

Excited state analysis in the quasi-PDF matrix elements

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*Lattice
Parton
Physics
Project*

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Large momentum effective theory

The light-cone PDF is defined by

$$q(x, \mu^2) = \int \frac{d\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P | \bar{\psi}(\xi^-) \gamma^+ \times \exp \left(-ig \int_0^{\xi^-} d\eta^- A^+(\eta^-) \right) \psi(0) | P \rangle$$

When nucleon is boosted:

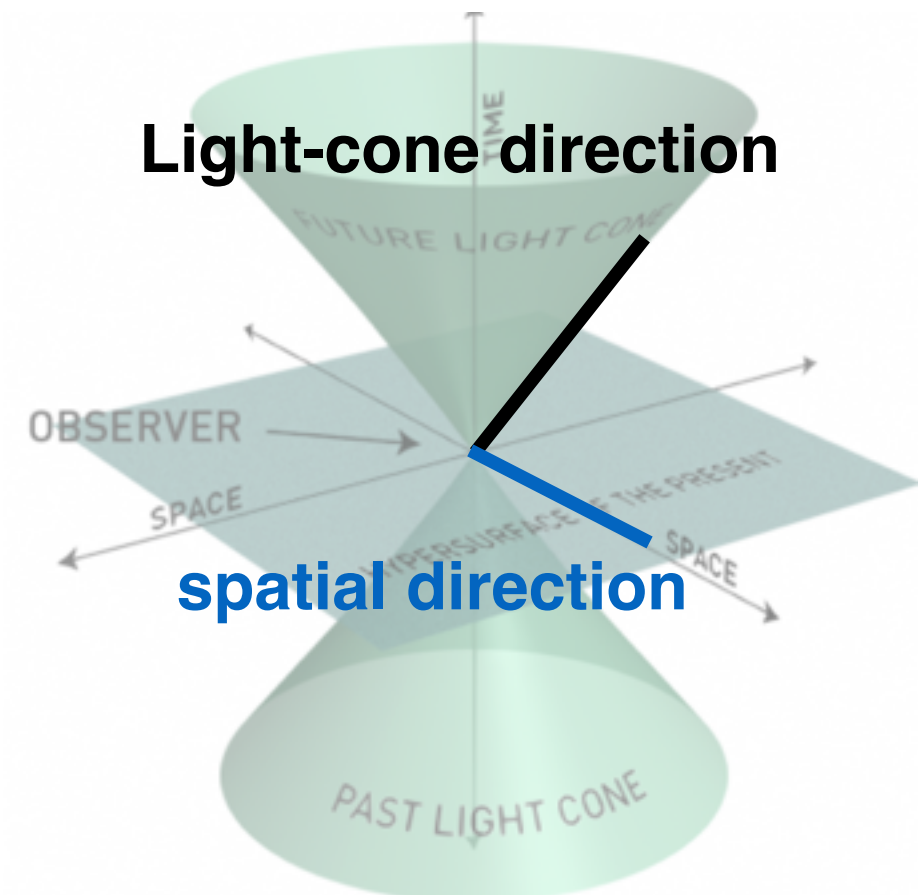
- The axial gauge conditions become the light-cone one,

and the **FT** of the **R/MOM renormalized quasi-PDF**,

$$\langle P | \bar{\psi}(z) \gamma_t U_z(z, 0) \psi(0) | P \rangle_{p^2=\mu_R^2, p_z=p_z^R}^R$$

becomes the light-cone PDF up to perturbative matching.

quasi-PDF



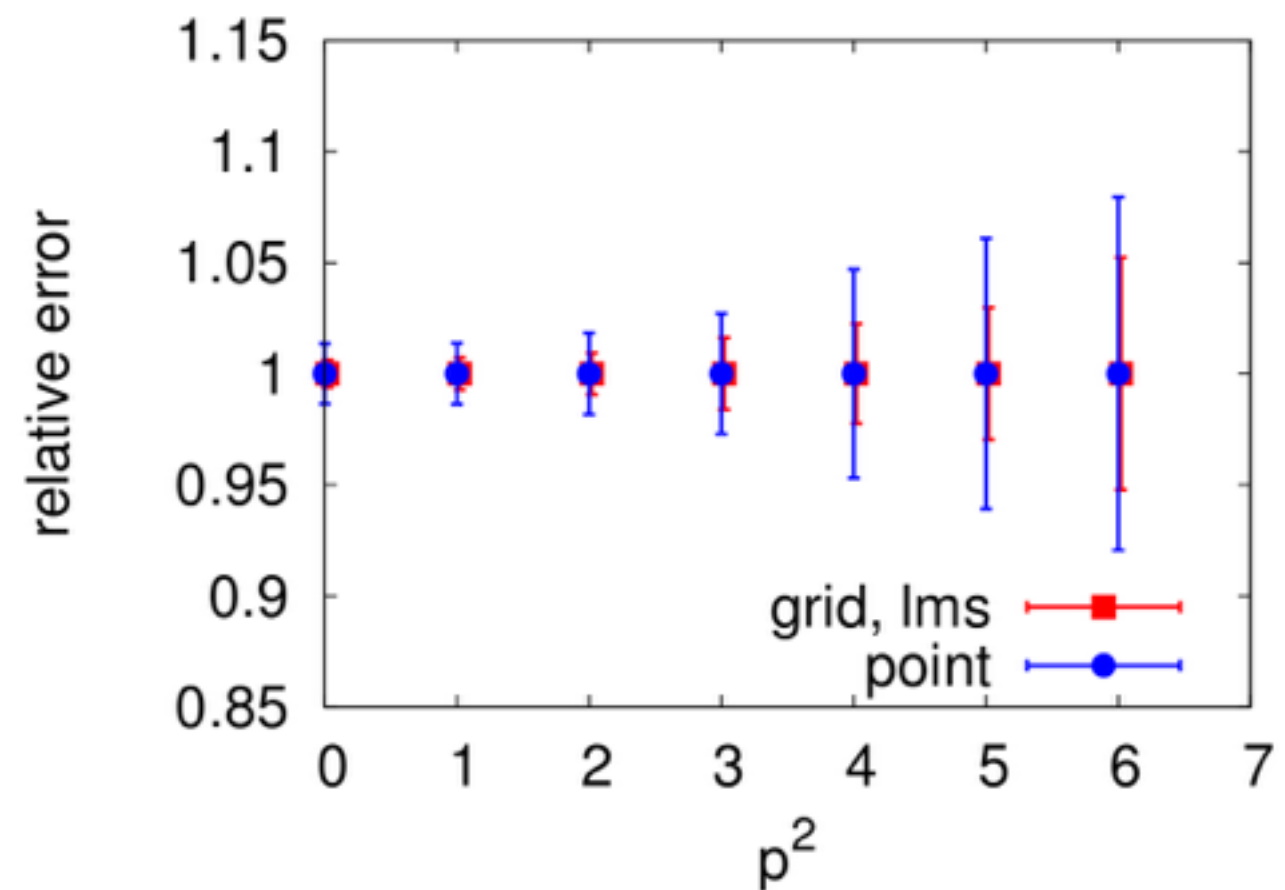
X. Ji, PRL 110 (2013) 262002, 1305.1539

X. Ji. SCPMA 57 (2014) 1407, 1404.6680

Large momentum...

Really possible?

- The expectation value of a moving hadron decays as $\sim e^{-Et}$, where $E = \sqrt{(m^2 + p^2)}$.
- Its statistical uncertainty decays as $\sim e^{-m_0 t}$, where $m_0 \propto m_\pi$ and $m_0 \leq m$.
- Seems to be hopeless to reach large momentum as required by LaMET.

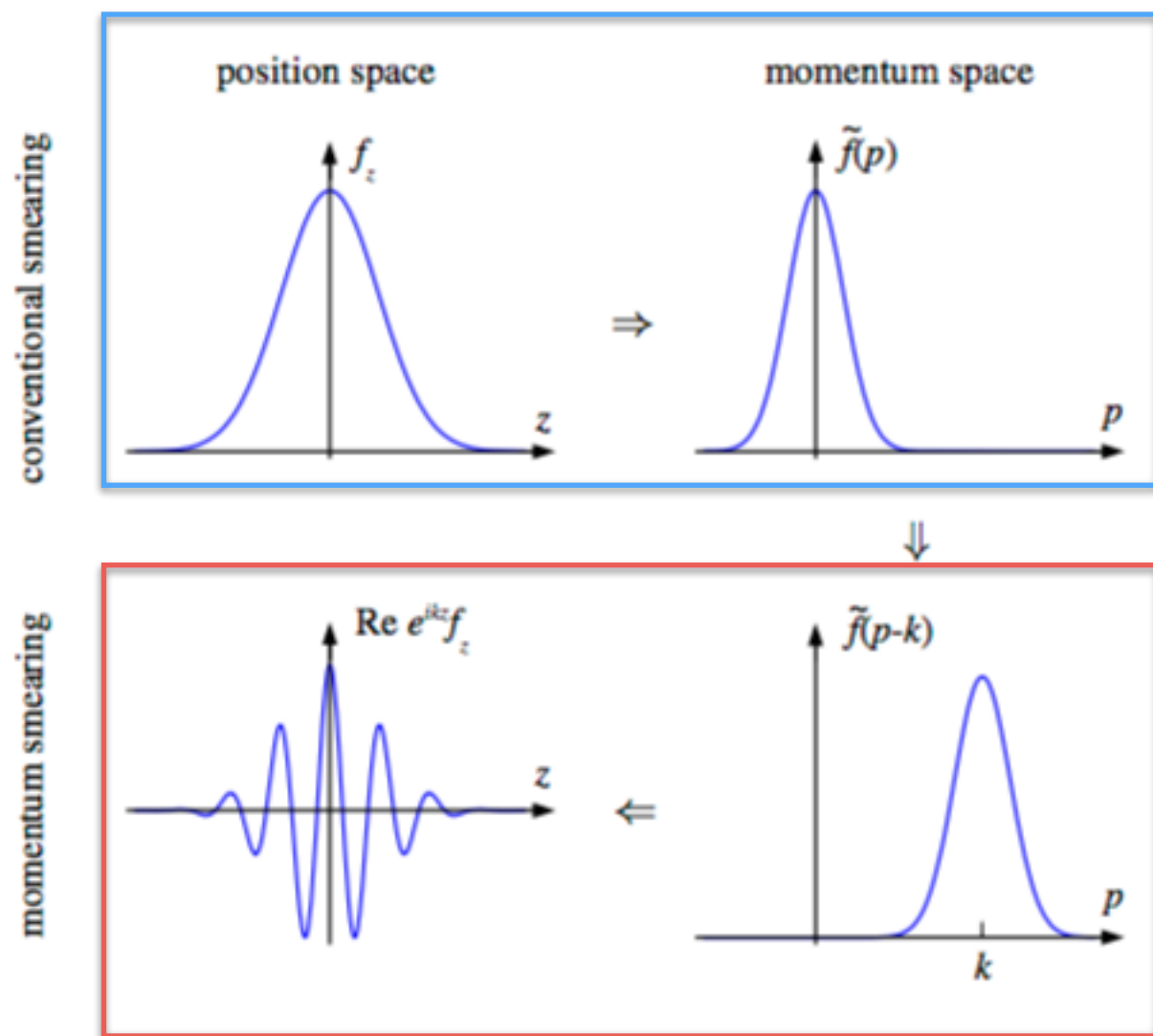


An example from: YBY, et al., χ QCD collaboration, PRD93 (2016) 034503, 1509.04616

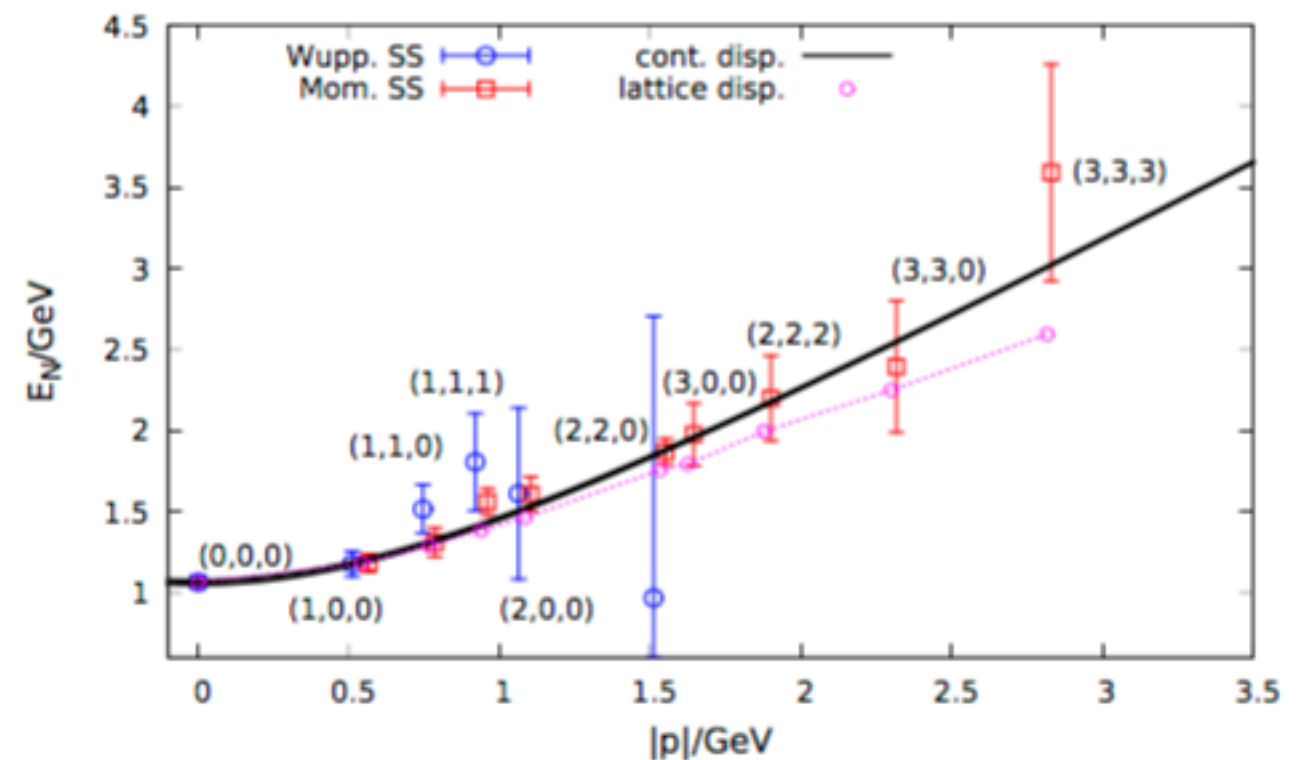
Momentum smearing

in 2pt

Wuppertal (Gaussian) Smearing



Momentum Smearing

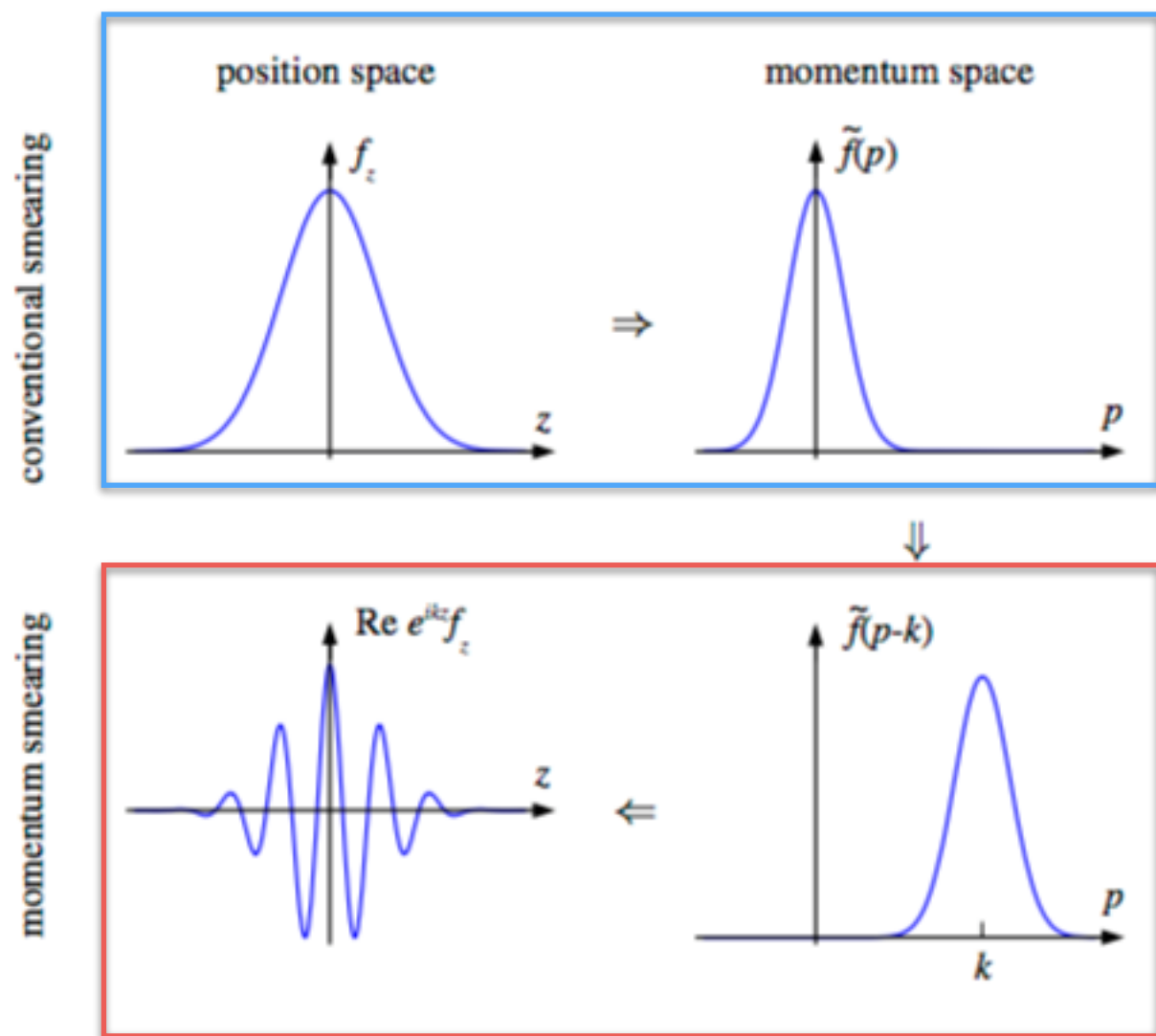


Momentum smearing can provide much better signal in 2pt!

Momentum smearing

Why?

Wuppertal (Gaussian) Smearing

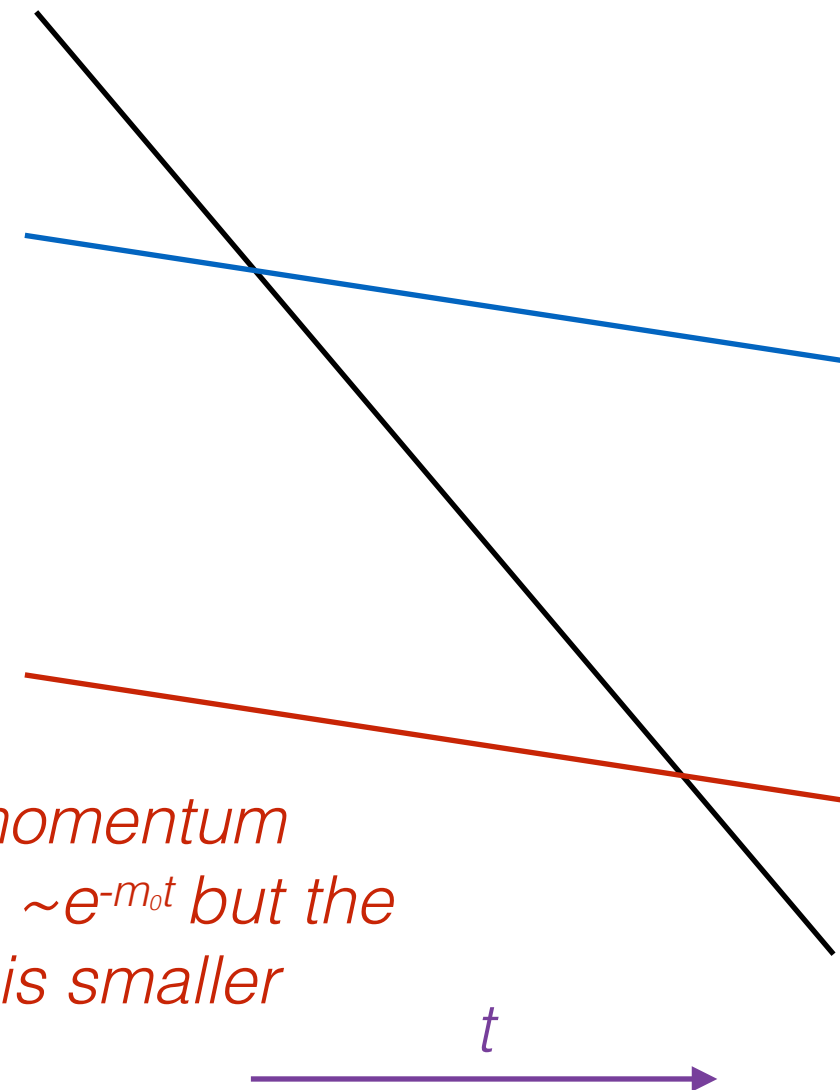


Momentum Smearing

The expectation value of the 2pt with momentum, $\sim e^{-Et}$

The statistical uncertainty with momentum smearing, $\sim e^{-m_0 t}$

That with momentum smearing, also $\sim e^{-m_0 t}$ but the coefficient is smaller

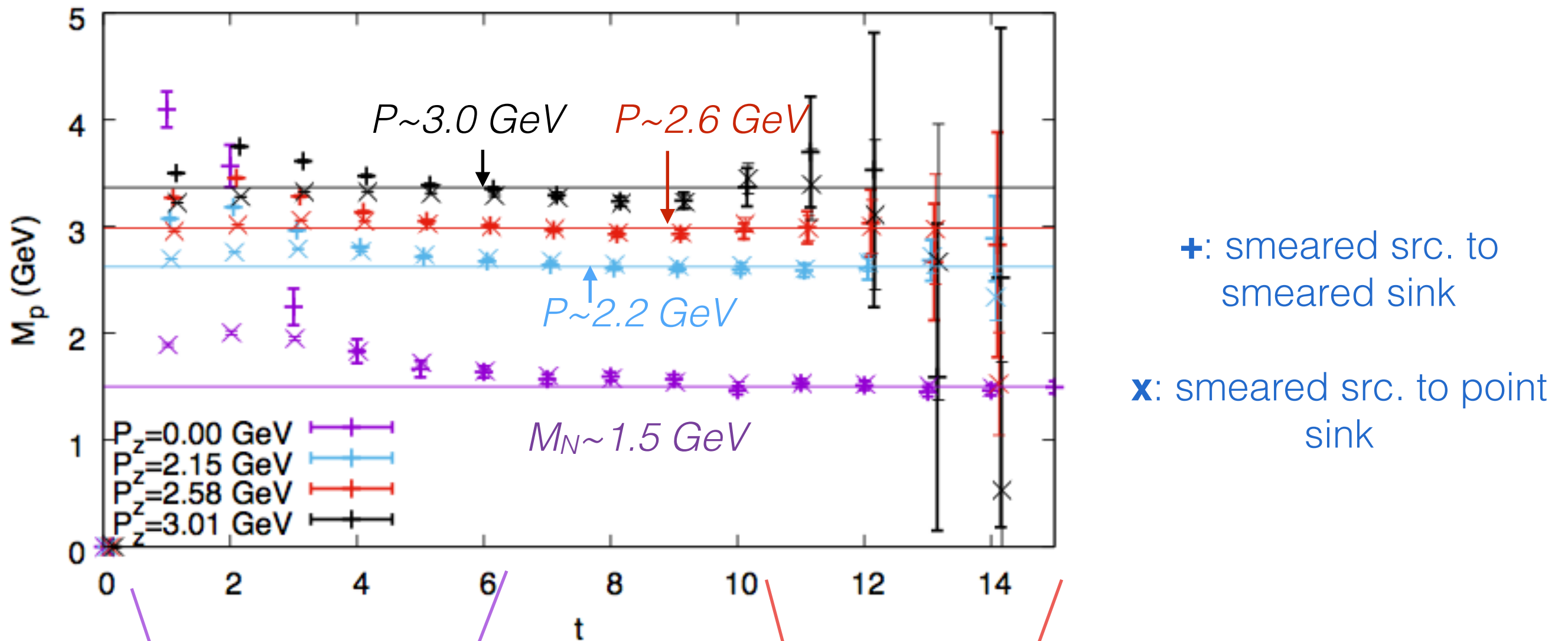


The momentum smearing is not a magic. Just effectively gain some statistics.

2pt

a09m310

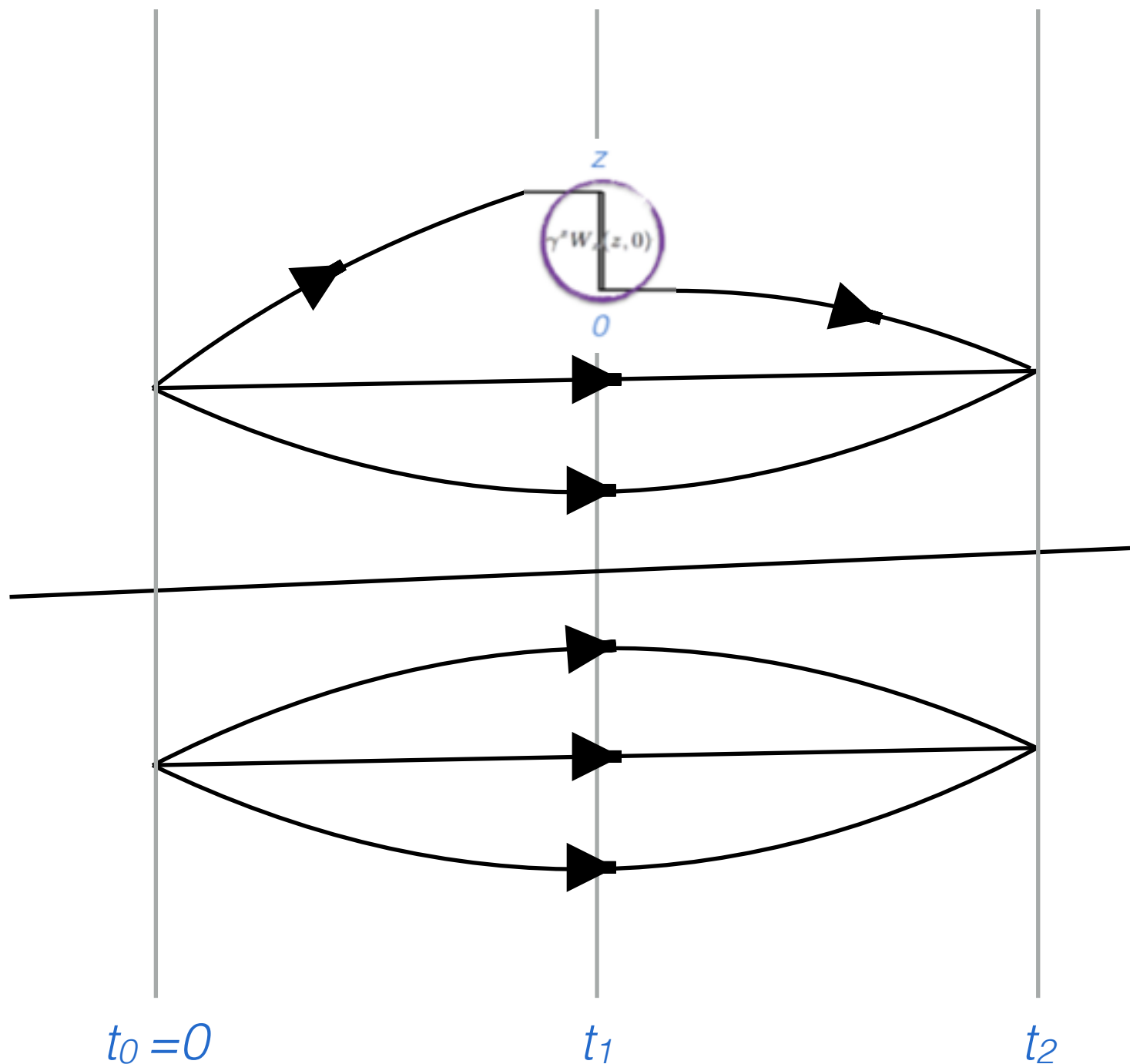
- $m_\pi \sim 670$ MeV, $m_\pi^{\text{sea}} \sim 310$ MeV, $a = 0.09$ fm;
- $L^3 \times T = 32^3 \times 96$;
- 1 step HYP smearing on everything.
- 1152 measurements = (288 configurations) \times (4 sources/configuration);



The signals with $P \neq 0$ are much better than that with $P = 0$, at small t

The condition is reversed at large t

Quasi-PDF matrix element

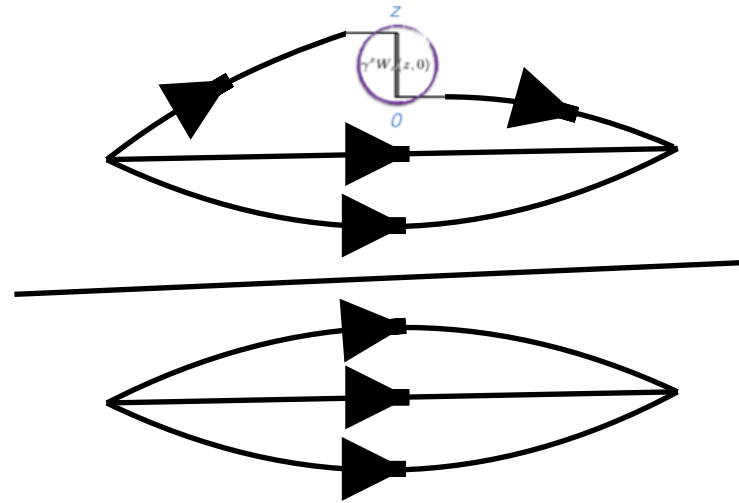


In practical, the following ratio is calculated on the lattice:

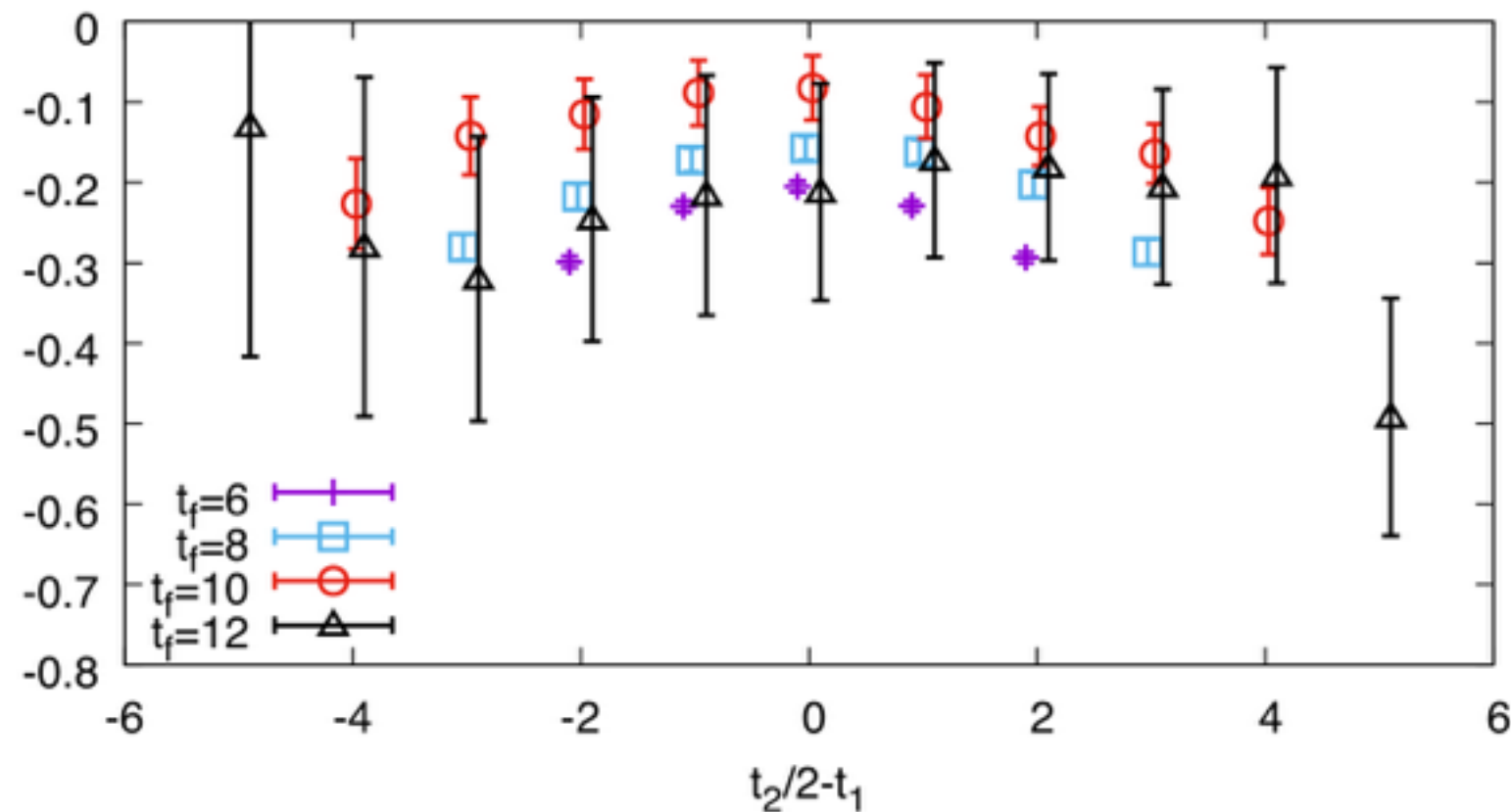
$$= \tilde{h}(z, P_z) + C_1(e^{-\Delta m t_1} + e^{-\Delta m(t_2 - t_1)}) + C_2 e^{-\Delta m t_2}$$

where the $C_{1,2}$ terms correspond to the excited state contaminations vanishing in the $t_2 \gg t_1 \gg 0$ limit.

Quasi-PDF matrix element



$$= \tilde{h}(z, P_z) + C_1(e^{-\Delta m t_1} + e^{-\Delta m(t_2 - t_1)}) + C_2 e^{-\Delta m t_2}$$

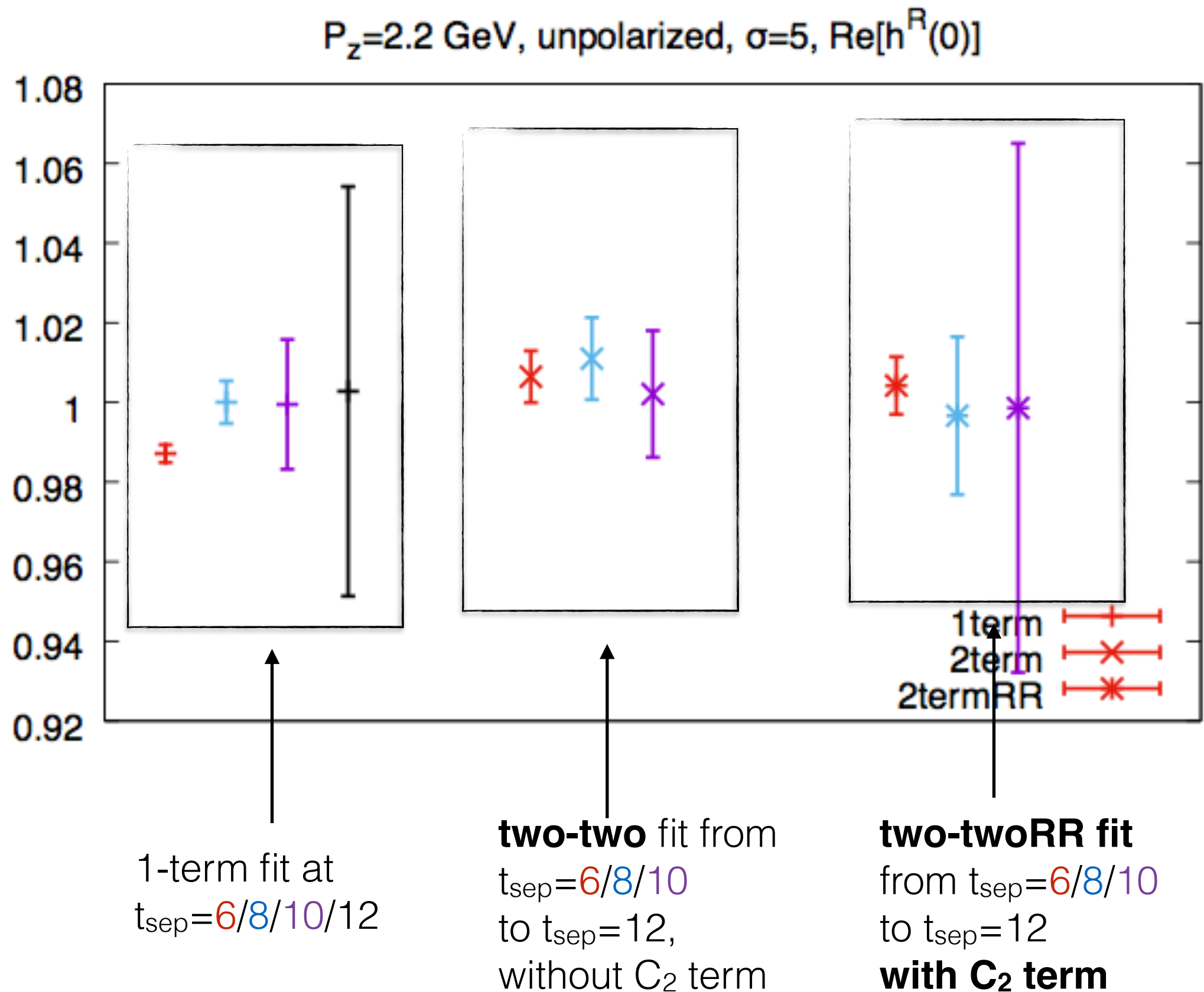


1-term fit: A constant fit around $t_1 \sim t_2/2$

2-term fit: drop the contribution from the C_2 term

2-termRR fit: including all the terms

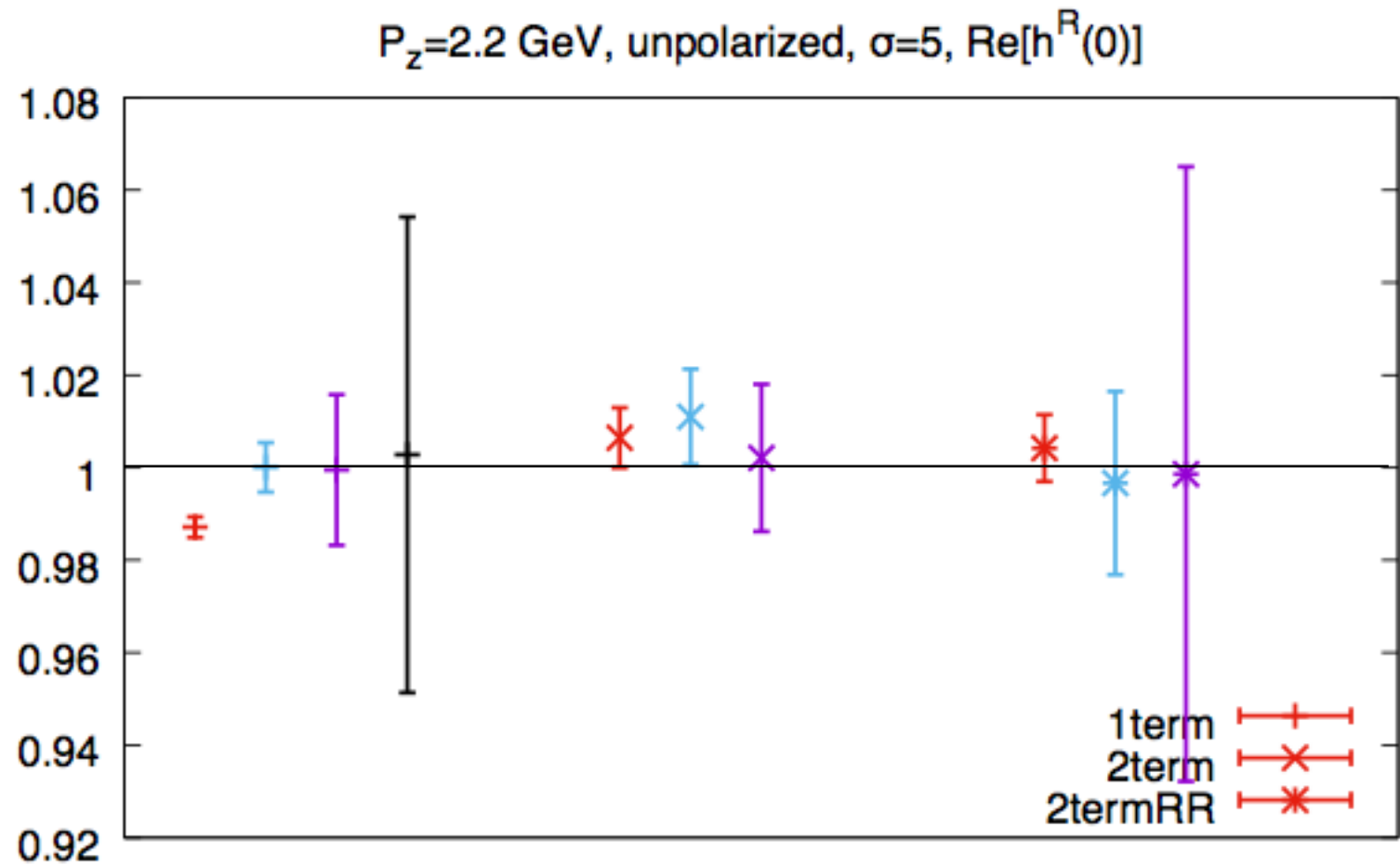
The fit results of the ME



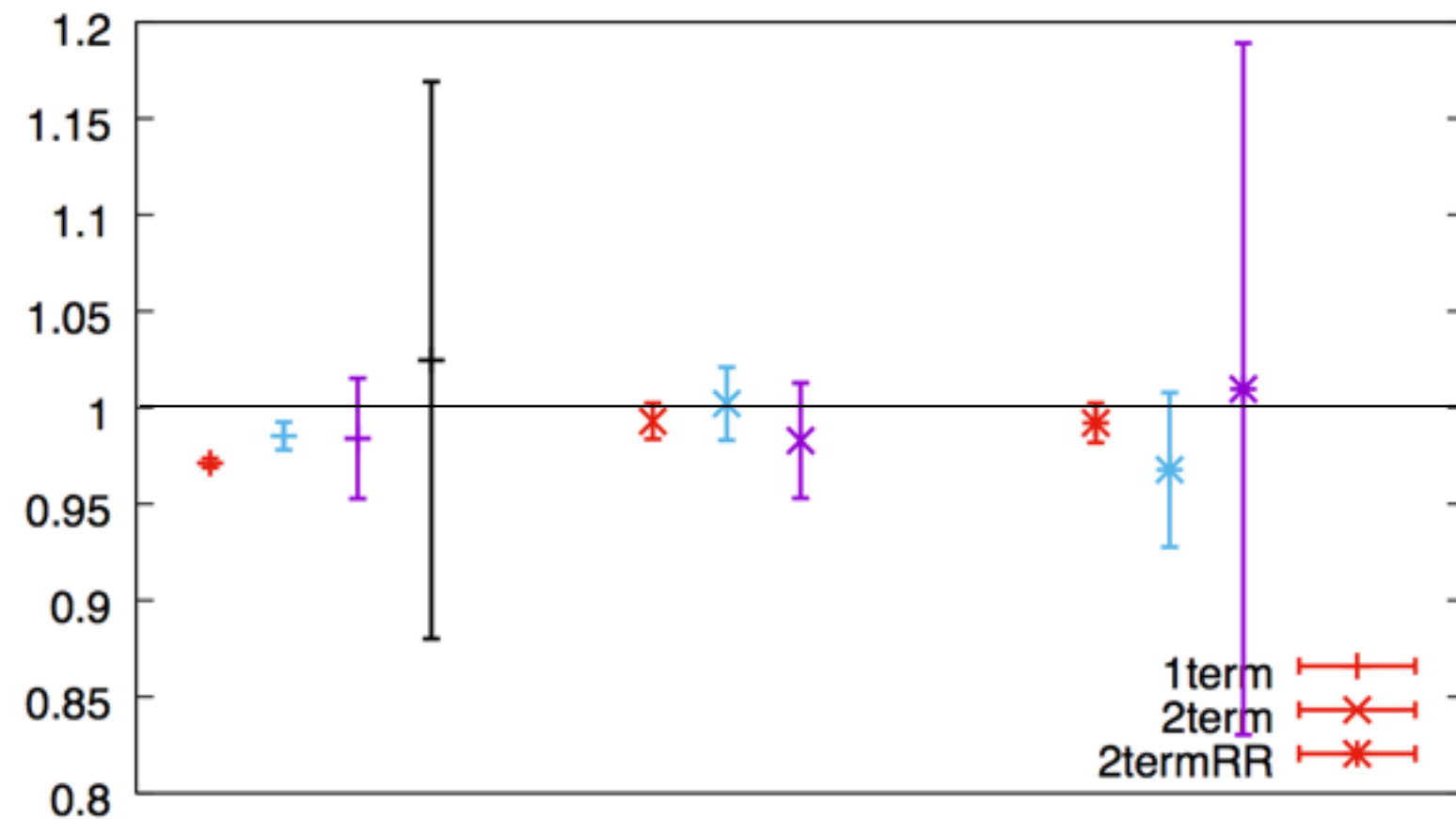
$z=0$, unpolarized

Three groups for three types of the fits

Different colors for different minimum separations.



$P_z=2.6$ GeV, unpolarized, $\sigma=5$, $\text{Re}[h^R(0)]$



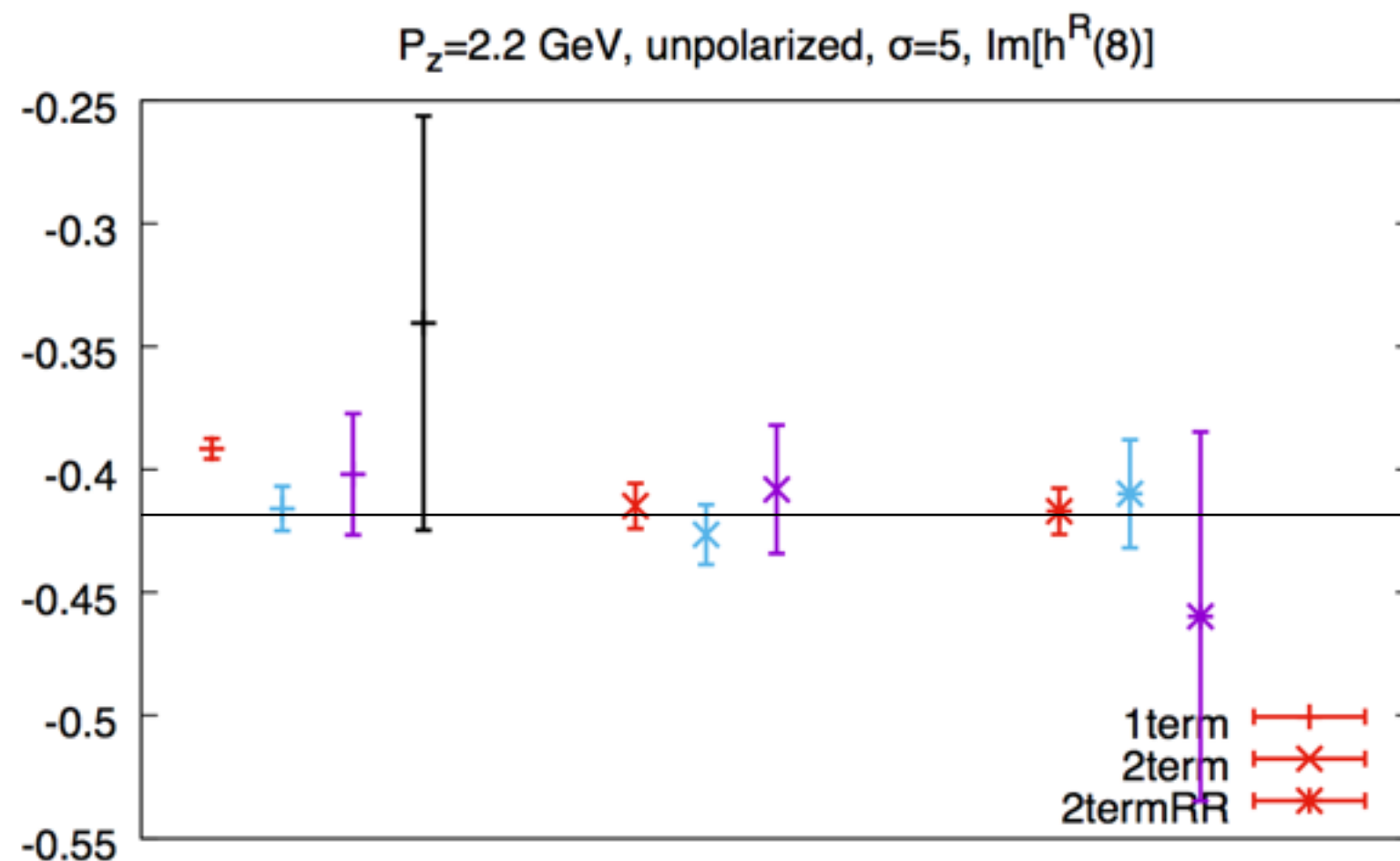
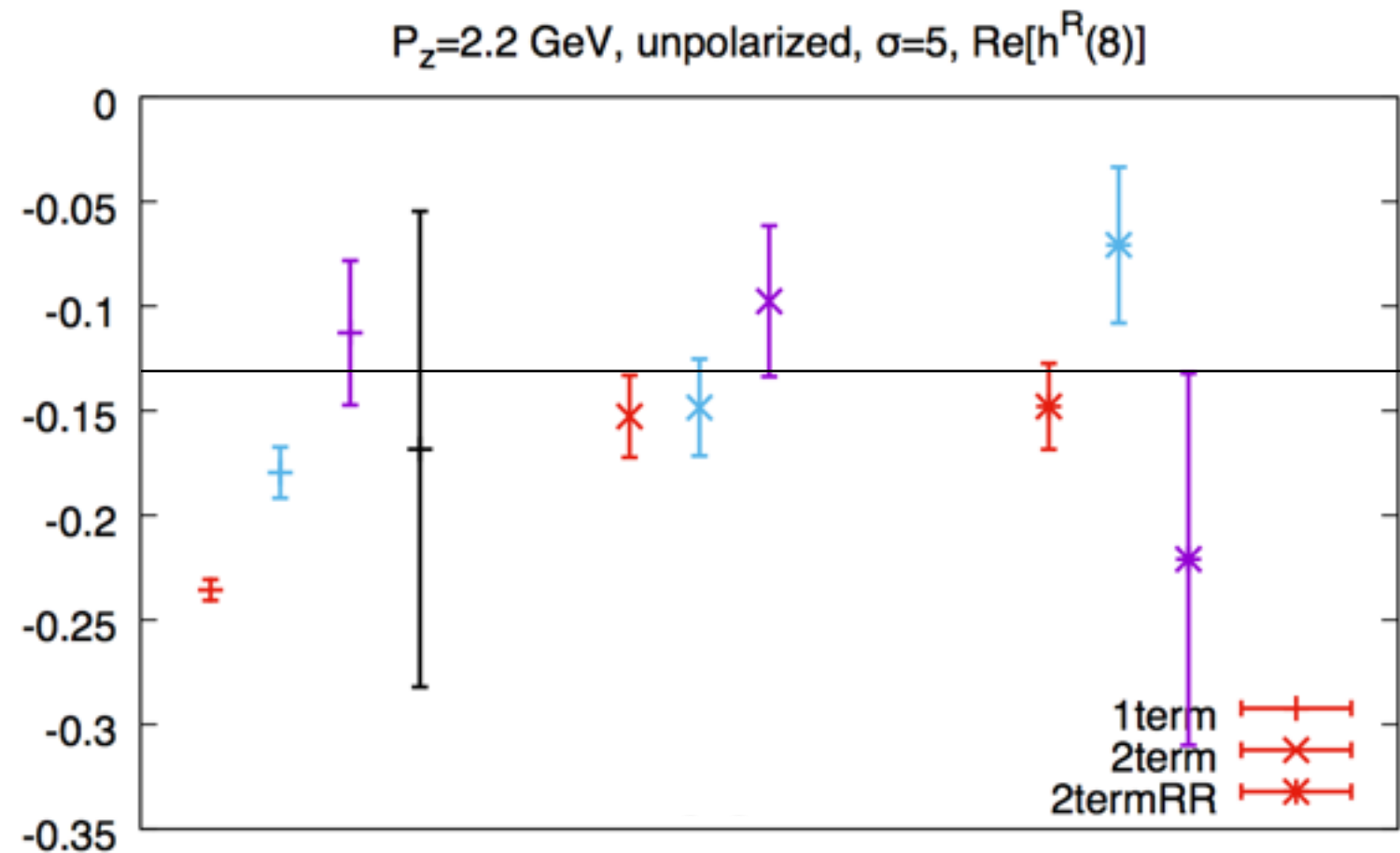
The results with different P_z are consistent with the same normalization at $z=0$.

The horizontal line with the same value are placed to guide your eyes.

$P_z=2.2$ GeV, $z=8$, unpolarized

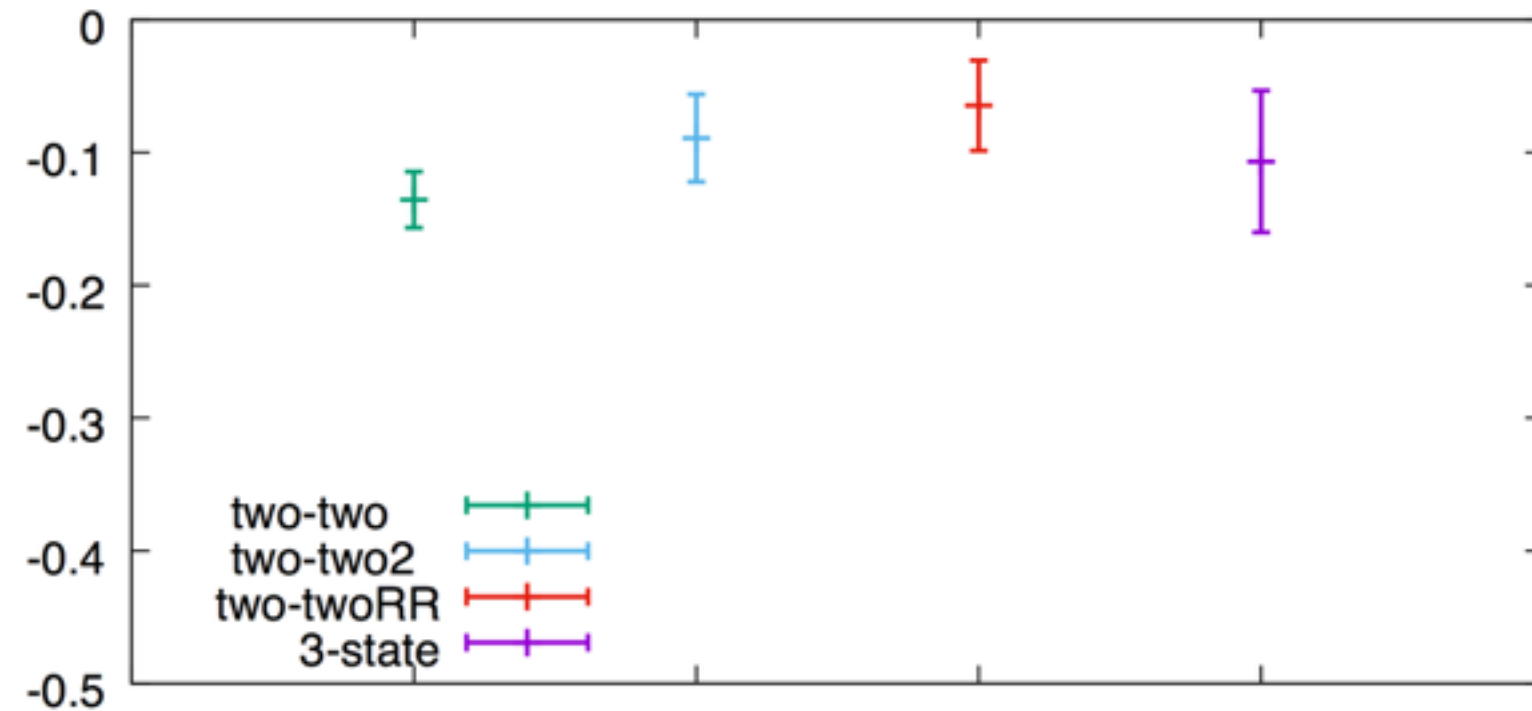
Three groups for three types of the fits

Different colors for different minimum separations.



At $z=8$, the fits for the imaginal part have good agreement but the real part require more statistics at large t_{sep} to get a solid conclusion.

Two-state fit vs. three-state fit

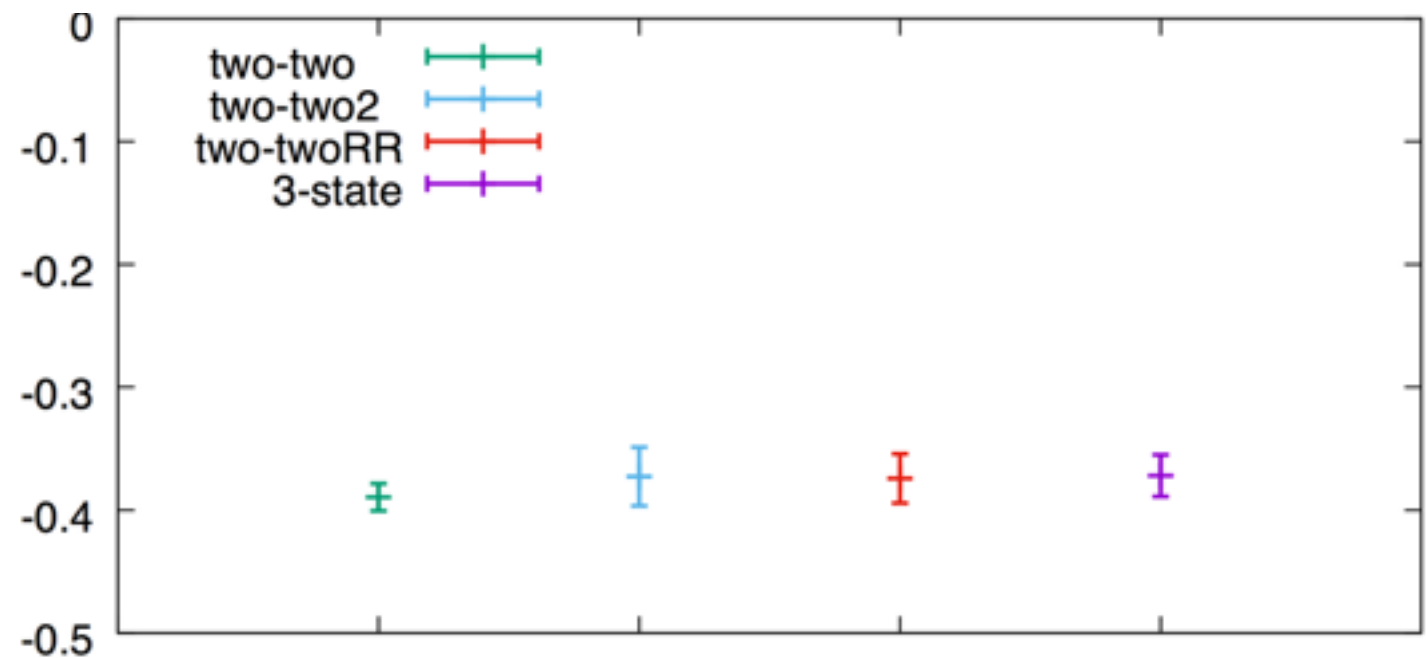


*Two-Two: $t_2=8, 10, 12$,
without RR*

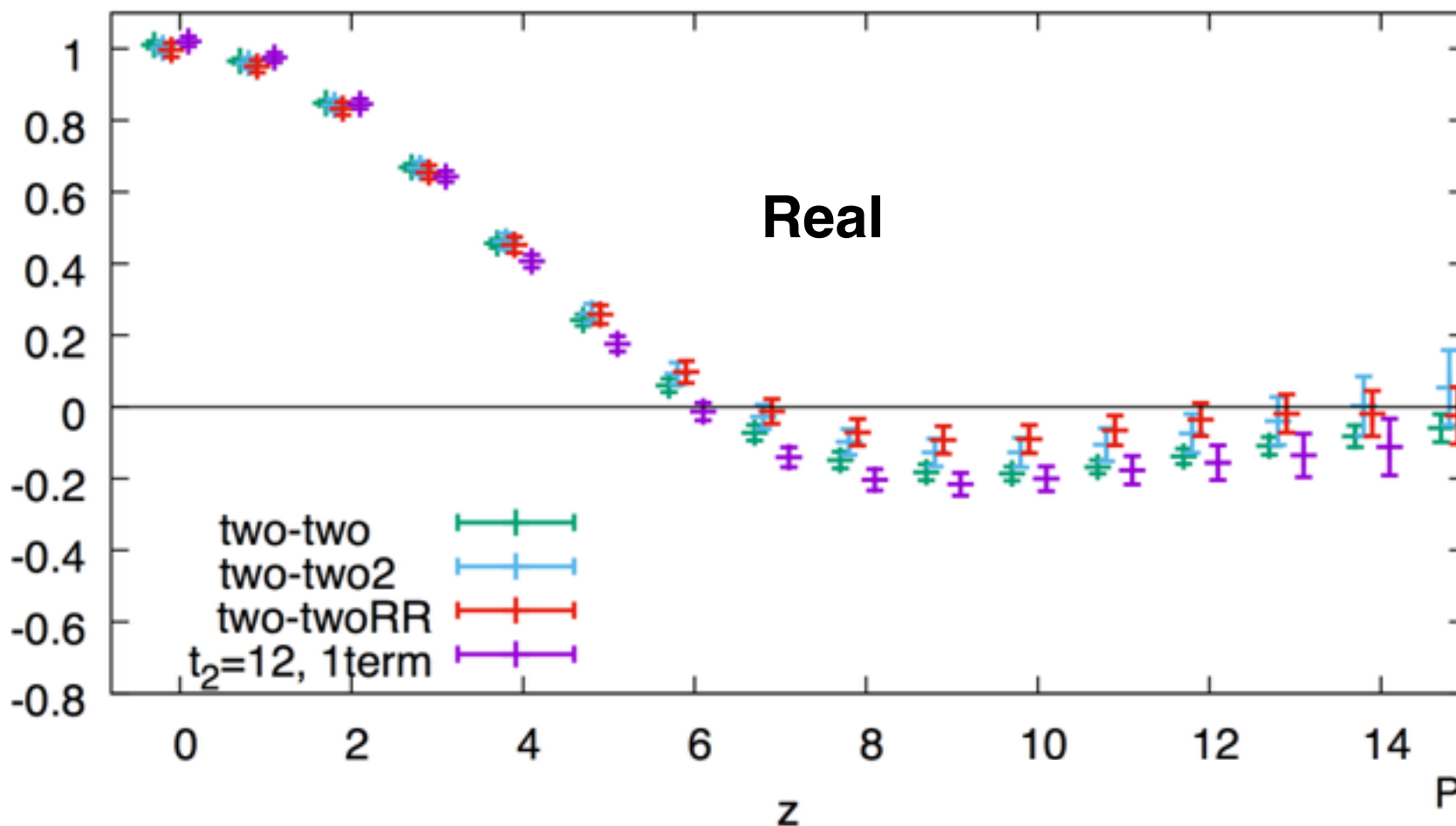
*Two-Two2: $t_2=10, 12$,
without RR*

*Two-TwoRR: $t_2=8, 10, 12$,
with RR*

*$t_2=6, 8, 10, 12$,
3-state fit*



$P_z=2.2$ GeV, unpolarized



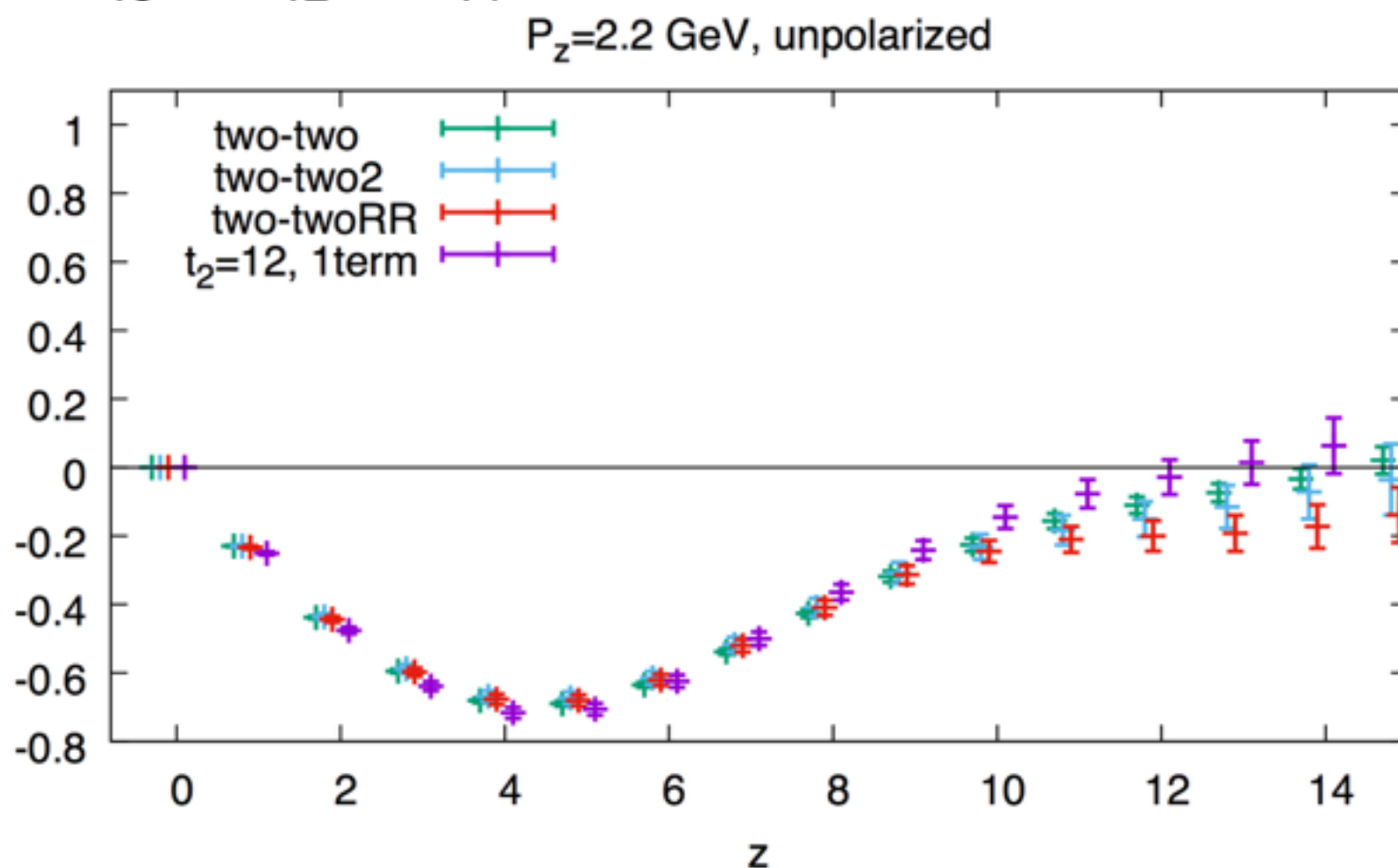
*Two-Two: $t_2=8, 10, 12$,
without RR*

*Two-Two2: $t_2=10, 12$,
without RR*

*Two-TwoRR: $t_2=8, 10, 12$,
with RR*

***16x statistics for
 $t_2=12$, 1-state fit***

The large statistics result
with $t=12$ agrees with that
based on two-state fits
ones using smaller
separations!



Summary

- The momentum smearing allow us to achieve good signal for the matrix elements with large hadron momentum, at small source-sink separation.
- The multi-state fit can provide a good subtraction on the excited state contamination with smaller source-sink separations.
- The production with another smearing size are ongoing to confirm the multi-state fit results.