Hypercubic effects in semileptonic decays of heavy mesons, toward $B \to \pi \ell \nu$ with Nf=2+1+1 Twisted fermions

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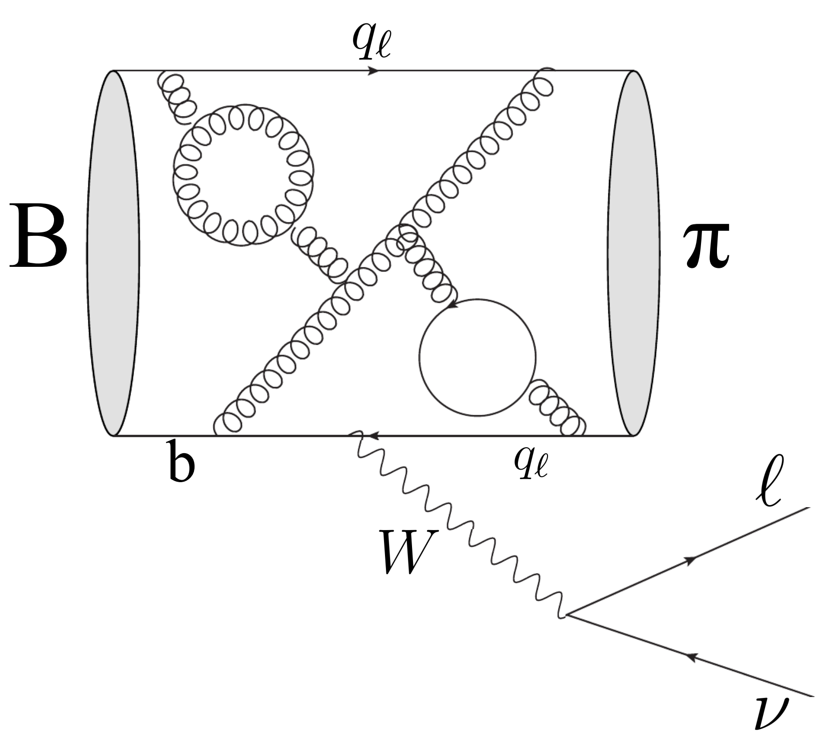
Lattice 2018 - 36th International Symposium on Lattice Field Theory, 22-28 July 2018 East Lansing (Mi) USA

Motivation

The hadronic contribution to the B → π semileptonic decay rate is regulated by the vector and scalar form factors f_0 and f_+ which are function of q^2



Decay rate in the SM

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} \frac{(q^2 - m_\ell^2)^2 |\vec{p}_\pi|}{q^4 M_B^2} \times$$

$$\times \left[\left(1 + \frac{m_\ell^2}{2q^2}\right) M_B^2 |\vec{p}_\pi|^2 |f_+^{B\pi}(q^2)|^2 + \frac{3m_\ell^2}{8q^2} (M_B^2 - M_\pi^2)^2 |f_0^{B\pi}(q^2)|^2 \right]$$

Motivation

In [Phys. Rev. D96 (2017) 054514] we found evidence of the breaking of Lorentz Symmetry due to hypercubic effects in the vector and scalar form factors of both the $D \to \pi$ and $D \to K$ semileptonic transitions

In [arXiv:1803.04807] we observed a similar breaking of Lorentz Symmetry in the tensor form factor of the same transitions

These effects may be relevant when we move to B-physics and have to be under control

motivation for this analysis:

- An exploratory study toward $B \to \pi$
- Gain a better understanding on the relevance of these effects as the mass of the father meson increases

Simulation Details

Details of the ensembles used in this N_f = 2+1+1 analysis

ensemble	β	V/a^4	$a\mu_{sea} = a\mu_\ell$	$a\mu_s$	$a\mu_c$	$a\mu_h$	M_π(MeV)	L(fm)	$M_\pi L$
A30.32	1.90	$32^3 \times 64$	0.0030	{0.0180,	{0.21256,	{0.34583, 0.40675,	275	2.84	3.96
A40.32			0.0040	0.0220,	0.25000,	0.47840, 0.56267,	315		4.53
A50.32			0.0050	0.0260}	0.29404}	0.66178, 0.77836,	351		5.04
A40.24		$24^3 \times 48$	0.0040			0.91546, 1.07672}	324	2.13	3.49
A60.24			0.0060				386		4.17
A80.24			0.0080				444		4.79
A100.24			0.0100				495		5.34
B25.32	1.95	$32^3 \times 64$	0.0025	{0.0155,	{0.18705,	{0.30433, 0.35794,	258	2.61	3.42
B35.32			0.0035	0.0190,	0.22000,	0.42099, 0.49515,	302		3.99
B55.32			0.0055	0.0225}	0.25875}	0.58237, 0.68495,	375		4.96
B75.32			0.0075			0.80561, 0.94752}	436		5.77
B85.24		$24^3 \times 48$	0.0085				467	1.96	4.63
D15.48	2.10	$48^3 \times 96$	0.0015	{0.0123,	{0.14454,	{0.23517, 0.27659,	220	2.97	3.31
D20.48			0.0020	0.0150,	0.17000,	0.32531, 0.38262,	254		3.83
D30.48			0.0030	0.0177}	0.19995}	0.45001, 0.52928,	308		4.65
						0.62252, 0.73217}			

Three values of the **lattice spacing**: 0.06 fm ÷ 0.09 fm

Different **volumes**: 2 fm ÷ 3 fm

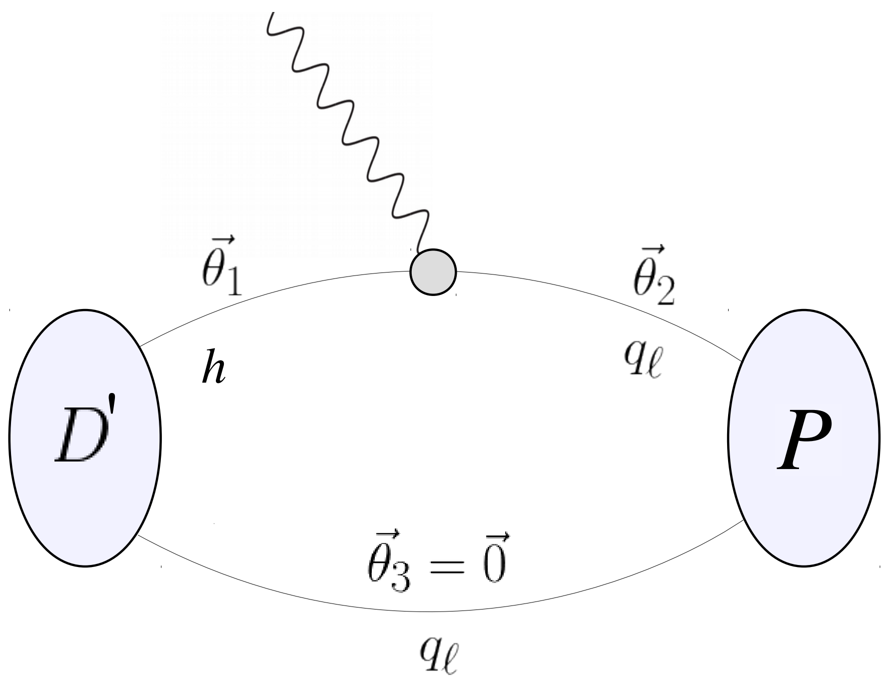
Pion masses in range 220 ÷ 440 MeV

heavy quark masses in range 1.5 ÷ 2.5 m_c

Lattice Spacings	
a(β = 1.90)	0.0885(36)fm
a(β = 1.95)	0.0815(30)fm
a(β = 2.10)	0.0619(18)fm

Simulation Details

To inject momenta we used non-periodic boundary conditions



β	V/a^4	θ
1.90	$32^3 \times 64$	$0.0, \ \pm 0.200, \ \pm 0.467, \ \pm 0.867$
	$24^3 \times 48$	$0.0, \ \pm 0.150, \ \pm 0.350, \ \pm 0.650$
1.95	$32^3 \times 64$	$0.0, \ \pm 0.183, \ \pm 0.427, \ \pm 0.794$
	$24^3 \times 48$	$0.0, \ \pm 0.138, \ \pm 0.321, \ \pm 0.596$
2.10	$48^3 \times 96$	$0.0, \ \pm 0.212, \ \pm 0.493, \ \pm 0.916$

$$\vec{p}_{D'} = \frac{2\pi}{L}\vec{\theta}_1 \qquad \vec{p}_P = \frac{2\pi}{L}\vec{\theta}_2 \qquad \vec{\theta} = \theta(1,1,1)$$

$$\boxed{p \quad = (0, \ \pm 151, \pm 353, \ \pm 656)\mathrm{MeV}}$$

<u>Momentum</u> range up to 0 ÷ 650 MeV

Both the D' and the π mesons can be either moving or at rest

Matrix Elements Extraction

The matrix elements V_0, V_i and S can be extracted for each ensemble and for different values of q^2 fitting the time dependence of ratios of 3-points and 2-points correlation functions

3-pts functions

$$C_{\widehat{\Gamma}}^{D'\pi}\,(t,\,t',\,\vec{p}_{D'},\,\vec{p}_{\pi})\;\xrightarrow[t\gg a\,,\,(t'-t)\gg a]{}\;\frac{Z_\pi Z_{D'}^*}{4E_\pi E_{D'}}\;\langle\pi(p_\pi)|\widehat{\Gamma}|D'(p_{D'})\rangle\;e^{-E_{D'}t}\,e^{-E_\pi(t'-t)}$$

2-pts functions

$$C_2(t,\vec{p}) = \xrightarrow[t\gg a]{} \frac{Z}{2E_0}\left(e^{-E_0 t}+e^{-E_0(T-t)}\right)$$

$$\tilde{C}_2(t,\vec{p}) = \frac{1}{2}\left[C_2(t,\vec{p})+\sqrt{C_2^D(t,\vec{p})^2-C_2^D(T/2,\vec{p})^2}\,\right]\xrightarrow[t\gg a]{}\frac{Z}{2E_0}e^{-E_0 t}$$

Ratios

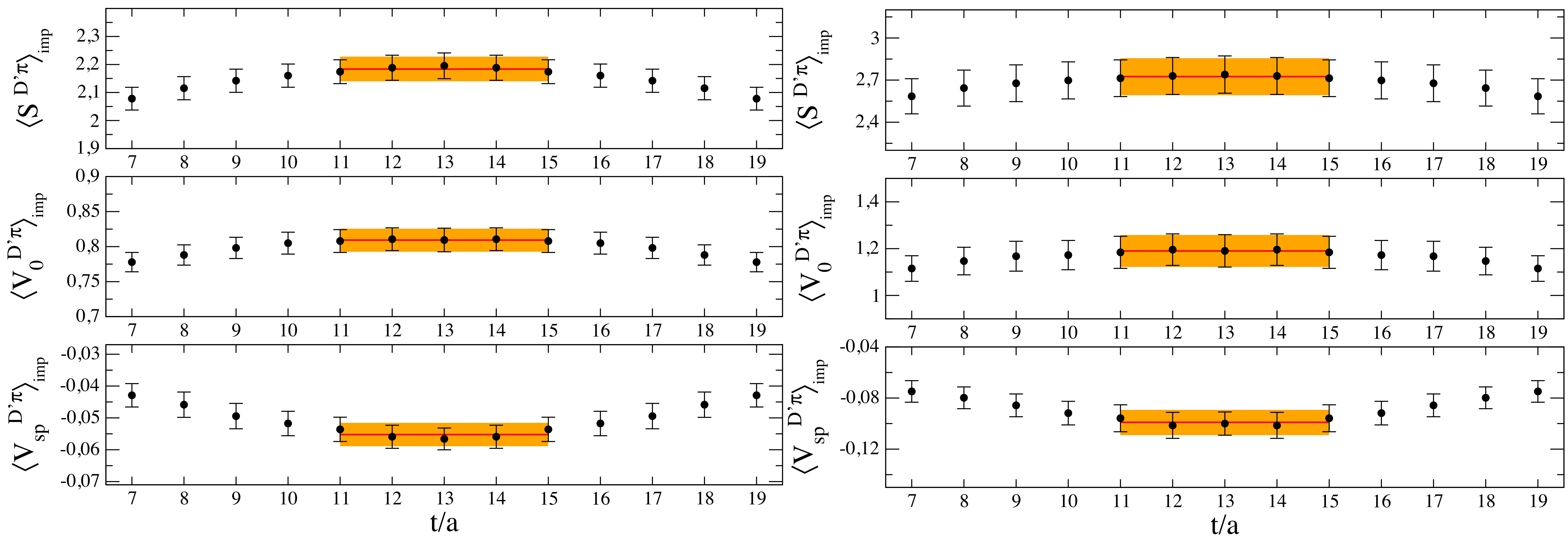
$$R_\mu(t,\vec{p}_{D'},\vec{p}_\pi)\equiv 4\,p_{D'\mu}\,p_{\pi\mu}\,\frac{C_{V_\mu}^{D'\pi}(t,t',\vec{p}_{D'},\vec{p}_\pi)\,C_{V_\mu}^{\pi D'}(t,t',\vec{p}_\pi,\vec{p}_{D'})}{C_{V_\mu}^{\pi\pi}(t,t',\vec{p}_\pi,\vec{p}_\pi)\,C_{V_\mu}^{D'D'}(t,t',\vec{p}_{D'},\vec{p}_{D'})}\quad\xrightarrow[t\gg a\;(t'-t)\gg a]{}\quad|\langle\pi(p_\pi)|\widehat{V}_\mu|D'(p_{D'})\rangle|^2$$

$$R_S(t,\vec{p}_{D'},\vec{p}_\pi)\equiv 4\,E_{D'}\,E_\pi\,\frac{C_S^{D'\pi}(t,t',\vec{p}_{D'},\vec{p}_\pi)\,C_S^{\pi D'}(t,t',\vec{p}_\pi,\vec{p}_{D'})}{\widetilde{C}_2^{D'}(t',\vec{p}_{D'})\;\widetilde{C}_2^{\pi}(t',\vec{p}_\pi)}\;,\quad\xrightarrow[t\gg a\;(t'-t)\gg a]{}\quad|\langle\pi(p_\pi)|S|D'(p_{D'})\rangle|^2$$

Matrix Elements Plateaux

Example of the time dependence of the ratio R and the extraction of the matrix elements

Ensemble D20.48



fit intervals:

β	V/a^4	$[t_{\min},\, t_{\max}]_{(\ell\ell,\ell s)}/a$	$[t_{\min},\, t_{\max}]_{(\ell c)}/a$	$[t_{\min},\, t_{\max}]_{(\ell h)}/a$	t'/a
1.90	$32^3 \times 64$	[12, 31]	[8, 16]	[8, 13]	18
	$24^3 \times 48$	[12, 23]	[8, 17]	[8, 14]	18
1.95	$32^3 \times 64$	[13, 31]	[9, 18]	[10, 16]	20
	$24^3 \times 48$	[13, 23]	[9, 18]	[10, 16]	20
2.10	$48^3 \times 96$	[18, 40]	[12, 26]	[13, 24]	26

p_D=-151 MeV
p_π=151 MeV
M_π=254 MeV
$M_{D'}$=2441 MeV

p_D=-151 MeV
p_π=151 MeV
M_π=254 MeV
$M_{D'}$=3736 MeV

From Matrix Elements To Form Factors

Using both matrix elements we over-constrain f_0 and f_+ and we get the determination of the form factors on each ensemble for the different values of q^2 from a combined fit

$$\langle \widehat{V}_0 \rangle_{\rm imp} = (E_{D'} + E_\pi) f_+(q^2) + (E_{D'} - E_\pi) \frac{M_{D'}^2 - M_\pi^2}{q^2} \left[f_0(q^2) - f_+(q^2) \right] + \mathcal{O}(a^2)$$

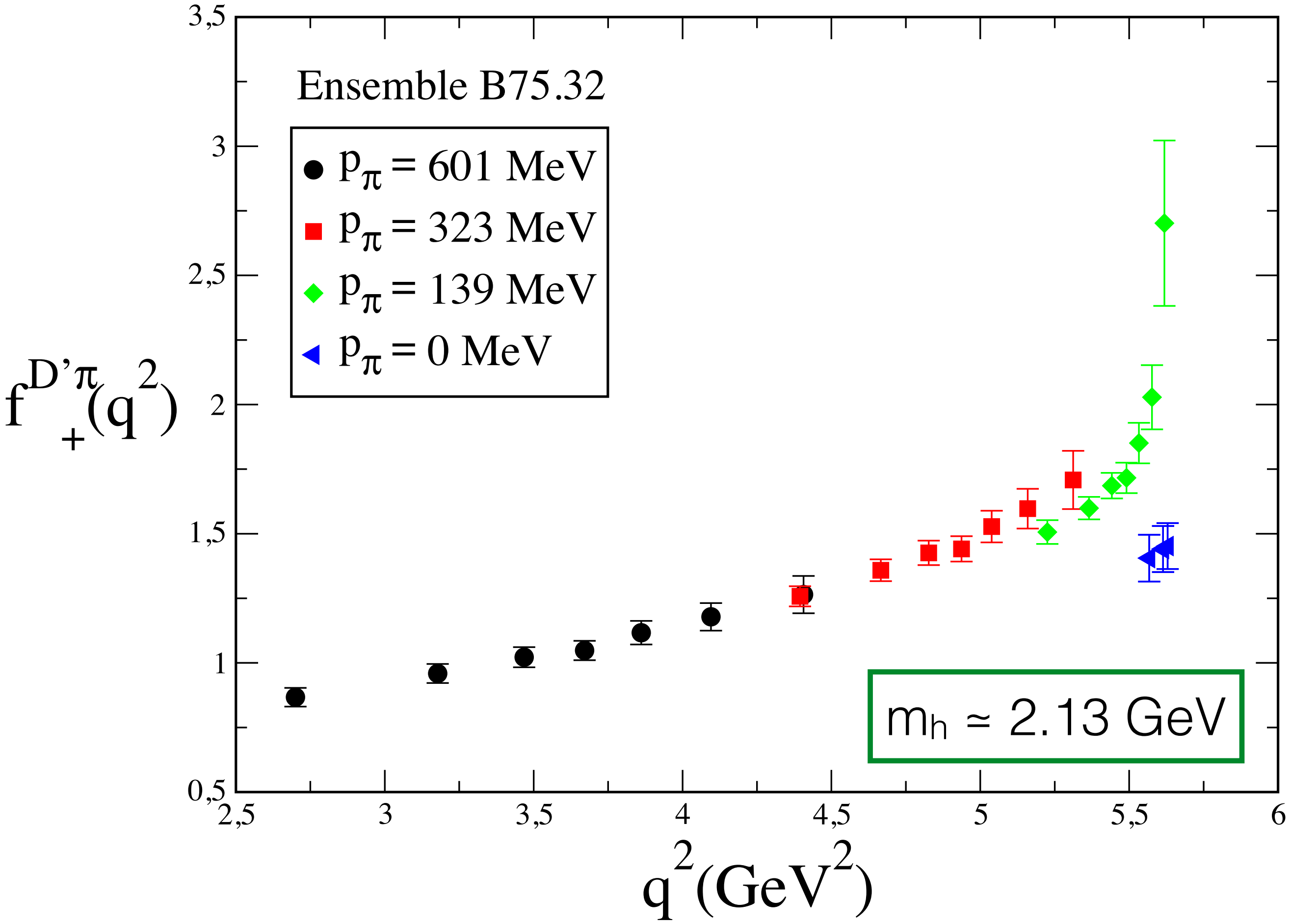
$$\langle \widehat{V}_{\rm sp} \rangle_{\rm imp} = \left\{ (p_{D'} + p_\pi) f_+(q^2) + (p_{D'} - p_\pi) \frac{M_{D'}^2 - M_\pi^2}{q^2} \left[f_0(q^2) - f_+(q^2) \right] \right\} + \mathcal{O}(a^2)$$

$$\langle S \rangle_{\rm imp} = \frac{M_{D'}^2 - M_\pi^2}{\mu_h - \mu_q} f_0(q^2) + \mathcal{O}(a^2)$$

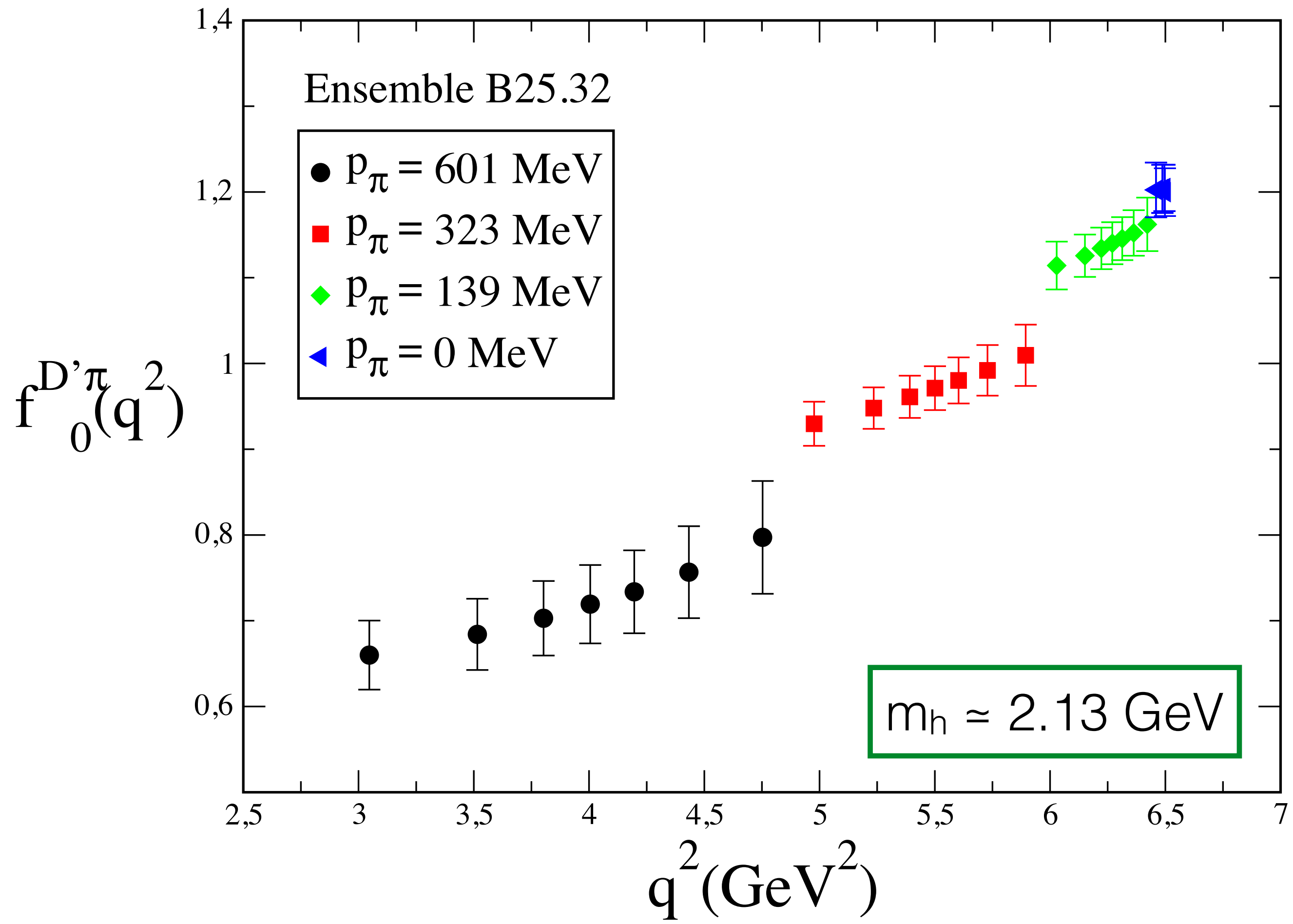
Evidence Of Hypercubic Effects

Hypercubic effects: breaking of the Lorentz invariance

M_π=436 MeV



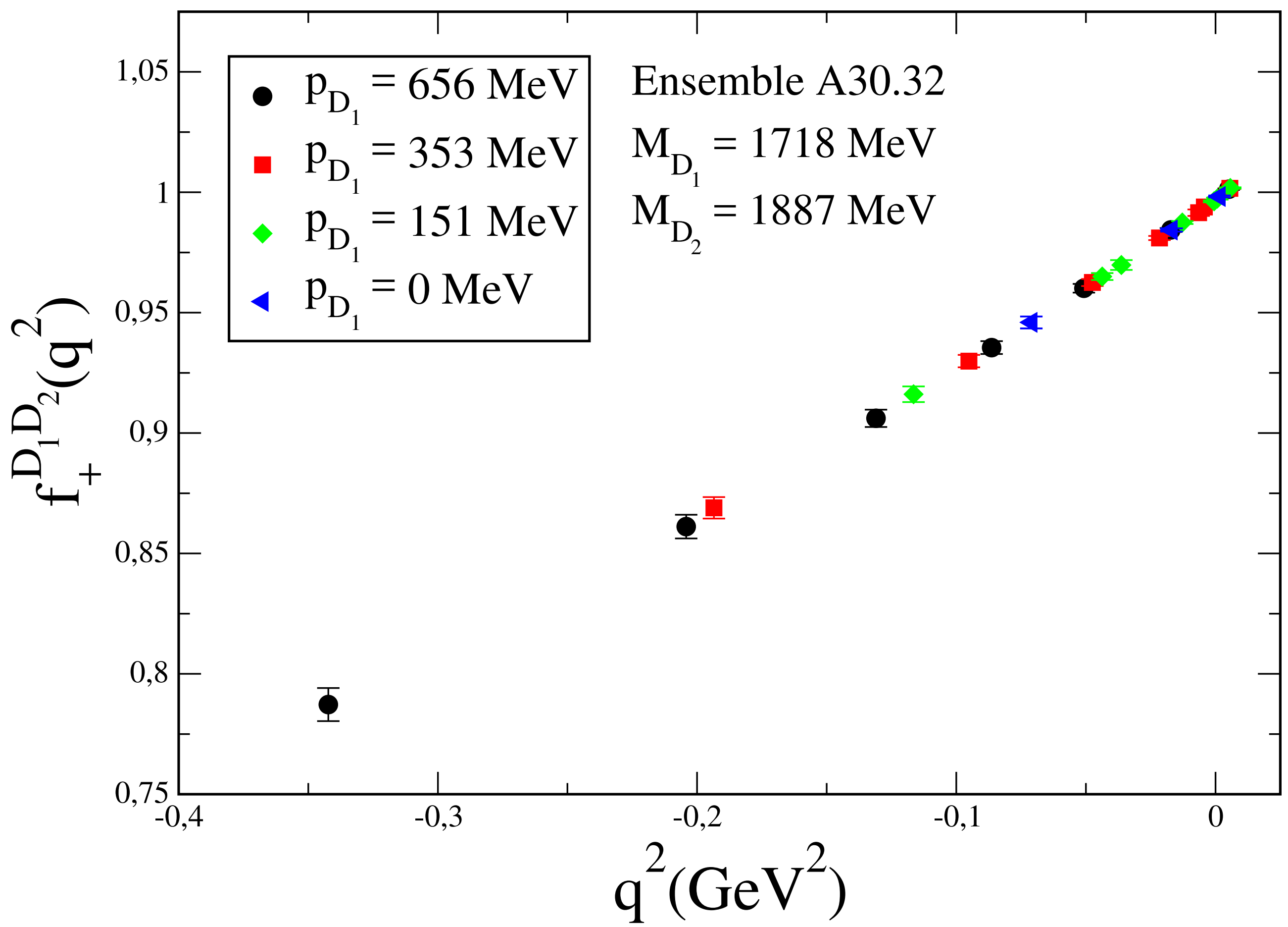
M_π=258 MeV

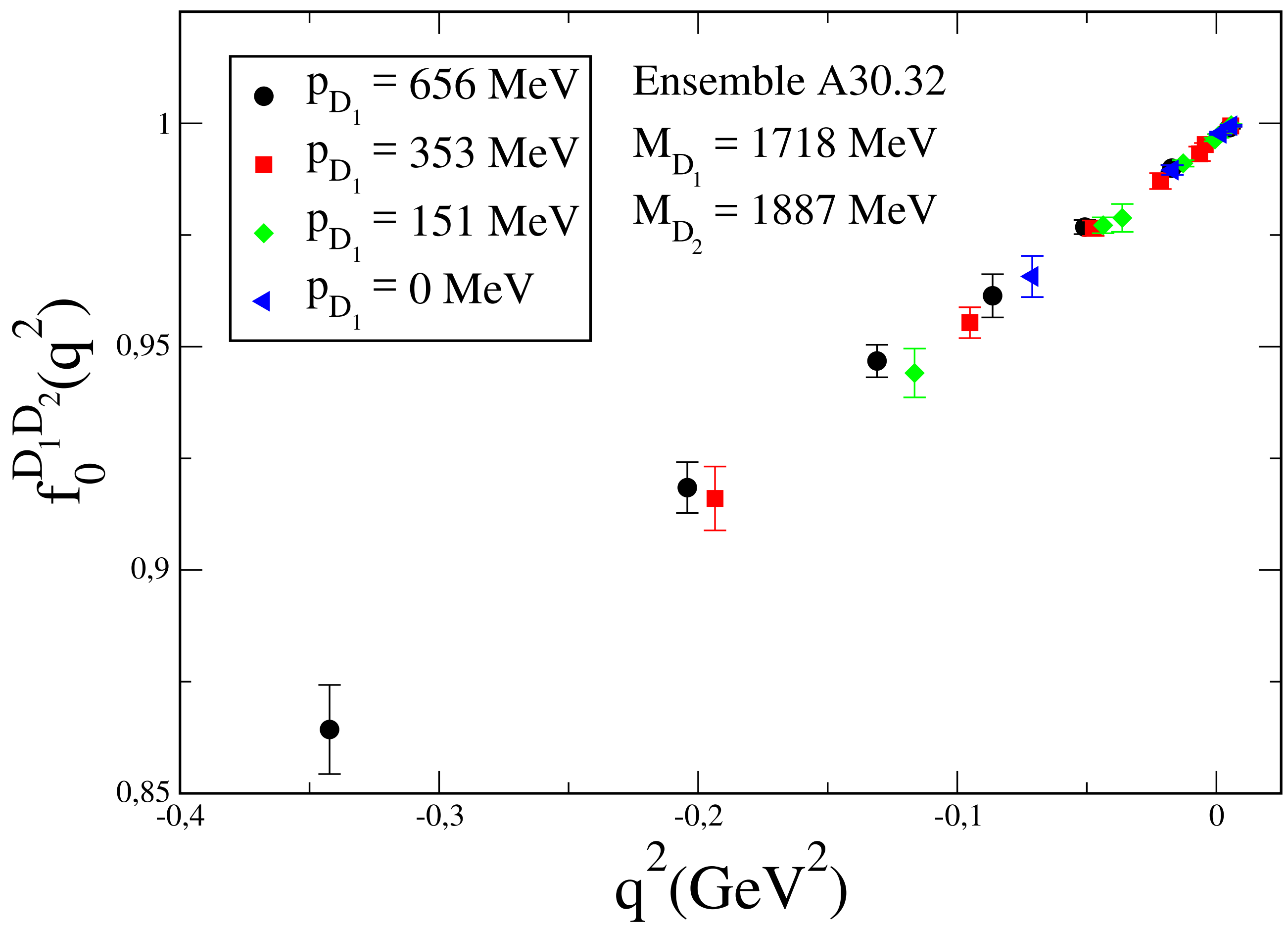


- Form factors are not function of q^2 only. We need to estimate and subtract these effects
- This subtraction cannot be done at the level of a single ensemble (with the present data). We need a global fit

Elastic Form Factor

Hypercubic effects seem to disappear in the M_1=M_2 limit

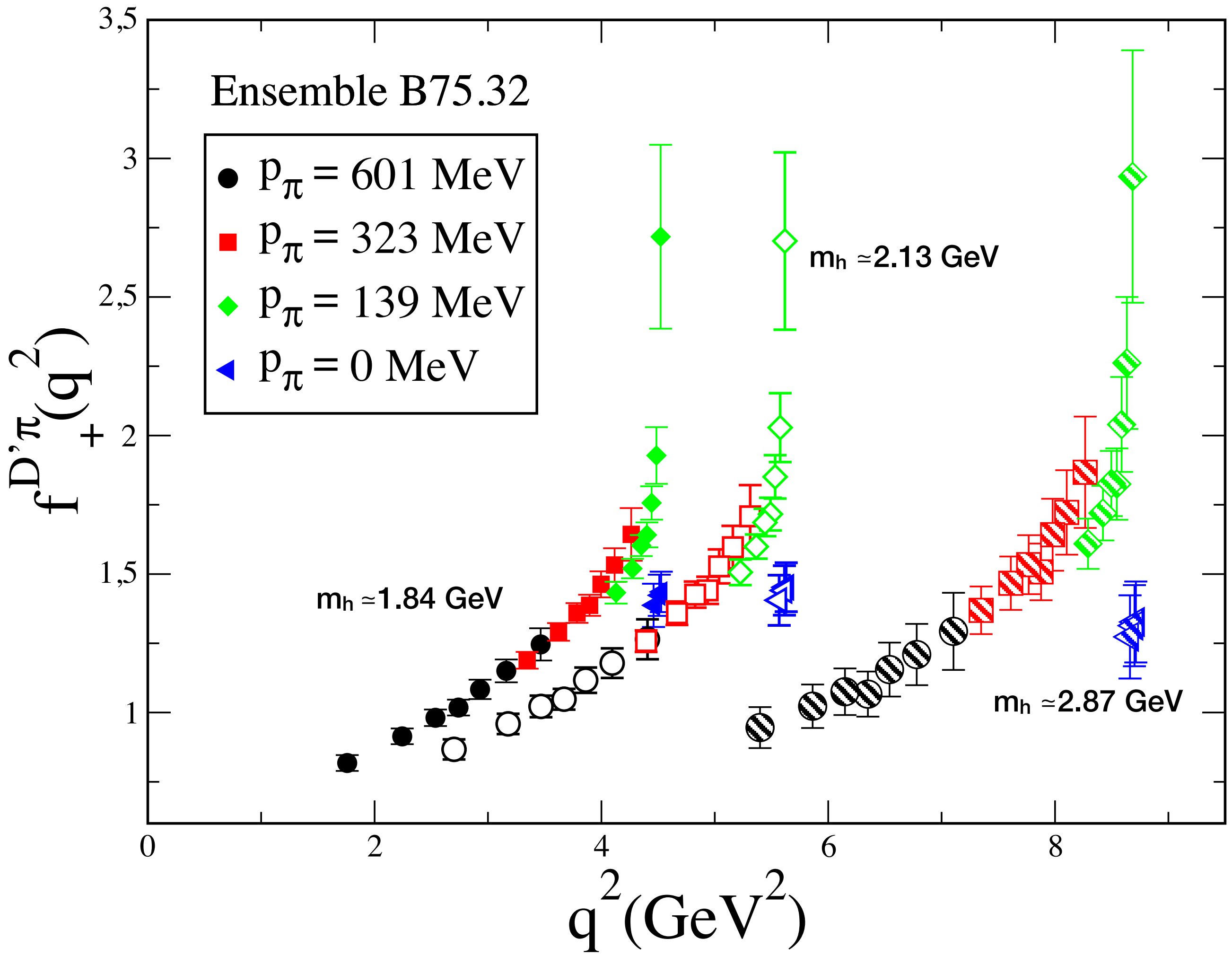




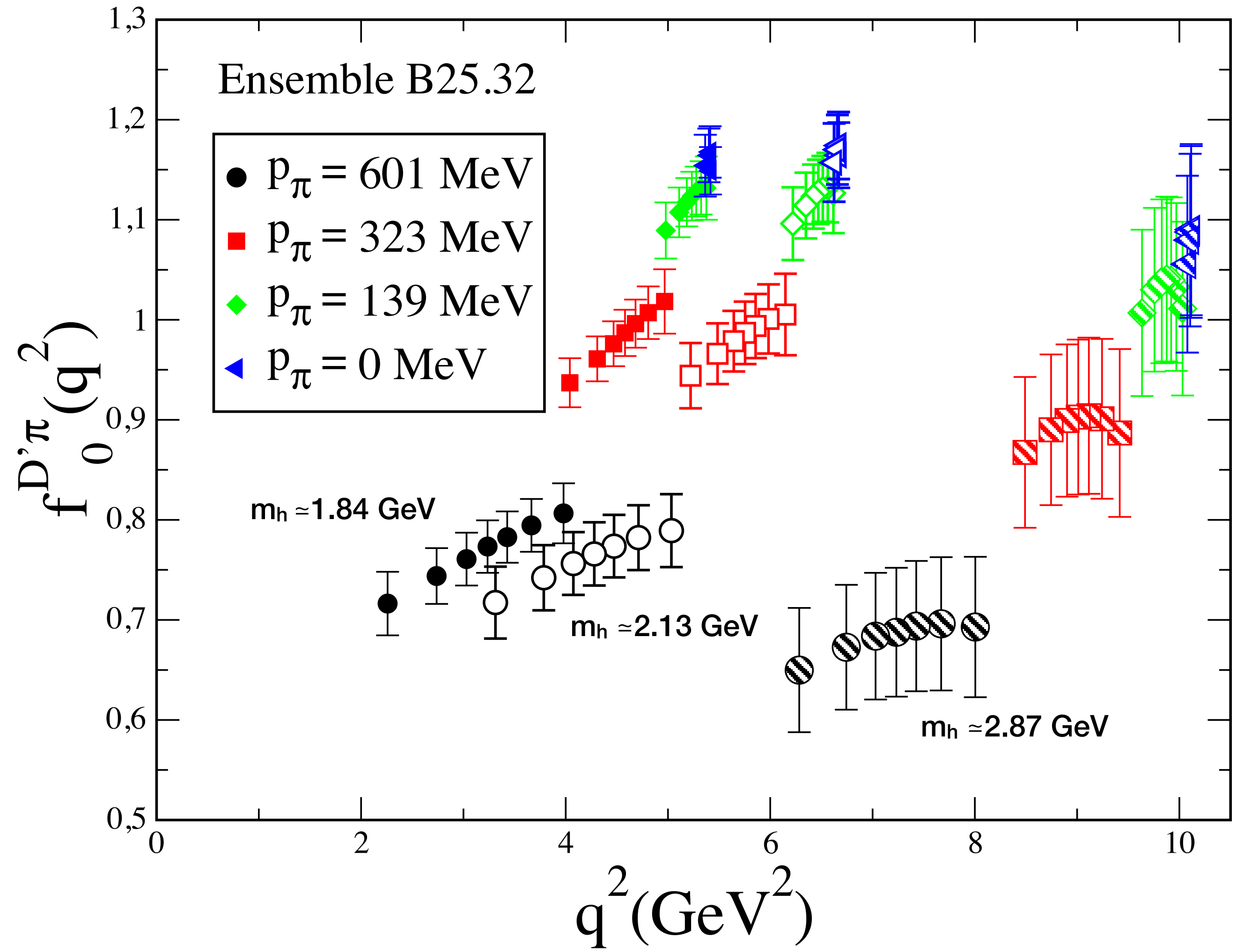
Evidence Of Hypercubic Effects

Is this getting worst as the meson gets heavier?

M_π=436 MeV



M_π=258 MeV



Global Fit

Global fit studying simultaneously a^2 m_l and q^2 dependencies to extract the physical form factors

ingredients:

- the matrix elements are decomposed in a Lorentz-covariant part + a Lorentz-breaking Hypercubic part proportional to a^2
- Lorentz-breaking part must transform properly under hypercubic rotation
- Lorentz-breaking part will contain additional form factors
- The physical form factors will be described with modified z-expansion

Global Fit

Hypercubic effects

$$q_\mu = (p_{D'} - p_\pi)_\mu$$

$$P_\mu = (p_{D'} + p_\pi)_\mu$$

$$q^E_\mu = (\vec{q},\ q_4) = (\vec{q},\ iq_0)$$

$$q^{[n]} P^{[m]} \equiv \sum_\mu \left(q^E_\mu\right)^n \left(P^E_\mu\right)^m$$

<u>V_μ matrix elements decompositions</u>

$$\langle \pi(p_\pi) | \widehat{V}^E_\mu | D'(p_{D'}) \rangle = \langle \widehat{V}^E_\mu \rangle_{\mathrm{Lor}} + \langle \widehat{V}^E_\mu \rangle_{\mathrm{hyp}}$$

$$\langle \widehat{V}^E_\mu \rangle_{\mathrm{Lor}} = P^E_\mu\ f_+(q^2, a^2) + q^E_\mu\ \frac{M^2_{D'} - M^2_\pi}{q^2} \left[f_0(q^2, a^2) - f_+(q^2, a^2) \right]$$

$$\langle \widehat{V}^E_\mu \rangle_{\mathrm{hyp}} = a^2 \left[\left(q^E_\mu\right)^3\ H_1 + \left(q^E_\mu\right)^2 P^E_\mu\ H_2 + q^E_\mu \left(P^E_\mu\right)^2\ H_3 + \left(P^E_\mu\right)^3\ H_4 \right]$$

$\langle V_\mu \rangle_{\mathrm{hyp}}$ is the most general structure up to $O(a^2)$ that transforms properly under hypercubic rotations

this decomposition implies that $f_{+,0}$ depends on q^2 and on the hypercubic invariants $q^{[4]}$, $q^{[3]}P^{[1]}$, $q^{[2]}P^{[2]}$, $q^{[1]}P^{[3]}$, $P^{[4]}$

H_i are assumed to depend only on q^2, $M_{D'}$ and M_π

$$H_i(z) = d^i_0 + d^i_1 z + d^i_2 z^2$$

Global Fit

[Phys. Rev. D82 (2010) 114506]

Modified z expansion

BCL
[Phys.Rev. D79 (2009) 013008]

$$f_+^{D'\to\pi}(q^2,a^2) = \frac{f^{D'\to\pi}(0,a^2) + c_+(a^2)\,(z-z_0)\left(1+\frac{z+z_0}{2}\right)}{1-\frac{q^2}{M_{D'}^2+\Delta^2}\,(1+P_+a^2)}$$

$$f_0^{D'\to\pi}(q^2,a^2) = \frac{f^{D'\to\pi}(0,a^2) + c_0(a^2)\,(z-z_0)\left(1+\frac{z+z_0}{2}\right)}{1-\frac{q^2}{M_S^2}}$$

$$z = \frac{\sqrt{t_+-q^2}-\sqrt{t_+-t_0}}{\sqrt{t_+-q^2}+\sqrt{t_+-t_0}}$$

$$t_+ = (M_{D'}+M_\pi)^2 \ ,$$

$$t_0 = (M_{D'}+M_\pi)\left(\sqrt{M_{D'}}-\sqrt{M_\pi}\right)^2$$

for $c_{+,0}$ we adopted a simple linear dependence in a^2

$M_{D'}$ is the PS mass calculated on the lattice and $\Delta^2 = M_{D^*}^2 - M_D^2$

M_S is left as a free parameter

Hard Pion SU(2) ChPT
Bijnens & Jemos
[Phys.Rev. D79 (2009) 013008]

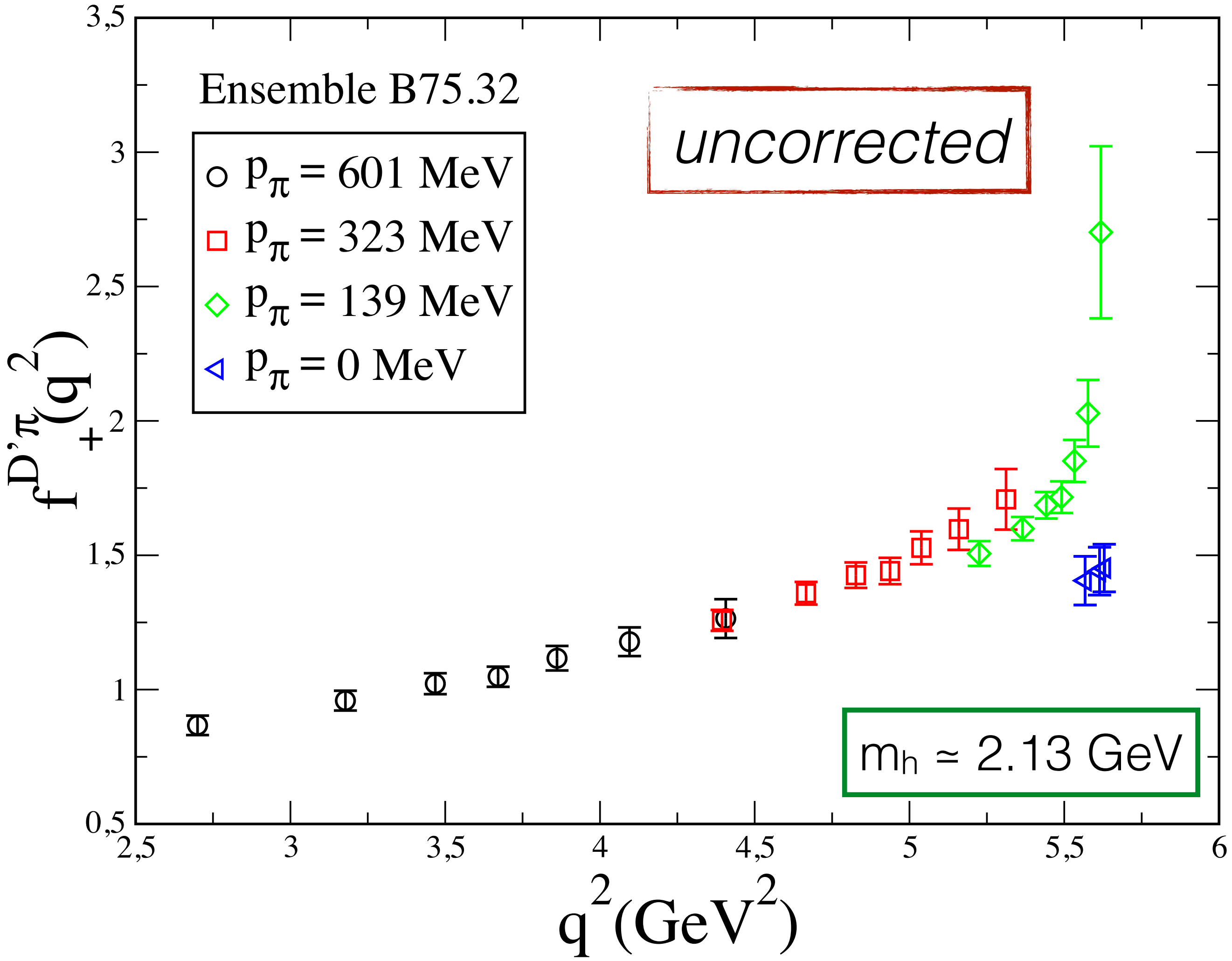
$$f^{D'\to\pi}(0,a^2) = F_+\left[1 + -\frac{3}{4}\left(1+3\hat{g}^2\right)\xi_\ell\log\xi_\ell + b_1\,\xi_\ell + b_2\,\xi_\ell^2 + b_3\,a^2\right]$$

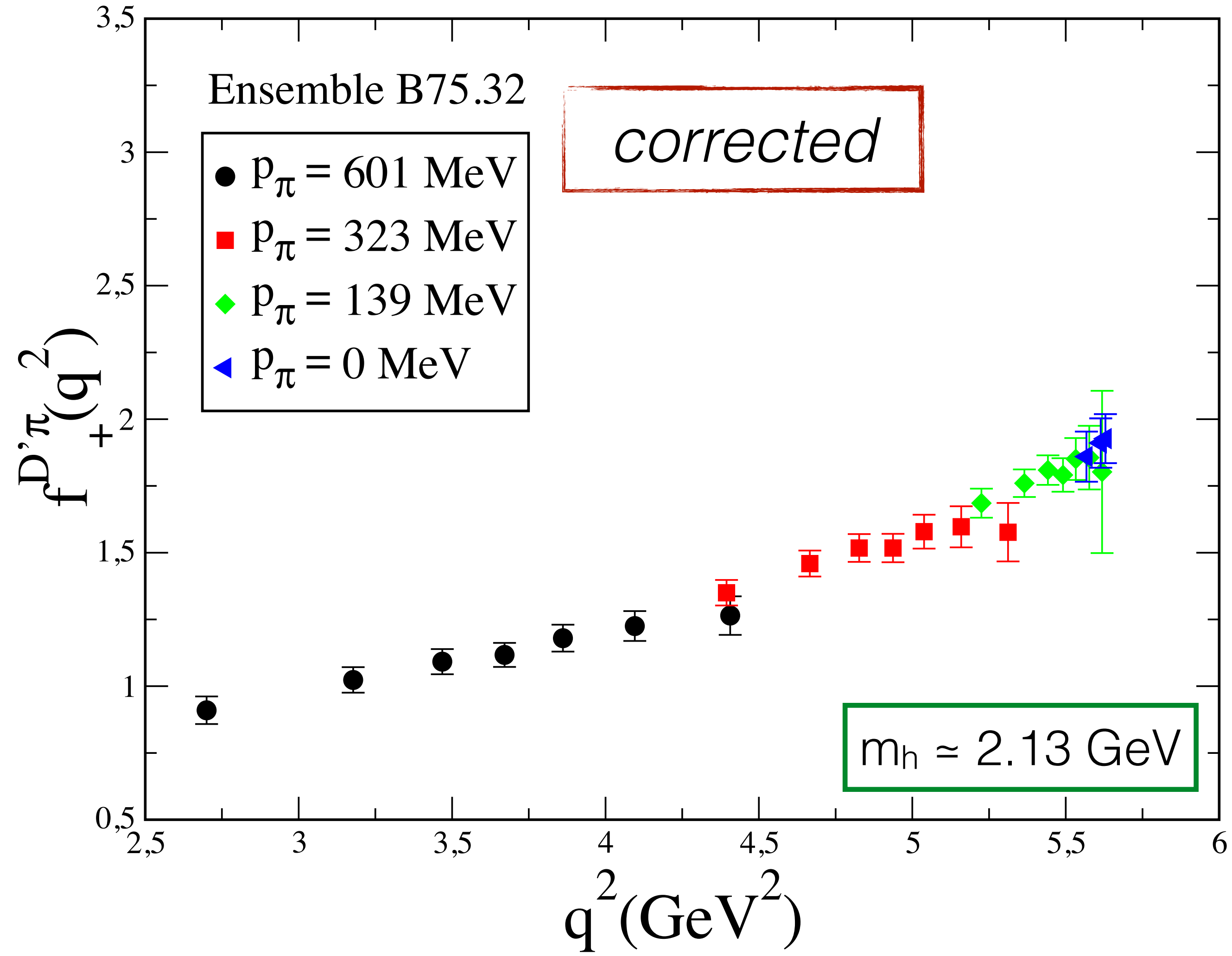
Results

Subtraction of the hypercubic effects in the D'→π vector form factor and restored q^2 dependence

f plus

M_π=258 MeV



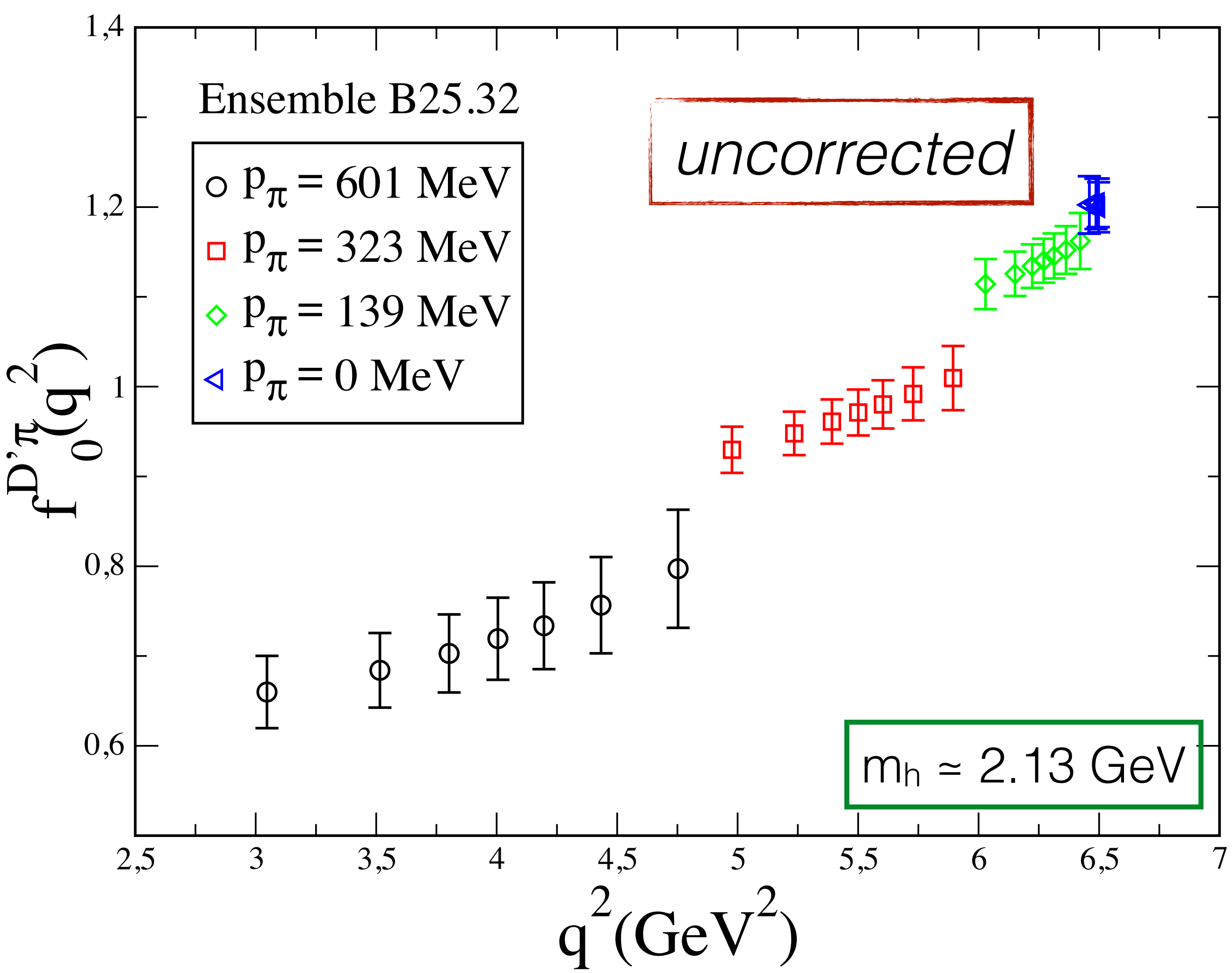


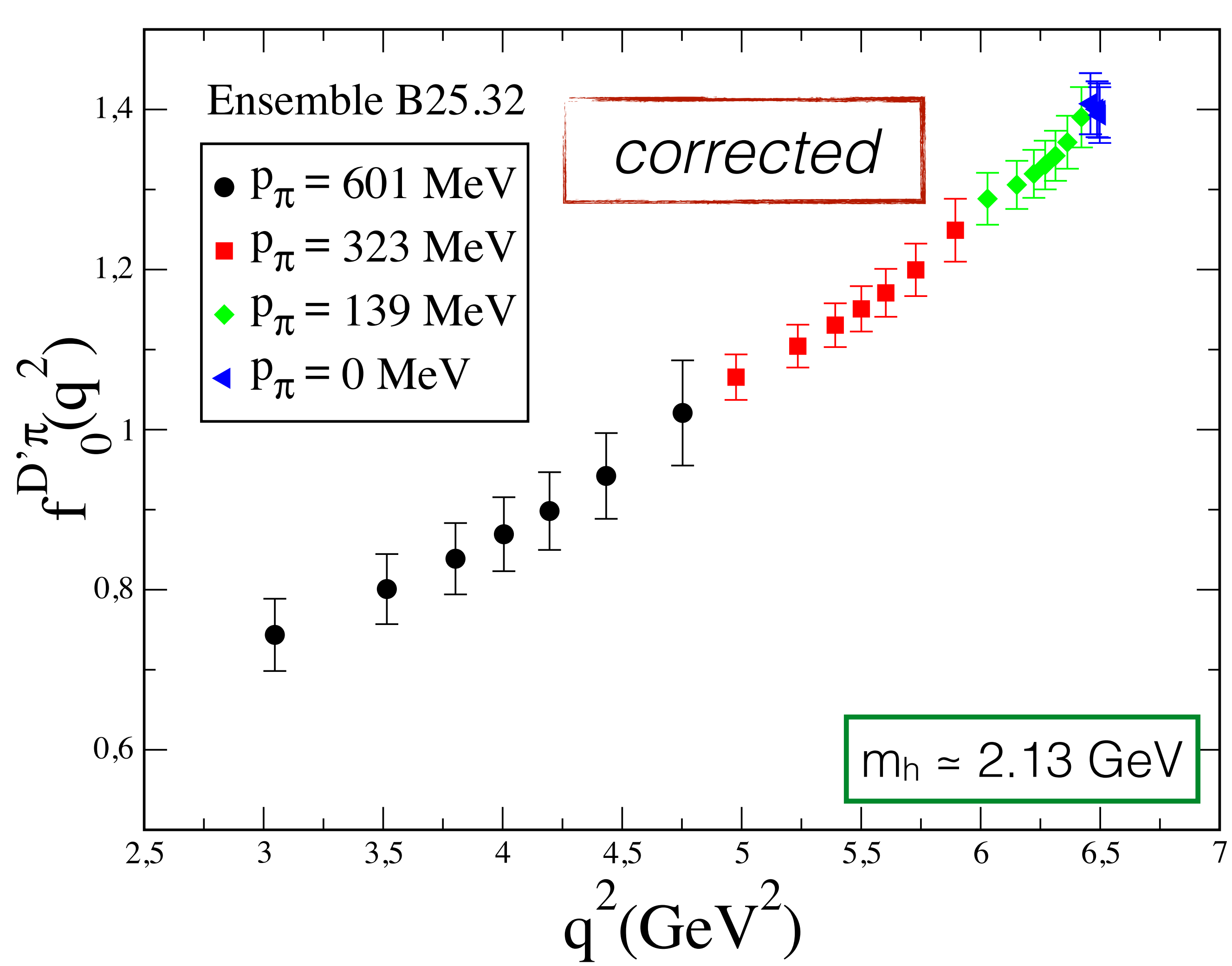
Results

Subtraction of the hypercubic effects in the D'→π vector form factor and restored q² dependence

f zero

M_π=436 MeV

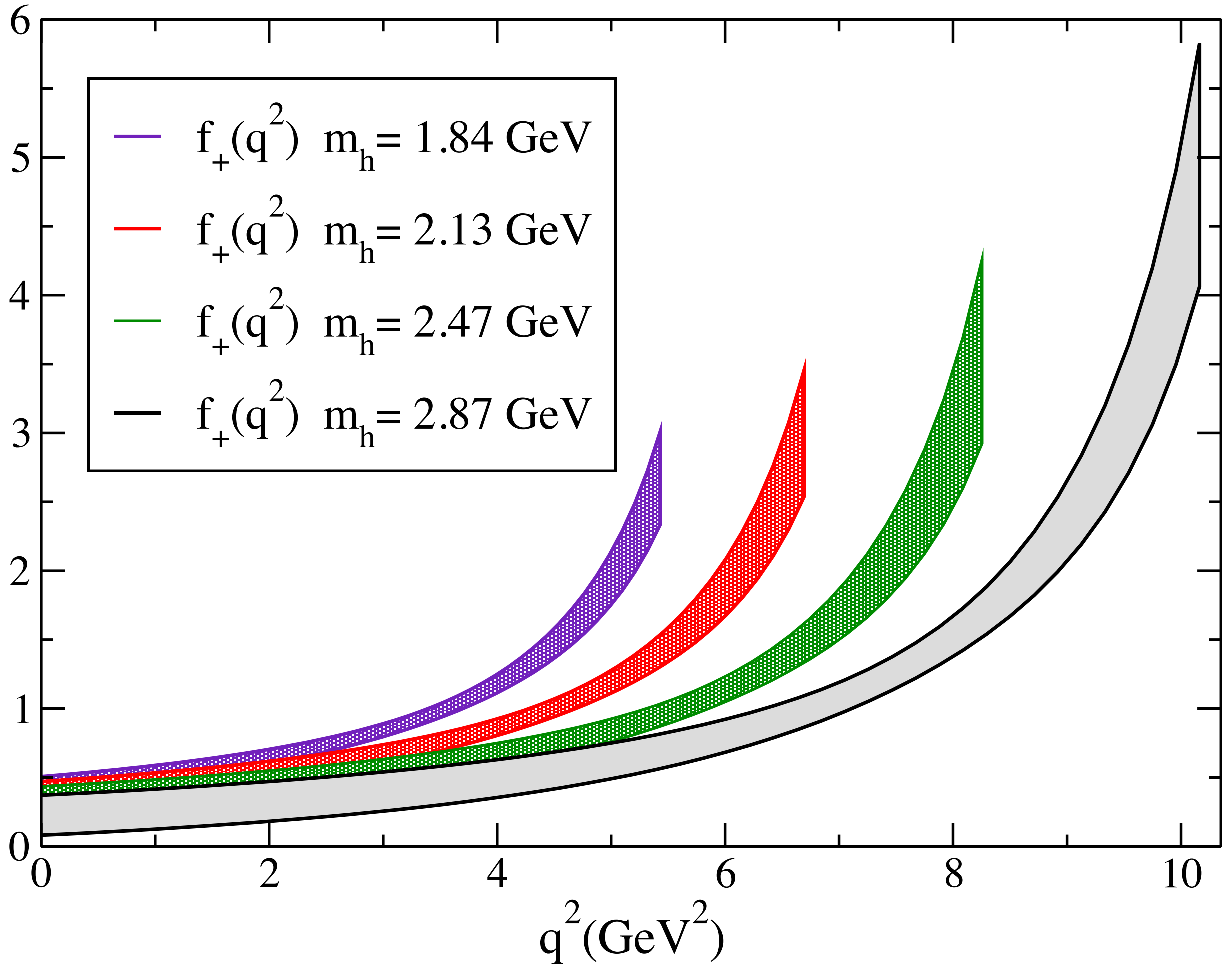




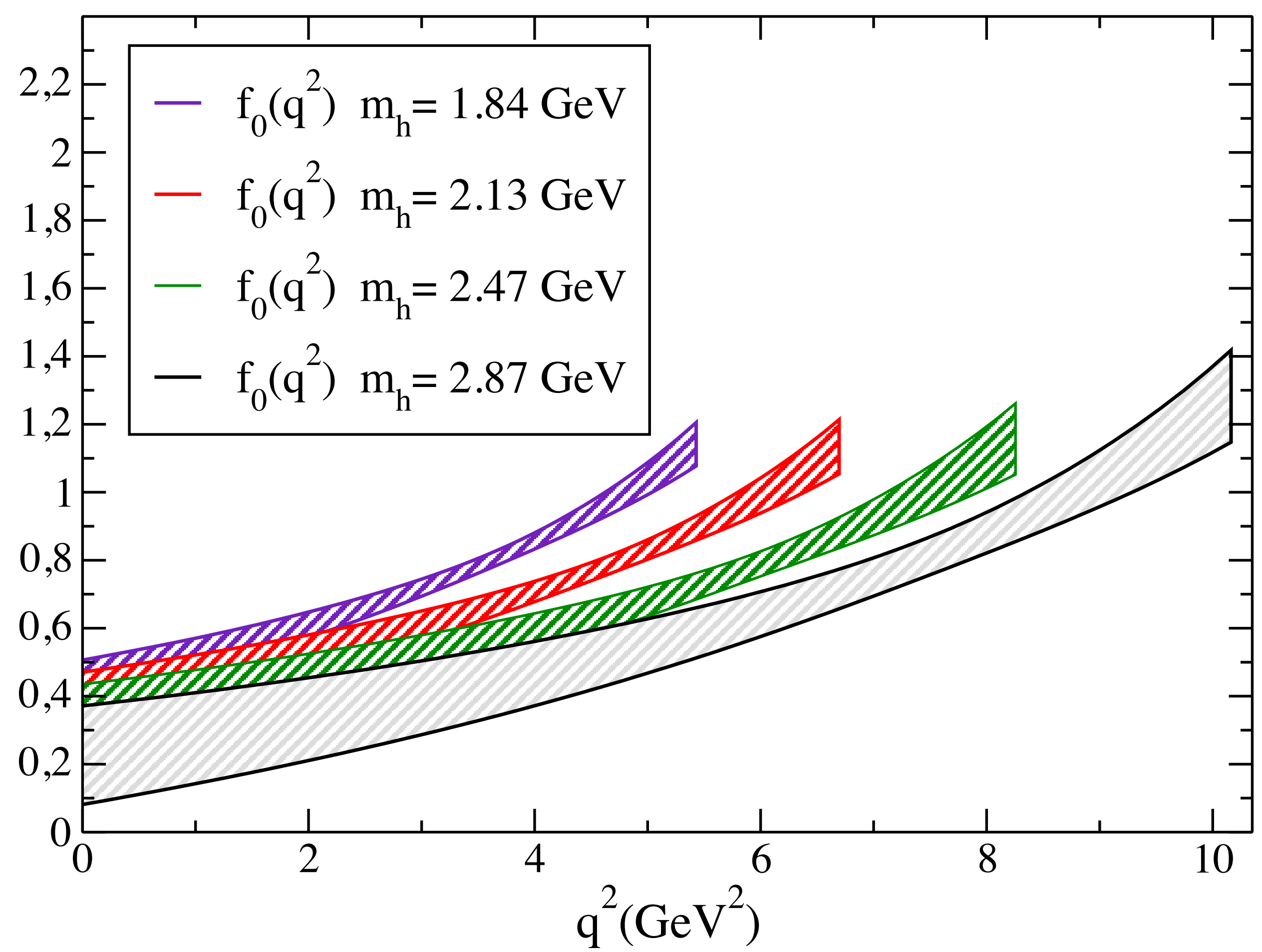
Results in the continuum

D' $\longrightarrow \pi$ vector and scalar form factors in the continuum as the D' mass gets heavier

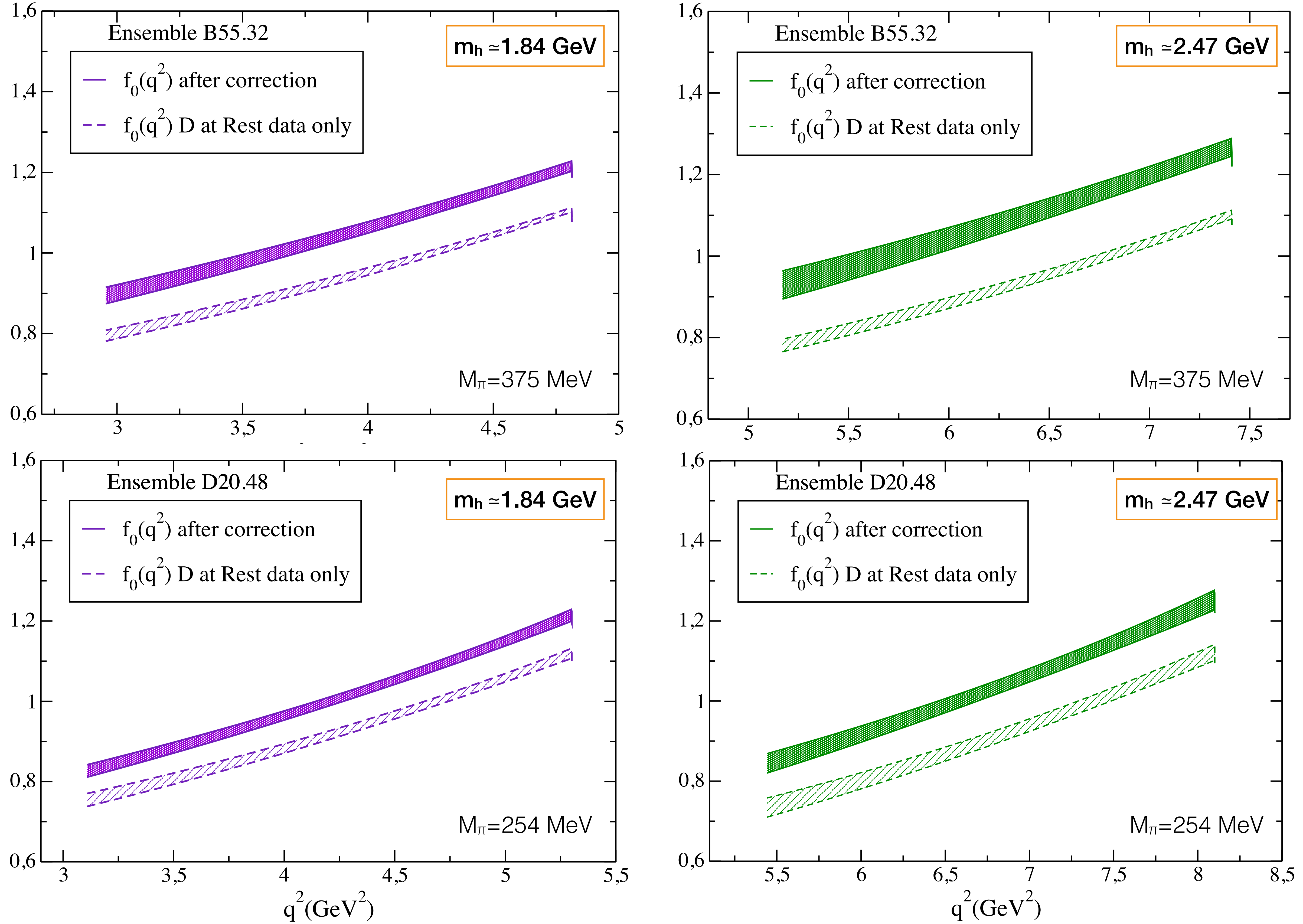
f plus



f zero



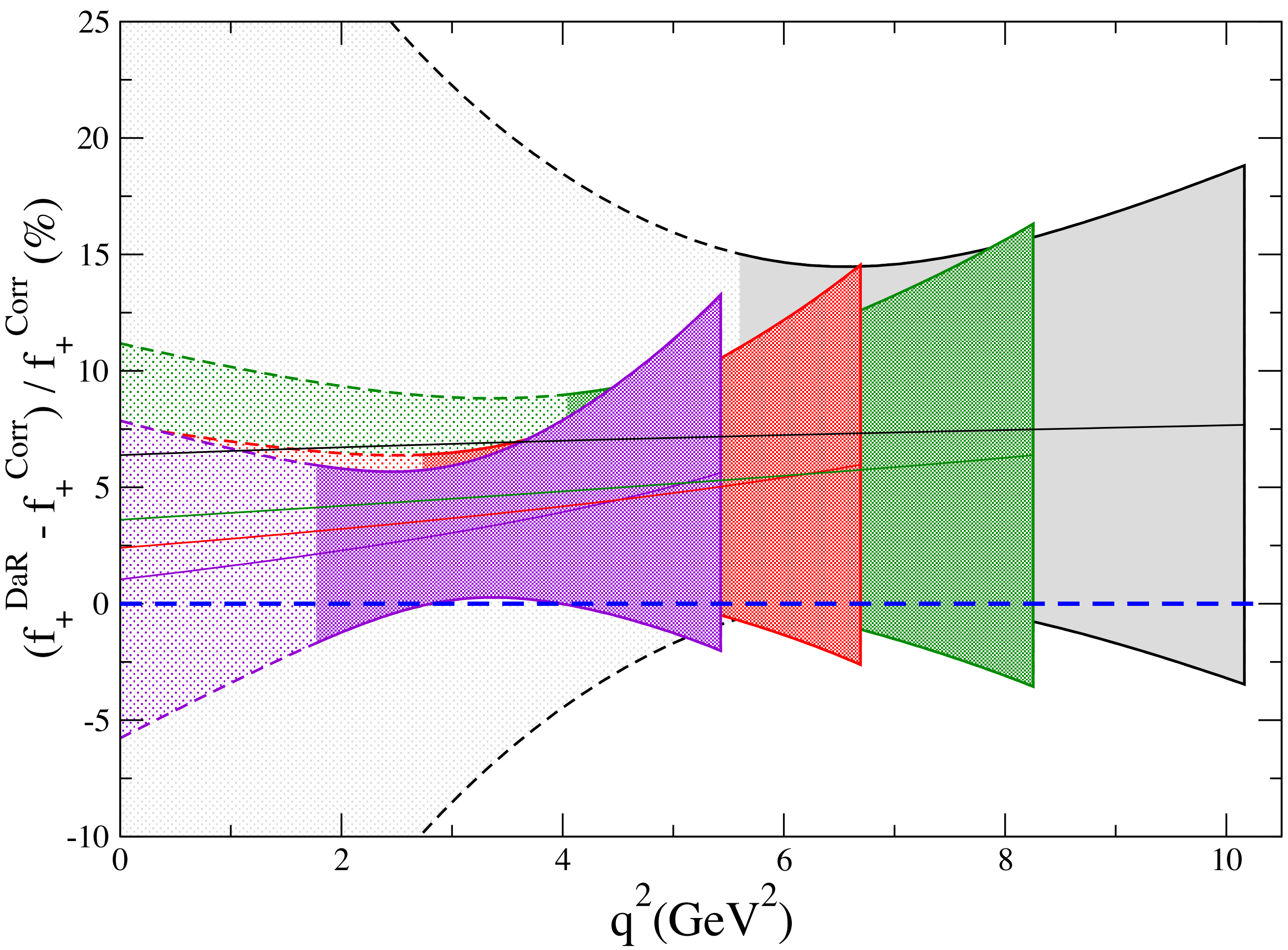
Comparison With Heavy Meson At Rest - on the lattice



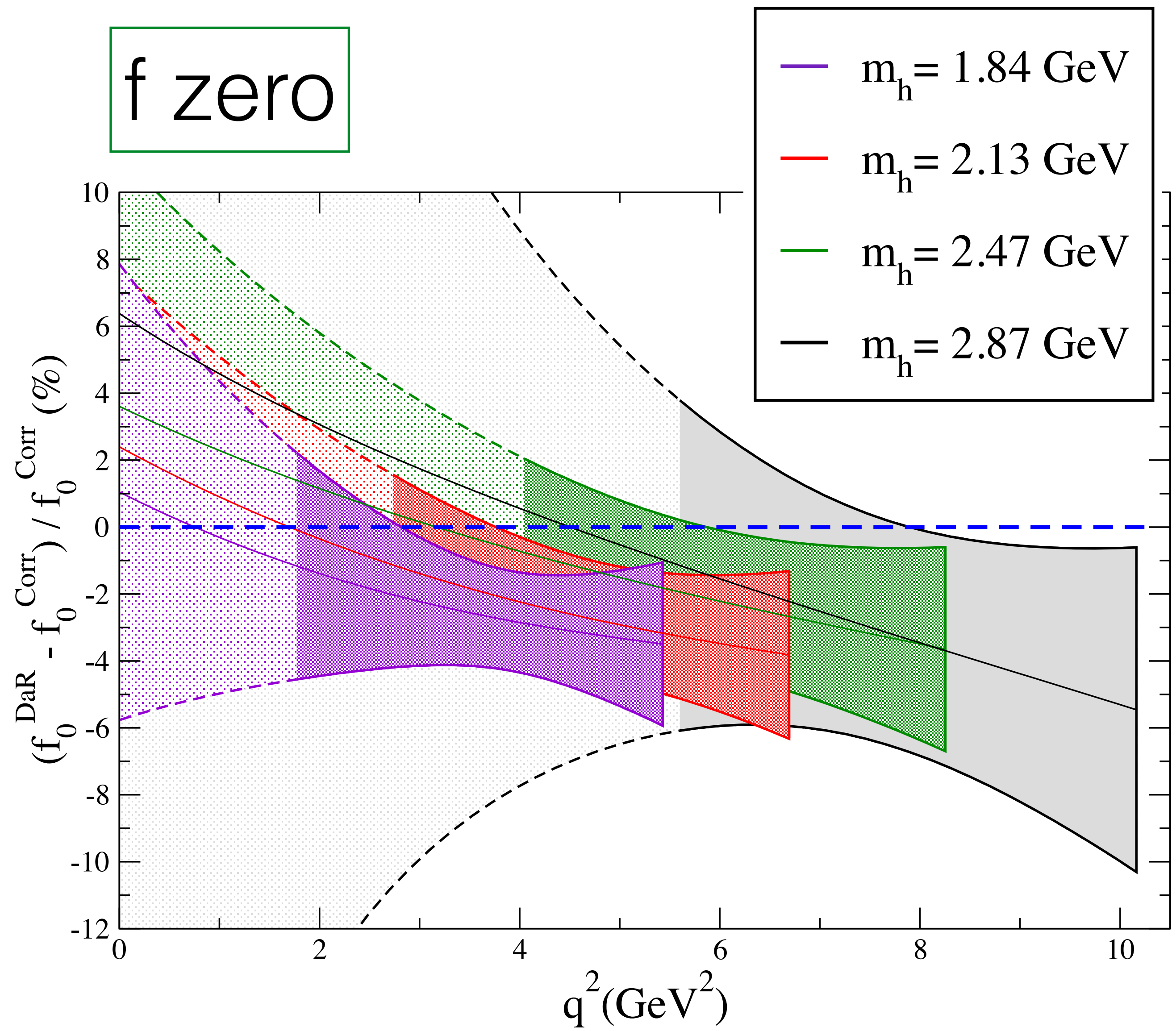
Comparison With Heavy Meson At Rest - in the continuum

Percentage difference between the form factors obtained in our analysis using all available data (corrected for hyper cubic effect) or using heavy meson at rest data (uncorrected for hyper cubic effect)

f plus

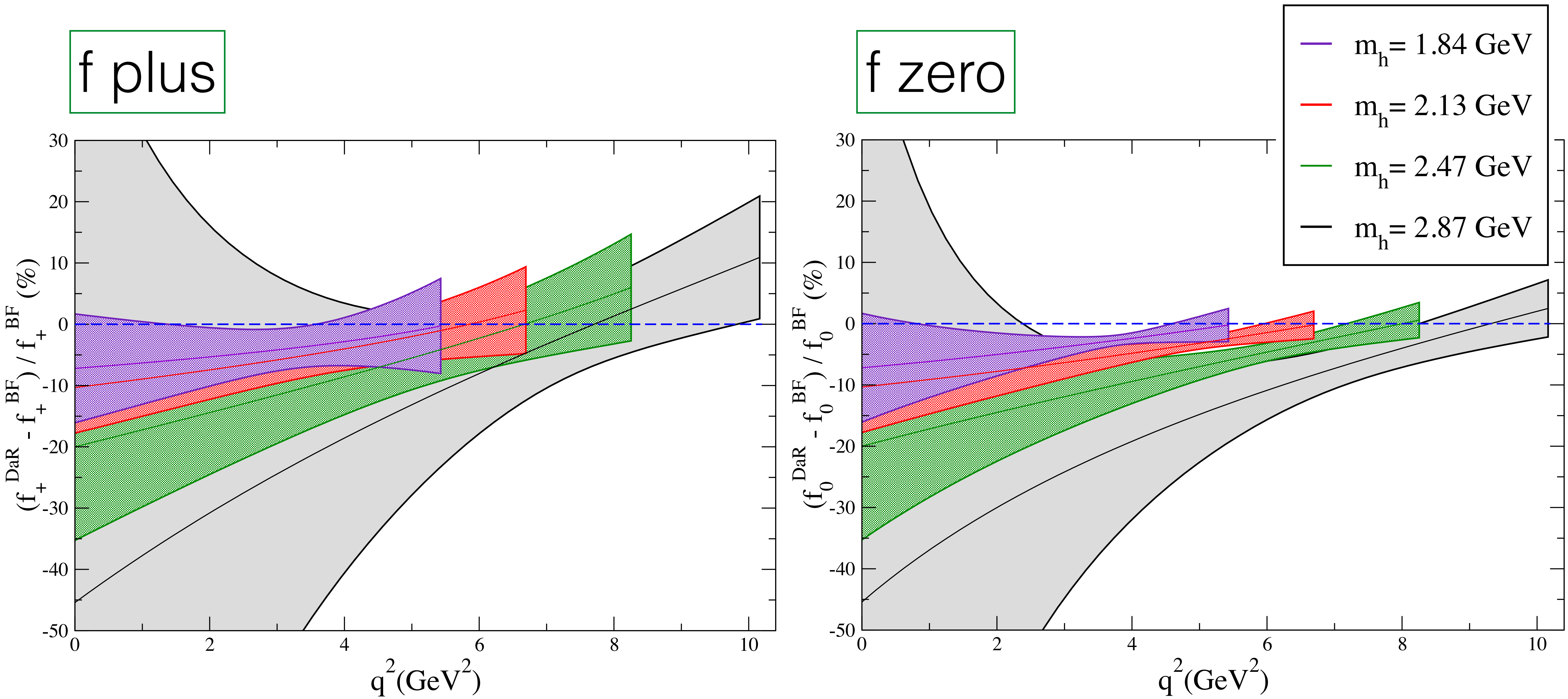


f zero



Comparison Between Different Kinematics

Percentage difference between the form factors obtained using only heavy meson at rest data or Breit Frame data



Conclusions

The breaking of the Lorentz symmetry due to hypercubic effects we observe in the vector, scalar and tensor form factor for the $D \to \pi(K)$ transitions is still there for transition with heavier father meson

However our precision rapidly degrades as we move to heavier masses

The next necessary step will be to improve the quality of the signals

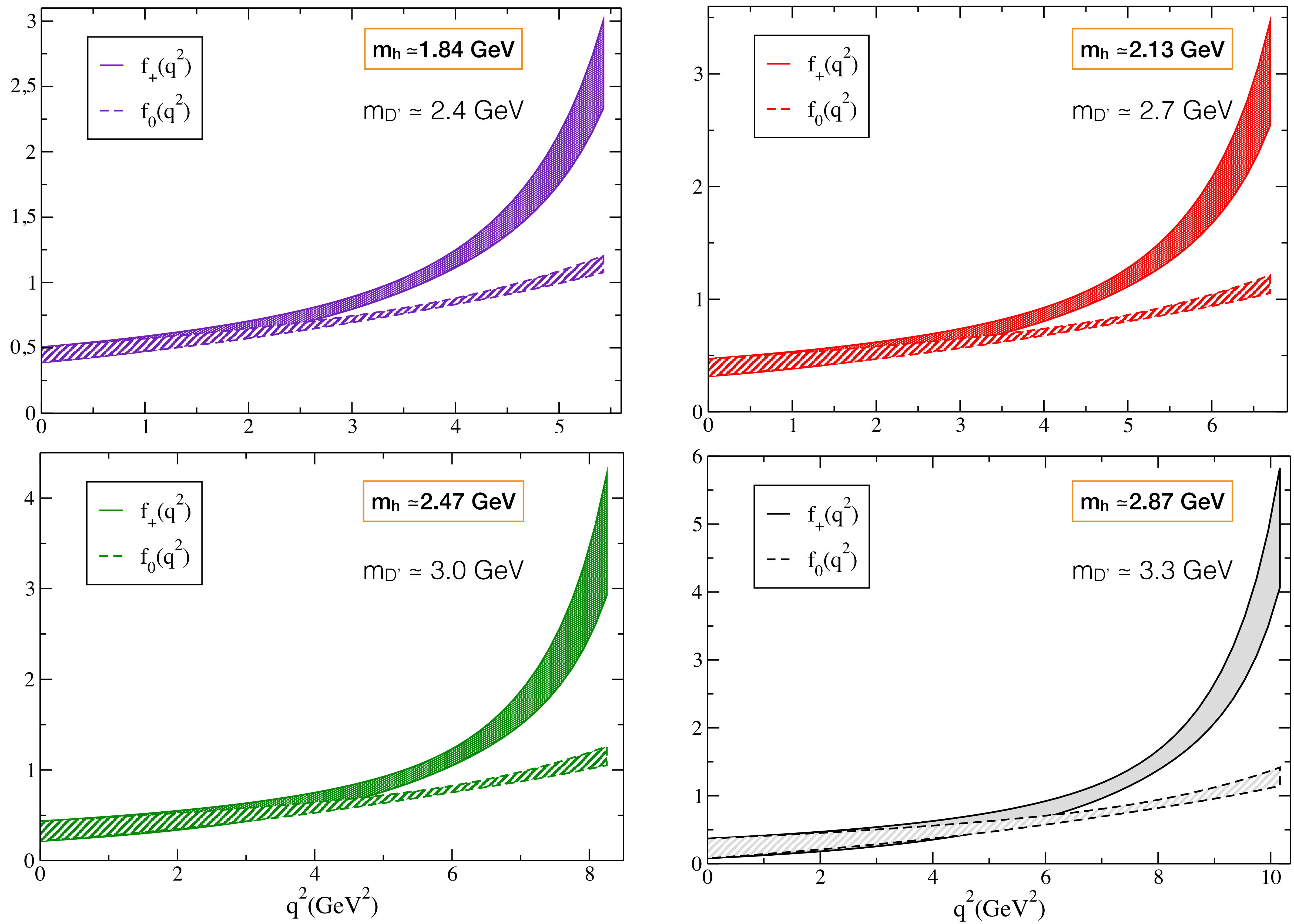
Backup slides

Simulation Details

Something on the action:

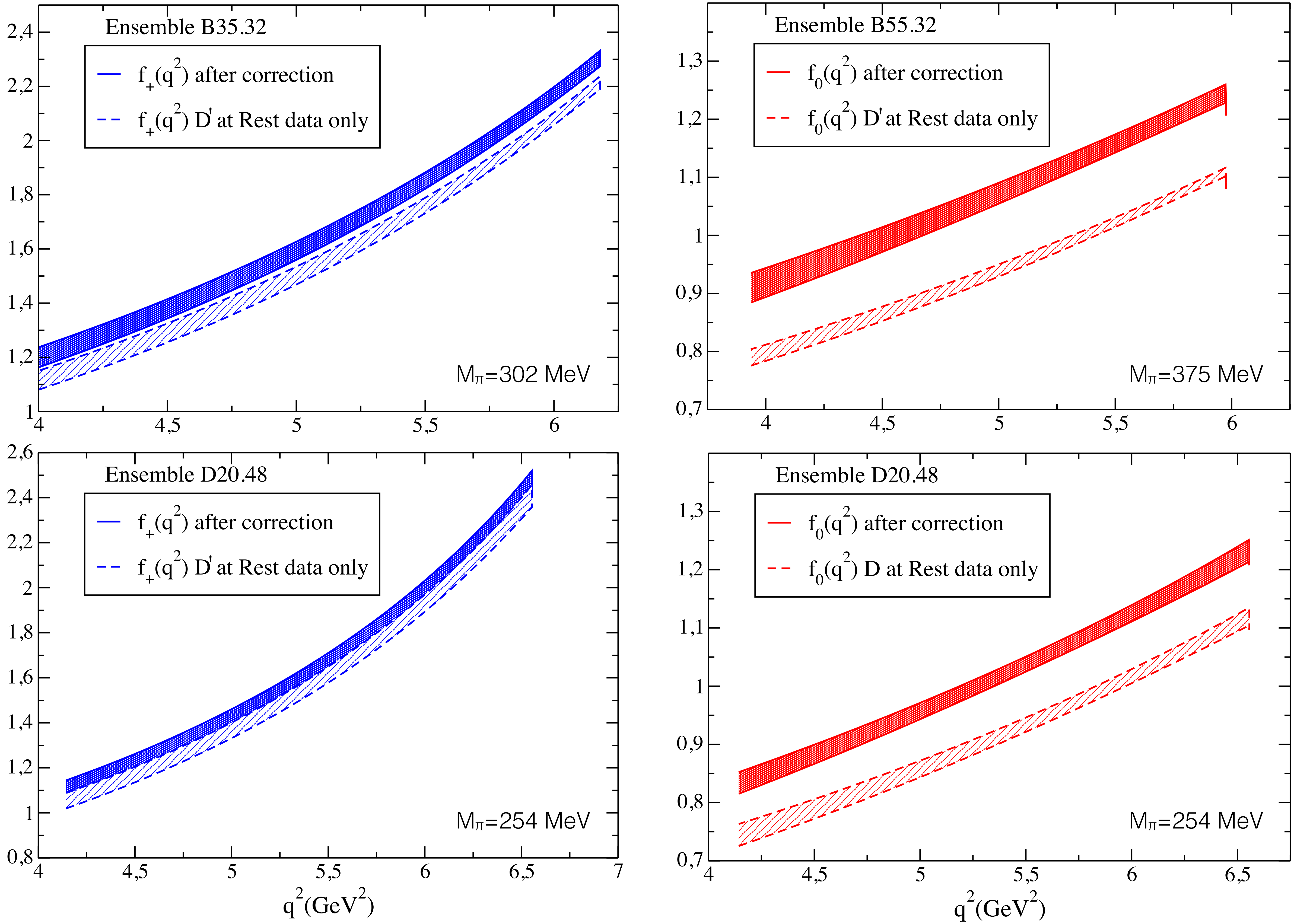
- Wilson Twisted Mass action at maximal twist with Nf=2+1+1 sea quarks
- Osterwalder-Seiler valence quark action
- Iwasaki gluon action

Results in the continuum



Comparison With Heavy Meson At Rest - on the lattice

$m_h \simeq 2.47$ GeV



Comparison With Heavy Meson At Rest - on the lattice

$m_h \simeq 2.47$ GeV