eBRST $SU(2)$ Gauge Theory on Lattice

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Outline :

- ➥ eBRST $SU(2)$
- ➥ Reduced model
- ➥ Nontrivial phase diagram
- ➥ Preliminary results

Introduction

Equivariant BRST gauge-fixing first proposed by Schaden for $SU(2)$.

[Phys Rev D, Vol 59, 014508]

Later, expanded to general $SU(N)$ with eBRST & anti-eBRST by Golterman & Shamir.

[Phys Rev D, Vol 70, 094506]

Basic idea $\rightarrow$ *gauge-fix in the coset leaving an Abelian subgroup invariant.*

Goals :

- ➪ Avoid no-go theorem [Neuberger]
- ➪ Prescription to study $SU(N)$ chiral gauge theories on the lattice.

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Our ongoing work

Numerical study of phase diagram of the pure $SU(2)$ gauge theory, gauge-fixed in the coset $SU(2)/U(1)$ with a resulting eBRST symmetry.

eBRST $SU(2)/U(1)$

$SU(2)/U(1)$ gauge-fixed Lagrangian in the continuum (integrating out the auxiliary field b)

$$\mathcal{L}_{gf} = \frac{1}{\xi g^2} \mathrm{tr}(\mathcal{D}_\mu(A) W_\mu)^2 + \mathcal{L}_{gh}^{(2)} + \xi g^2 \mathcal{L}_{gh}^{(4)}$$

$$\mathcal{L}_{gh}^{(2)} = -2\mathrm{tr}(\overline{C}\mathcal{D}_\mu(A)\mathcal{D}_\mu(A)C) + 2\mathrm{tr}([W_\mu, \overline{C}][W_\mu, C])$$

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Action on the lattice

$$\begin{aligned}\mathcal{S}_{gf} =& \frac{1}{\xi g^2}\mathrm{tr}\sum_x(\mathcal{D}_\mu^- \mathcal{W}_{x\mu})^2 - \xi g^2 \mathrm{tr}\sum_x(\widetilde{X}^2) \\ & - 2\mathrm{tr}\sum_x\left(\left[U_{x\mu}T_3U_{x\mu}^\dagger, \mathcal{D}_\mu^+\overline{C}_x\right]\left[T_3, \mathcal{D}_\mu^+ C_x\right] + i\mathcal{W}_{x\mu}\left\{\overline{C}_x, \mathcal{D}_\mu^+ C_x\right\}\right)\end{aligned}$$

where $\mathcal{W}_{x\mu} = -[U_{x\mu}T_3U_{x\mu}^\dagger, T_3] = W_{x\mu} + O(V^2)$, $T_a = \sigma_a/2$,
lattice covariant derivatives $\mathcal{D}_\mu^+\Phi_x = U_{x\mu}\Phi_{x+\mu}U_{x\mu}^\dagger - \Phi_x$,
$\mathcal{D}_\mu^-\Phi_x = \Phi_x - U_{x-\mu,\mu}^\dagger\Phi_{x-\mu}U_{x-\mu,\mu}$.

Action on the lattice; Ghost matrix

➪ The 4-ghost term is tackled by introducing an **auxiliary field** $\rho \in \mathcal{H}$.

$$\mathcal{S}_{gf} = \tilde{\kappa} \sum_{x\alpha} (\mathcal{D}_\mu^- \mathcal{W}_{x\mu})_\alpha^2 + \tilde{\kappa} \sum_x \rho_x^2 + \sum_{xy\alpha\beta} \overline{C}_{x\alpha} M_{x\alpha,y\beta} C_{y\beta}, \quad \tilde{\kappa} = \frac{1}{2\xi g^2}$$

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- The ghost matrix is implemented in the HMC algorithm in the following way.

$$\int \mathcal{D}C\mathcal{D}\overline{C} \exp(-\overline{C}MC) = \det M = |\det M| \operatorname{sign}(\det M)$$

  $|\det M|$ can be simulated using HMC by introducing a real "pseudo-ghost" field φ,

$$|\det M| = \sqrt{\det(MM^T)} = \int \mathcal{D}\varphi \exp\left(-(1/2)\varphi^T (MM^T)^{-1} \varphi\right).$$

Algorithm : Discussions

The partition function :

$$Z \equiv \int \mathcal{D}U \mathcal{D}\varphi \mathcal{D}\rho \exp\left(-[\mathcal{S}_W + \mathcal{S}'_{gf} + \frac{1}{2}\varphi^T (M^T M)^{-1} \varphi]\right) \operatorname{sign}(\det M)$$

Simulate with Z' without the sign. Need to track sign of $\det M$.

Stochastic Tunneling HMC (a kind of stochastic deflation) is currently being tried.

[Phys Rev D 76, 094512 (2007)]

Present simulation with HMC.

We try to explore as much of the phase diagram as possible.

Reduced model

Gauge transf. : $U_{x,\mu} \to U^{\phi}_{x,\mu} = \phi_x U_{x,\mu} \phi^{\dagger}_{x+\mu}$.
The *lgdofs* ϕ are explicitly present in $\mathcal{S}_{gf}$ $\to$ radially frozen scalar fields.

$$Z = \int \mathcal{D}U \exp\left(-\mathcal{S}_W(U)\right) \tilde{Z}(U), \qquad \textbf{Higgs picture}$$

$$\text{where} \quad \tilde{Z}(U) = \int \mathcal{D}C\mathcal{D}\overline{C}\mathcal{D}\phi \exp\left(-[\mathcal{S}_{gf}(U^{\phi}, C, \overline{C})]\right)$$

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Apart from eBRST & U(1), local $SU(2)_R$ symmetry present.

$$U_{x,\mu} \to g_x U_{x,\mu} g^{\dagger}_{x+\mu}, \quad \phi_x \to \phi_x g^{\dagger}_x, \quad g_x \in SU(2)_R$$

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Reduced model of the theory is the limit $g \to 0$ or $U = 1$
$\Rightarrow$ only ϕ fields and ghosts.

$SU(2)_R$ becomes global

Spontaneous symmetry breaking

From strong coupling and mean field techniques,

- ➪ Due to strong dynamics, SSB takes place in the reduced model as $SU(2)_R \to U(1)$.
- ➪ In the full eBRST theory, by a Higgs mechanism, the W_1 and W_2 gauge fields gain a mass.
- ➪ eBRST is expected to be unbroken with the gauge boson mass generated being balanced by a ghost mass.

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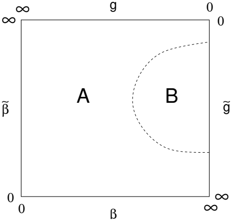

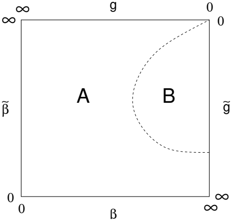

$\tilde{\kappa} \propto 1/\tilde{g}^2 = \tilde{\beta}$

A : Usual confining phase

B : Higgs/broken phase

[Golterman & Shamir, Phys. Rev. D 87, 054501 (2013)]

Invariance Theorem

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Possibility of interesting physics $\rightarrow$ break eBRST explicitly by a symmetry breaking seed.

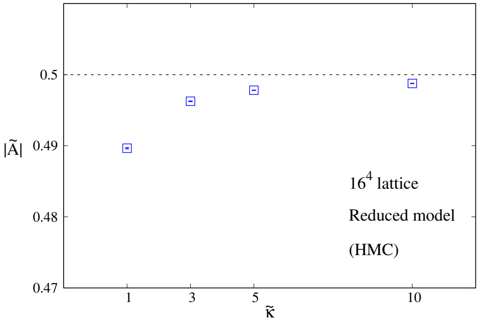
- ➪ Study SSB and Higgs mechanism
- ➪ Take infinite volume limit
- ➪ Turn off symmetry breaking seed
- ➪ See if new phase appears

Reduced model [Preliminary results]

Order parameter $\tilde{A} = \langle \phi_x^\dagger \tau_3 \phi_x \rangle$ for $SU(2)_R \to U(1)$.

Broken phase for range of $\tilde{\kappa}$.

Ongoing work of study of spectrum.



Invariance theorem [Preliminary result]

eBRST symmetry also allows a mass term, which helps in CG inversion

$$\mathcal{S}_m = m^2 \sum_x \Big[-4\,\tilde{\kappa}\,\mathrm{tr}(U_{x\mu}\tau_3 U^\dagger_{x\mu}\tau_3) + 2\,\mathrm{tr}(\overline{C}_x C_x) \Big]$$

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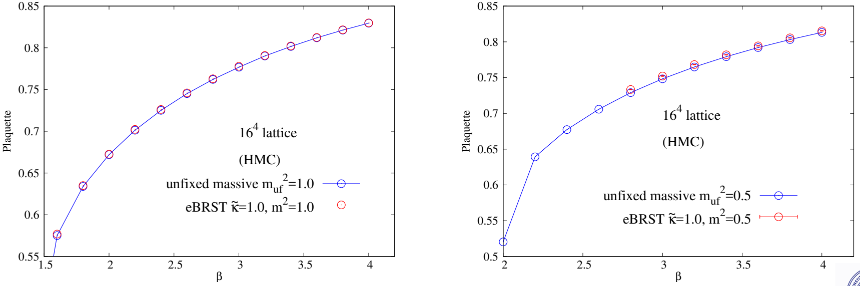
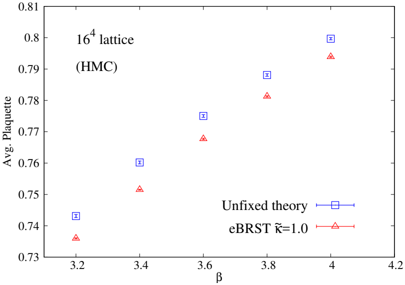
Validation of invariance theorem with mass term



Plaquette vs β

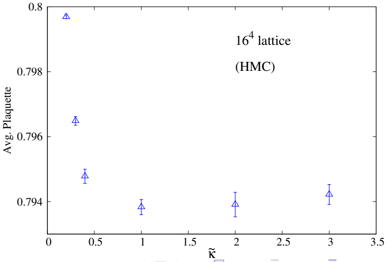
Invariance theorem [Preliminary result]



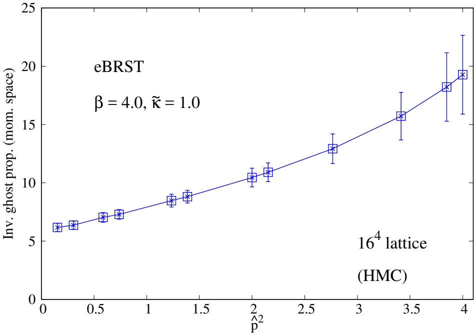
Invariance theorem without mass term

Possible breaking of eBRST symmetry and/or Higgs phase of the theory!

Preliminary indication of validation of invariance theorem with varying $\tilde{\kappa}$



Ghost propagator [Preliminary result]



The inverse ghost propagator in momemtum space has non-zero mass intercept.

W propagator is still noisy and need more statistics.

Conclusion

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- Coding is a challenging task since keeping track of the sign of the determinant will be a very difficult thing as it essentially boils down to tracking the zero crossing of the smallest eigenvalues. We intend to use some kind of deflation techniques with HMC.
- Ultimately, the abelian part of the theory has to be gauge-fixed by the HD action described in the previous talk.

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