Lattice 2018

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Confront lattice energy levels with chiral effective field theory

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Outline:

1. Background & Introduction

2. ChPT amplitudes and Finite-volume effects

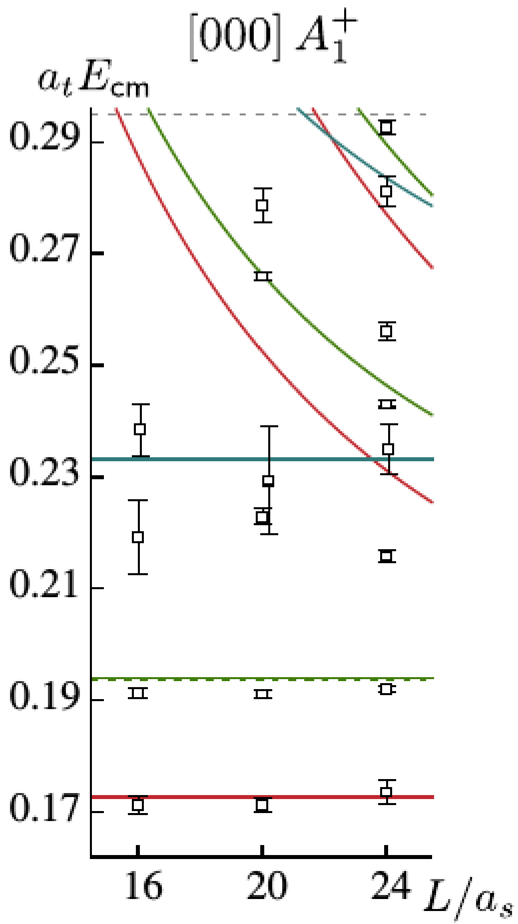
3. Results and Discussions

4. Summary

Background & Introduction

Remarkable progress in lattice simulation: finite-volume spectra in meson-meson scattering

Eigenenergies in finite box



An example:
pi-eta, K-Kbar, pi-eta'
coupled-channel scattering

[Dudek,Edwards,Wilson, PRD'16]

Length of the box

Conventional approach to relate the finite-volume spectra with infinite-volume amplitudes: Luscher function + K matrix

$$\det[\mathbf{1} + i\rho \cdot \mathbf{t} \cdot (\mathbf{1} + i\mathcal{M})] = 0$$

Kinematical factor

K matrix: polynomial + pole terms

Luscher's finite-volume functions (complex objects)

- **Free parameters in K matrix are determined by the finite-volume spectra. Then one can determine amplitudes in infinite volume.**

- **K matrix does not automatically respect the QCD symmetries, such as the chiral symmetry. It could be problematic for chiral extrapolation.**

Our approach:

Step 1: Put chiral perturbation theory (ChPT) in finite volume.

Step 2: The free parameters in ChPT, which are indepdent of quark masses and volumes, are fitted to the finite-volume energy levels obtained at (un)physical quark masses.

Step 3: Perform the chiral extrapolation and give the predictions in infinite volume with physical quark masses, including phase shifts, inelasticities, resonance poles, etc.

I will mainly focus on the pi-eta, K-Kbar and pi-eta' coupled-channel S-wave scattering and a_0(980) in this talk.

Preliminary results for the D-pi, D-eta and Ds-Kbar scattering will be also presented.

Status of lowest QCD scalars

$f_0(500)/\sigma$: precise $\pi\pi$ scattering data + dispersive technique + chiral EFT. Well determined pole positions !

[Xiao,Zheng, NPA01] [Caprini,Colangelo,Leutwyler,PRL06] [Garcia-Martin, et al., PRD11]

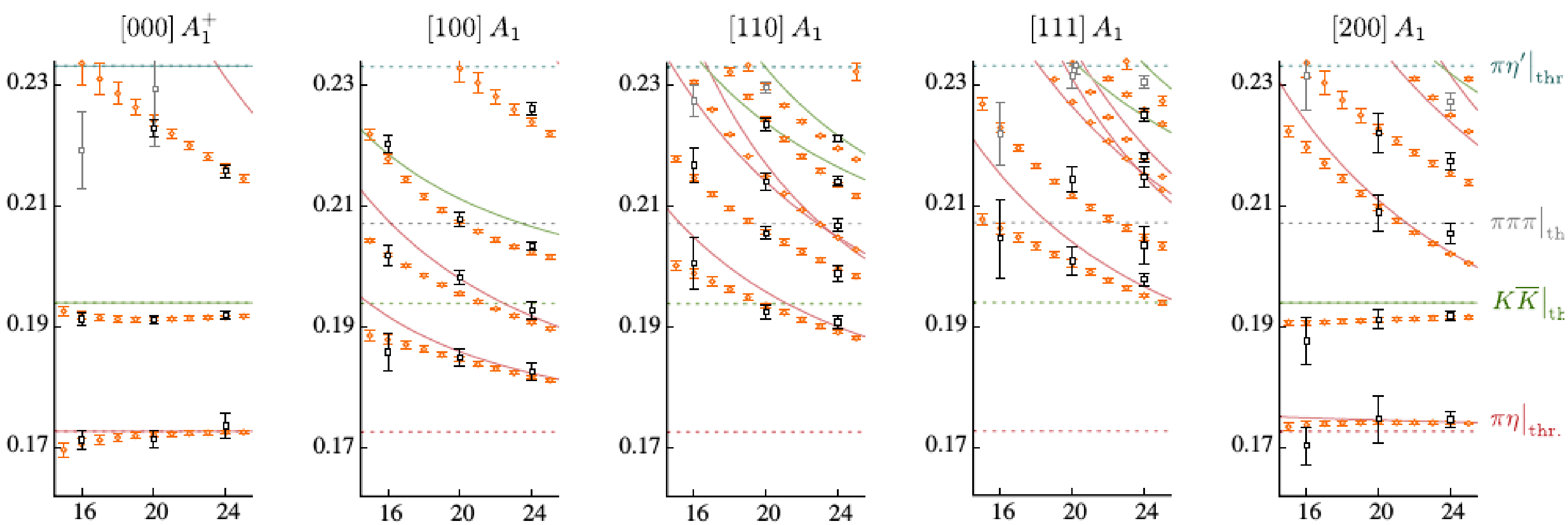
$f_0(980)$: $\pi\pi$ scattering data + (dispersive technique,Unitarized chiral EFT). Confirmed pole in the complex energy plane !
[Garcia-Martin, et al., PRD11] [Oller, Oset, Pelaez, PRD99]

$K^*_0(800)/\kappa$: πK scattering data+ (dispersive technique,Unitarized chiral EFT). Confirmed pole in the complex energy plane !
[Zheng, Zhou, et al.,] [Descotes-Genon, Moussallam, EPJC06] [Pelaez, Rodas, PRD16]

$a_0(980)$: Absence of the $\pi\eta$ scattering data.
Still controversial: resonance pole or cusp effect ?
[Oller, Oset, PRD99] [Albaladejo, Moussallam, EPJC16] [Guo, Oller, PRD11]

Alternative way: Lattice QCD

The first lattice calculation for πη scattering: [Dudek,Edwards,Wilson, PRD16]



- Many finite-volume energy levels are obtained in CM & moving frames
- Lattice box is big enough ($m_\pi L$ > 3.8)
- Only one large m_π is used in the simulation (m_π = 391 MeV)

Unitarized ChPT and its finite-volume effects

Three relevant coupled channels: $\pi\eta$, K-Kbar, $\pi\eta'$

In this case, it is essential to generalize from SU(3) to U(3) ChPT

$$\begin{pmatrix} \frac{\pi^0}{\sqrt{2}}+\frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}}+\frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 \end{pmatrix} \rightarrow \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0+\frac{1}{\sqrt{6}}\eta_8+\frac{1}{\sqrt{3}}\eta_0 & \pi^+ & K^+ \\ \pi^- & \frac{-1}{\sqrt{2}}\pi^0+\frac{1}{\sqrt{6}}\eta_8+\frac{1}{\sqrt{3}}\eta_0 & K^0 \\ K^- & \bar{K}^0 & \frac{-2}{\sqrt{6}}\eta_8+\frac{1}{\sqrt{3}}\eta_0 \end{pmatrix}$$

Leading order:

$$\mathcal{L}_2 = \frac{F^2}{4}\langle u_\mu u^\mu\rangle + \frac{F^2}{4}\langle \chi_+\rangle + \boxed{\frac{F^2}{3}M_0^2 \ln^2 \det u}$$

Leads to a massive η_0

$$\begin{aligned} \eta_8 &= c_\theta \overline{\eta} + s_\theta \overline{\eta}' \,, \\ \eta_0 &= -s_\theta \overline{\eta} + c_\theta \overline{\eta}' \,, \end{aligned}$$

$$\sin\theta = -\left(\sqrt{1+\frac{\left(3M_0^2-2\Delta^2+\sqrt{9M_0^4-12M_0^2\Delta^2+36\Delta^4}\right)^2}{32\Delta^4}}\right)^{-1} \qquad \Delta^2 = \overline{m}_K^2 - \overline{m}_\pi^2$$

Instead of using higher order local counterterms, resonance saturations are assumed in our study.

$$\mathcal{L}_S = c_d \langle\, S_8 u_\mu u^\mu \,\rangle + c_m \langle\, S_8 \chi_+ \,\rangle + \widetilde{c}_d S_1 \langle\, u_\mu u^\mu \,\rangle + \widetilde{c}_m S_1 \langle\, \chi_+ \,\rangle$$

$$\mathcal{L}_V = \frac{iG_V}{2\sqrt{2}} \langle V_{\mu\nu} [u^\mu, u^\nu] \rangle \,,$$

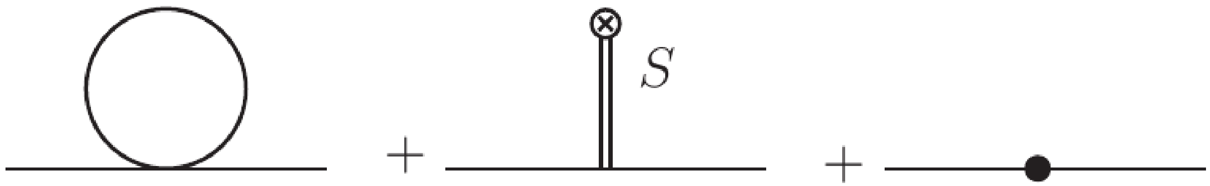
[Ecker, Gasser, Pich, de Rafael, NPB'89]

One important local U(3) operator is also considered:

$$-\Lambda_2 \frac{F^2}{12} \langle\, U^+ \chi - \chi^+ U \,\rangle \ln \det u^2$$

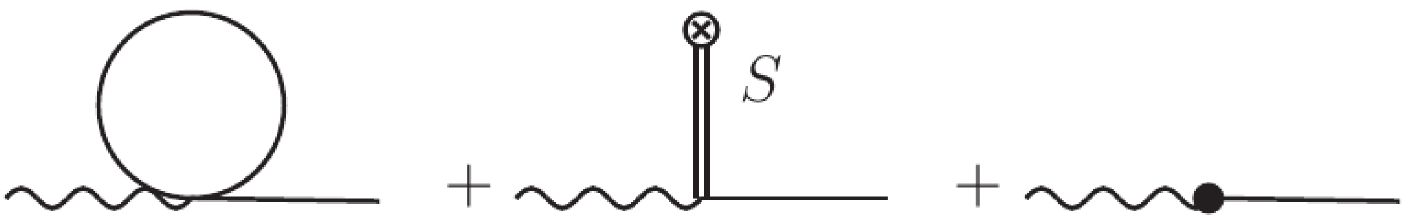
Meson-meson scattering: πη → πη, πη → KKbar, πη → πη' ......

Self energy :



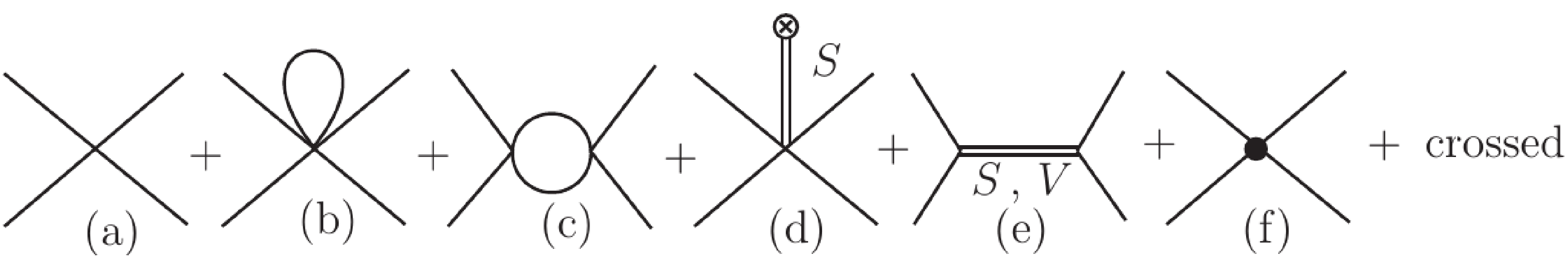
m_π, Z_π, …

Goldstone decay constant :



F_π

Scattering amplitude :



Leading order amplitudes:

$$
\begin{aligned}
T_{J=0}^{I=1,\pi\eta\to\pi\eta}(s)^{(2)} &= \frac{(c_\theta-\sqrt{2}s_\theta)^2 m_\pi^2}{3F_\pi^2}\,,\\
T_{J=0}^{I=1,\pi\eta\to K\bar{K}}(s)^{(2)} &= \frac{c_\theta(3m_\eta^2+8m_K^2+m_\pi^2-9s)+2\sqrt{2}s_\theta(2m_K^2+m_\pi^2)}{6\sqrt{6}F_\pi^2}\,,\\
T_{J=0}^{I=1,\pi\eta\to\pi\eta'}(s)^{(2)} &= \frac{(\sqrt{2}c_\theta^2-c_\theta s_\theta-\sqrt{2}s_\theta^2)m_\pi^2}{3F_\pi^2}\,,\\
T_{J=0}^{I=1,K\bar{K}\to K\bar{K}}(s)^{(2)} &= \frac{s}{4F_\pi^2}\,,\\
T_{J=0}^{I=1,K\bar{K}\to\pi\eta'}(s)^{(2)} &= \frac{s_\theta(3m_{\eta'}^2+8m_K^2+m_\pi^2-9s)-2\sqrt{2}c_\theta(2m_K^2+m_\pi^2)}{6\sqrt{6}F_\pi^2}\,,\\
T_{J=0}^{I=1,\pi\eta'\to\pi\eta'}(s)^{(2)} &= \frac{(\sqrt{2}c_\theta+s_\theta)^2 m_\pi^2}{3F_\pi^2}\,,
\end{aligned}
$$

Unitarization: Algebraic approximation of N/D (a variant version of K-matrix)

$$T_J(s) = \frac{N(s)}{1 + G(s)\ N(s)}$$

- **The s-channel unitariy is exact. The crossed-channel dyanmics is included in a perturbative manner.**

- **Unitarity condition:** $$\mathrm{Im}G(s) = -\rho(s)$$

$$G(s) = \boxed{a^{SL}(s_0)} - \frac{s - s_0}{\pi} \int_{4m^2}^{\infty} \frac{\rho(s')}{(s' - s)(s' - s_0)} ds'$$

- ***N*(*s*): given by the partial wave chiral amplitudes**

$$\mathcal{V}^{(S,I)}_{J,D_1\phi_1\to D_2\phi_2}(s) = \frac{1}{2}\int_{-1}^{+1} \mathrm{d}\cos\varphi P_J(\cos\varphi) V^{(S,I)}_{D_1\phi_1\to D_2\phi_2}(s, t(s,\cos\varphi))$$

Finite-volume effects

Two types of finite volume dependence of scattering amplitudes:

- **Exponentially suppressed type $\propto exp(-m_p L)$: *s, t, u* channels**
- **Power suppressed type $\propto 1/L^3$: only *s* channel**

We ignore the exponentially suppressed terms, indicating that finite-volume effects only enter through s channel.

$$T_J(s) = \frac{N(s)}{1 + G(s)\ N(s)}$$

I.e. We only consider the finite-volume corrections for G(s).

$$G(s) = i\int \frac{\mathrm{d}^4q}{(2\pi)^4}\frac{1}{(q^2-m_1^2+i\epsilon)[(P-q)^2-m_2^2+i\epsilon]}\,, \qquad s\equiv P^2$$

Sharp momentum cutoff to regularize G(s)

$$G(s)^{\rm cutoff} = \int^{|\vec{q}|<q_{\rm max}} \frac{\mathrm{d}^3\vec{q}}{(2\pi)^3} I(|\vec{q}|)\,, \quad \begin{aligned} I(|\vec{q}|) &= \frac{w_1+w_2}{2w_1w_2\,[E^2-(w_1+w_2)^2]}\,, \\ w_i &= \sqrt{|\vec{q}|^2+m_i^2}\,, \quad s=E^2 \end{aligned}$$

G(s) in a finite box of length L with periodic boundary condition

$$\widetilde{G} = \frac{1}{L^3}\sum_{\vec{n}}^{|\vec{q}|<q_{\rm max}} I(|\vec{q}|)\,, \qquad \vec{q} = \frac{2\pi}{L}\vec{n}, \qquad \vec{n}\in\mathbb{Z}^3$$

Finite-volume correction ΔG

[Doring, Meissner, Oset, Rusetsky, EPJA11]

$$\Delta G = \widetilde{G} - G^{\rm cutoff}$$

$$= \left\{\frac{1}{L^3}\sum_{\boldsymbol{q}}^{|\boldsymbol{q}|<q_{\rm max}} - \int^{|\boldsymbol{q}|<q_{\rm max}}\frac{\mathrm{d}^3\boldsymbol{q}}{(2\pi)^3}\right\}\frac{1}{2\omega_1(\boldsymbol{q})\,\omega_2(\boldsymbol{q})}\,\frac{\omega_1(\boldsymbol{q})+\omega_2(\boldsymbol{q})}{E^2-(\omega_1(\boldsymbol{q})+\omega_2(\boldsymbol{q}))^2}$$

Finite-volume effects in the moving frames

Lorentz invariance is lost in finite box. One needs to work out the explicit form of the loops when boosting from one frame to another.

transforming $\vec{q}_{i=1,2}$ to $\boxed{\vec{q}^{\,*}_{i=1,2}}$ $\longrightarrow$ **CM quantities**

$$\vec{q}^{\,*}_i = \vec{q}_i + \left[\left(\frac{P^0}{E} - 1\right)\frac{\vec{q}_i \cdot \vec{P}}{|\vec{P}|^2} - \frac{q_i^0}{E}\right]\vec{P}$$

moving frame with total four-momentum $P^\mu = (P^0, \vec{P})$ $\quad s = E^2 = (P^0)^2 - |\vec{P}|^2$

Impose on-shell condition $\quad q_i^{*\,0} = \sqrt{|\vec{q}^{\,*}_i|^2 + m_i^2}$

$$q_i^0 = \frac{q_i^{*\,0} E + \vec{q}_i \cdot \vec{P}}{P^0} \longrightarrow q_i^0 = \sqrt{|\vec{q}_i|^2 + m_i^2}$$

G function in the moving frame

$$\int^{|\vec{q}_1|^* < q_{\max}} \frac{d^3\vec{q}^{\,*}_1}{(2\pi)^3} I(|\vec{q}^{\,*}_1|) \Longrightarrow \widetilde{G}^{\rm MV} = \frac{E}{P^0 L^3} \sum_{\vec{q}_1}^{|\vec{q}^{\,*}_1| < q_{\max}} I(|\vec{q}^{\,*}_1(\vec{q}_1)|) \qquad \vec{q}_1 = \frac{2\pi}{L}\vec{n}\,, \quad \vec{n} \in \mathbb{Z}^3\,, \qquad \vec{P} = \frac{2\pi}{L}\vec{N}\,, \quad \vec{N} \in \mathbb{Z}^3$$

Finite-volume correction $\Delta G^{\rm MV}$: $\quad \Delta G^{\rm MV} = \widetilde{G}^{\rm MV} - G^{\rm cutoff}$

Mixing of different partial waves in finite volume

The mixing is absent in the infinite volume:

$$\int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta d\theta Y_{\ell m}(\theta,\phi) Y^*_{\ell' m'}(\theta,\phi) = \delta_{\ell\ell'}\delta_{mm'}$$

The mixing appears in finite-volume case, due to the absence of the general orthogonal conditions.

Finite-volume correction to G function:

$$\Delta G^{\rm MV}_{\ell m} = \widetilde{G}^{\rm MV}_{\ell m} - G^{\rm cutoff}$$

$$\widetilde{G}^{\rm MV}_{\ell m} = \sqrt{\frac{4\pi}{2\ell+1}}\frac{1}{L^3}\frac{E}{P^0}\sum_{\vec{n}}^{|\vec{q}^*|<q_{\rm max}} \left(\frac{|\vec{q}^*|}{|\vec{q}^{\rm on*}|}\right)^{\ell} Y_{\ell m}(\hat{q}^*)\, I(|\vec{q}^*|)$$

Final expression for the G function:

$$\widetilde{G}_{\ell m} = G^{\rm Infinite\ volume} + \Delta G^{\rm MV}_{\ell m}$$

To determine the energy levels in different frames with only S and P waves:

A_1^+ (0,0,0) : $\det[I + N_0(s)\cdot \widetilde{G}_{00}] = 0$

T_1^- (0,0,0): $\det[I + N_1(s)\cdot (\widetilde{G}_{00} + 2\widetilde{G}_{20})] = 0$

A_1 (0,0,1): $\det[I + N_{0,1}\cdot \mathcal{M}^{A_1}_{0,1}] = 0\,,\quad N_{0,1} = \begin{pmatrix} N_0 & 0 \\ 0 & N_1 \end{pmatrix},\quad \mathcal{M}^{A_1}_{0,1} = \begin{pmatrix} \widetilde{G}_{00} & i\sqrt{3}\widetilde{G}_{10} \\ -i\sqrt{3}\widetilde{G}_{10} & \widetilde{G}_{00} + 2\widetilde{G}_{20} \end{pmatrix}$

……

Results and Discussions

for pi-eta, KKbar, pi-eta' scattering

Fits to lattice finite-volume energy levels

[Dudek,Edwards,Wilson, PRD16]

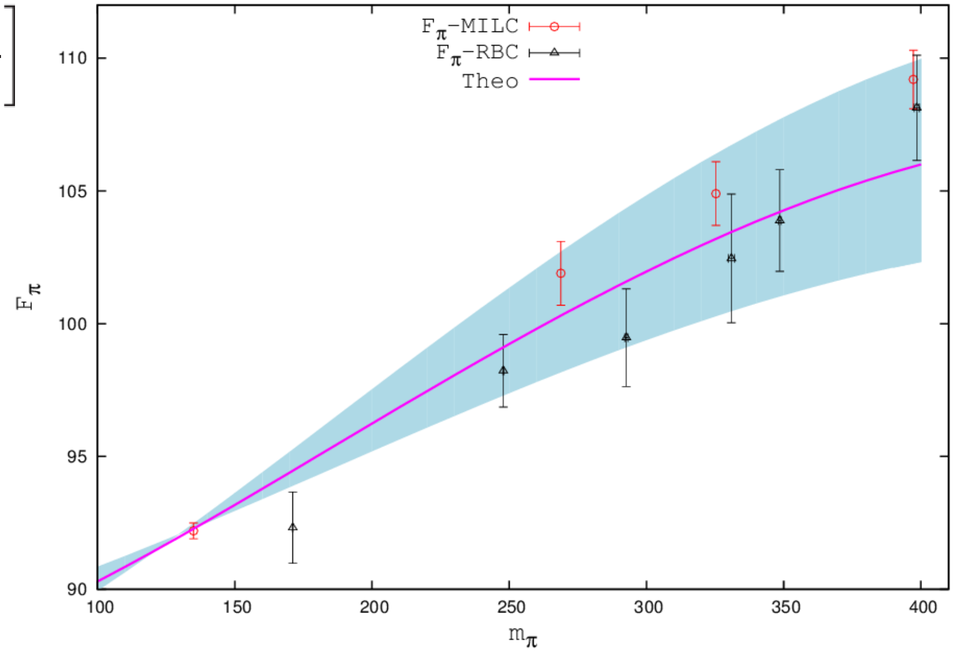
$$m_\pi = 391.3\pm0.7\ \text{MeV}\ ,\ \ m_K = 549.5\pm0.5\ \text{MeV}\ ,\ \ m_\eta = 587.2\pm1.1\ \text{MeV}\ ,\ \ m_{\eta'} = 929.8\pm5.7\ \text{MeV}$$

Our estimate of the leading order η-η' mixing angle at unphysical masses

$$\theta = (-10.0 \pm 0.1)^\circ \ \ (\ \theta^{\text{phys}} = -16.2^\circ \ )$$

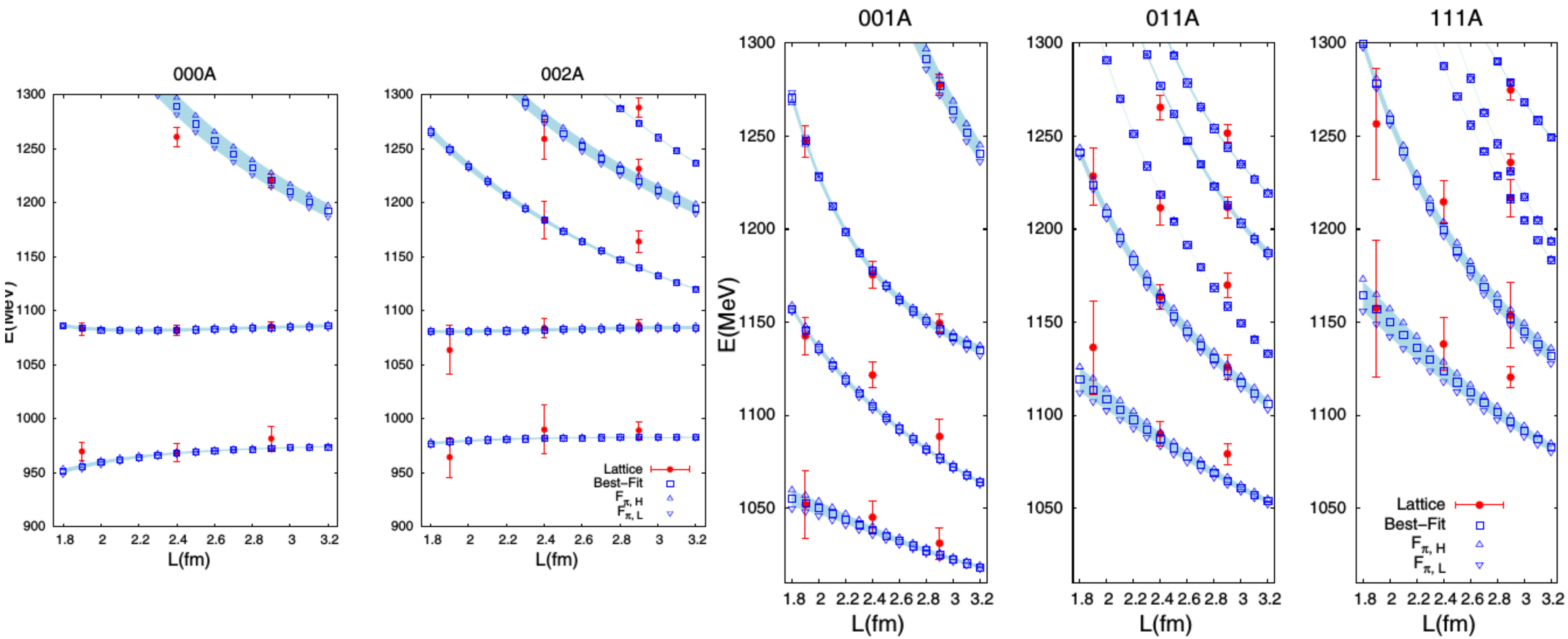
We also need to estimate F_π at the unphysical meson masses.

$$F_\pi = F\Big\{1 - \frac{1}{16\pi^2F^2}\Big[m_\pi^2 \ln\frac{m_\pi^2}{\mu^2} + \frac{m_K^2}{2}\ln\frac{m_K^2}{\mu^2}\Big] + \Big[\frac{4\widetilde{c}_d\,\widetilde{c}_m(m_\pi^2+2m_K^2)}{F^2M_{S_1}^2} - \frac{8c_d\,c_m\,(m_K^2-m_\pi^2)}{3F^2M_{S_8}^2}\Big]\Big\}$$



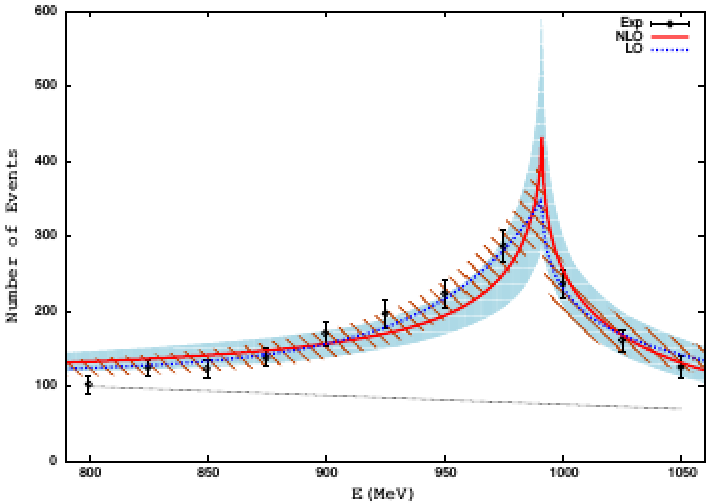
Leading order Fit (Only LO amplitudes are included in the N(s) function.)

[ZHG,Liu,Meissner,Oller,Rusetsky, PRD'17]

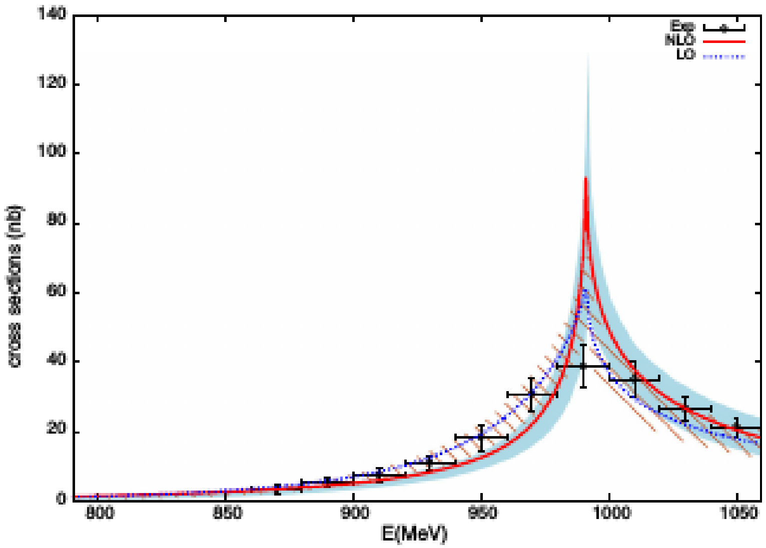


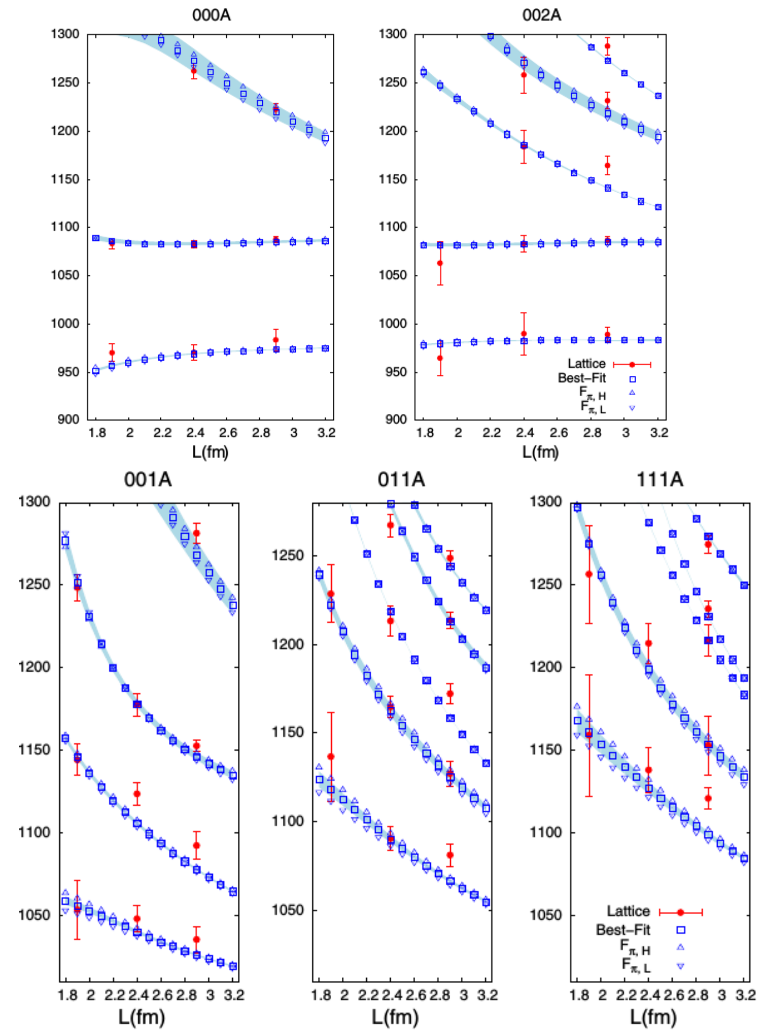
Remark: there is only one free parameter in the fits, i.e. the common subtraction constant !

$$\frac{dN_{\pi\eta}}{dE_{\pi\eta}} = q_{\pi\eta}|c_1 D^{-1}(s)_{\pi\eta\to\pi\eta} + c_2 D^{-1}(s)_{\pi\eta\to K\bar{K}}|^2$$



$$\sigma(s) = \frac{\alpha^2 q_{\pi\eta}}{2s^{3/2}}|c_1' D^{-1}(s)_{\pi\eta\to\pi\eta} + c_2' D^{-1}(s)_{\pi\eta\to K\bar{K}}|^2$$



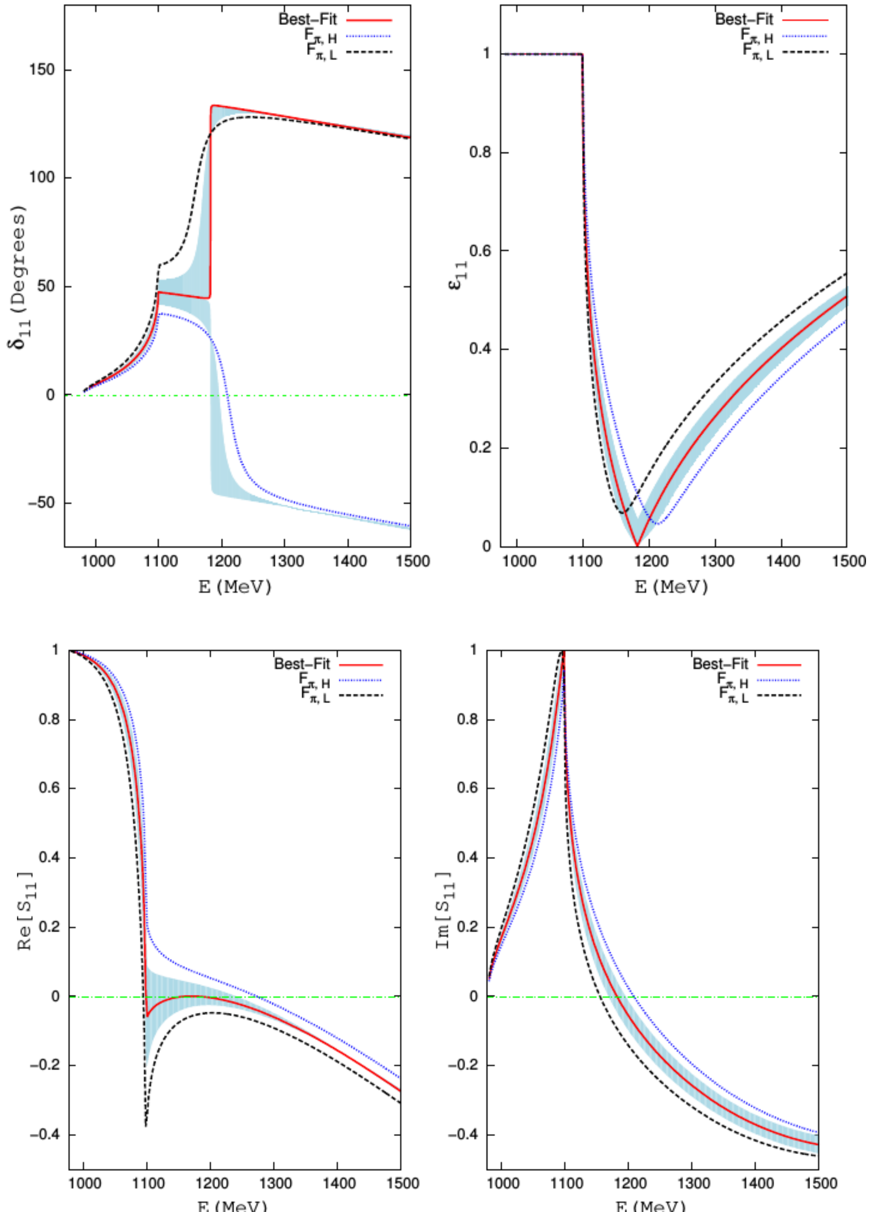


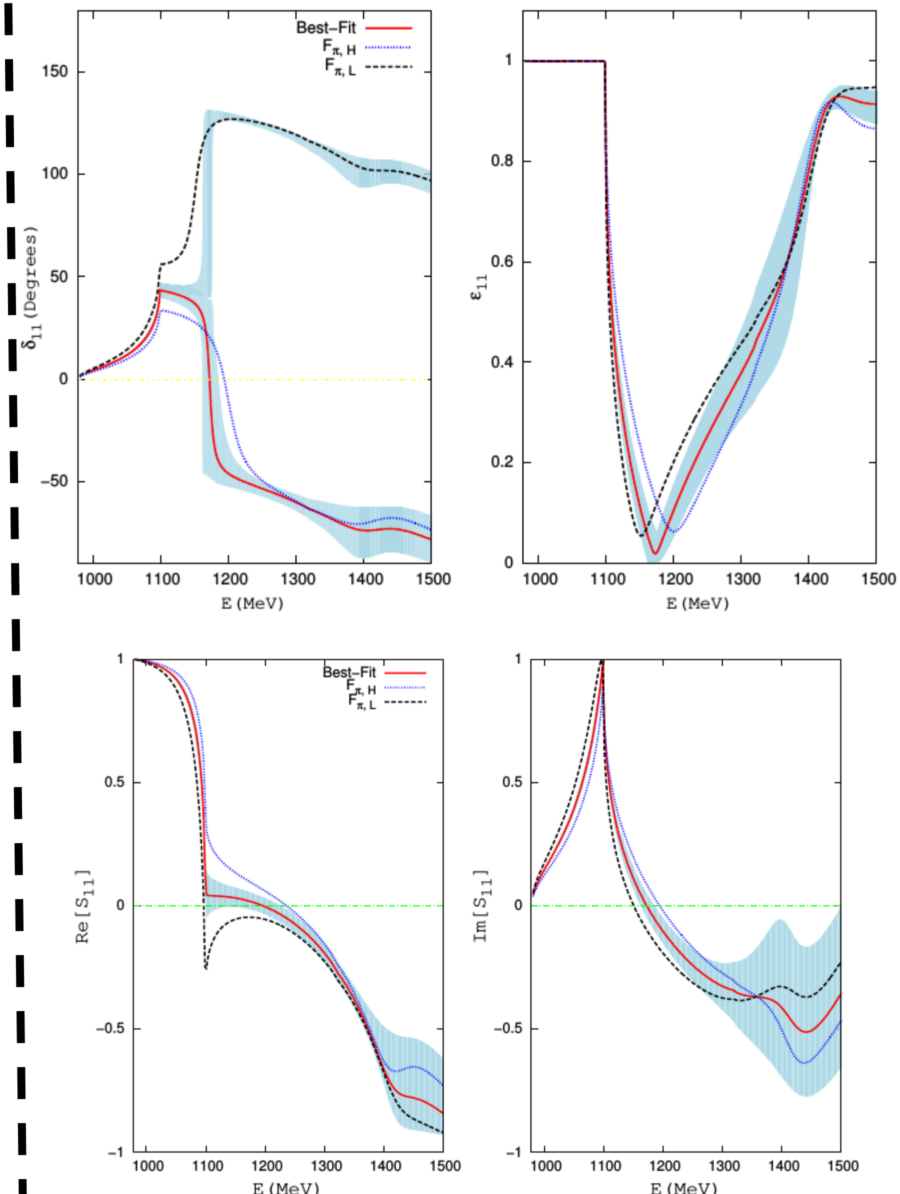
Next-to-Leading order Fit (Both loops and resonance exchanges are included in the N(s) function.) **Similar fit quality from the two cases.**

Phase shifts and inelasticities at unphysical meson masses

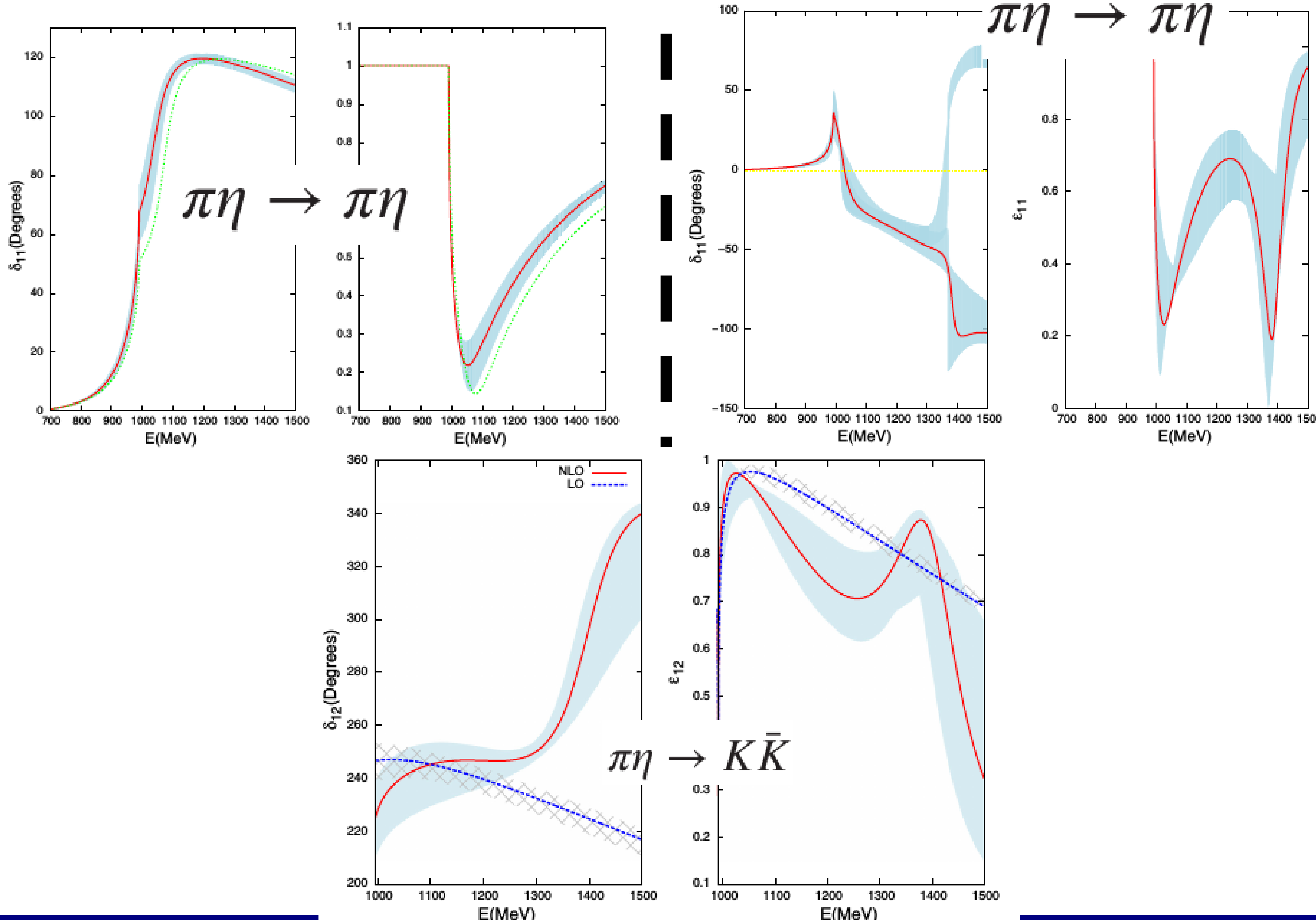
$$S = 1 + 2i\sqrt{\rho(s)} \cdot \mathcal{T}(s) \cdot \sqrt{\rho(s)}$$

$$S_{kk} = \varepsilon_{kk} e^{2i\delta_{kk}}\,, \qquad S_{kl} = i\varepsilon_{kl} e^{i\delta_{kl}}$$





Phase shifts and inelasticities at physical meson masses



Pole positions and residues at physical meson masses

Resonances are uniquely characterized by their poles and residues in the complex energy plane.

Resonance	RS	Mass (MeV)	Width/2 (MeV)	$\lvert\text{Residue}\rvert^{1/2}_{\pi\eta}$ (GeV)	Ratios	
LO						
$a_0(980)$	II	1037^{+17}_{-14}	44^{+6}_{-9}	$3.8^{+0.3}_{-0.2}$	$1.43^{+0.03}_{-0.03}$ $(K\bar{K}/\pi\eta)$	$0.05^{+0.01}_{-0.01}$ $(\pi\eta'/\pi\eta)$
NLO						
$a_0(980)$	IV	1019^{+22}_{-8}	24^{+57}_{-17}	$2.8^{+1.4}_{-0.6}$	$1.8^{+0.1}_{-0.3}$ $(K\bar{K}/\pi\eta)$	$0.01^{+0.06}_{-0.01}$ $(\pi\eta'/\pi\eta)$
$a_0(1450)$	V	1397^{+40}_{-27}	62^{+79}_{-8}	$1.7^{+0.3}_{-0.4}$	$1.4^{+2.4}_{-0.6}$ $(K\bar{K}/\pi\eta)$	$0.9^{+0.8}_{-0.2}$ $(\pi\eta'/\pi\eta)$

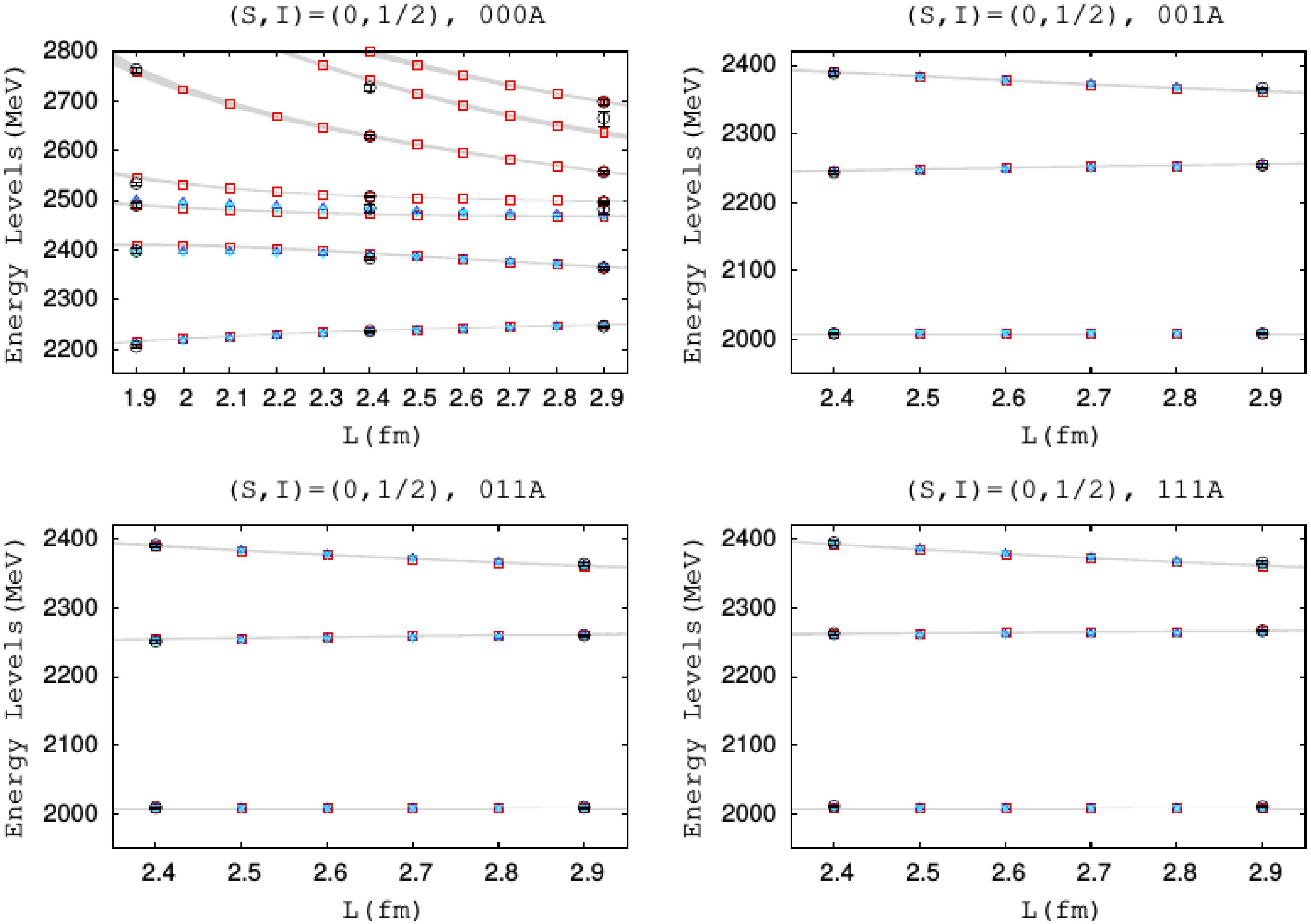
[ZHG,Liu,Meissner,Oller,Rusetsky, PRD'17]

Preliminary results for

the D-pi, D-eta, Ds-Kbar scattering

Reproduction of the finite-volume energy levels

[Moir,Peardon,Ryan,Thomas, Wilson, JHEP'16]



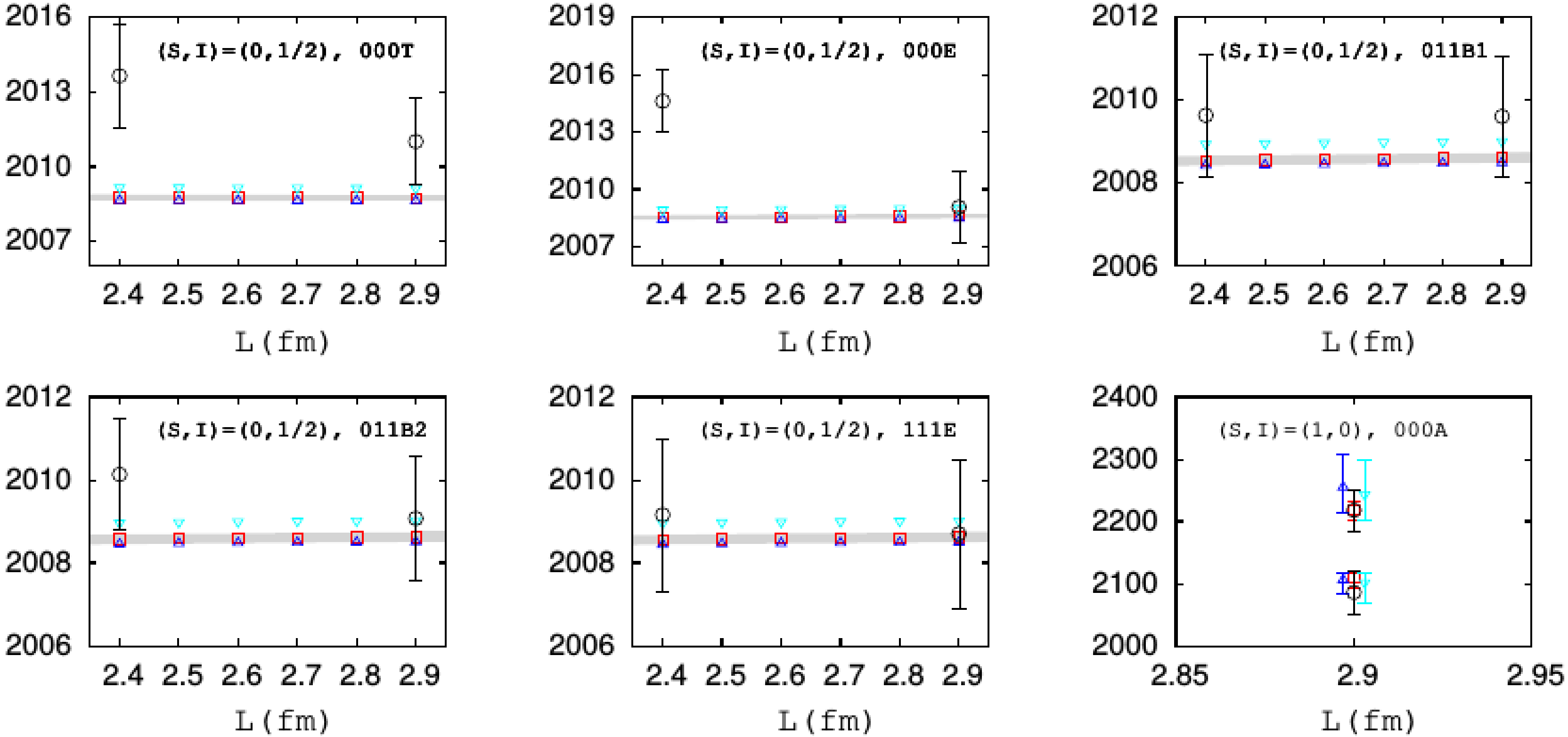
Preliminary

[ZHG, et al., in preparation]

Reproduction of the finite-volume energy levels

[Moir,Peardon,Ryan,Thomas, Wilson, JHEP'16]

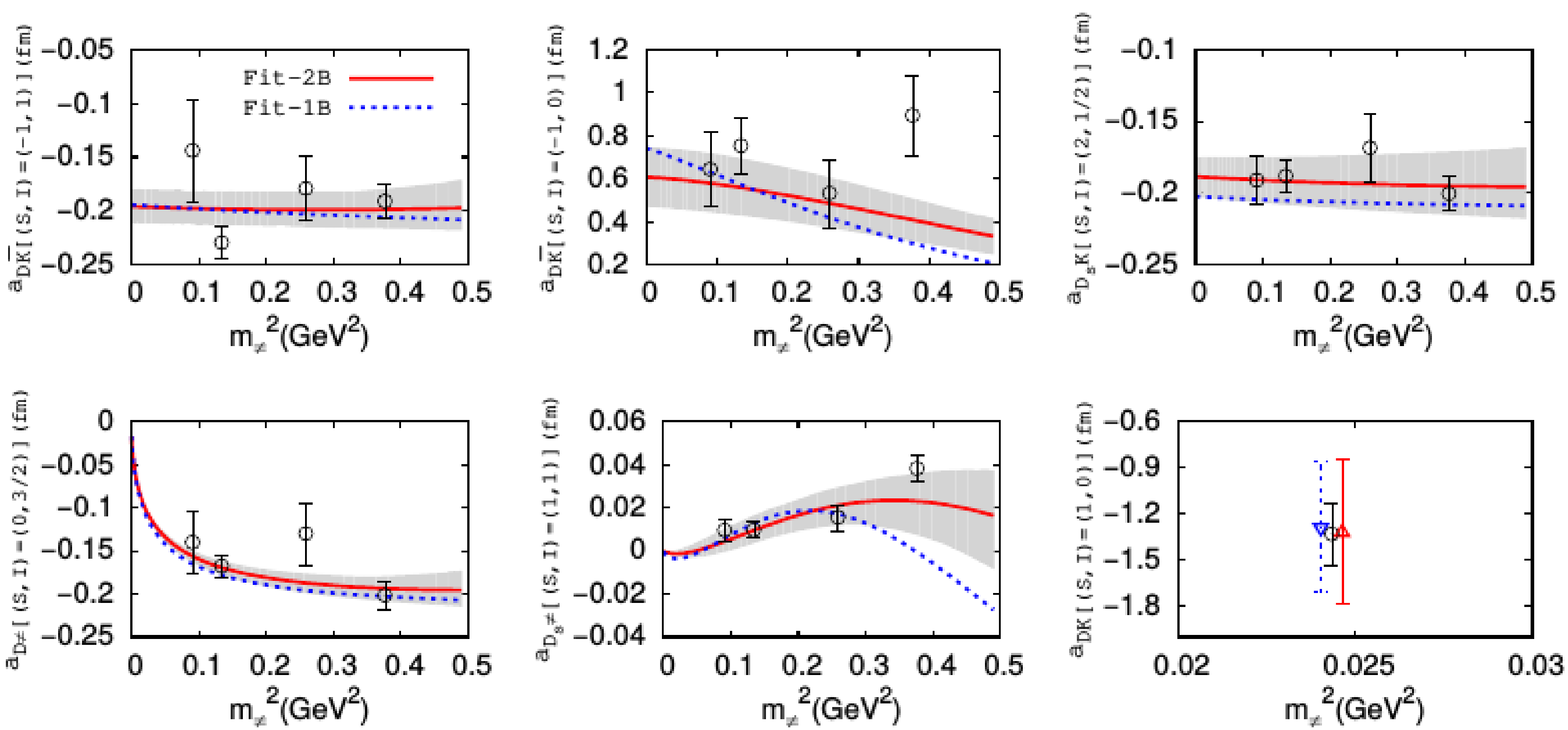
[Lang,Leskovec,Mohler,Prelovsek,Woloshyn,PRD'14]



Preliminary

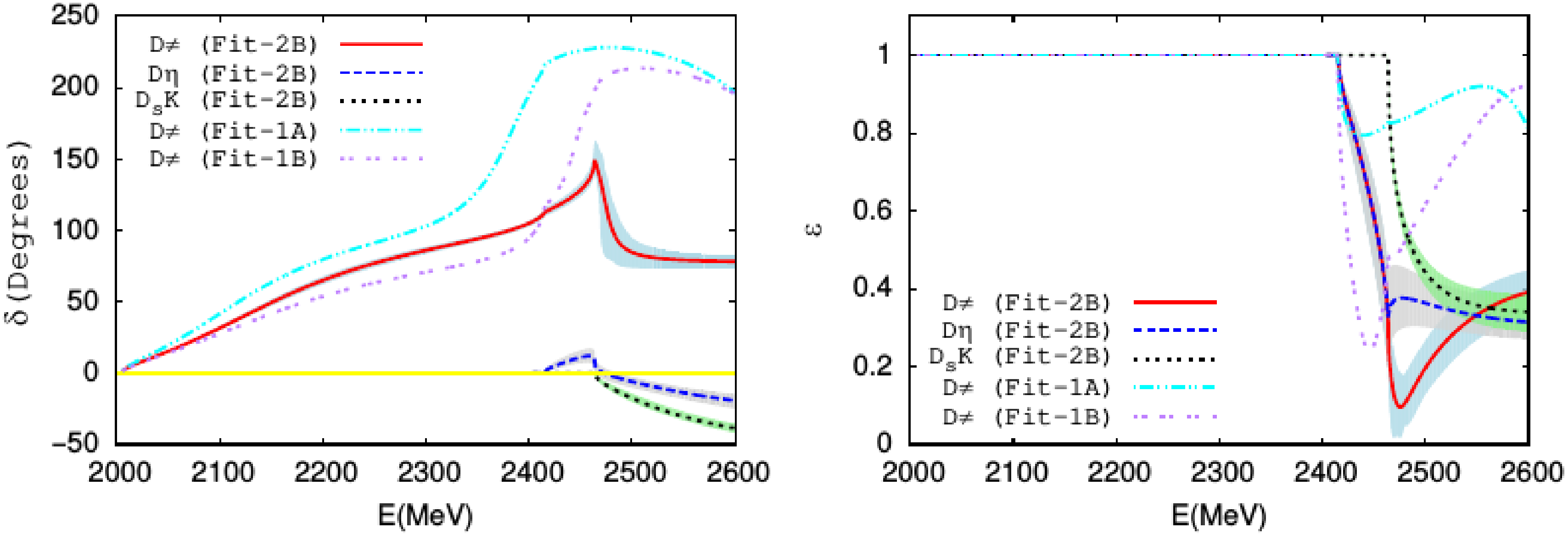
Reproduction of scattering lengths

[L.Liu,Orginos,F.K.Guo,Meissner, PRD'13]
[Lang,Leskovec,Mohler,Prelovsek,Woloshyn,PRD'14]



Preliminary

Preliminary results of the D-pi phase shifts and inelasticities at physical meson masses



[ZHG, et al., in preparation]

Summary

- The chiral approach illustrated in this talk provides an efficient way to study the finite-volume energy levels.

- It can build a bridge to connect the lattice eigenenergies in finite box obtained at unphysical masses with the physical observables, such as phase shifts, inelasticities, at physical meson masses.

- We have successfully applied this approach to the pi-eta, K-Kbar and pi-eta' coupled-channel scattering.

- Similar study in other systems is in progress.

Thanks for your attention!