Jefferson Lab

# Structure of pion and kaon from lattice QCD

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In collaboration with:

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Lattice 2018, Michigan
24th July, 2018

# Motivation

- Huge progress in tackling hadronic light-cone observables on the lattice

- "Light-like" separated hadonic tensor- K-F Liu et al Phys. Rev. Lett. 72 1790 (1994) ,
  Phys. Rev. D62 (2000) 074501
  A Chambers et.al (2017) 1703.01153
- Ioffe Time Pseudo Distribution Methods –
  Quasi PDF: X. Ji, Phys.Rev.Lett. 110, (2013)
  J.-W. Chen et.al. (2018) 1803.04393
  C Alexandrou et.al. (2018) 1803.02685

  Pseudo PDF : A. Radyushkin Phys.Lett. B767 (2017)
  K. Orginos et. al. (2017) 1706.05373

- Another new concept – "Good" lattice cross-sections
  Y.-Q. Ma J.-W. Qiu (2014) 1404.6860
  Y.-Q. Ma, J.-W. Qiu (2017) 1709.03018

# PDFs from "Good"Lattice Cross-sections

"Good" lattice cross sections – Ma & Qiu, Phys.Rev.Lett. 120 (2018) no.2, 022003

If we can form single hadron matrix elements of renormalizable nonlocal operators such that -

(1) It is calculable in lattice- QCD with an Euclidean time,

$$\sigma_n(\omega, \xi^2, P^2) = \langle P|T\{\mathcal{O}_n(\xi)\}|P\rangle$$

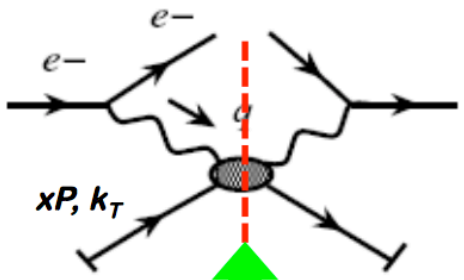
(2) It has a well-defined continuum limit at the lattice spacing zero limit,

(3) It has the same and factorizable logarithmic collinear (CO) divergences as that of PDFs.

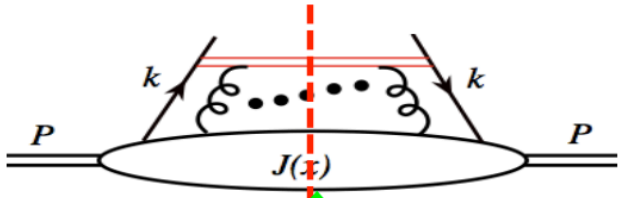
$$\mathcal{O}_{j_1 j_2}(\xi) \equiv \xi^{d_{j_1}+d_{j_2}-2}\, Z_{j_1}\, Z_{j_2} j_1(\xi)\, j_2(0)$$

Then …

$$\sigma_{\text{tot}}^{\text{DIS}} :$$




$$\otimes$$




$$+ O\left(\frac{1}{QR}\right)$$

**Factorization**

**Hard-part**
**Probe**

**Parton-distribution**
**Structure**

**Power corrections**
**Approximation**

If $\xi^2$ is sufficiently small, the lattice cross-section constructed from two renormalizable currents can be factorized into PDFs

$$\sigma_n(\nu, z^2, p^2) = \sum_a \int_{-1}^{1} \frac{dx}{x} f_a(x, \mu^2) K_n^a(x\nu, z^2, x^2p^2, \mu^2) + O(z^2\Lambda_{\text{QCD}}^2)$$

$$f_{\bar{a}}(x, \mu^2) = -f_a(-x, \mu^2)$$

Ma & Qiu, Phys.Rev.Lett. 120 (2018) no.2, 022003

$$\mathcal{O}_{j_1 j_2}(\xi) \equiv \xi^{d_{j_1}+d_{j_2}-2}\, Z_{j_1}\, Z_{j_2} j_1(\xi)\, j_2(0)$$

What currents we can use?

$$\mathcal{O}_S(\xi) = \xi^4 Z_S^2 [\overline{\psi}_q \psi_q](\xi)\, [\overline{\psi}_q \psi_q](0)\,,$$

$$\mathcal{O}_V(\xi) = \xi^2 Z_V^2 [\overline{\psi}_q \not{\xi} \psi_q](\xi)\, [\overline{\psi}_q \not{\xi} \psi_q](0)\,,$$

$$\mathcal{O}_{\widetilde{V}}(\xi) = -\frac{\xi^4}{2} Z_V^2 [\overline{\psi}_q \gamma_\nu \psi_q](\xi)\, [\overline{\psi}_q \gamma^\nu \psi_q](0)\,,$$

$$\mathcal{O}_{V'}(\xi) = \xi^2 Z_{V'}^2 [\overline{\psi}_q \not{\xi} \psi_{q'}](\xi)\, [\overline{\psi}_{q'} \not{\xi} \psi_q](0)\,, \ldots,$$

$$\mathcal{O}_{\mu\nu}(\xi) = \xi^4 Z_V^2 [\overline{\psi}_q \gamma_\mu \psi_q](\xi)\, [\overline{\psi}_q \gamma_\nu \psi_q](0)$$

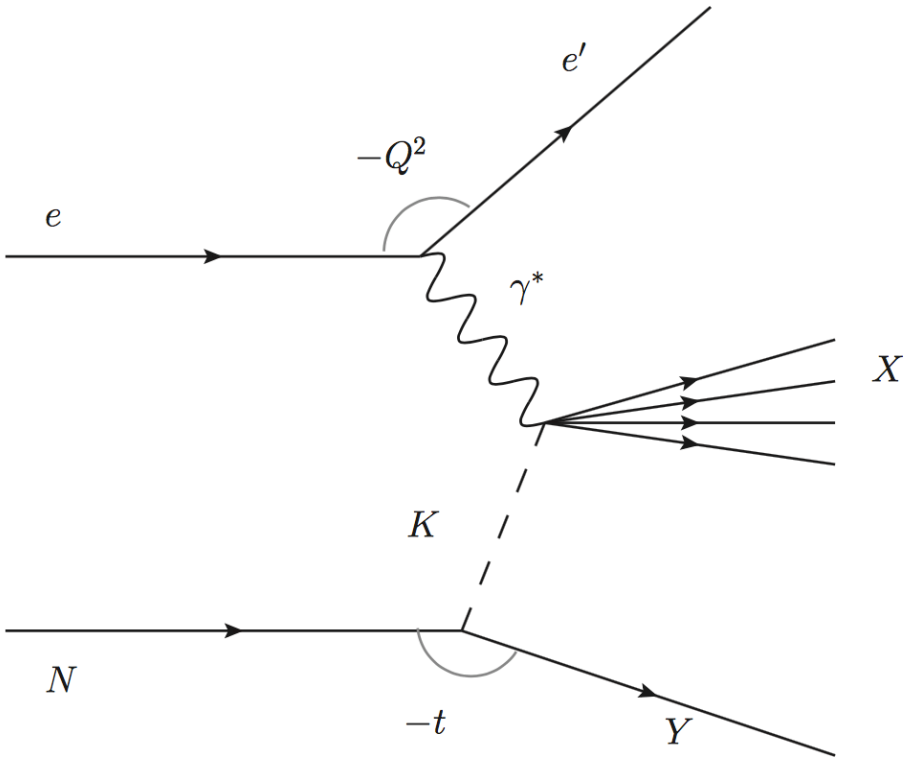
Including flavor changing and gluonic currents with any single hadronic state

Raza Sufian's talk: Valence quark distribution in pion

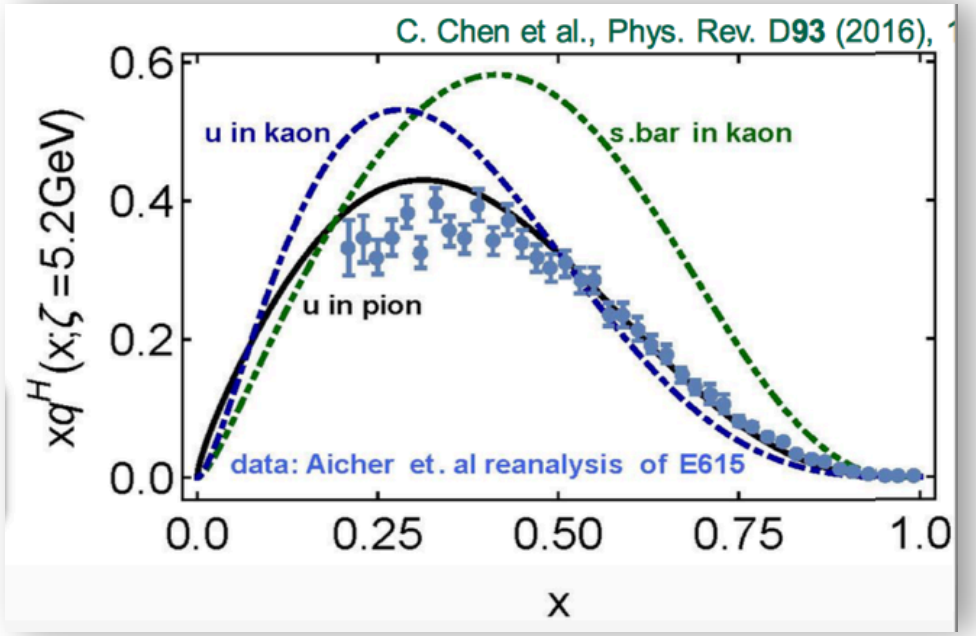
# Motivation for Kaon PDF calculation

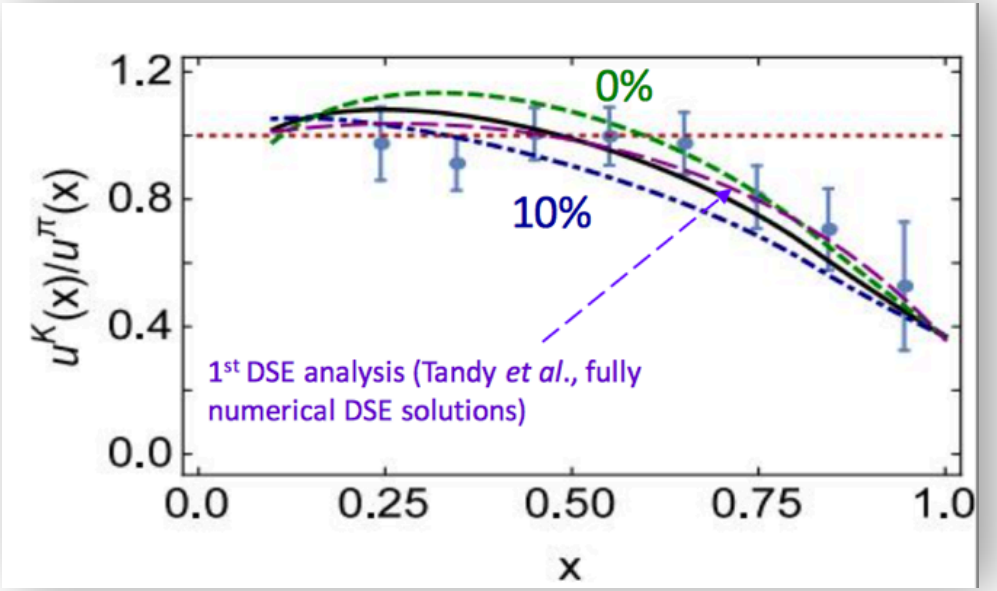
- Limited Old data from Drell-Yan – no "Global" fit exists for Kaon PDF
- JLab 12 GeV: approved pion-TDIS (π-TDIS, PR12-15-006) will also measure

  Kaon structure functions for the first time ever at small x
- Long-searched-for Sullivan process for accessing the kaon structure function
- EIC will add large (x,$Q^2$) landscape for both pion and kaon!



- Recent DSE calculations of pion and Kaon PDFs and ratios

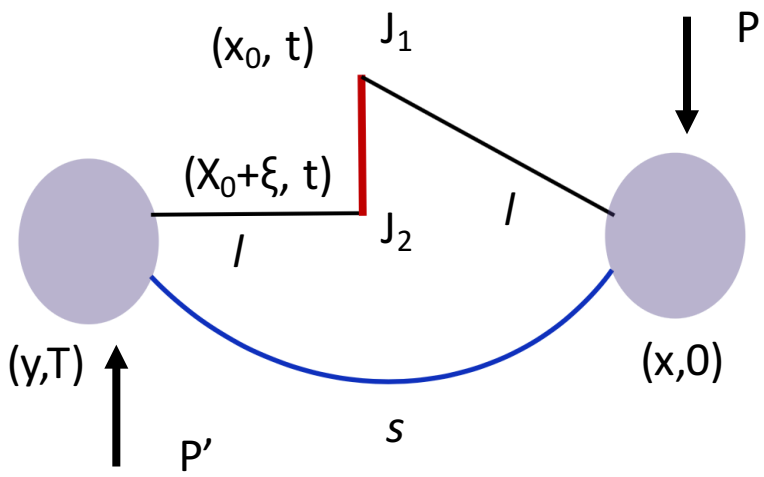


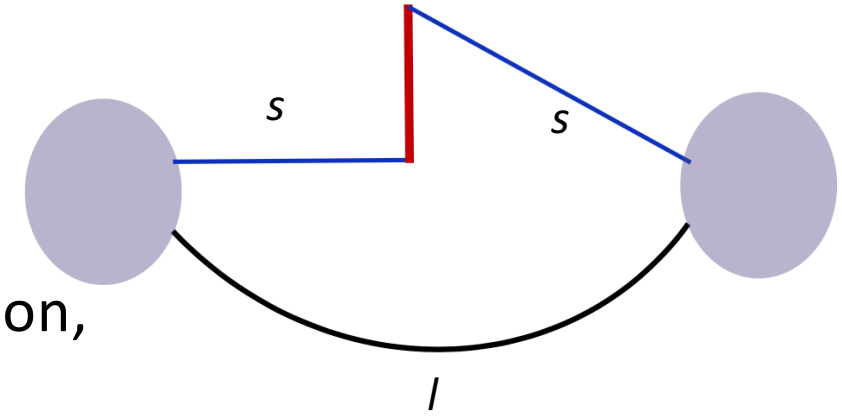




First-principle lattice QCD calculation of needed – first direct calculation of Kaon PDF on the lattice

# Lattice simulation

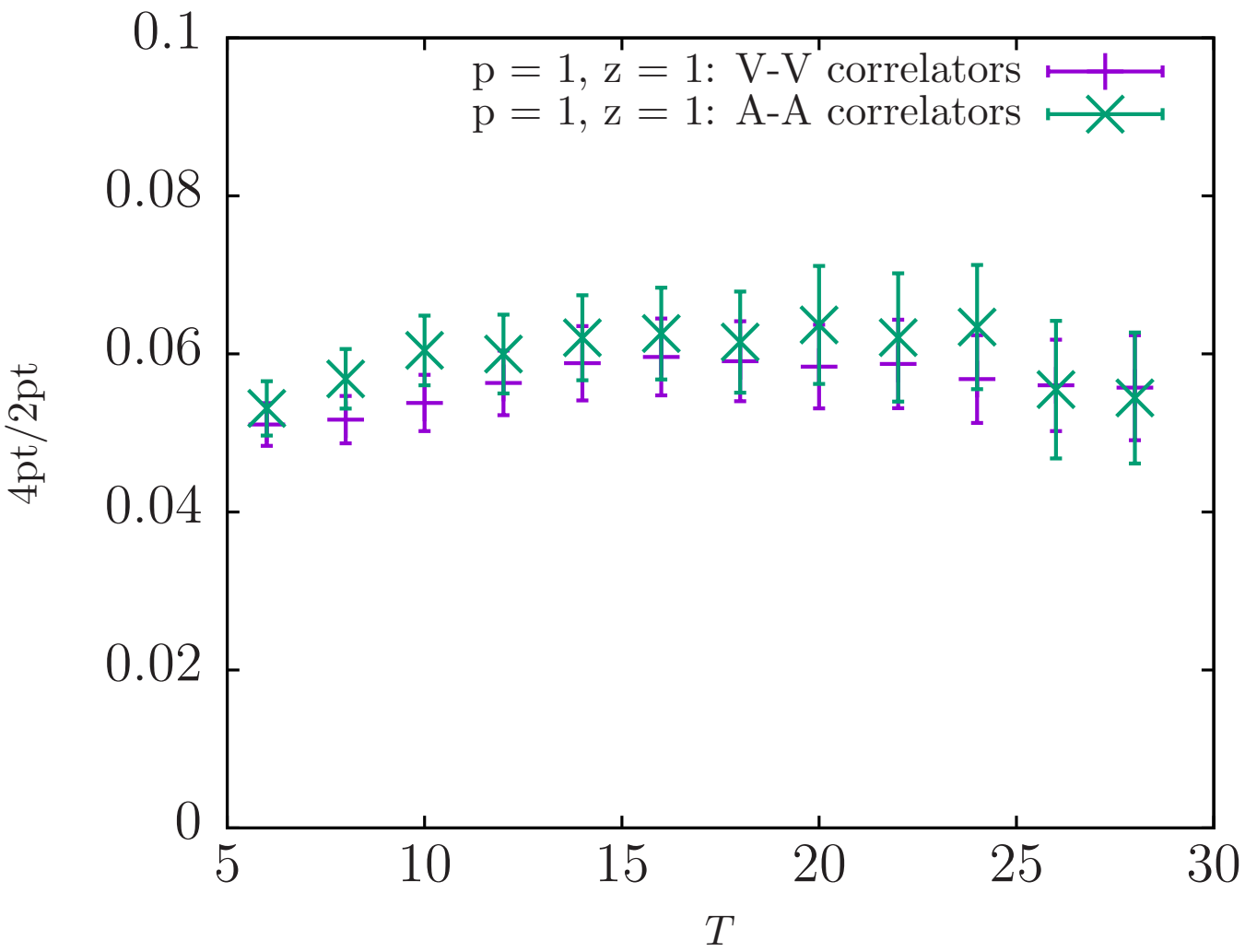
- Need calculation of four point functions, with 'red' being different heavier quarks [similar technique by Bali et. al. 1807.03073]
- Isoclover configurations $32^3 \times 96$, lattice spacing 0.127 fm
- Pion mass around 405 MeV
- Analysis shown on 100 configurations, 1 time source
- We will increase the statistics 20-30 times
- Current renormalisations known from previous calculation, arXiv:1611.07452
- Two-exponential fits for ratios of 4pt and 2pt functions



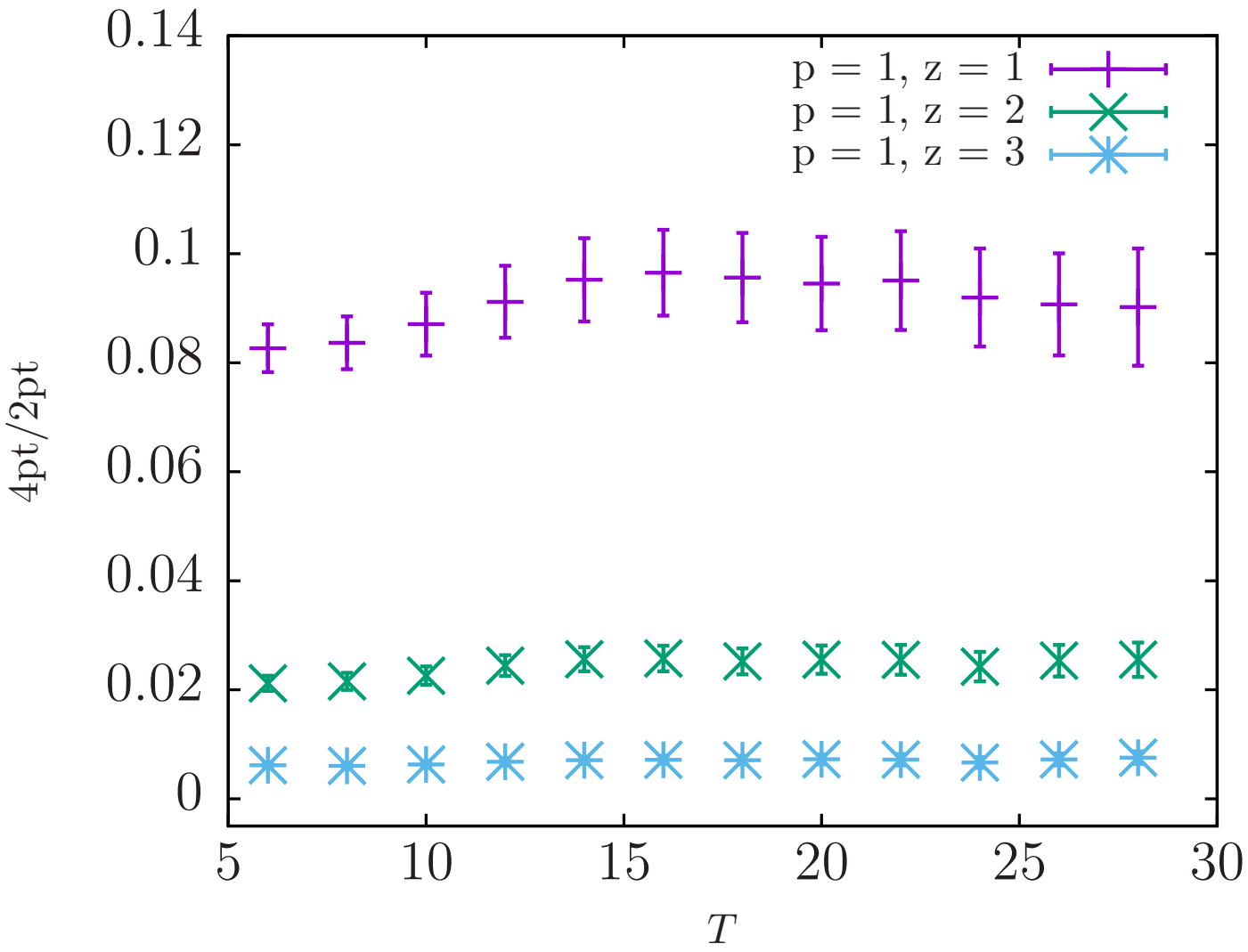

$\xi$ could be on/off axis

# (Very) preliminary lattice results

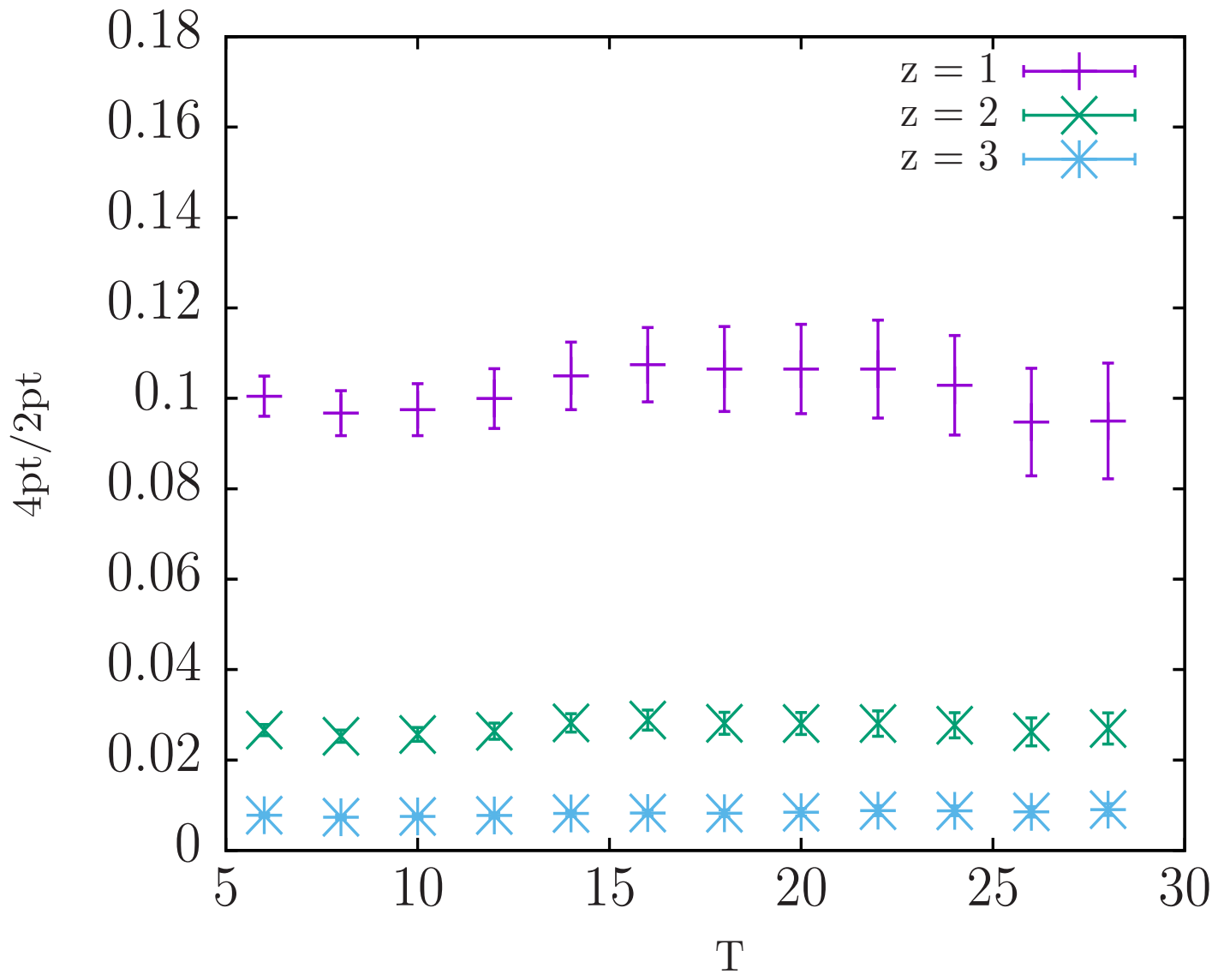



T is the source sink separation

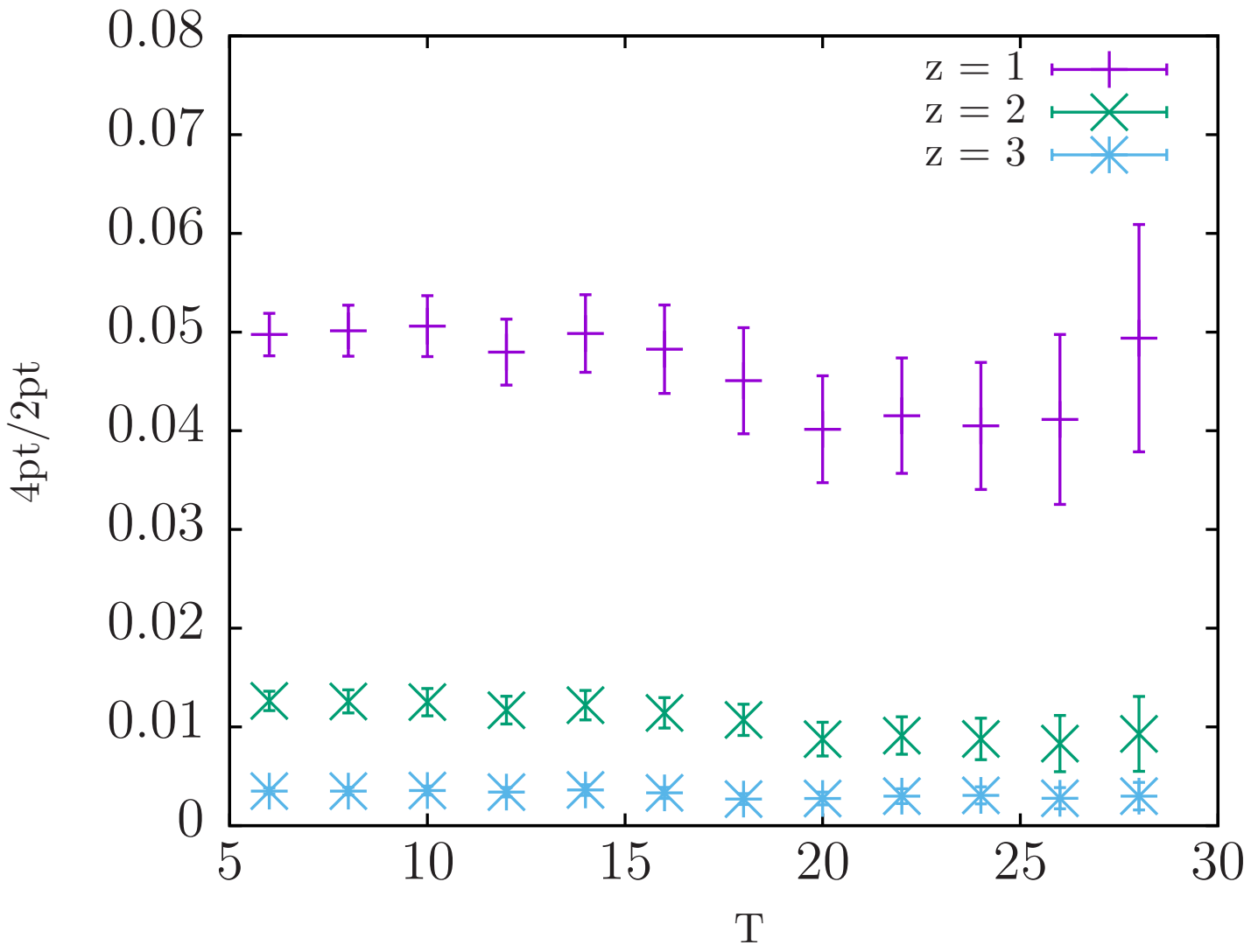
- Comparison between:
($J_1$ = V, $J_2$ = V) and ($J_1$ = A, $J_2$ = A)
- *$P_z$= 1 (~0.3 GeV)*
- After renormalisation both ratios agree
- Real part of the 4pt correlator
- Strange quark is the active quark




- Comparison between:
($J_1$ = V, $J_2$ = V) and different ξ = z
- *$P_z$= 1 (~0.3 GeV)*
- Real part of the 4pt correlator
- Strange quark is the active quark
- No signal after ξ = 6



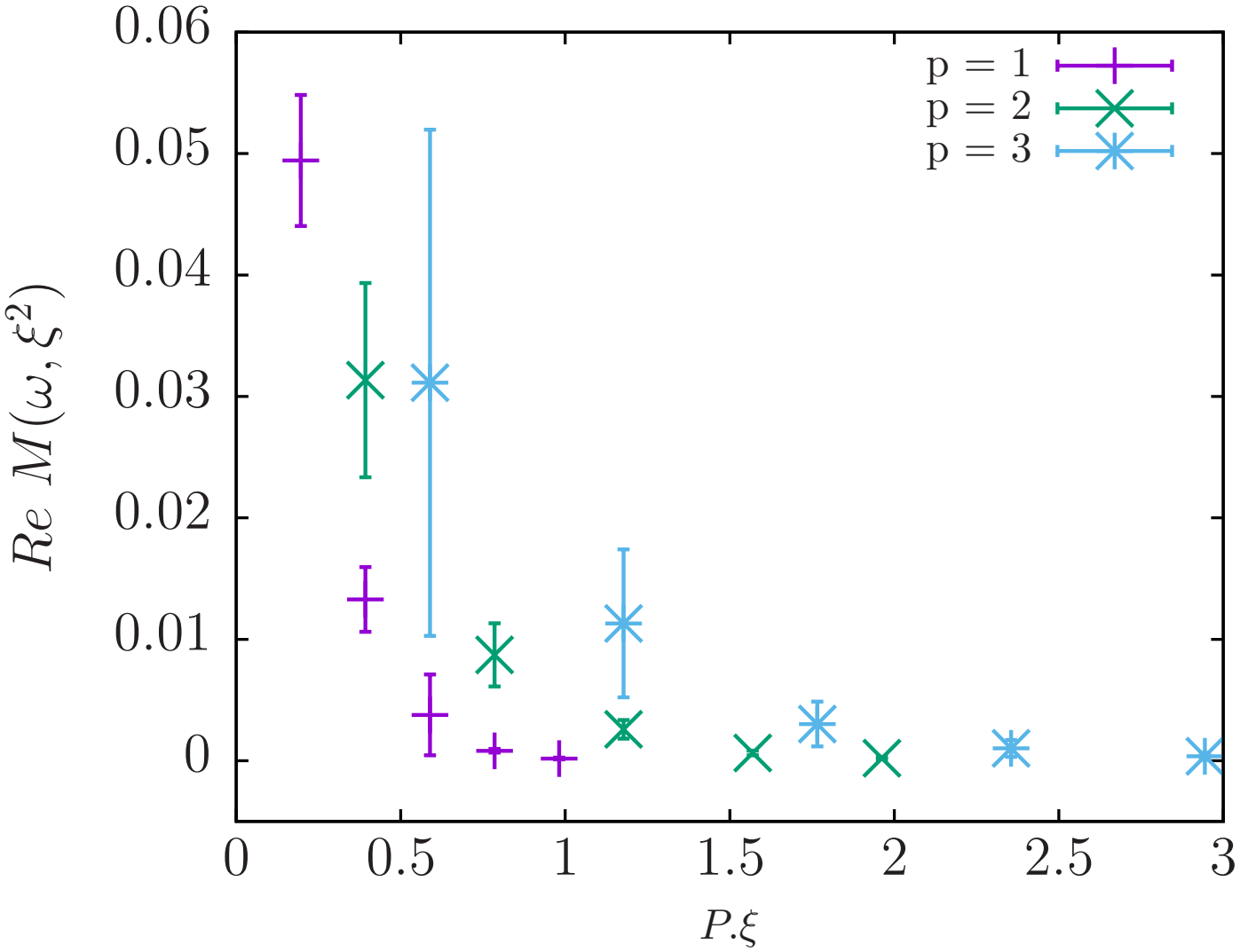

- Comparison between:
($J_1$ = V, $J_2$ = A and vice versa) and different ξ = z
- *$P_z$= 1 (~0.3 GeV)*
- Real part of the 4pt correlator
- Strange quark is the active quark
- No signal after ξ = 6



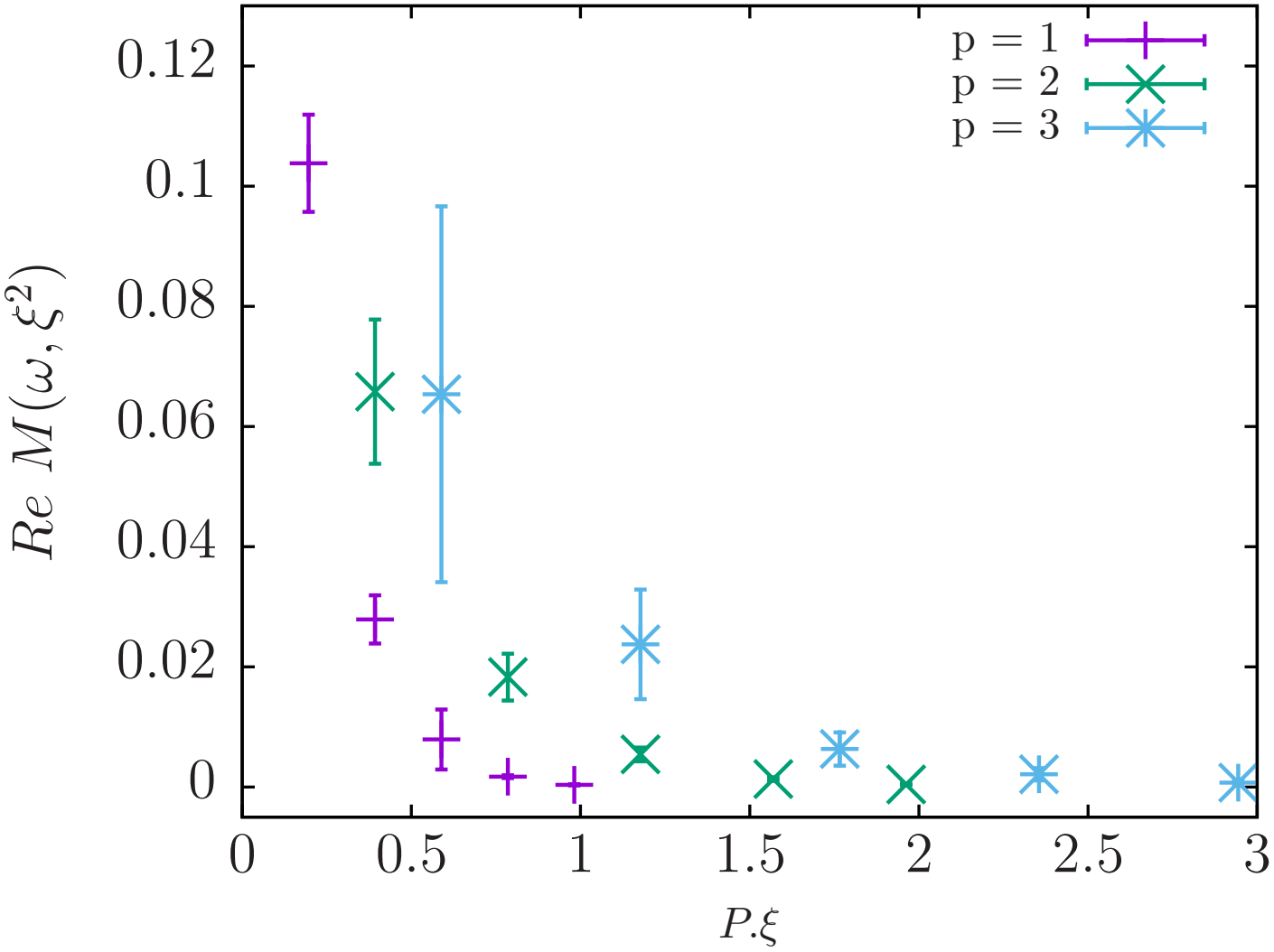

- Comparison between:
($J_1$ = V, $J_2$ = A and vice versa) and different ξ = z
- *$P_z$= 1 (~0.3 GeV)*
- Real part of the 4pt correlator
- Light quark is the active quark
- No signal after ξ = 6

Ioffe time distribution,
Lorentz invariant ν = P.ξ

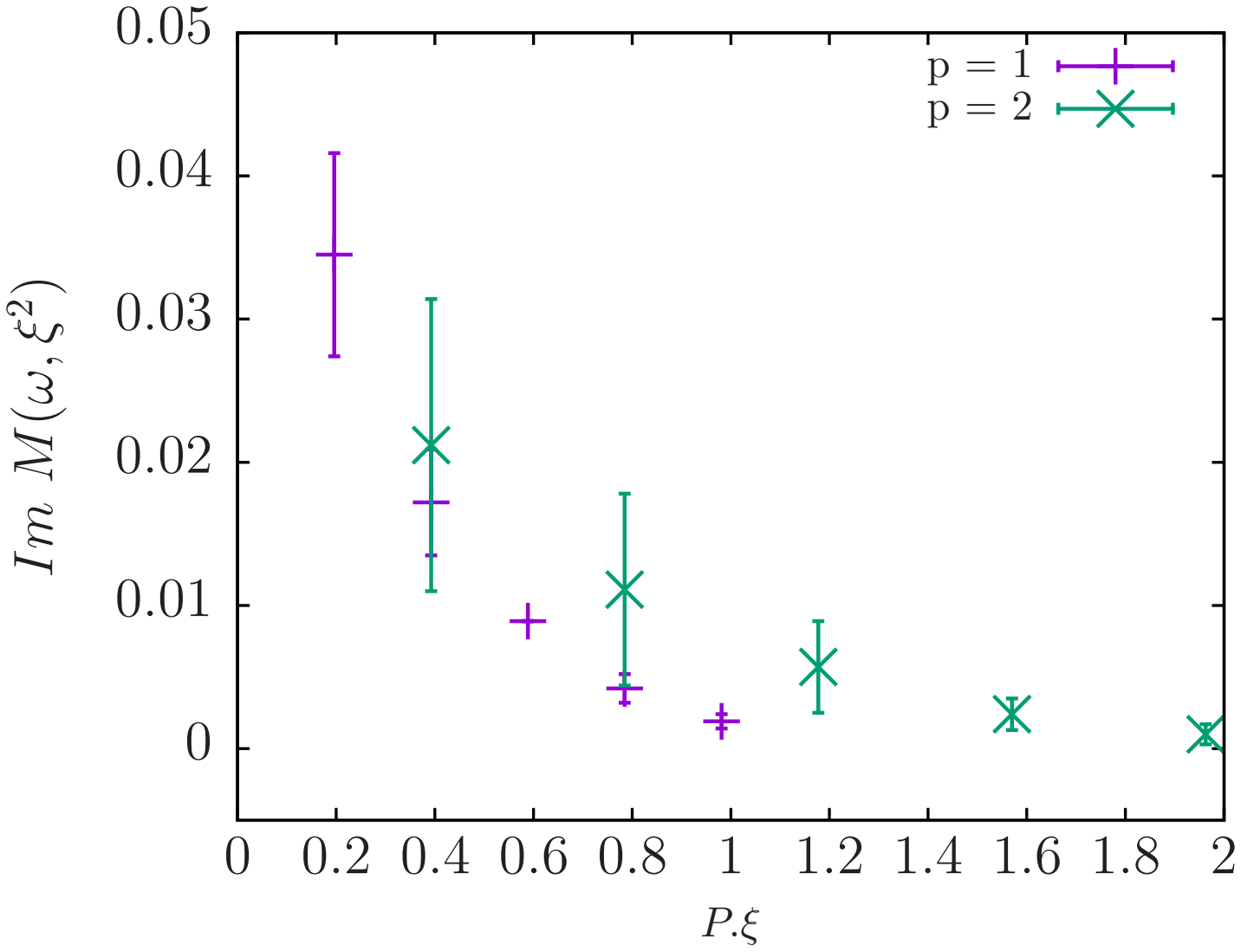
B. L. Ioffe, Phys. Lett. 30B, 123 (1969)




- Comparison between:
($J_1$ = V, $J_2$ = A and vice versa) and different P.ξ
- *$P_z$= 1-3 (~0.3 – 0.9 GeV)*
- Real part of the 4pt correlator
- Light quark is the active quark
- No signal after P. ξ = 3




- Comparison between:
($J_1$ = V, $J_2$ = A and vice versa) and different P.ξ
- *$P_z$= 1-3 (~0.3 – 0.9 GeV)*
- Real part of the 4pt correlator
- Strange quark is the active quark
- No signal after P. ξ = 3



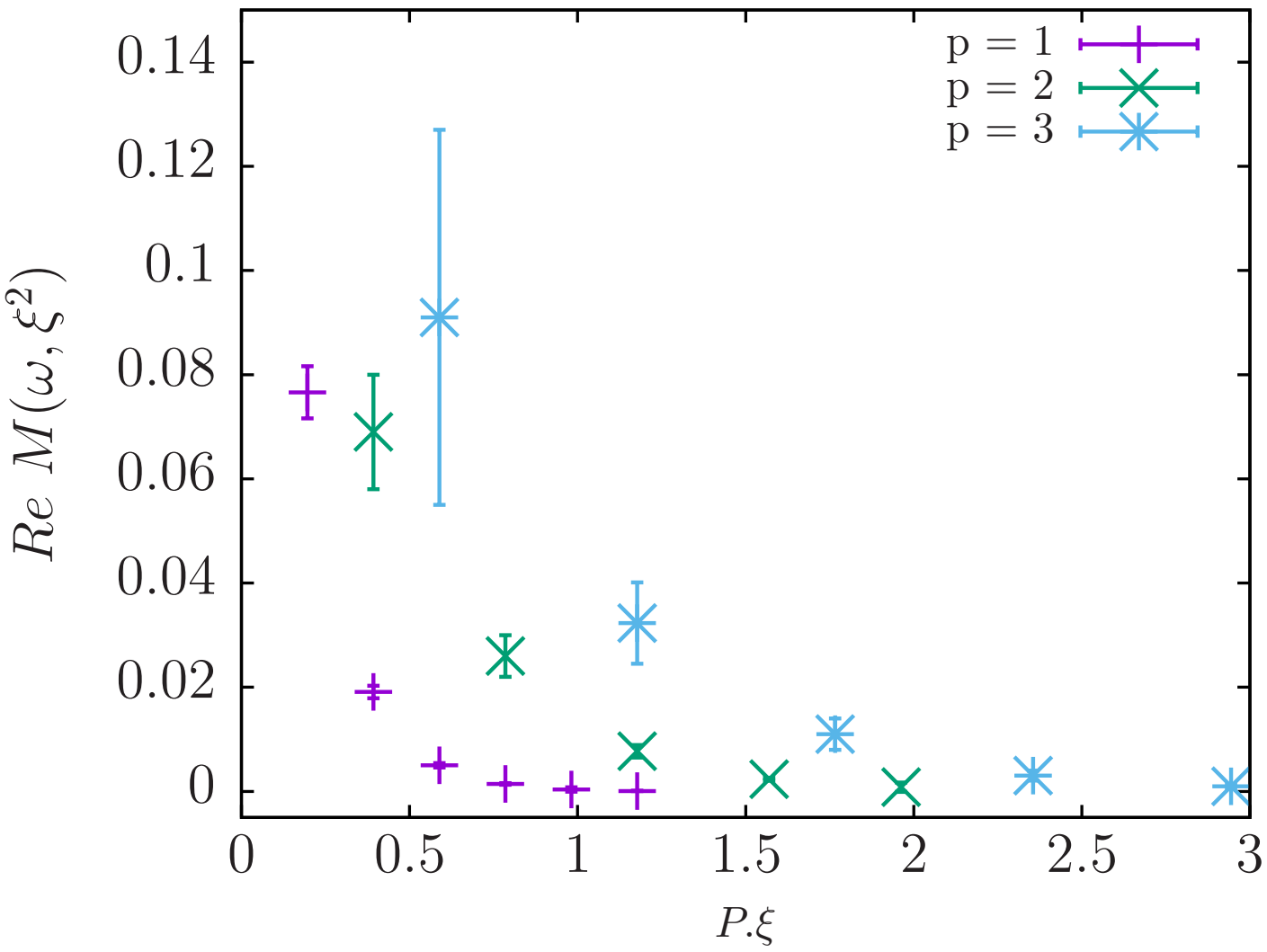

- Comparison between:
($J_1$ = V, $J_2$ = A and vice versa) and different P.ξ

- *$P_z$= 1-3 (~0.3 – 0.9 GeV)*

- Imaginary part of the 4pt correlator

- Strange quark is the active quark

- No signal after P. ξ = 3




- Comparison between:
($J_1$ = V, $J_2$ = V and vice versa) and different P.ξ

- *$P_z$= 1-3 (~0.3 – 0.9 GeV)*

- Real part of the 4pt correlator

- Strange quark is the active quark

- No signal after P. ξ = 3

# "Global" fit for lattice data - technique

$$\sigma_n(\nu, z^2, p^2) = \sum_a \int_{-1}^{1} \frac{dx}{x} f_a(x, \mu^2) K_n^a(x\nu, z^2, x^2p^2, \mu^2) + O(z^2\Lambda_{\rm QCD}^2)$$

Many such lattice "cross-sections" To be calculated

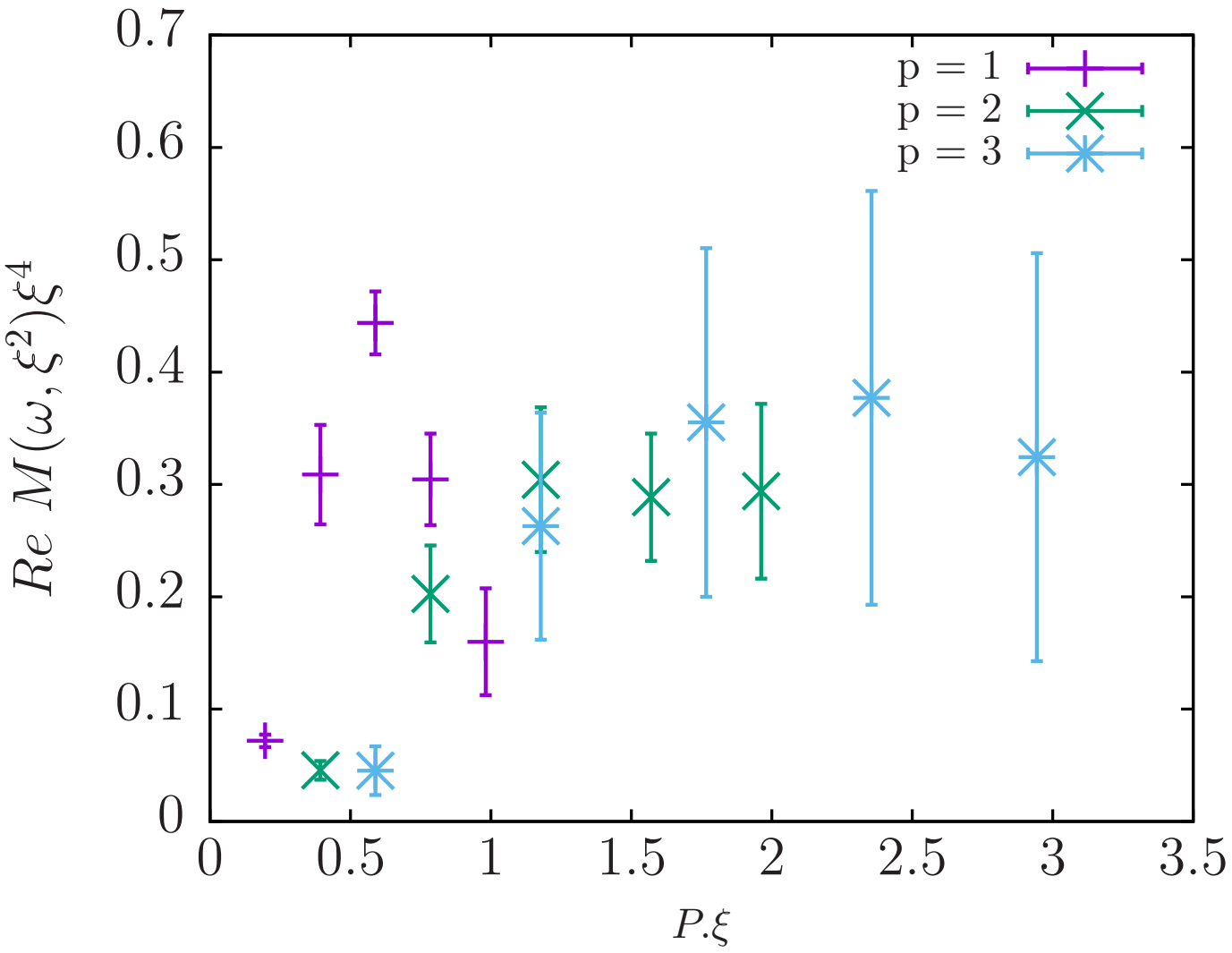
We use $f_{\bar{a}}(x, \mu^2) = -f_a(-x, \mu^2)$ to obtain valence quark and anti-quark distribution

Perturbative Kernel known, Oscillatory

- We can get around ill posed inverse Fourier Transform
- Need many P.ξ data to have a good global fit for f(x)
- Fit forms – not clear for Kaon, however, similar to pion
- A model fit form [JAM Collaboration, arXiv:1804.01965 ]

- Regge : a = - 0.5
- Quark counting : b > 2
- Small corrections for particular meson

$$f_{abcd}(x) = N_{abcd}\ x^a(1-x)^b(1 + c\sqrt{x} + d\,x\,)$$




- Need lot more P.ξ data points for a realistic f(x) fit
- And with higher P values
- Real part of the V-V 4pt correlator multiplied by vector current renormalisation and a factor $\xi^4$
- Strange quark is the active quark

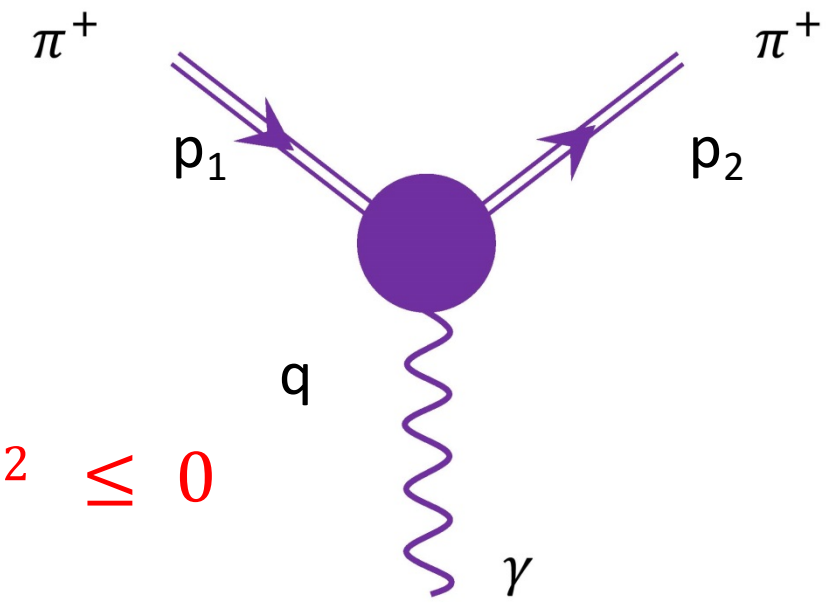
## Ongoing…

- Getting higher momenta data to obtain many P.ξ values
- Testing momentum smearing [Bali et. al. (2016)]
- To calculate at different lattice spacings, quark masses and volumes
- We have demonstrated that this method works
- Corresponding momentum space matrix elements are also good lattice cross sections
- Generalisation of other methods – pseudo, quasi, T33
- We can also get polarised pdf and other correlation functions of other hadrons

# Pion electromagnetic form factor at high $Q^2$ from lattice QCD

# Definition

Simplest hadrons – pion, kaon




Space like "$q$":

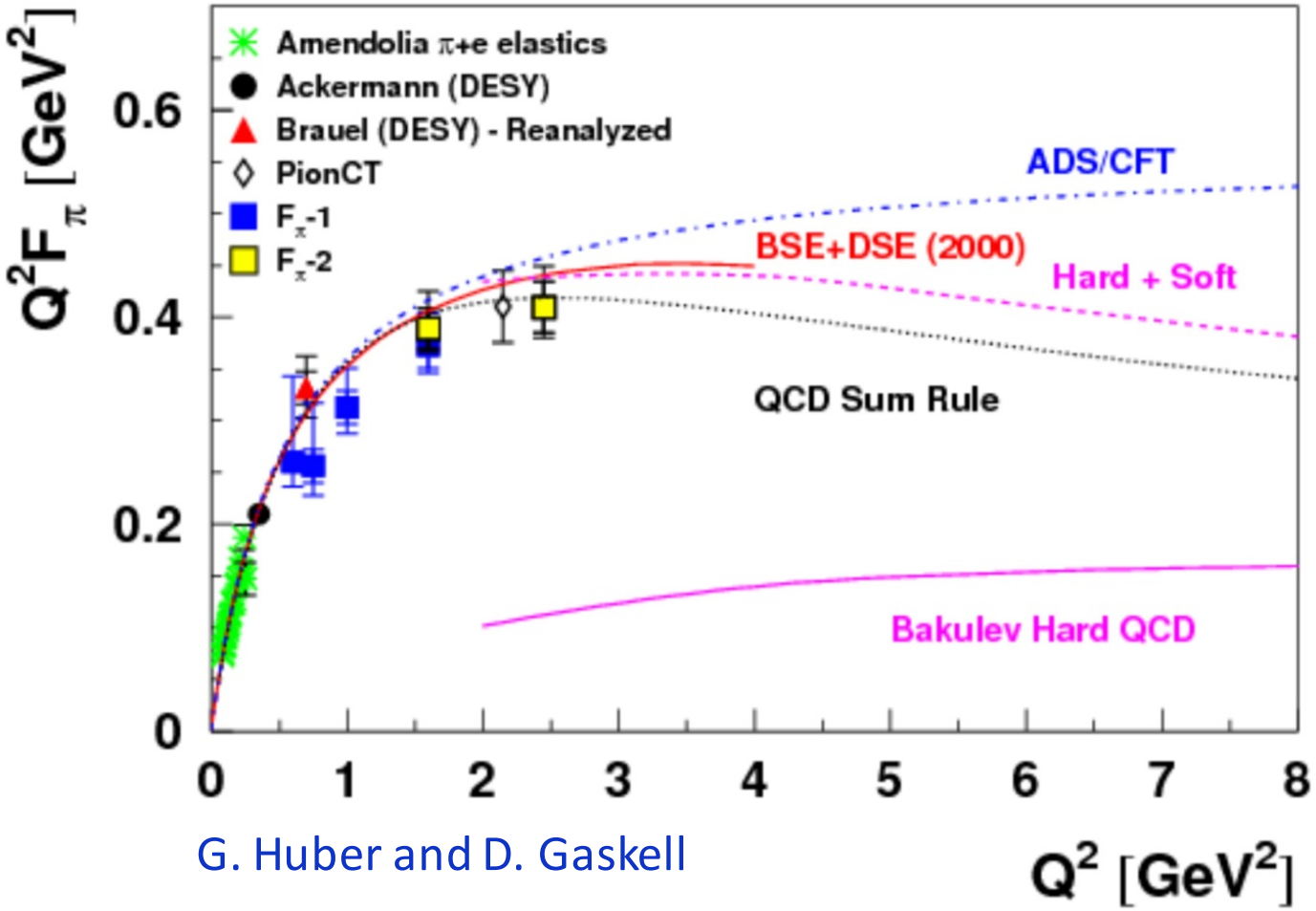
$$q^2 = (p_2 - p_1)^2 \leq 0$$

$$Q^2 = -q^2$$

$$\langle \pi^+(\vec{p}\,') | j^\mu | \pi^+(\vec{p}) \rangle = (p + p')^\mu \, F_\pi(Q^2)$$

(in units of '$e$')

# Interplay between hard and soft scales




G. Huber and D. Gaskell

Hard tail ($Q^2 \to \infty$) from pQCD:

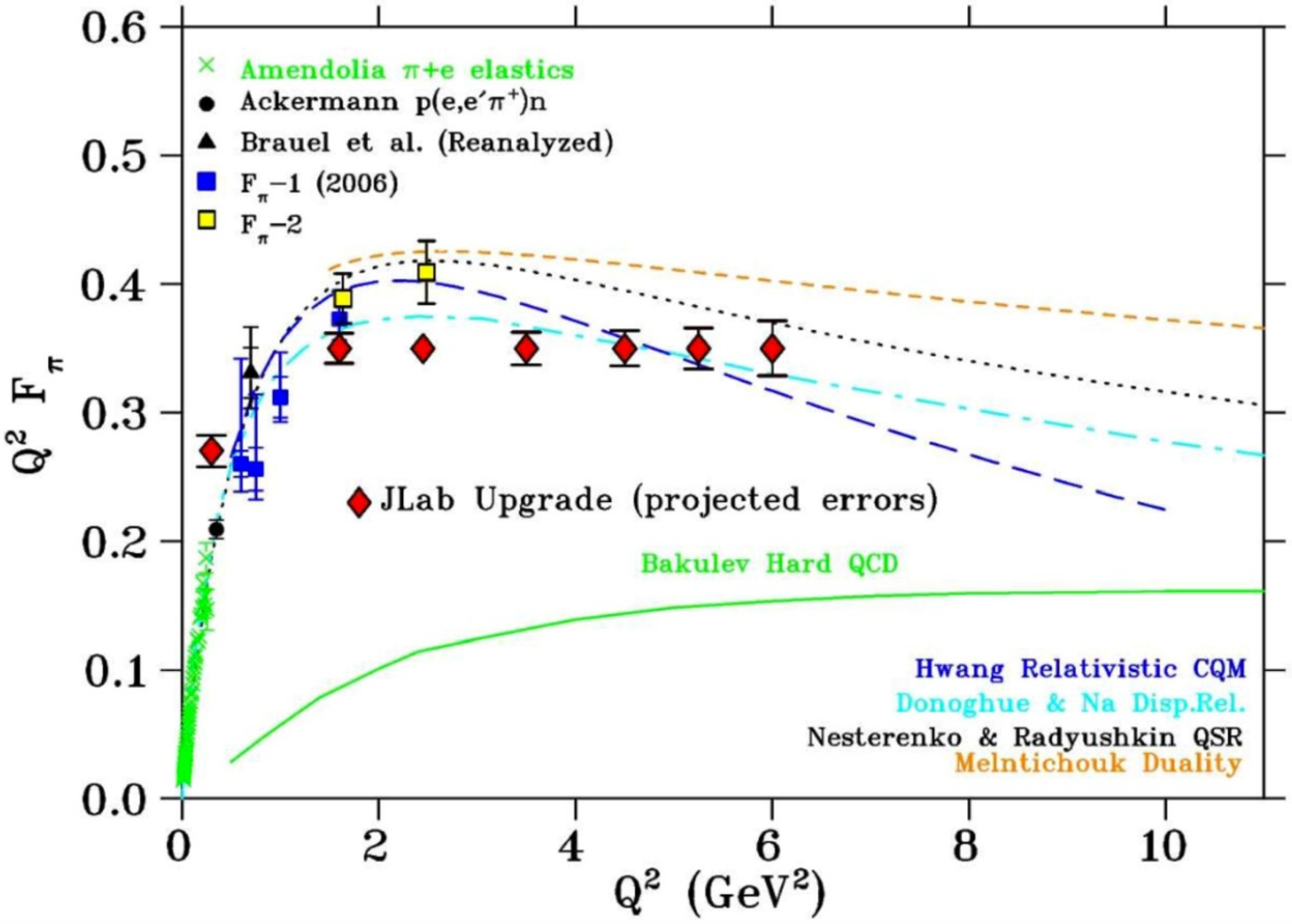
$$F_\pi(Q^2) \to \frac{16\pi\alpha_s(Q^2) f_\pi{}^2}{Q^2}$$

G. P. Lepage, S.J.Brodsky, Phys. Lett. 87B(1979)359

Soft part ($Q^2 < 1$ GeV$^2$):
vector meson dominance with $F_\pi(0) = 1$,
data fits well

Need better understanding of the transition to the asymptotic region

# JLAB 12 GeV upgrade

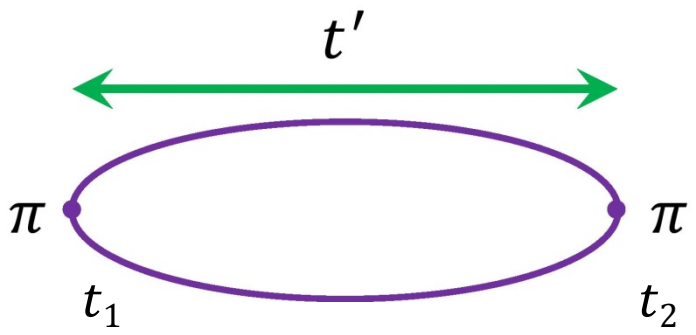



G. Huber and D. Gaskell

$F_\pi$ measurements at $Q^2 \sim 6 - 10$ GeV$^2$: E12-06-101 at JLAB Hall C,
For EIC, $Q^2 \sim 30$ GeV$^2$

Can we get some insight from first principles lattice QCD calculations to the question - where does the transition to pQCD happen?

# Two-point correlator construction




$$C_{ij}(t) = \langle 0|\mathcal{O}_i(t)\mathcal{O}_j^\dagger(0)|0\rangle$$

- Basis of operators

$$\mathcal{O} \sim \bar{\psi}\Gamma \overleftrightarrow{D} \cdots \overleftrightarrow{D}\psi$$

- Optimized operator for state $|n>$

$$\Omega_{\mathfrak{n}}^\dagger = \sum_i w_i^{(\mathfrak{n})} \mathcal{O}_i^\dagger$$

in a variational sense by solving generalized eigenvalue problem-

$$C(t)\, v^{(\mathfrak{n})} = \lambda_{\mathfrak{n}}(t)\, C(t_0)\, v^{(\mathfrak{n})}$$

- Diagonalize the correlation matrix – eigenvalues

$$\lambda_n(t) = \exp[-E_n(t - t_0)]$$

# Two-point correlator construction

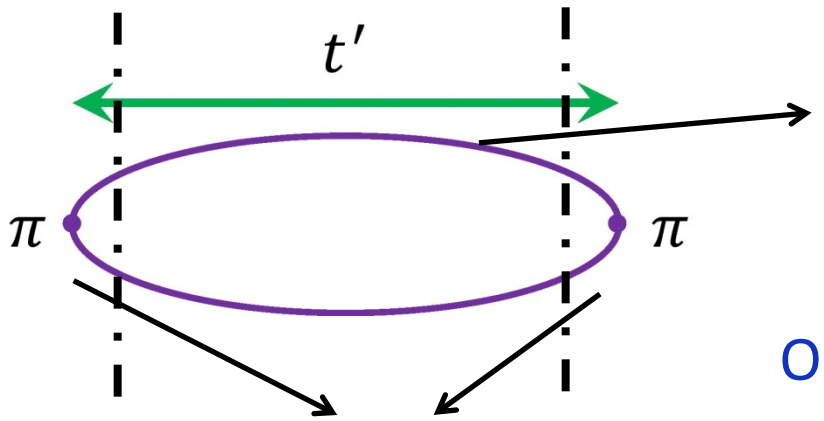
Correlator Construction: smearing of quark fields - 'distillation' with

$$\Box_{\vec{x}\vec{y}}(t) = \sum_{n=1}^{N_D} \xi_{\vec{x}}^{(n)}(t)\, \xi_{\vec{y}}^{(n)\dagger}(t)$$

→ Extraction of low lying hadron states

Meson creation operator :

$$\mathcal{O}^\dagger(\vec{p}) = \bar{\psi}_{\vec{x}} \Box_{\vec{x}\vec{y}}\, e^{-i\vec{p}\cdot\vec{y}}\, \mathbf{\Gamma}_{\vec{y}\vec{z}}\, \Box_{\vec{z}\vec{w}} \psi_{\vec{w}}$$



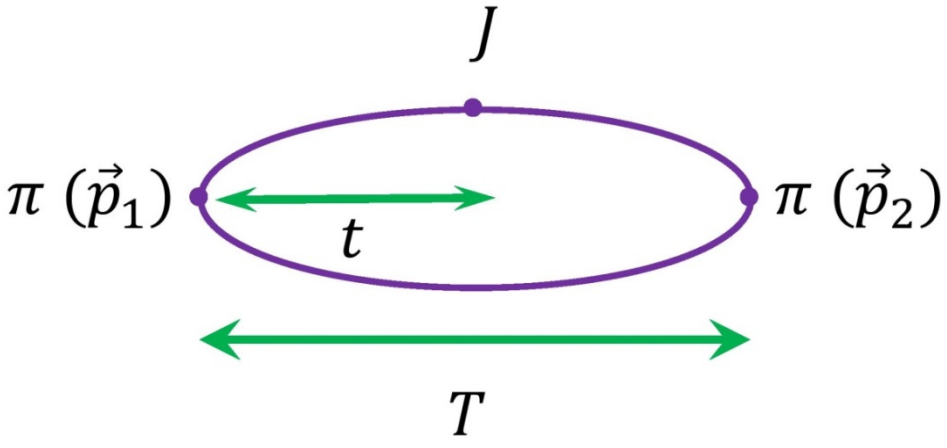

Parambulators by inverting the Dirac matrix

+

Operator construction with momentum projection

# Form factor calculation

Need three-point correlator – calculated with Weighted operators



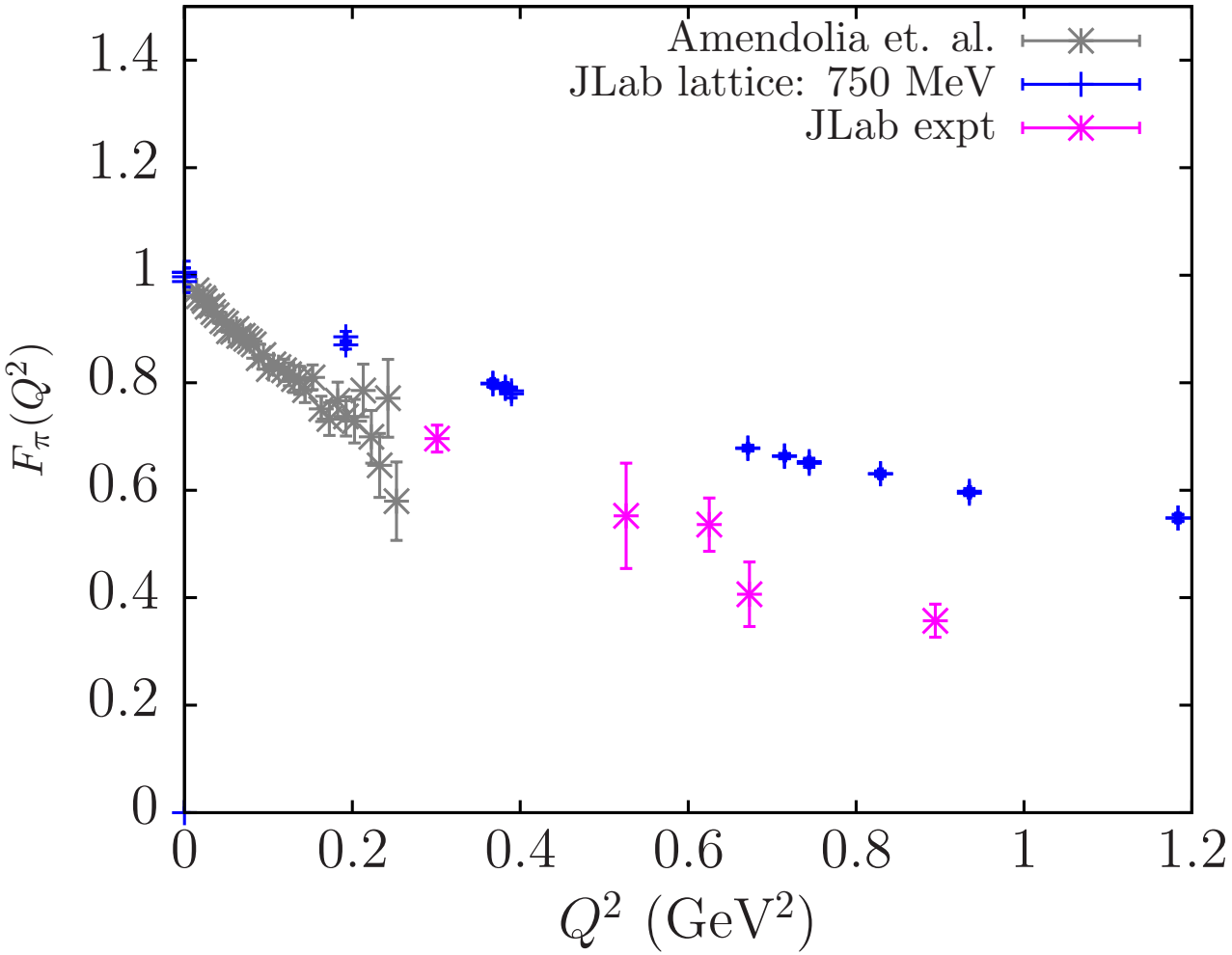

$$C_{\mathrm{f}\mu\mathrm{i}}(\Delta t, t) = \langle 0 | \mathcal{O}_{\mathrm{f}}(\Delta t)\, j_\mu(t)\, \mathcal{O}_{\mathrm{i}}^\dagger(0) | 0 \rangle$$

$$Z_V < \pi^+(p_2) | J^\mu{}_\pi(0) | \pi^+(p_1) > = e(p_1 + p_2)^\mu F_\pi(q^2)$$

Clover discretised fermion action

$Z_V$ calculated using $F_\pi(q^2 = 0) = 1$

# Pion electromagnetic form factor: up to $Q^2 = 1$ GeV$^2$




$m_\pi = 750$ MeV

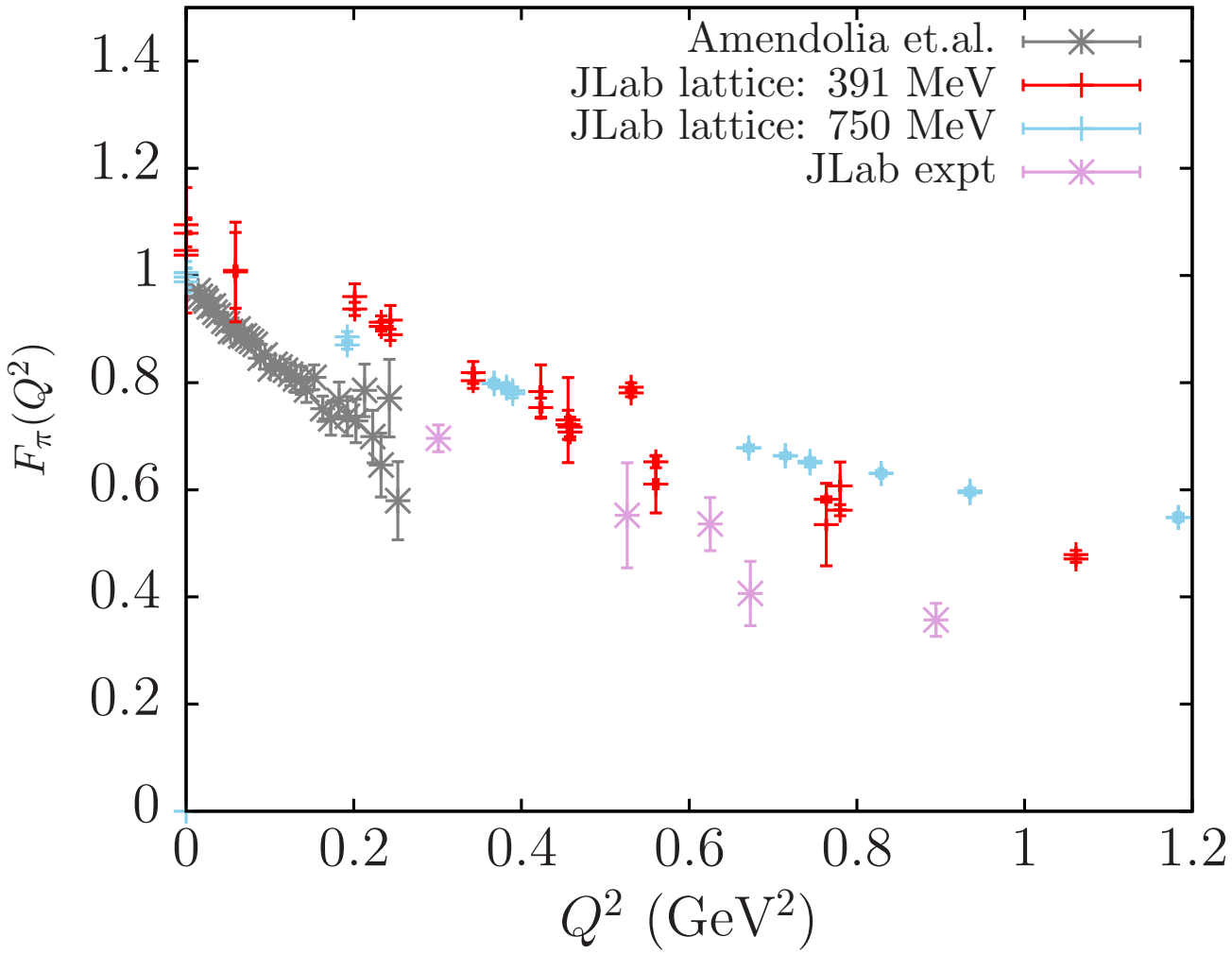
Phys.Rev. D91 (2015)

Anisotropy $\xi = 3.44$

$$(L/a_s)^3 \times (T/a_t) = 20^3 \times 128$$

$$a_s = 0.12 \text{ fm}, \quad \frac{a_s}{a_t} = 3.44$$

# Pion electromagnetic form factor: up to $Q^2 = 1$ GeV$^2$




$m_\pi = 750$ MeV

Phys.Rev. D91 (2015)

$m_\pi = 390$ MeV

(Paper in preparation)

In agreement with recent lattice result
from HPQCD
(up to 0.25 GeV$^2$)
Phys.Rev. D93 (2016)

Anisotropy

$\xi = 3.44$

$(L/a_s)^3 \times (T/a_t) = 20^3 \times 128$

$a_s = 0.12$ fm, $\dfrac{a_s}{a_t} = 3.44$

## Pion charge radius:

$$\langle r^2 \rangle \equiv -6 \frac{d}{dQ^2} F(Q^2)\Big|_{Q^2=0}$$

Parametrising $Q^2$ dependence $Q^2 < 0.3\,\mathrm{GeV}^2$

$$F_\pi(Q^2) = F(0)\,\frac{1}{1+Q^2/m^2}$$

Lightest vector meson mass

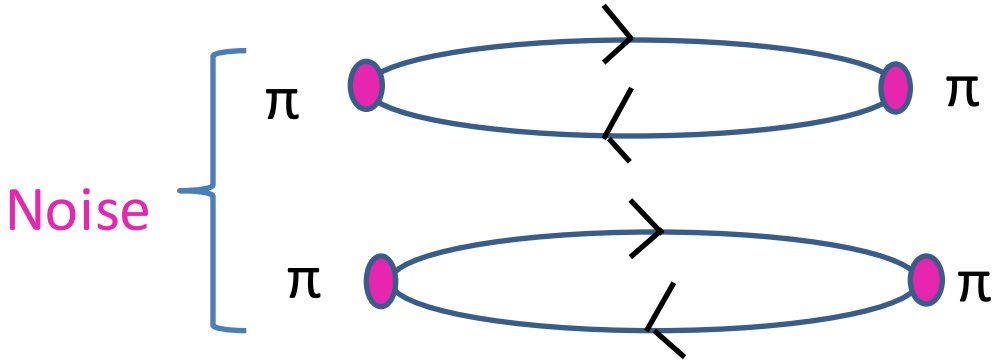
$m_\pi = 750$ MeV : 0.47(6) fm

$m_\pi = 390$ MeV : 0.55 (10) fm

# Towards higher $Q^2$

More difficult on lattice for higher momenta

Signal-to-noise ratio:




- 2-point correlators :

$$\exp[-(E_\pi(p) - 2m_\pi)t]$$

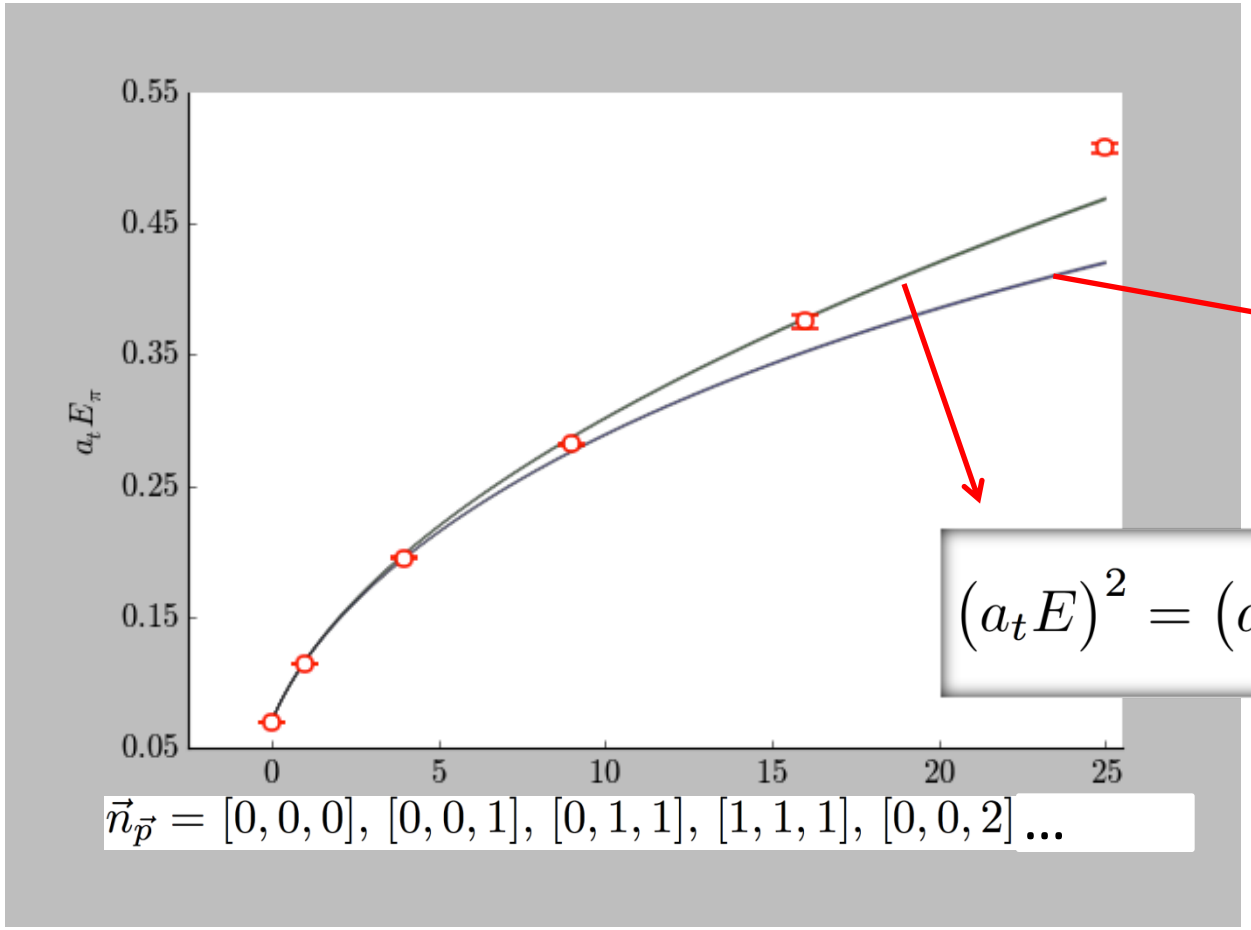
- 3-point correlators :

$$\exp[-(E_\pi(pi) + E_\pi(pf) - 2m_\pi)t/2]$$

in the middle of the plateau

Minimize energies
for a given $Q^2$
to get better signal

# Towards higher $Q^2$



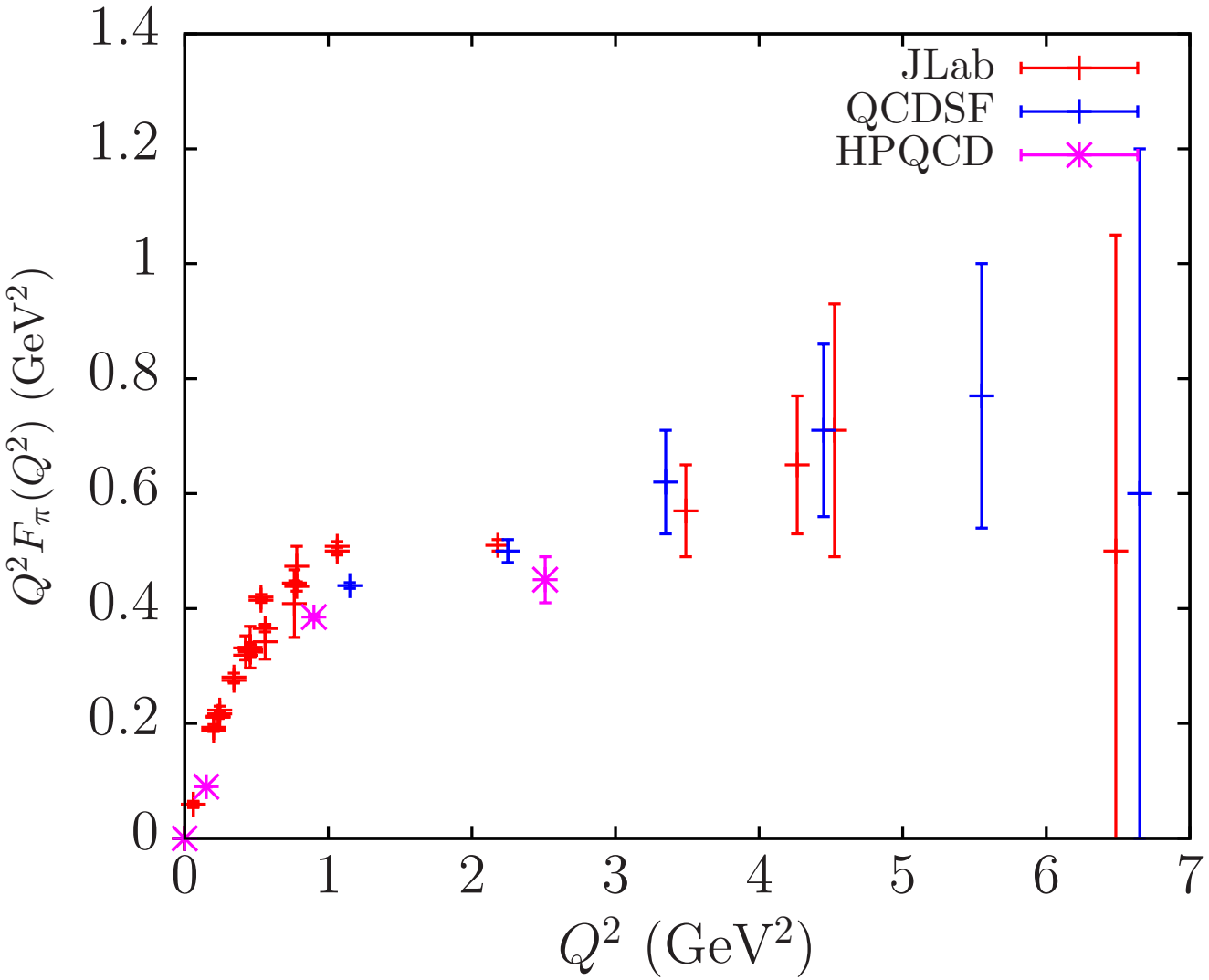

Dispersion relation:

$$E^2 = m^2 + p^2$$

$$(a_t E)^2 = (a_t m)^2 + \left(\frac{2\pi}{\xi(L/a_s)}\right)^2 |\vec{n}_{\vec{p}}|^2$$

Achieve maximum $Q^2$ by using Breit frame : $\overrightarrow{P_f} = -\overrightarrow{P_i}$

# Comparison of lattice results




Our paper in preparation

QCDSF, Phys.Rev. D96 (2017)

HPQCD, EPJ Web Conf. 175 (2018)

# Collaborators

PDF calculation

R. Edwards
C. Egerer
J. Karpie
K. Orginos
J. Qiu
D. Richards
R. Sufian

Pion form factor calculation

R. Briceno
R. Edwards
D.Richards

Thank you

# Outlook

- Tremendous improvement in last couple of years for pion/kaon structure on lattice
- Many new concepts, great progress with non-perturbative renormalisation
- Different techniques such as momentum smearing

- Need to include lighter pion masses , multiple volumes, multiple lattice spacings
- Higher momentum
- Finite volume Corrections needed
- Inclusion of higher loops in matching

- We (JLab lattice hadrons structure group) will present our first results of cross section method in lattice conference, Pseudo-pdf with dynamical quarks underway, pion/kaon form factors on multiple ensemble