D Meson Semileptonic Decay Form Factors at $q^2 = 0$

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Outline

- Current status of CKM matrix elements $|V_{cd}|, |V_{cs}|$
- $f_+^{\pi/K}(q^2=0)$ using HISQ quarks
  - Lattice setup
  - Chiral-continuum extrapolations
  - Error budgets
- Preliminary results and outlook

Current status of $|V_{cd}|$, $|V_{cs}|$

- Test of CKM matrix
  - More precise results help provide evidence or/and constraint on new physics BSM.
  - Need reduced errors from theory

	$\|V_{cd}\|$	$\|V_{cs}\|$
Semileptonic [1]	0.2140(29)(93)	0.967(7)(25)
Leptonic [2][10]*	0.2152(49)(5)(6)	1.001(16)(2)(3)

Error in red is from the theoretical uncertainty of the form factor

- Error order is experiment, lattice, structure dependent electro-magnetic

D semileptonic decay form factors

- Hadronic D semileptonic decay

$$\langle P(p) | \bar{l}\gamma_\mu c | D(p') \rangle$$

$$= f_+(q^2)\left[(p'+p)_\mu - \frac{m_D^2 - m_P^2}{q^2} q_\mu\right] + f_0(q^2)\frac{m_D^2 - m_P^2}{q^2} q_\mu \ ,$$

with $q = p' - p$

- Scalar current

$$f_0(q^2) = \frac{m_c - m_l}{m_D^2 - m_P^2}\langle P(p) | \bar{l}c | D(p') \rangle$$

- Kinematic constraint $f_0(0) = f_+(0)$

MILC N_f = 2+1+1 HISQ ensembles[3]

V	$a\ (fm)$	m_l/m_s	$N_{conf} \times N_{src}$	T_{ext}	m_c^{val}/m_c^{tun}
64^3*192	0.042	0.2	431 x 12	40	1.00
96^3*192	0.06	Phys.	866 x 6	31,39,40	1.01
48^3*144	0.06	0.2	942 x 8	34,41,48	1.11
64^3*96	0.09	Phys.	905 x 8	23,27,32	1.00
48^3*96	0.09	0.1	840 x 8	23,27,32	1.02
32^3*96	0.09	0.2	645 x 4	23,27,32	1.04
48^3*64	0.12	Phys.	942 x 4	15,18,20	0.98
32^3*64	0.12	0.1	992 x 4	15,18,20	1.02
24^3*64	0.12	0.2	1050 x 4	15,18,20	1.00
40^3*64	0.12	0.1	1018 x 8	15,18,20	1.02
24^3*64	0.12	0.1	1001 x 8	15,18,20	1.02

T_{ext} is the source-sink Euclidian time separation of the D and daughter meson

Correlators & fit functions

- Correlators
  - Wall sources and point sinks; twisted boundary conditions for nonzero quark-momentum $\vec{p} = p_i(1,1,1)$
  - Blocked data to eliminate effects from autoccorelations and stabilize fit error sizes
  - Simultaneous 2pt & 3pt fits using Bayesian technique, including various T_{ext}'s
  - Jackknife re-sampling on form factors

- Fit functions and parameters
  - Use same number of channels for normal & oscillating states, except for zero-momentum pion propagator (no oscillating states needed)
  - Include up to second excited state fits on most ensembles, or third excited state on (a few) others
  - Aligned on various ensembles $t_{min} \approx 0.36$ fm (daughter meson), 0.45 fm (D meson)

Correlators & fit functions

- Meson propagators, 3-pt correlators

$$\langle \overline{M}(t) | M(0) \rangle \approx \sum_{n=0}^{N} \Big[R_n^{+2} (e^{-E_n^+ t} + e^{-E_n^+ (T-t)}) + (-1)^t R_n^{-2} (e^{-E_n^- t} + e^{-E_n^- (T-t)}) \Big]$$

$$\doteq \sum_{n=0}^{N} \Big[R_n^{+2} e^{-E_n^+ t} + (-1)^t R_n^{-2} e^{-E_n^- t} + refl(T) \Big],\ R_n^-(M=\pi, \vec{p}=0) = 0$$

$$\langle \overline{P}(0) | \bar{l} c(t_c) | D(T_{ext}) \rangle \approx \sum_{n=0}^{N_P} \sum_{m=0}^{N_D} [V_{n,m}^{ee} R_n^+ \tilde{R}_m^+ e^{-E_n^+ t_c - \tilde{E}_m^+ (T_{ext} - t_c)} + V_{n,m}^{oe} (-1)^{t_c} R_n^- \tilde{R}_m^+ e^{-E_n^- t_c - \tilde{E}_m^+ (T_{ext} - t_c)}$$

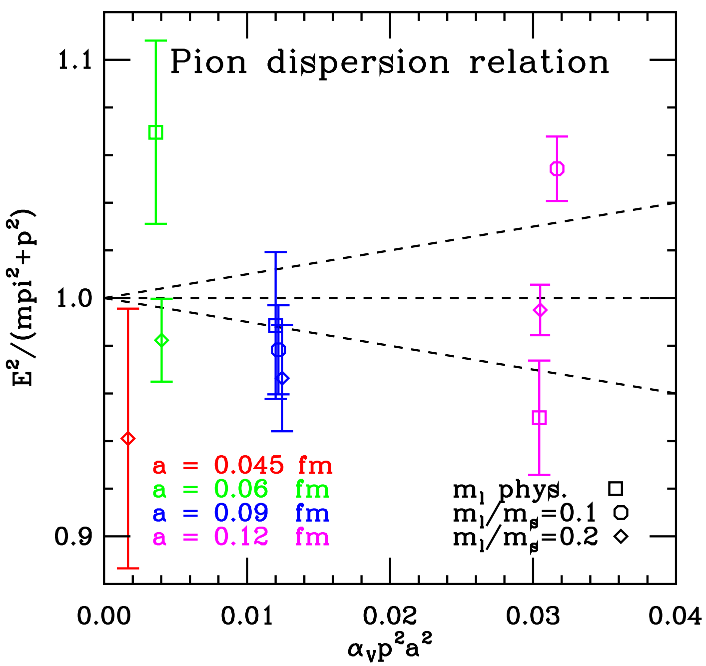
$$+ V_{n,m}^{eo} (-1)^{T_{ext} - t_c} R_n^+ \tilde{R}_m^- e^{-E_n^+ t_c - \tilde{E}_m^- (T_{ext} - t_c)} + V_{n,m}^{oo} (-1)^{T_{ext}} R_n^- \tilde{R}_m^- e^{-E_n^- t_c - \tilde{E}_m^- (T_{ext} - t_c)}$$

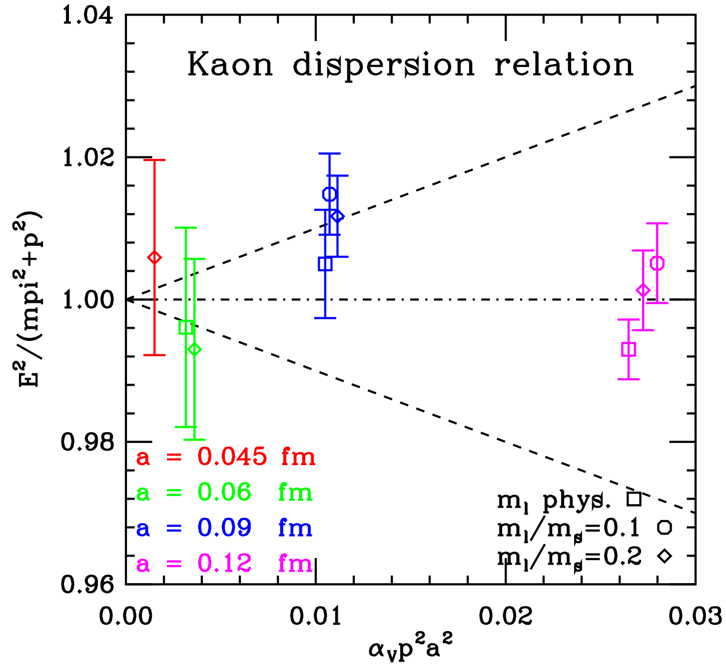
$$+ refl(T)]$$

$$f_0 = 2 V_{0,0}^{ee} \big(m_c - m_l \big) \sqrt{E_0^+ \tilde{E}_0^+} \Big/ \Big(\tilde{E}_0^{+2} - E_0^+ (\vec{p}=0)^2 \Big)$$

Energy dispersion relations

- Dispersion relation violations
  - Combined effects of lattice discretizations and statistical errors





$f_0^{\pi/K}(q^2 = 0)$ at physical point

- Chiral-continuum extrapolation
  - SU(3) hard-pion(kaon) HMrSChPT[4][5][6][7]

$$f_0^P = \frac{f_{p4s}C_0}{f_\pi}\left(1 + \delta f_{cl} + C_a\chi_{a^2} + C_l\chi_l + C_q\chi_{q^2} + C_s\chi_{sea} + C_e\chi_e \ldots\right)$$

$$\chi_{a^2} = \left(8\pi^2 f_\pi^2\right)^{-1}\overline{\Delta}(a)$$

$$\chi_l = \left(4\pi^2 f_\pi^2\right)^{-1} m_{u(d),s}^{,val}\mu(a)$$

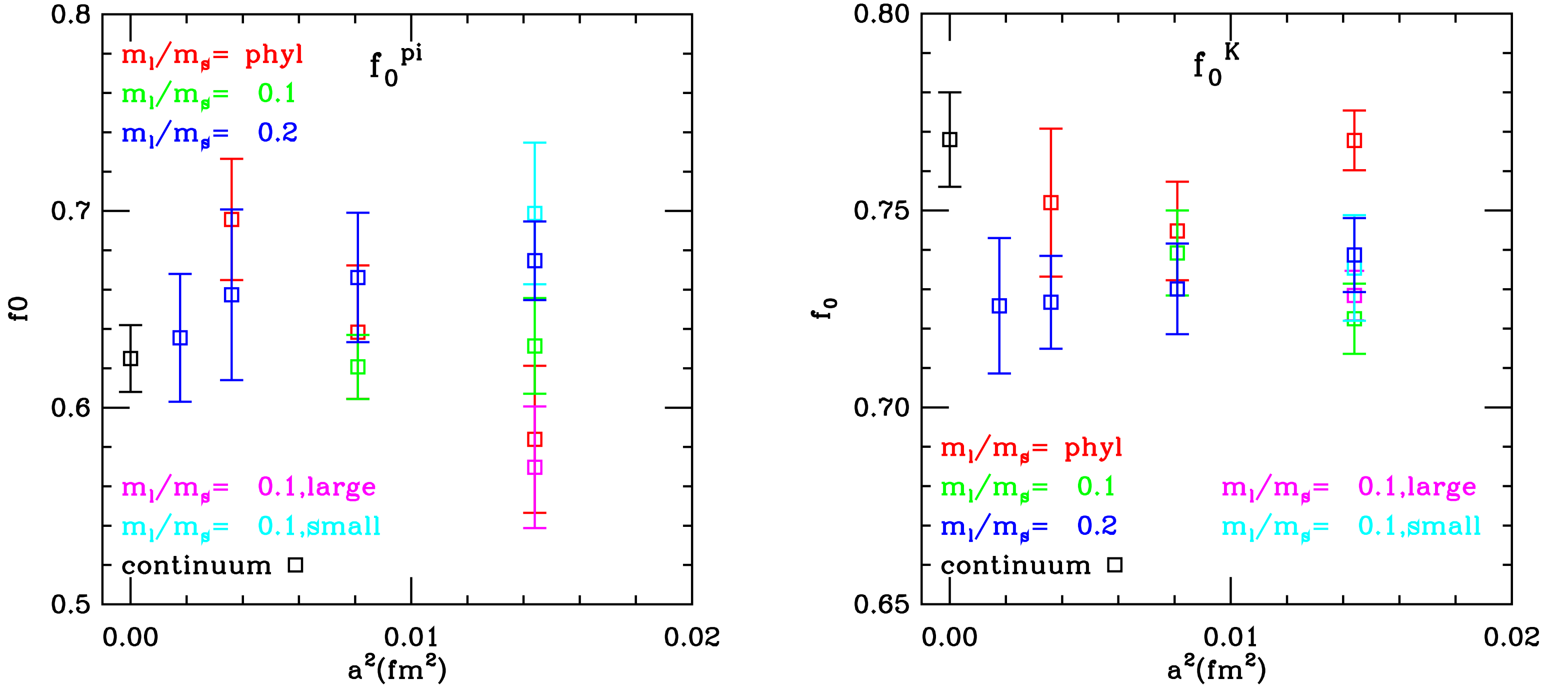
$$\chi_{q^2} = \left(8\pi^2 f_\pi^2\right)^{-1} q^2$$

$$\chi_{sea} = \left(8\pi^2 f_\pi^2\right)^{-1}\left(2m_{u(d)} + m_s^{sea}\right)\mu(a)$$

$$\chi_e = \sqrt{2}\left(4\pi f_\pi\right)^{-1} E_P$$

Others:
- d: pion/kaon energy discretization
- p: pion/kaon spatial momentum
- n: strange valence & sea quark mass difference
- t: charm quark mass mistuning
- NNLO terms

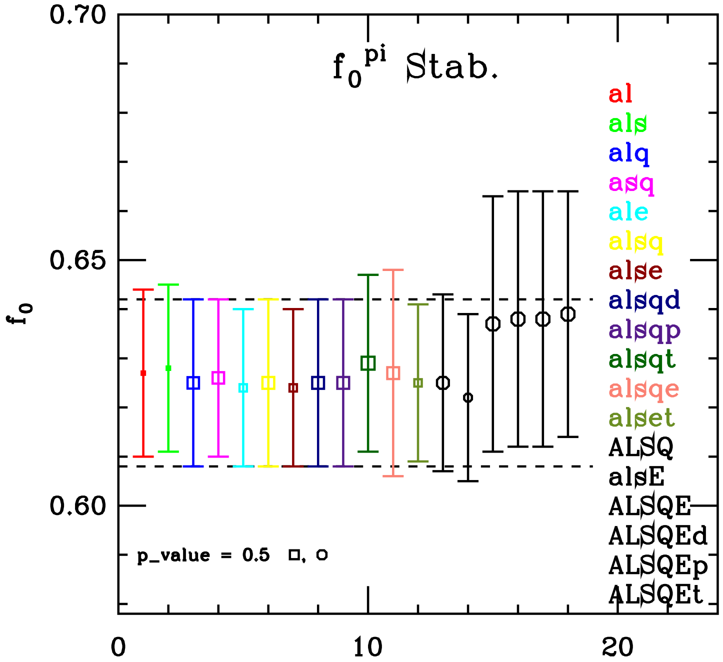
- Fitted $f_0^{pi/K}$ on each ensemble and at the continuum physical limit



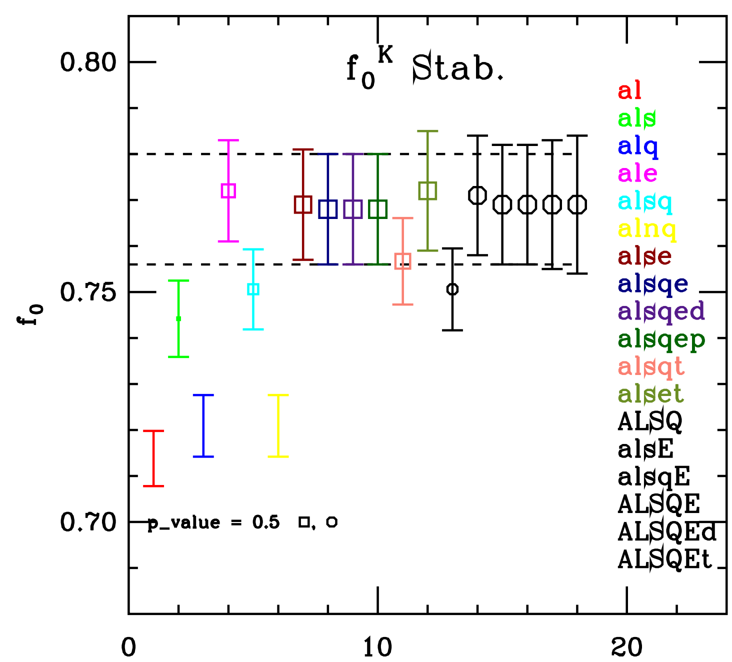
- Data on coarse ensembles with smaller and larger than average spatial volumes are averaged after FV (finite volume) corrections for an estimate of FV QCD effects

Preliminary

- Chiral-continuum fits stability



Central fit *alsq* χ^2/dof = 0.75



Central fit *alsqe* χ^2/dof = 0.26

- Include up to NNLO analytic terms (letters in capital) for a systematic error estimate
- Better stabilities with q^2 term in f^{pi} than f^K
- Stability errors are estimated from comparing central values of various fits to central fits

Systematic errors

- 1-loop partial quenching effects
  - Calculate using staggered ChPT, compare between $m_s = m_s^{val}, m_s^{sea}$
- Lattice discretizations at $O(\alpha_s^2 a^2)$
  - Light quark momentum, light quark masses
  - Exclude charm quark mass discretizations
- Nonequilibrated topological charge corrections[8] at $a \approx 0.042\,fm$
  - $f_0 = f_0(\theta) - f_0''(0)\left(2\chi_T V\right)^{-1}\left(1 - \left\langle Q^2 \right\rangle (\chi_T V)^{-1}\right)$

    $f_0''(0) = -1/4\left(m_l m_s / m_y\right)^2 \left(m_l + 2m_s\right)^{-2}$

    $m_{l,s}$ are sea quark masses; m_y is the active light valence quark mass; χ_T is the topological susceptibility; V is the lattice spatial volume
- Others

Preliminary

Error budgets, results

Systematic errors	f_0^π (pct.)	f_0^K (pct.)
Fit stab.	2.15	1.47
PQ effects	N/A	0.12
Lattice scale	0.05	0.03
Finite volume QCD[9]	0.04	<0.04
Nonequil. topological charge	0.005	<0.005
Total	2.15	1.48

- Lattice results

	$f_+^\pi(q^2=0)$	$f_+^K(q^2=0)$
This work	0.625(17)(13)	0.768(12)(11)
Average[2] *	0.637(20)	0.745(15)

* Averaged results from recent LQCD calculations

Preliminary

Estimate of $|V_{cd}|$, $|V_{cs}|$, outlook

- $|V_{cd}|$, $|V_{cs}|$, with inputs from recent experiments[1]

$\lvert V_{cd}\rvert$	$\lvert V_{cs}\rvert$
0.2280(31)(78)	0.941(7)(20)

- CKM matrix second row unitarity $|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2$

This work	0.939(44)
Semileptonic, Ave. [2]	1.005(53)
Leptonic, FNAL/MILC [2][10]	1.050(32)

- Outlook
  - D semileptonic vector form factors at nonzero q^2

References

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Thank you!

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Fit parameters

Ensembles a, ml/ms	Block size D2pi, D2K	#P-p_e-chnl / #D-p_e-chnl	Fits t_{min} * pi/D, K/D	Thin 2pt P/D D2pi, D2K
0.042, 0.2	4, 4	3/3, 3/3	8/10, 8/10	5/5, 5/5
0.06, phys.	3, 3	3/3, 4/4	6/8, 6/7	/
0.06, 0.2	4, 3	3/3, 3/3	6/8, 6/7	5/3, 5/3
0.09, phys.	3, 3	3/3, 3/4	4/5, 4/5	/
0.09, 0.1	3, 3	3/3, 3/3	4/5, 4/5	/
0.09, 0.2	3, 3	3/3, 3/3	4/5, 4/5	/
0.12, phys.	3, 2	3/3, 3/3	3/4, 3/4	/
0.12, 0.1	3, 4	3/3, 3/3	3/4, 3/4	/
0.12, 0.2	3, 3	3/3, 3/3	3/4, 3/4	/
0.12, 0.1, V_s	3, 4	3/3, 3/3	3/4, 3/4	/
0.12, 0.1, V_l	3, 4	3/3, 3/3	3/4, 3/4	/

* 3pt t_{min} adjusted to also exclude data with large statistical errors