Neutron Electric Dipole Moment from Beyond the Standard Model

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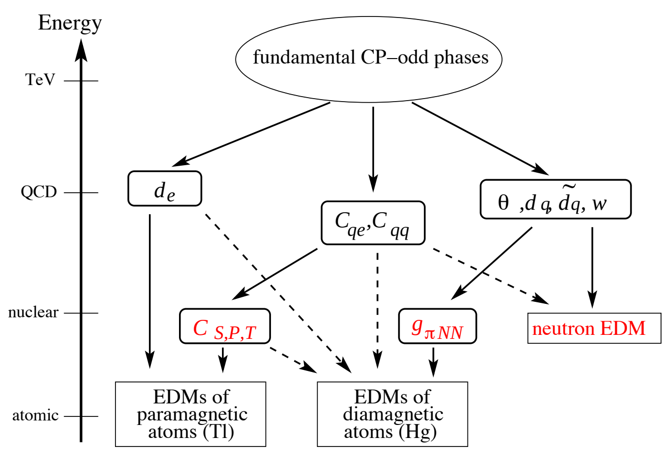
Introduction
BSM Operators

Standard model CP violation in the weak sector.
Strong CP violation from dimension 3 and 4 operators anomalously small.

- Dimension 3 and 4:
  - CP violating mass $\bar{\psi}\gamma_5\psi$.
  - Toplogical charge $G_{\mu\nu}\tilde{G}^{\mu\nu}$.
- Suppressed by $v_{\rm EW}/M^2_{\rm BSM}$:
  - Electric Dipole Moment $\bar{\psi}\Sigma_{\mu\nu}\tilde{F}^{\mu\nu}\psi$.
  - Chromo Dipole Moment $\bar{\psi}\Sigma_{\mu\nu}\tilde{G}^{\mu\nu}\psi$.
- Suppressed by $1/M^2_{\rm BSM}$:
  - Weinberg operator (Gluon chromo-electric moment): $G_{\mu\nu}G_{\lambda\nu}\tilde{G}_{\mu\lambda}$.
  - Various four-fermi operators.

Introduction

Effective Field Theory



Pospelov and Ritz, *Ann. Phys.* **318** (2005) 119.

Introduction
Form Factors

Vector form-factors
Dirac F_1, Pauli F_2, Electric dipole F_3, and Anapole F_A

Sachs electric $G_E \equiv F_1 - (q^2/4M^2)F_2$ and magnetic $G_M \equiv F_1 + F_2$

$$\langle N|V_\mu(q)|N\rangle = \overline{u}_N \left[\gamma_\mu \, F_1(q^2) + i\frac{[\gamma_\mu,\gamma_\nu]}{2}q_\nu \, \frac{F_2(q^2)}{2m_N} + (2i\, m_N\gamma_5 q_\mu - \gamma_\mu\gamma_5 q^2)\, \frac{F_A(q^2)}{m_N^2} + \frac{[\gamma_\mu,\gamma_\nu]}{2}q_\nu\gamma_5 \, \frac{F_3(q^2)}{2m_N}\right] u_N$$

- The charge $G_E(0) = F_1(0) = 0$.
- $G_M(0)/2M_N = F_2(0)/2M_N$ is the (anomalous) magnetic dipole moment.
- $F_3(0)/2m_N$ is the electric dipole moment.
- F_A violates PT; F_3 violates CP.

Lattice Calculation

Technique

nEDM due to the quark EDM operator is given by the tensor charge, g_T.

See talk by Rajan Gupta.

The quark chromo-EDM operator is a quark bilinear and can be handled by the Schwinger source method

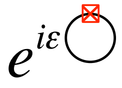
$$\not{D} + m - \frac{r}{2}D^2 + c_{sw}\Sigma^{\mu\nu}G_{\mu\nu} \longrightarrow \not{D} + m - \frac{r}{2}D^2 + \Sigma^{\mu\nu}(c_{sw}G_{\mu\nu} + i\epsilon\tilde{G}_{\mu\nu})$$

The fermion determinant gives a 'reweighting factor'

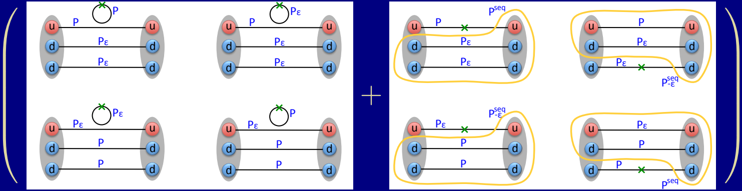
$$\begin{aligned}
&\frac{\det(\not{D} + m - \frac{r}{2}D^2 + \Sigma^{\mu\nu}(c_{sw}G_{\mu\nu} + i\epsilon\tilde{G}_{\mu\nu})}{\det(\not{D} + m - \frac{r}{2}D^2 + c_{sw}\Sigma^{\mu\nu}G_{\mu\nu})} \\
&= \quad \exp\operatorname{Tr}\ln\left[1 + i\epsilon\,\Sigma^{\mu\nu}\tilde{G}_{\mu\nu}(\not{D} + m - \frac{r}{2}D^2 + c_{sw}\Sigma^{\mu\nu}G_{\mu\nu})^{-1}\right] \\
&\approx \quad \exp\left[i\epsilon\operatorname{Tr}\Sigma^{\mu\nu}\tilde{G}_{\mu\nu}(\not{D} + m - \frac{r}{2}D^2 + c_{sw}\Sigma^{\mu\nu}G_{\mu\nu})^{-1}\right].
\end{aligned}$$

Lattice Calculation

Three-point function



$\times$



The chromoEDM operator is dimension 5.

Uncontrolled divergences unless $\epsilon \lesssim 4\pi a \Lambda_{\rm QCD} \sim 1$.

Need to check linearity.

Two point functions

Ensembles

Tests with Clover fermions on MILC 2+1+1 HISQ ensembles.

- $a = 0.1207(11)$ fm, $M_\pi^{\rm sea} = 305.3(4)$ MeV, $M_\pi^{\rm val} = 310.2(2.8)$ MeV, $L^3 \times T = 24^3 \times 64$, $\kappa \approx 0.1272103$, $c_{\rm SW} = 1.05094$, $u_P^{HYP} = 0.9358574(29)$. 400 Configurations, 64 LP + 8 HP calculations/configuration.

Use two CP violating operators that mix under renormalization.

- CEDM: $a^2 \bar\psi\, \tilde G \cdot \Sigma\, \psi$
- P: $\bar\psi \gamma_5 \psi$

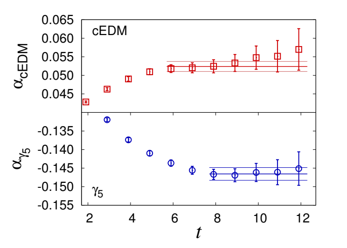
Two point functions

Neutron Propagator

Neutron operator

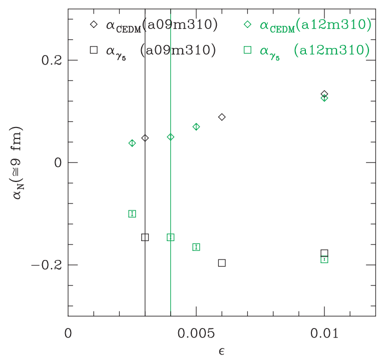
$$\epsilon_{abc}\left[\bar{d}^{aT}\gamma_0\gamma_2\gamma_5\left(\frac{1\pm\gamma_4}{2}\right)u^b\right]e^{i\alpha_N\gamma_5}d^c$$

- α_N chosen to make the asymptotic "free" fields have the standard parity
- α_N state dependent
- PT symmetry $\Rightarrow \alpha_N$ is real



Preliminary; Connected Diagrams Only

Two point functions
Linearity



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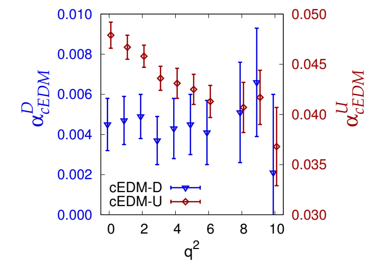
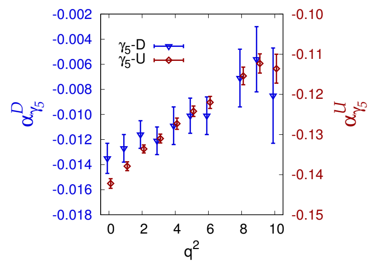
Use $\epsilon \approx \frac{a}{30\text{fm}} \approx 6.6\text{MeV}\, a \approx 0.36\, ma$ for experiments.

Two point functions

$O(a)$ effects

α is a Lorentz scalar, independent of momentum.
On the lattice, see a momentum dependence.





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Three point functions
Projection

The three point function we calculate is

$$N' \equiv \bar{d}^c \gamma_5 \frac{1+\gamma_4}{2} u \; d = e^{-i\alpha_N \gamma_5} N$$

$$\langle \Omega | N'(\vec{0},0) V_\mu(\vec{q},t) \overline{N'}(\vec{p},T) | \Omega \rangle =$$

$$\sum_{n,n'} e^{-i\alpha_n \gamma_5} u_n e^{-m_n t} \, \langle n | V_\mu(q) | n' \rangle \; e^{-E_{n'}(T-t)} \overline{u}_{n'} e^{-i\alpha^*_{n'} \gamma_5}$$

We project onto only one spinor component with

$$\mathcal{P} = \frac{1}{2}(1+\gamma_4)(1+i\gamma_5\gamma_3)$$

We isolate the matrix elements using the combination

$$\frac{C_{3\text{pt}}(q;\tau,t)}{C_{2\text{pt}}(\tau;0)}\sqrt{\frac{C_{2\text{pt}}(t;0)C_{2\text{pt}}(\tau;0)C_{2\text{pt}}(\tau-t;-q)}{C_{2\text{pt}}(t;-q)C_{2\text{pt}}(\tau;-q)C_{2\text{pt}}(\tau-t;0)}}$$

where

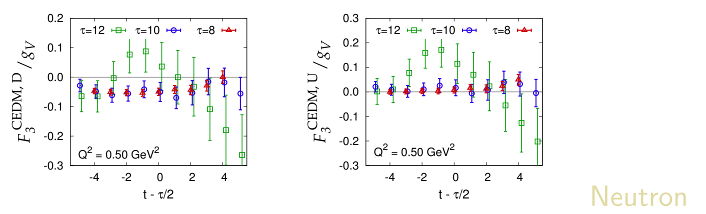
- $C_{3\text{pt}}$ are the projected 3pt functions
- $C_{2\text{pt}}$ are the real parts of projected 2pt functions

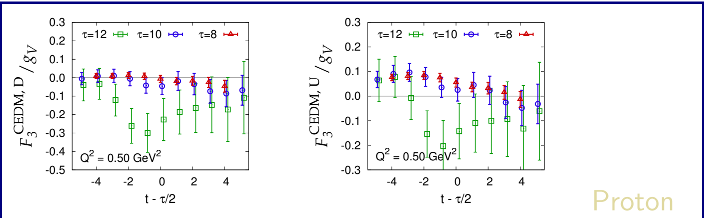
One then has

$$F_3 = -\frac{2m\sqrt{2E}}{\cos\alpha[q_3\sqrt{E+m}(q_1^2+q_2^2)(E+m\cos 2\alpha)]}\times$$
$$\times\Big[\cos\alpha(E+m)^2(q_1\Re V_1+q_2\Re V_2)+$$
$$+q_3\sin\alpha[(E+m)(q_2\Re V_1-q_1\Re V_2)+(q_1^2+q_2^2)\Re V_4]\Big]$$

Three point functions

F_3 Form factor from CEDM





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Three point functions

Variance Reduction

Many combinations are zero by continuum symmetries. For example,

$$
\begin{aligned}
z_0 &= cEDM(\epsilon = 0) && (1)\\
z_1 &= -(E-m)\Im V_4 + q_1 \Re V_1 + q_2 \Re V_2 + q_3 \Re V_3 && (2)\\
z_2 &= \tan\alpha(q_1 \Re V_2 - q_2 \Re V_1) - (E+m)\Re V_3 - q_3 \Im V_4 && (3)\\
z_3 &= -(E-m)\Re V_4 + q_1 \Im V_1 + q_2 \Im V_2 + q_3 \Im V_3 && (4)\\
z_4 &= q_2 \Im V_1 - q_1 \Im V_2 && (5)\\
z_5 &= q_3 \Im V_1 - q_1 \Im V_2 && (6)
\end{aligned}
$$

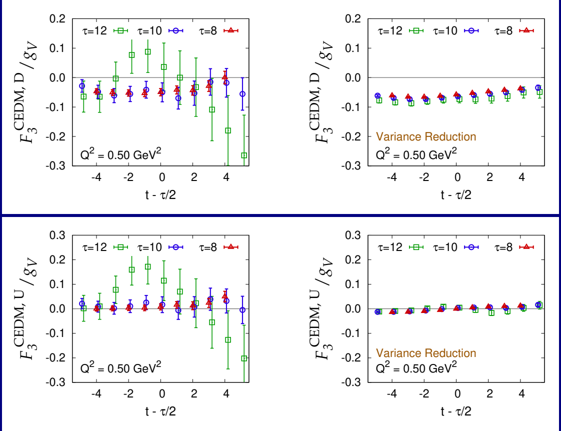
But these may have non-zero correlations with F_3. If σ^2_{FF} and σ^2_{ii} are the variances and σ^2_{Fi} and σ^2_{ij} are the covariances, then

$$F_3 - \sigma^2_{Fi}(\sigma^{-2})_{ij} z_j$$

has lower variance.

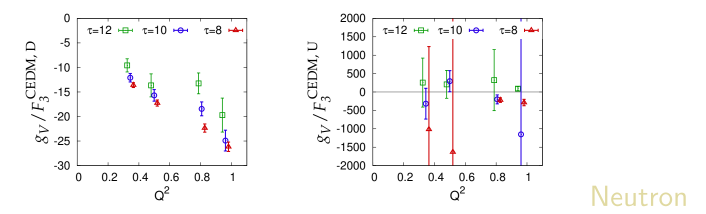
Currently, showing results only with z_0.

- For cEDM, the coefficient $=$ 0.99
- For Pseudoscalar, the coefficient $=$ 0.90



Preliminary; Connected Diagrams Only; Neutron

Three point functions
nEDM from cEDM



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Future

Next steps in progress

- Disconnected diagrams, fits, and extrapolations
- Mixing with lower dimensional operators
- Renormalization (gradient flow?)
- Weinberg operator
- Reduce variance

Future

Mixing with lower dimensional operators

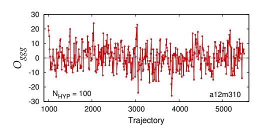
- cEDM mixes with $a^{-2}\bar{\psi}\gamma_5\psi$ even in a theory with chiral symmetry.
- Mixing structure different when chiral symmetry is broken, but no other operators.
- RI-sMOM scheme worked out, but complicated (More difficult if no chiral symmetry).
- In Gradient Flow scheme, smearing cEDM allows smooth $a \to 0$ limit at fixed physical smearing t.
- Continuum calculation mixes matrix elements of cEDM and $t^{-1}\bar{\psi}\gamma_5\psi$ to obtain cEDM in other schemes.

Future

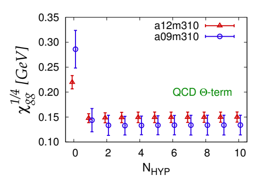
Weinberg Operator

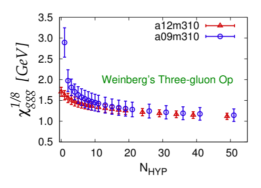
Weinberg operator mixes with the Θ-term.

Large fluctuations from configuration to configuration.



Smearing reduces the ultraviolet fluctuations; scale dependence remains.

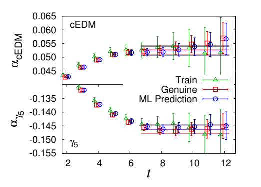




Future
Machine learning

Variance reduction due to correlation between different observables: using different momentum channels to get further reductions.

Can also use the correlations to measure quantities: similar to reweighting or unraveling.



Predicting imaginary part from real part