πN P-wave resonant scattering from lattice QCD

Srijit Paul

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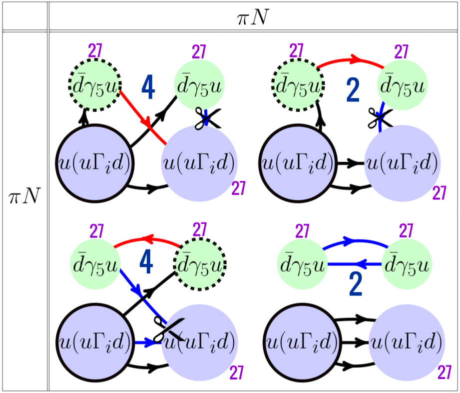

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N_s	N_t	β	$am_{u,d}$	am_s	c_{sw}	$a(fm)$	$L(fm)$	$m_\pi(MeV)$	$m_\pi L$
24	48	3.31	-0.0953	-0.040	1.0	0.116	2.8	254	3.6

BMW Ensemble, S. Dürr et al., JHEP 1108, 148 (2011)

Lüscher Methodology

$$\det\left(\mathbb{1} + it_\ell(s)(\mathbb{1} + i\mathcal{M}^{\vec{P}})\right) = 0,$$

where $t_\ell(s) = \frac{1}{\cot\delta_\ell(s) - i}$.

[Lüscher(1991)]

General Lüscher Methodology

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For Baryons

$$\det(\boxed{M_{Jl\mu,J'l'\mu'}} - \delta_{JJ'}\delta_{ll'}\delta_{\mu\mu'} \cot \boxed{\delta_{Jl}}) = 0$$

[Göckeler et.al.(2012)]

In the above formula $M_{Jl\mu,J'l'\mu'}$ can be simplified by basis transformations as block diagonal by,

$$\langle \Gamma\alpha J l n | \hat{M} | \Gamma'\alpha' J' l' n' \rangle = \sum_{\mu\mu'} c^{\Gamma\alpha n\,*}_{Jl\mu}\, c^{\Gamma'\alpha' n'}_{J'l'\mu'}\, M_{Jl\mu,J'l'\mu'}$$

Example $M_{Jl\mu,J'l'\mu'}$ calculation: C_{4v}^D

J = 1/2	J = 3/2
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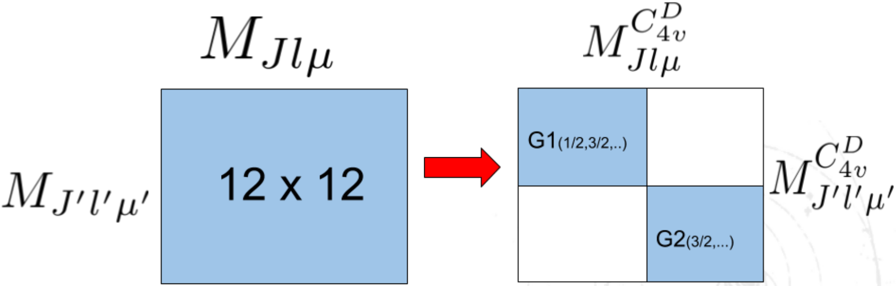

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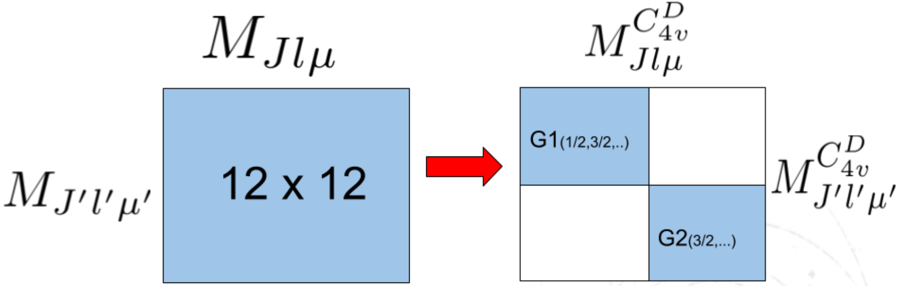

$$\det\left(\boxed{M^{G1}_{Jl\mu,J'l'\mu'}} - \delta_{JJ'}\delta_{ll'}\delta_{\mu\mu'} \cot \boxed{\delta^{G1}_{Jl}} \right) = 0$$

$$\det\left(\boxed{M^{G2}_{Jl\mu,J'l'\mu'}} - \delta_{JJ'}\delta_{ll'}\delta_{\mu\mu'} \cot \boxed{\delta^{G2}_{Jl}} \right) = 0$$

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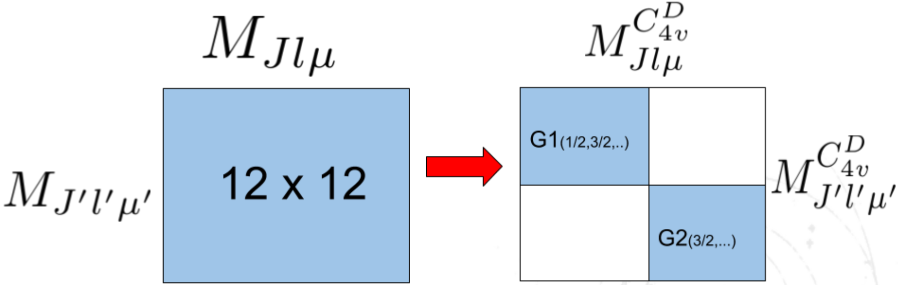

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Resonances

Narrow resonances in scattering are characterised by Breit Wigner

$$t_\ell(s) = \frac{\sqrt{s}\,\Gamma(s)}{m_R^2 - s - i\sqrt{s}\,\Gamma(s)}$$

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Decay Width form for $\Delta(1232)$.

$$\Gamma^{LO}_{EFT} = \frac{\boxed{g^2_{\Delta-\pi N}}}{48\pi}\boxed{\frac{E_N + m_N}{E_N + E_\pi}\frac{p^{*3}}{m_N^2}}$$

[V. Pascalutsa and M. Vanderhaeghen, Phys.Rev. D73 ,034003 (2006)]

Used in lattice QCD for the first time by,

[Alexandrou, Negele, Petschlies, Strelchenko, Tsapalis , Phys.Rev. D88 (2013)]

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$$m_\pi = 258.3(1.1)\,\text{MeV}$$

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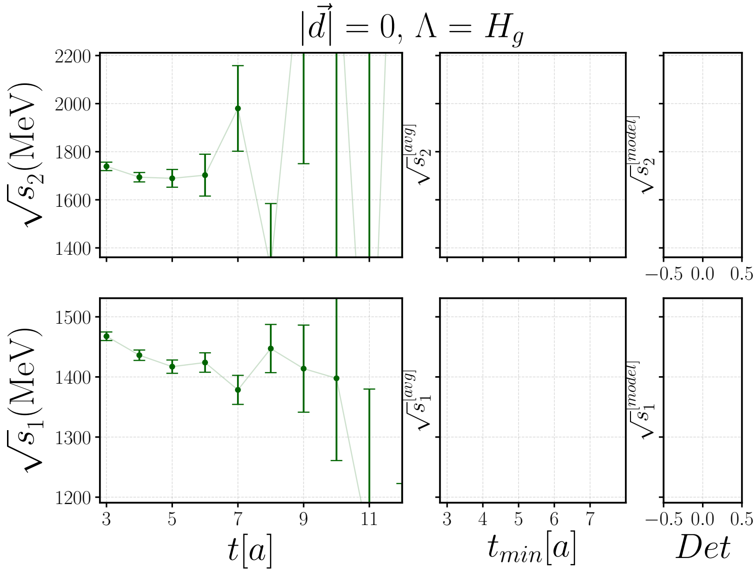
- Find the values of s for which the determinant condition is satisfied.

Lüscher Analysis v/s Inverse Lüscher



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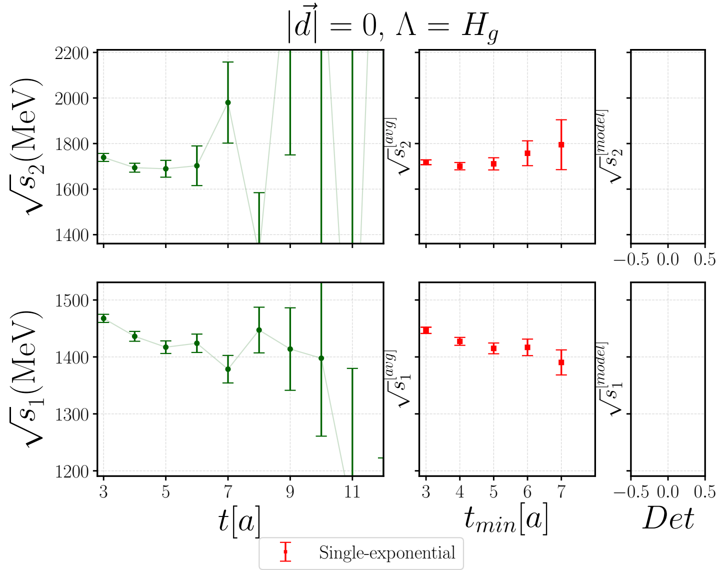

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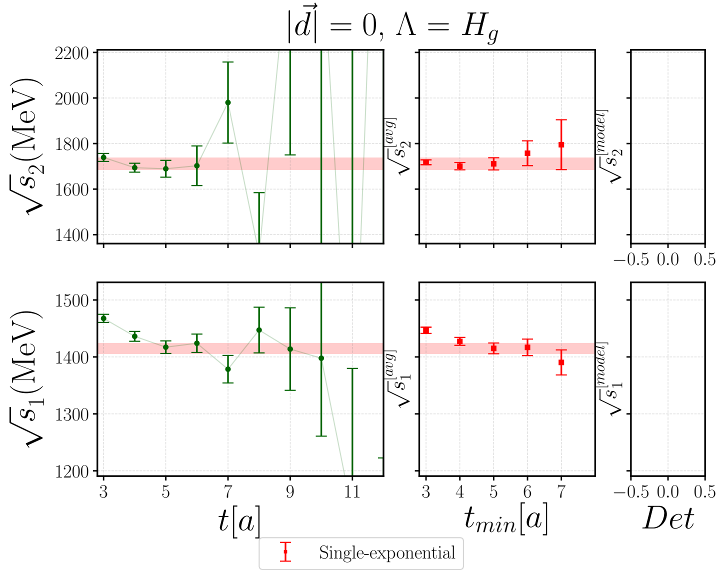

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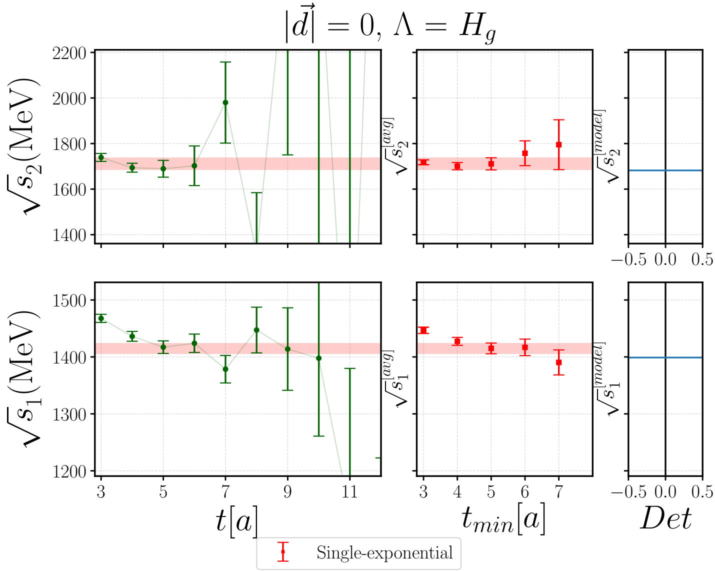

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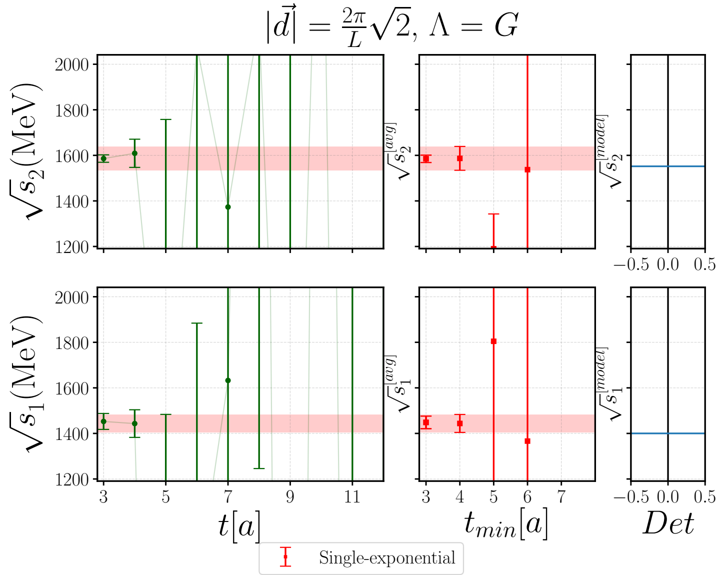

χ^2 with a model fit

$$\chi^2 = \sum_{\vec{P},\Lambda,n} \sum_{\vec{P}',\Lambda',n'} \left(\boxed{\sqrt{s_n^{\Lambda,\vec{P}}}^{[avg]}} - \boxed{\sqrt{s_n^{\Lambda,\vec{P}}}^{[model]}} \right) [C^{-1}]_{\vec{P},\Lambda,n;\vec{P}',\Lambda',n'} \left(\boxed{\sqrt{s_{n'}^{\Lambda',\vec{P}'}}^{[avg]}} - \boxed{\sqrt{s_{n'}^{\Lambda',\vec{P}'}}^{[model]}} \right),$$

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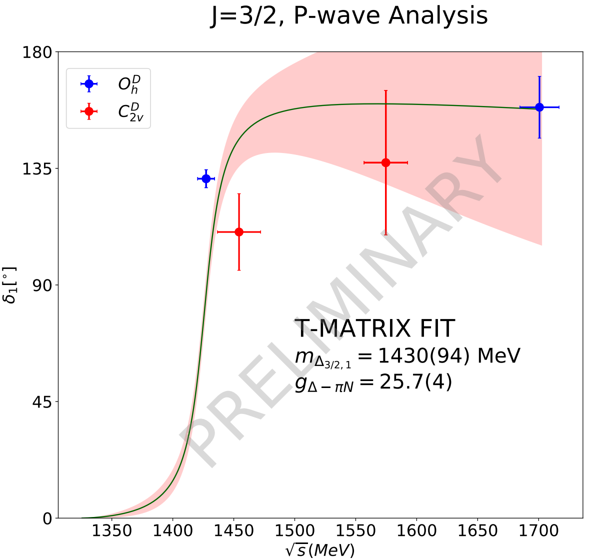
m_Δ	1430 MeV	(94)
$g_{\Delta-\pi N}$	25.7	(4)

Phase Shift plot



Contemporary results

Collab.	m_π(MeV)	Methodology	m_Δ(MeV)	coupling
Verduci(2014)	266(3)	(WC)Distillation, Lüscher	1396(19)	19.9(83)
Alexandrou et.al. (2013)	360	(DW)Michael, McNeile	-	26.7(0.6)(1.4)
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Our result(Preliminary)	258.3(1.1)	(WC)Src-smear, Lüscher	1430(94)	25.7(4)
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We differ from the above calculations, in terms of analysis methods, in the following ways:

- Tuned Smearing parameters,
  $$W\left[U_{2-HEX}\right] D^{-1} W\left[U_{2-HEX}\right]^{\dagger}$$
  $$(N, \alpha_{WUP}) = (45, 3.0)$$
- We use direct t-matrix fits, for estimating the resonance parameters.

Outlook

- We have done the calculation on only 1/3rd of total configurations available.

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- With the application of t-matrix fits, we can deal with non-resonant contributions from other J's and l's.
- Only one Breit Wigner model was taken into account, more models need to be taken with more data.

Collaborators

Constantia Alexandrou (University of Cyprus / The Cyprus Institute)

Stefan Krieg (Forschungszentrum Jülich / University of Wuppertal)

Giannis Koutsou (The Cyprus Institute)

Luka Leskovec (University of Arizona)

Stefan Meinel (University of Arizona / RIKEN BNL Research Center)

John Negele (MIT)

Marcus Petschlies (University of Bonn / Bethe Center for Theoretical Physics)

Andrew Pochinsky (MIT)

Gumaro Rendon (University of Arizona)

Giorgio Silvi (Forschungszentrum Jülich / University of Wuppertal)

Sergey Syritsyn (Stony Brook University / RIKEN BNL Research Center)