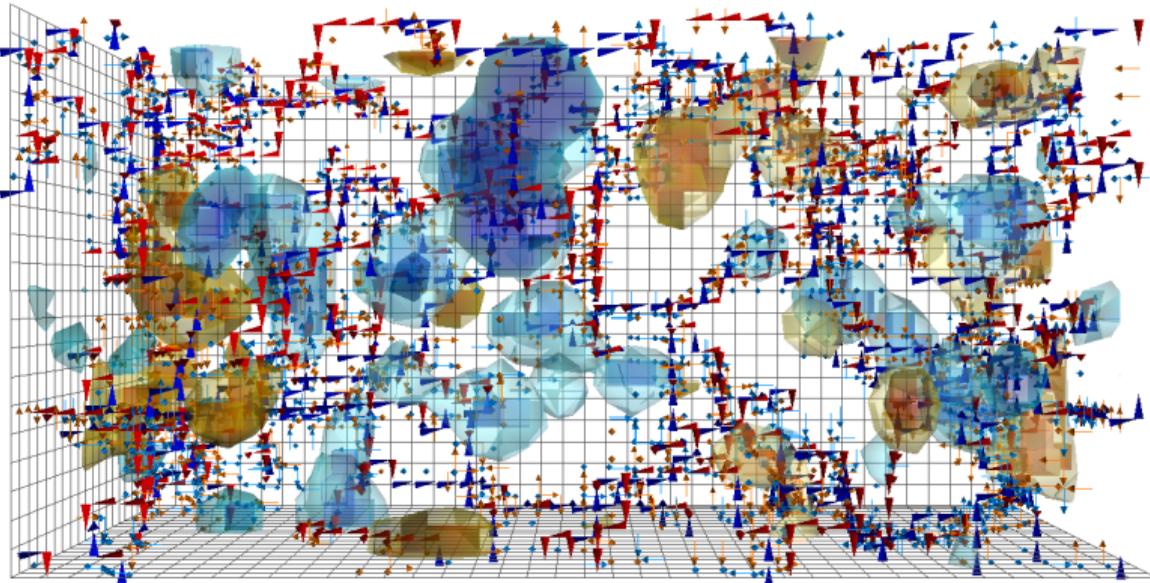
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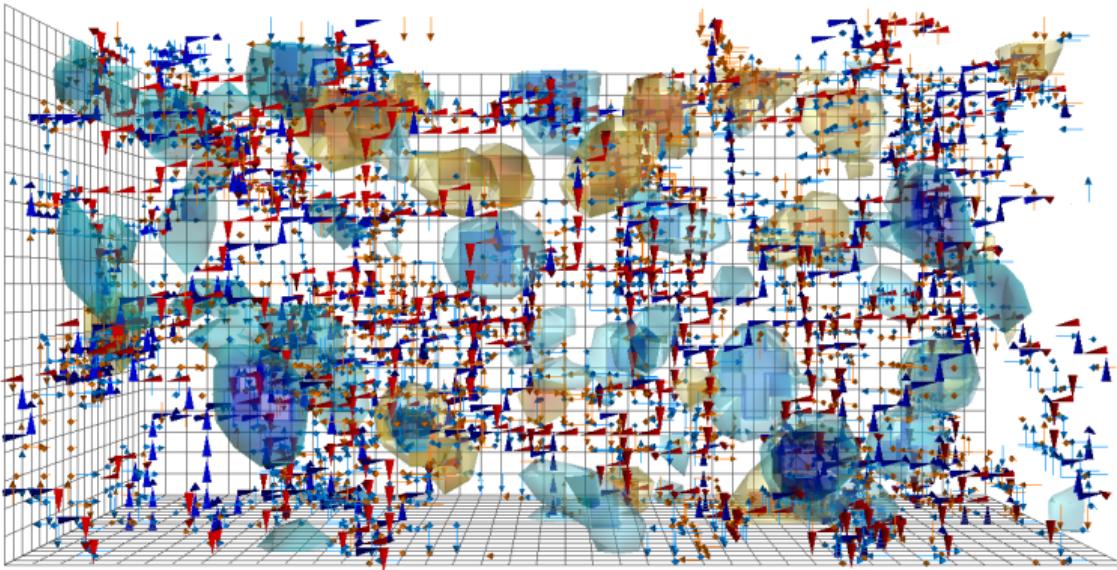
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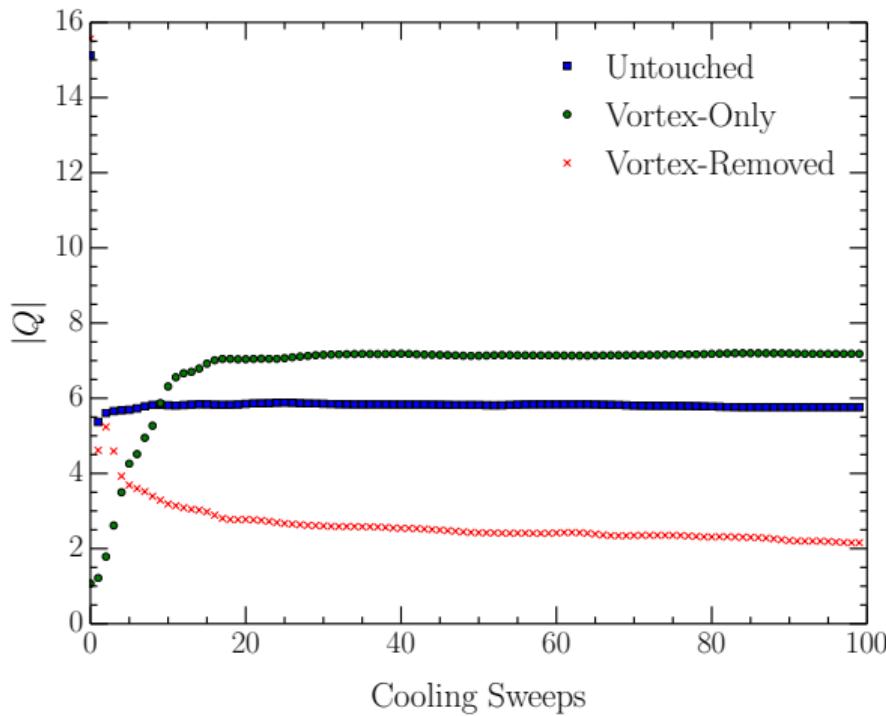
- Search for a plaquette with a jet, and a parallel link on the corner.
- Any colour combination and link orientation is fine.
- Here site n is a singular point.



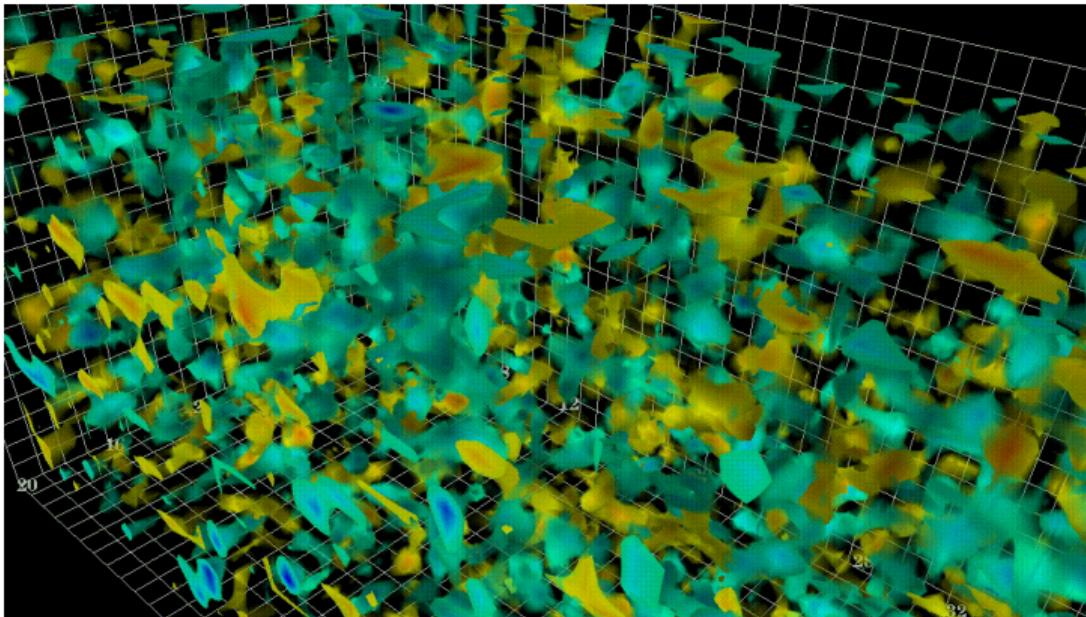




Evolution of $|Q|$ under cooling averaged over 100 configurations



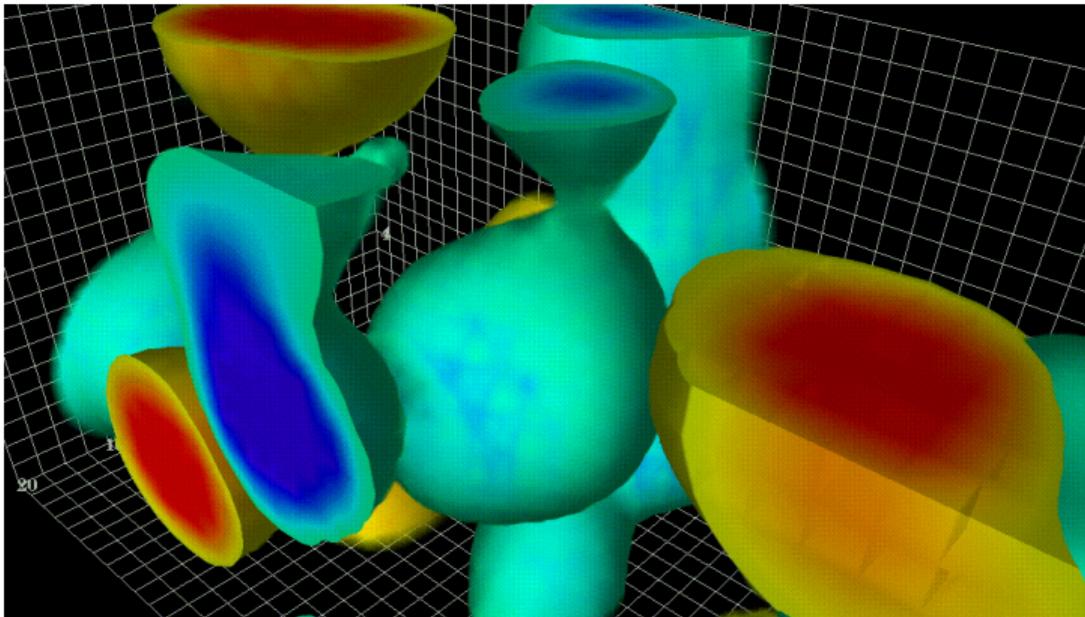
Original Configurations under $\mathcal{O}(a^4)$ -improved Cooling



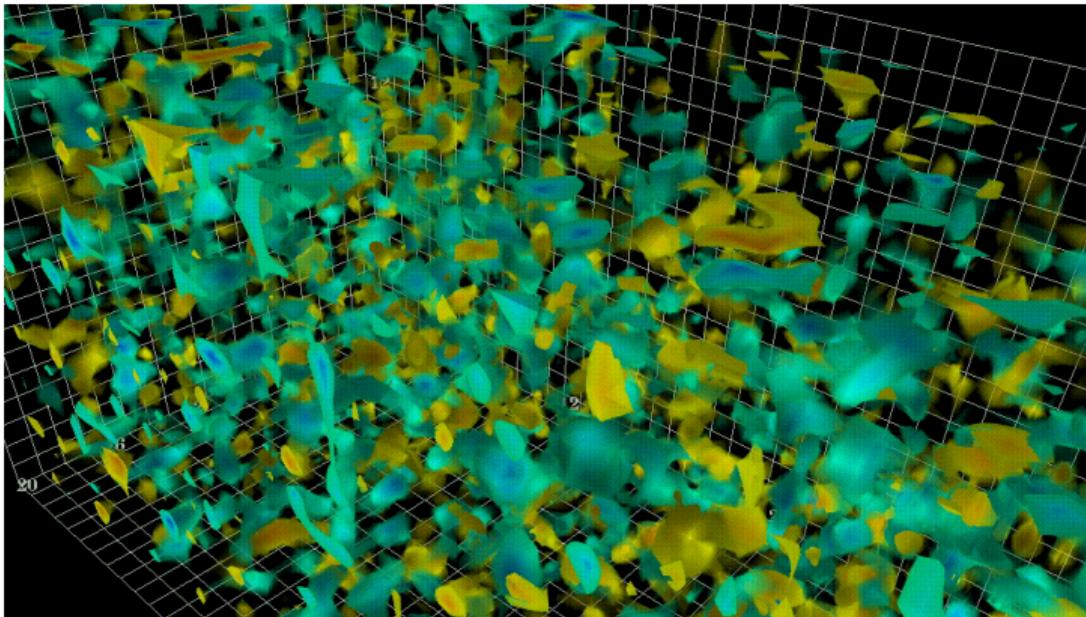
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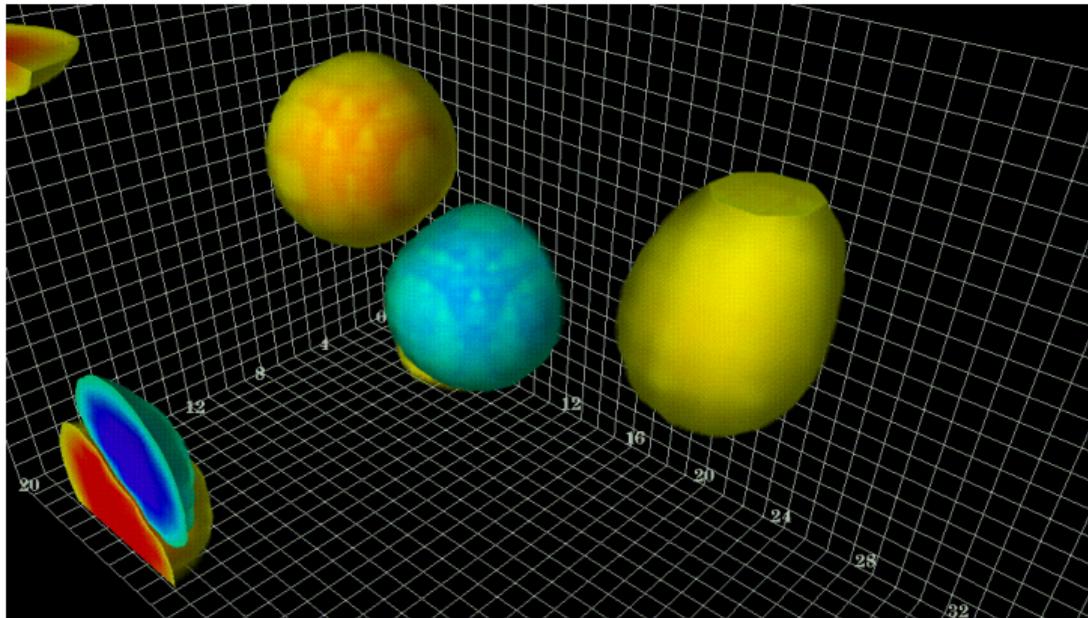
Vortex Removed Configurations under $\mathcal{O}(a^4)$ -improved Cooling



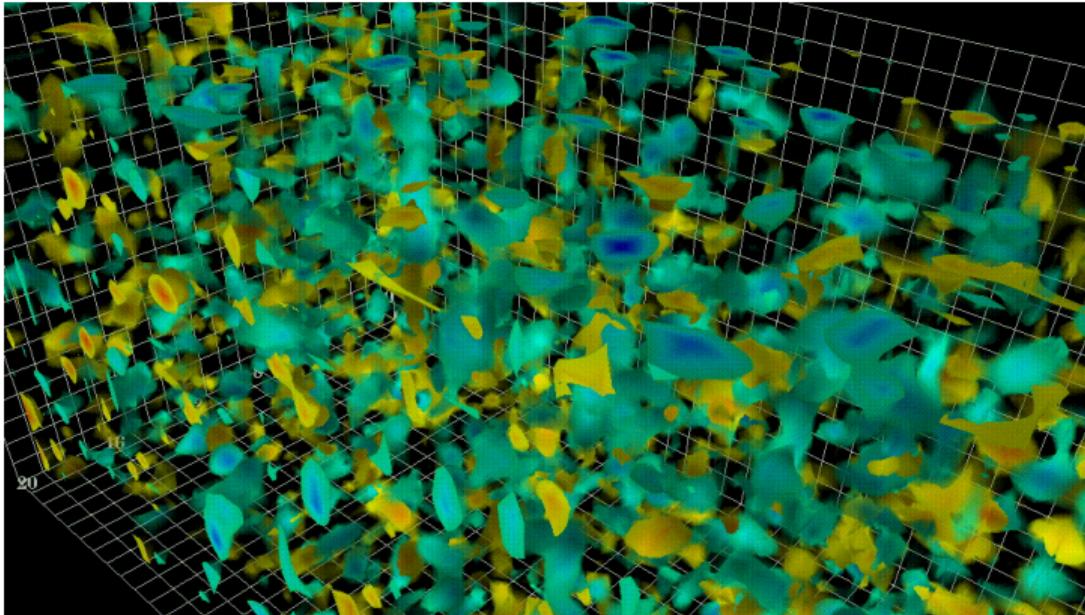
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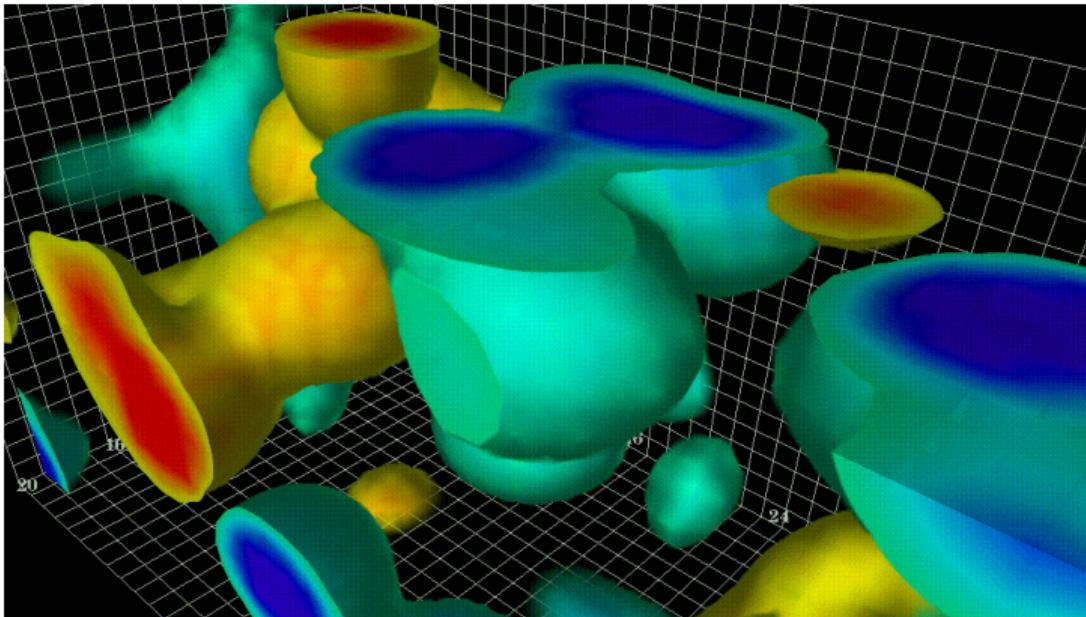
Vortex Only Configurations under $\mathcal{O}(a^4)$ -improved Cooling



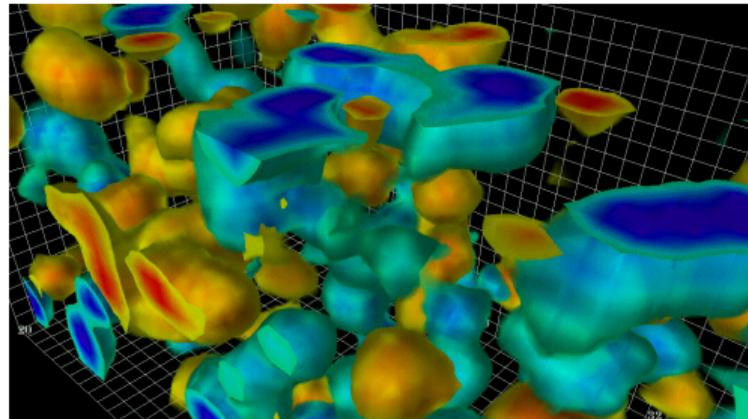
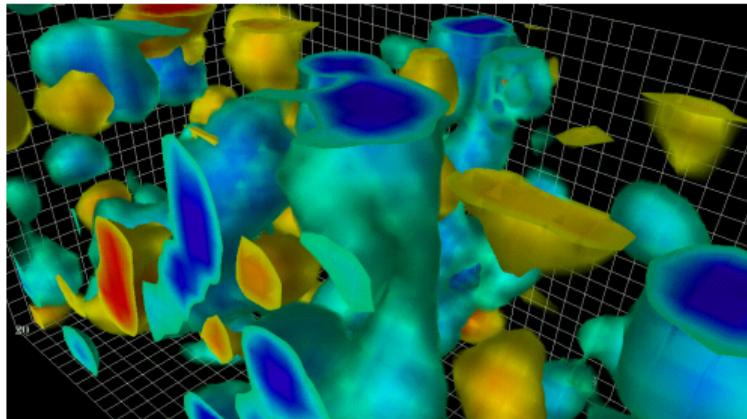
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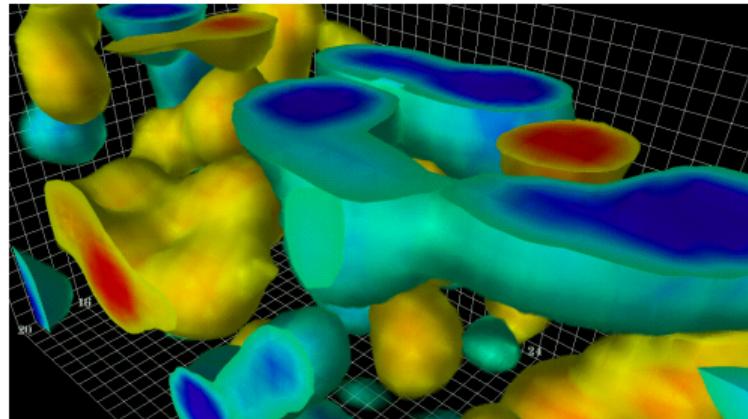
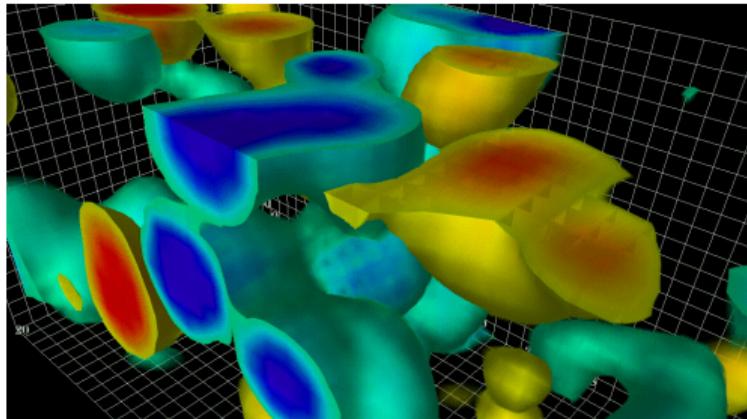
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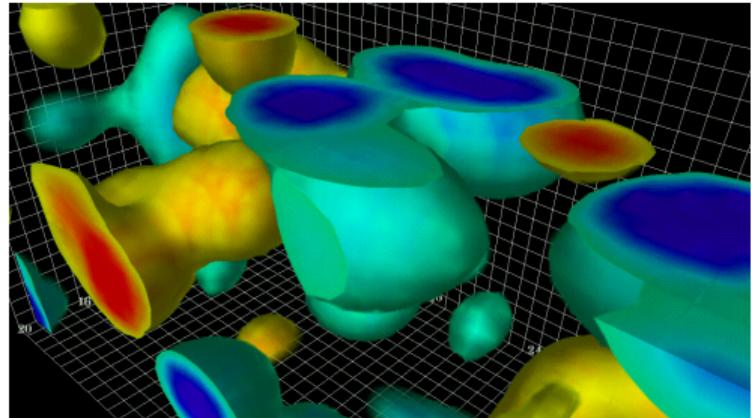
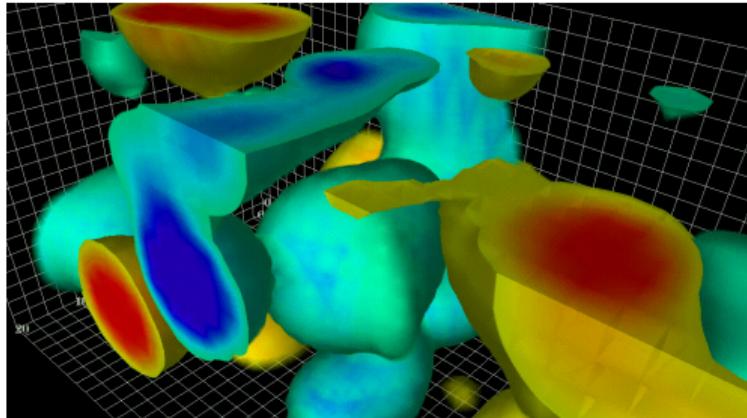
Original and Vortex Only Configuration Comparison (10 sweeps)

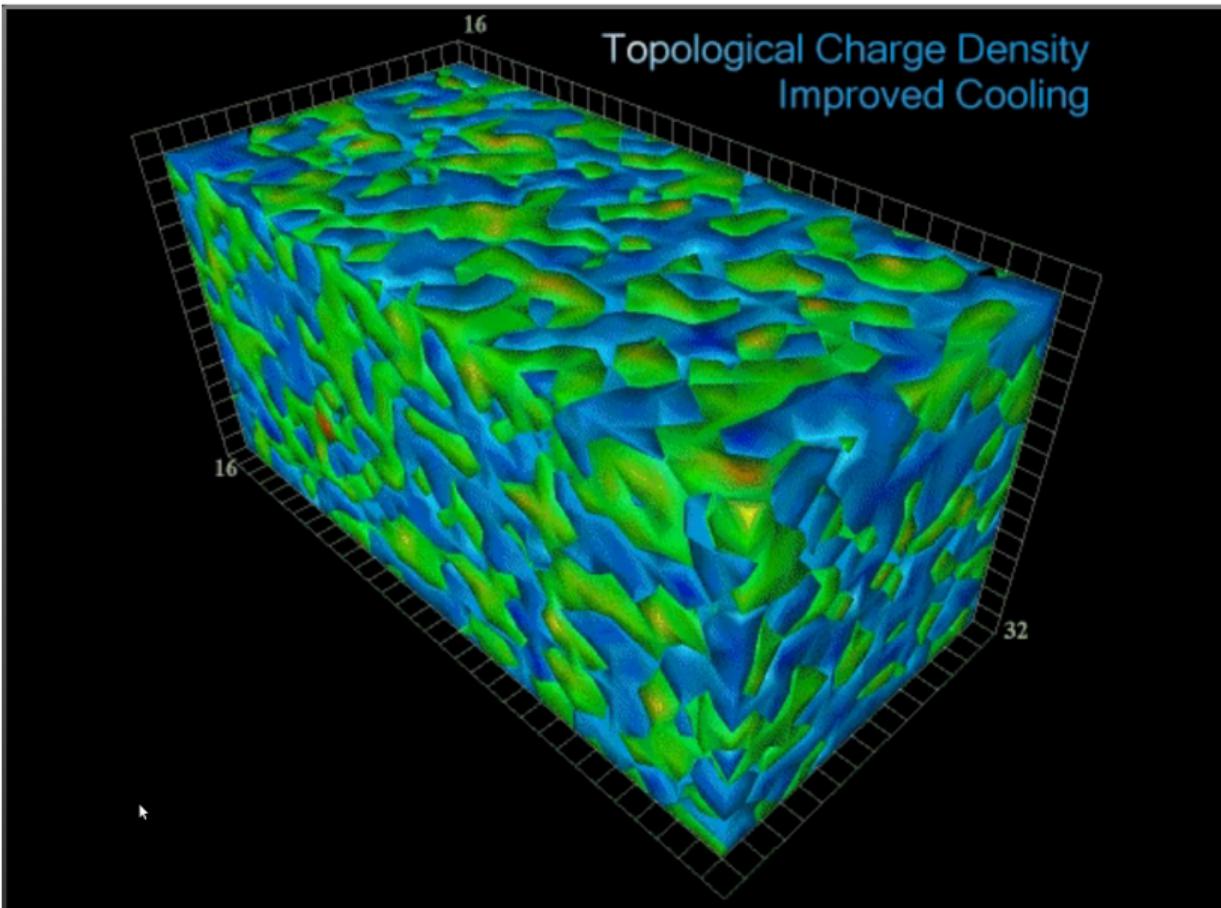


Original and Vortex Only Configuration Comparison (40 sweeps)



Original and Vortex Only Configuration Comparison (80 sweeps)

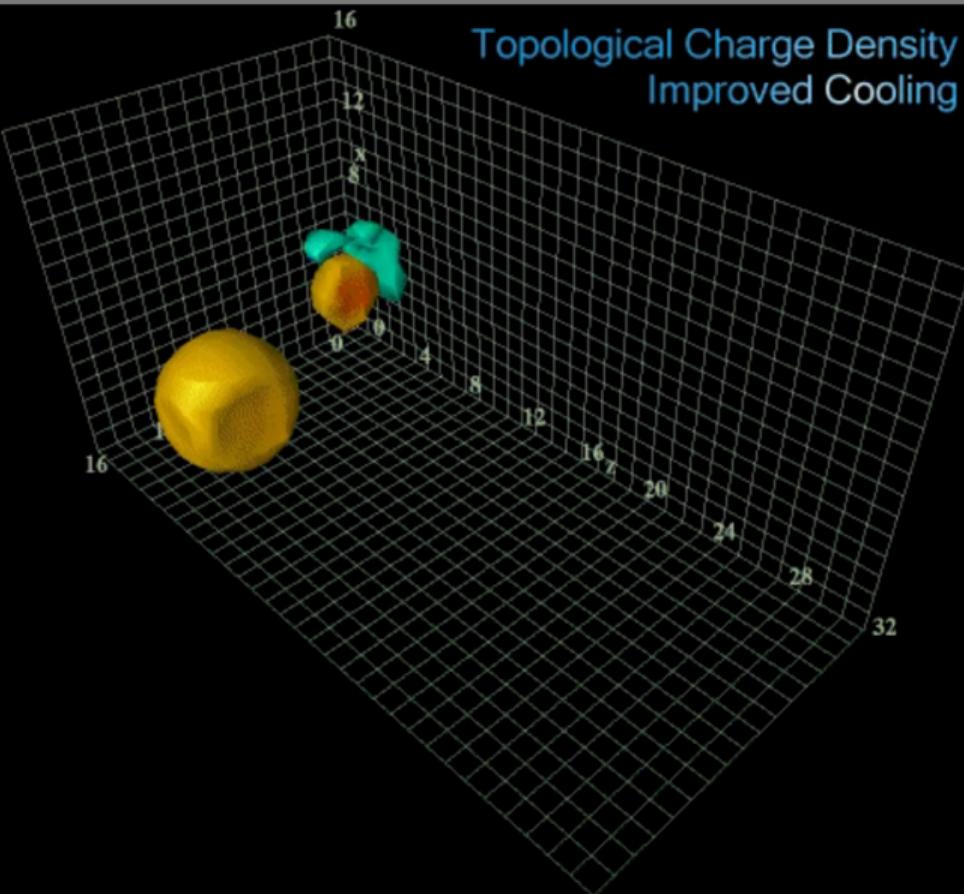






Buffing 56%

Topological Charge Density Improved Cooling



Conclusions and Future Directions

- The Centre Vortex structure of $SU(3)$ gauge fields is complicated.
- Qualitative idea of a 2D vortex sheet in 4 dimensions passing through a 3D visualisation is realised.
- Ample evidence of monopole - anti-monopole dynamics.
- Projected vortices are associated with the positions of topological charge.
- However, singular points creating topological charge density in the vortex configuration are not as abundant as needed.
- Future Directions
 - Visualisations of Gribov-copy issues in vortex identification.
 - Explore the approach to the continuum limit.
 - Create/explore alternative methods for the recreation of thick vortices.
 - Understand the impact of dynamical fermions.

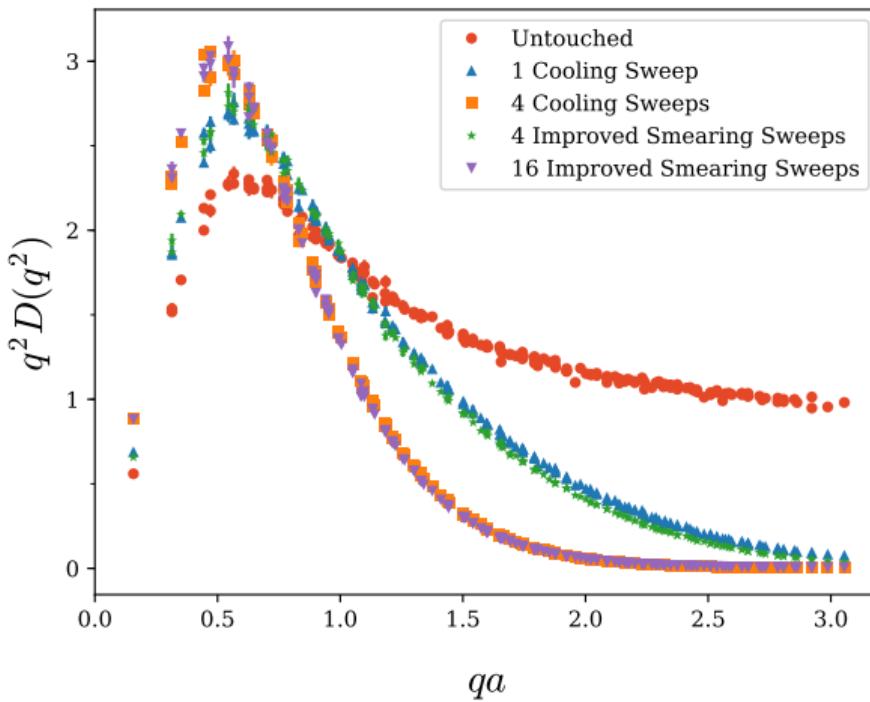
Interactive 3D Visualisation Techniques

- Rendered in AVS Express Visualisation Edition.
<http://www.avs.com/solutions/express/>
- Exported in VRML.
- Converted to U3D format via pdf3d ReportGen.
<https://www.pdf3d.com/products/pdf3d-reportgen/>
- Imported into L^AT_EX via media9 package.
- Viewed in Adobe acroread (Linux, use 9.4.1 when 3D support was maintained).
<ftp://ftp.adobe.com/pub/adobe/reader/unix/9.x/9.4.1/>

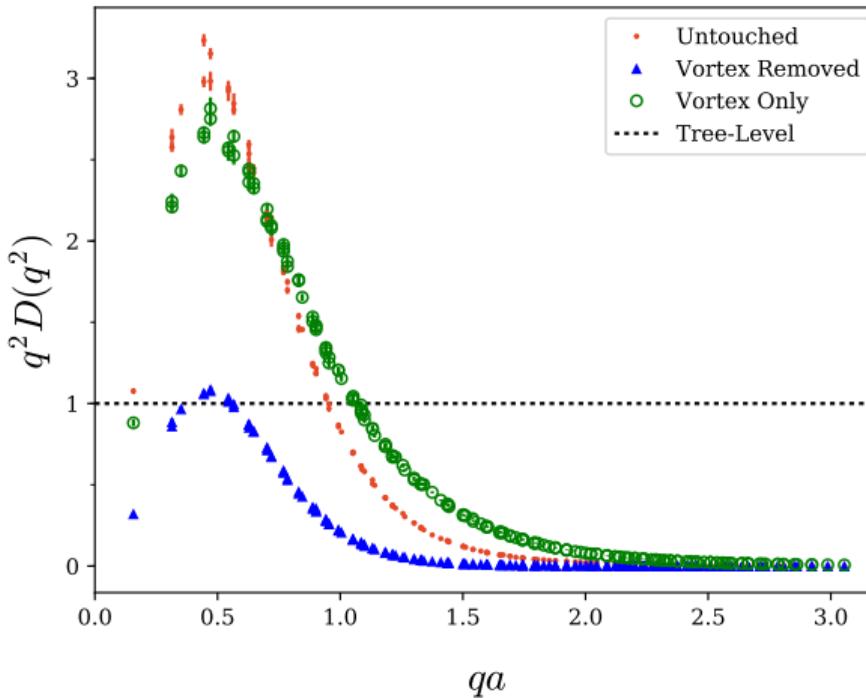
Supplementary Information

The following slides provide additional information which may be of interest.

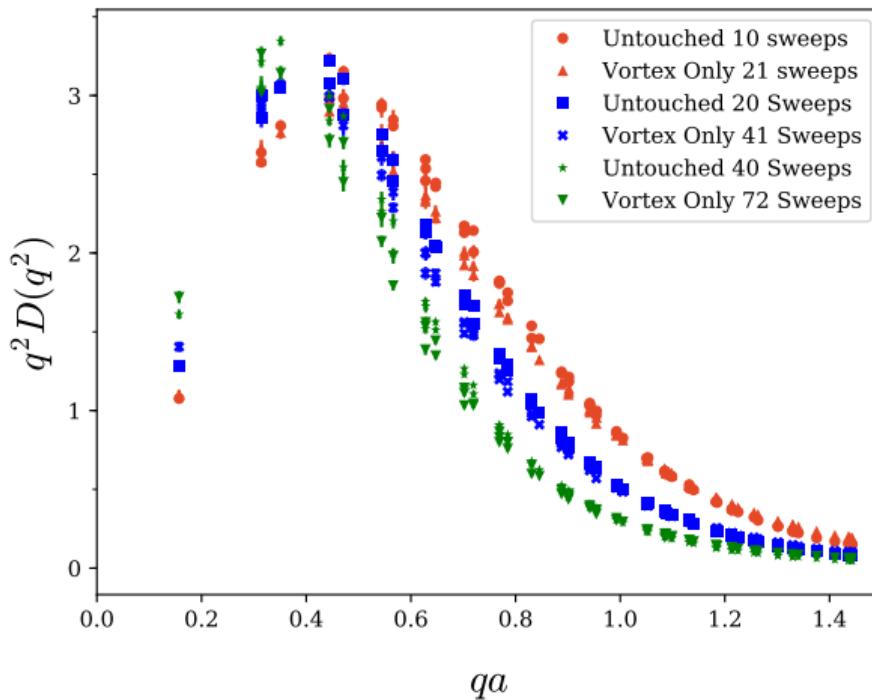
Gluon Propagator – Smearing versus Cooling



Vortex-Only Gluon Propagator with $\mathcal{O}(a^4)$ -improved cooling



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Relationship to Topological Charge Density

- The topological charge density is

$$q(x) = \frac{g^2}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} F_{\mu\nu}^{ab}(x) F_{\rho\sigma}^{ba}(x) \sim \vec{E}^{ab}(x) \cdot \vec{B}^{ba}(x).$$

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- An $\mathcal{O}(a^4)$ -improved field-strength tensor is considered.
- Constructed from the sum of Clover contributions $C_{\mu\nu}^{m \times n}$ for $m \times n$ loops:

$$F_{\mu\nu}^{\text{Imp}} = k_1 C_{\mu\nu}^{(1 \times 1)} + k_2 C_{\mu\nu}^{(2 \times 2)} + k_3 C_{\mu\nu}^{(1 \times 2)} + k_4 C_{\mu\nu}^{(1 \times 3)} + k_5 C_{\mu\nu}^{(3 \times 3)},$$

where

$$k_1 = 19/9 - 55 k_5, \quad k_2 = 1/36 - 16 k_5, \quad k_3 = 64 k_5 - 32/45, \quad k_4 = 1/15 - 6 k_5.$$

- k_5 is a tunable free parameter, governing $\mathcal{O}(a^6)$ errors.

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- An $\mathcal{O}(a^4)$ -improved field-strength tensor is considered.
- Constructed from the sum of Clover contributions $C_{\mu\nu}^{m \times n}$ for $m \times n$ loops:

$$F_{\mu\nu}^{\text{Imp}} = k_1 C_{\mu\nu}^{(1 \times 1)} + k_2 C_{\mu\nu}^{(2 \times 2)} + k_3 C_{\mu\nu}^{(1 \times 2)} + k_4 C_{\mu\nu}^{(1 \times 3)} + k_5 C_{\mu\nu}^{(3 \times 3)},$$

where

$$k_1 = 19/9 - 55 k_5, \quad k_2 = 1/36 - 16 k_5, \quad k_3 = 64 k_5 - 32/45, \quad k_4 = 1/15 - 6 k_5.$$

- k_5 is a tunable free parameter, governing $\mathcal{O}(a^6)$ errors.
- We consider $k_5 = 1/180$, as the 5-loop-improved $F_{\mu\nu}$.

$$k_1 + k_3 \rightarrow 95.6\%, \quad k_4 + k_5 \rightarrow 1.7\%.$$