Towards lattice-assisted hadron physics calculations based on gauge-fixed n-point functions

André Sternbeck

Friedrich-Schiller-Universität Jena, Germany

in collaboration with

G. Eichmann (Lisbon) and M. Leutnant (Jena)

Lattice 2018, East Lansing, Michigan (US)

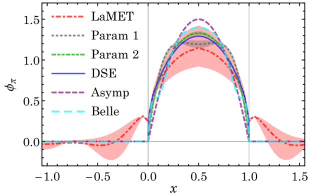
Motivation

Research in hadron physics / QCD thermodynamics

- lattice QCD currently preferred tool to provide theoretical estimates
- full control over systematic error, hard/expensive in practice
- New: PDFs and PDAs available via quasi-function/amplitudes (due to Ji (2013) and collaborators since then)
- Many new studies recently, requires much effort (see, e.g., LaMET, ETMC or RQCD approach)

Example:

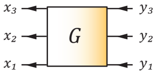
LaMET: Zhang et al. (2017)



Lattice is not the only nonperturbative framework

- Bound-state / Dyson-Schwinger equations
- Functional Renormalization group
- Pros/Cons different to lattice
- Input: nonperturbative n-point functions (fixed gauge)
- **Problem**: truncation of infinite system of equations / of effective action
- Systematic error difficult without external input / guidance

Hadron physics calculations



Hadron properties encoded in QCD's *n*-point functions

[follow review Eichmann et al., Prog.Part.Nucl.Phys 91 (2016) 1]

- information contained in many *n*-point functions, effort to get them varies
- bound states / resonances = color singlets, poles in *n*-point functions
- Example: quark-antiquark 6-point function

$$G^{\alpha\beta\gamma}_{\delta\eta\rho}(x_1,x_2,x_3|y_1,y_2,y_3) \;:= \left\langle 0\right| T\psi_\alpha(x_1)\psi_\beta(x_2)\psi_\gamma(x_3)\;\bar\psi_\delta(y_1)\bar\psi_\eta(y_2)\bar\psi_\rho(y_3)\left|0\right\rangle$$

spectral decomposition in momentum space

$$G^{\alpha\beta\gamma}_{\delta\eta\rho}(p_f,q_f,P|p_i,q_i,P) \;\simeq\; \sum_n \frac{\Psi^{(n)}_{\alpha\beta\gamma}(p_f,q_f,P)\,\bar\Psi^{(n)}_{\delta\eta\rho}(p_i,q_i,P)}{P^2+m_n^2} + \ldots$$

- p, q … relative momenta, P … total momentum
- G and $\Psi^{(n)}$ may be gauge-dependent, but **poles** $P^2=-m_n^2$ gauge-**independent**
- Pole residue = Bethe-Salpeter wave function $\Psi^{(n)}$

(coordinate space)

$$\Psi^{(n)}_{\alpha\beta\gamma}(x_1,x_2,x_3,P) = \left\langle 0\right| T\psi_\alpha(x_1)\psi_\beta(x_2)\psi_\gamma(x_3)\left|n\right\rangle$$

Hadron physics calculations

Spectroscopy with lattice QCD

- define gauge-invariant interpolating fields $h(x)$ and $\bar{h}(y)$ at

$$x_1 = x_2 = x_3 = x \quad \text{and} \quad y_1 = y_2 = y_3 = y$$

- extract poles from 2-point correlator

$$C(x-y) = \langle 0|\, T \underbrace{[\Gamma^{\alpha\beta\gamma}\psi_\alpha\psi_\beta\psi_\gamma](x)}_{h(x)} \underbrace{[\bar{\Gamma}^{\delta\eta\rho}\bar{\psi}_\delta\bar{\psi}_\eta\bar{\psi}_\rho](y)}_{\bar{h}(y)} |0\rangle$$

- Spectral decomposition

$$C(\vec{P},t) = \int \frac{d^3\vec{x}}{(2\pi)^4}\, e^{ixP}\, C(\vec{x},t) \quad \xrightarrow{t\gg 0} \quad \frac{e^{-E_0|t|}}{2E_0} |r_0|^2 u_0(\vec{P})\, \bar{u}_0(\vec{P}) \quad + \quad \cdots$$

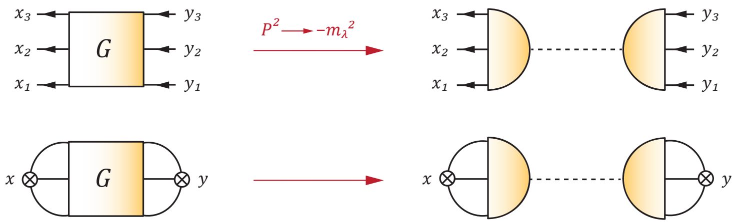
- time-like pole in momentum space = exponential Euclidean time decay
- baryon mass from exponential decay of $C(\vec{P},t)$
- Pole residues are simple:

$$\Gamma^{\alpha\beta\gamma}\Psi^{(n)}_{\alpha\beta\gamma}(x,x,x,P) = \langle 0|h(x)|n\rangle = \langle 0|h(0)|n\rangle\, e^{-ixP} = r_n\, u_n(\vec{P})\, e^{-ixP}$$

Hadron physics calculations

Spectroscopy with lattice QCD (put into graphs)

quark-antiquark 6-point function residue at pole



2-point correlator (lattice) pole of C(x,y) = decay in Euclidean t

Figure from [Eichmann et al. Prog.Part.Nucl.Phys.91 (2016) 1, Fig.3.5]

Graphs

- Square = 6-point quark-antiquark function
- half circle = BS wave function (residue)

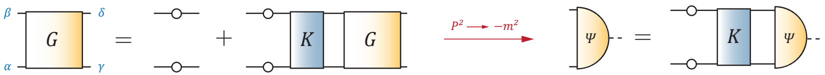
Hadron physics calculations

Functional approach (solving a bound-state equation)

- could apply same approach as lattice QCD
- much simpler: solve self-consistent relation for hadron wave functions Ψ
  (Ψ = residue of pole of n-point function, full information about hadron on its pole)
- resulting equations known as hadron bound-state equations
  Bethe-Salpether / Faddeev equations (for mesons / baryons)

Meson (4-point function = two-particle bound state)

- Dyson equation: $G = G_0 + KG$ $\xrightarrow{P^2 \to -m^2}$ Ψ satisfies BS equation
  (compact notation)



[Eichmann et al. Prog.Part.Nucl.Phys.91 (2016) 1, Fig.3.7]

- G_0 ... nonperturbative quark and antiquark propagators (no interaction)
- K ... 4-quark scattering kernel (interaction)

Bethe-Salpether equation for meson amplitude

Meson-BSE amplitude

- amputated wave function fulfills $\Gamma = KG_0\Gamma$, i.e.,

$$\Gamma_{\alpha\beta}(p,P) = \int \frac{d^4q}{(2\pi)^4} K_{\alpha\gamma,\delta\beta}(p,P;q) \left\{S(q_+)\,\Gamma(P,q)\,S(-q_-)\right\}_{\gamma\delta}$$

- $\Gamma = 4 \times 4$ Dirac matric, for mesons (J^P) with $J > 0$: $\Gamma \to \Gamma^{\mu_1 \cdots \mu_n}$
- S = nonperturbative quark propagators

Can solve it at least in some truncation (e.g., rainbow-ladder)

- Eigenvalue problem: $\Gamma = \lambda(P^2)\, KG_0\Gamma$
- For all P_n^2 with $\lambda(P_n^2) = 1$ read off mass: $m_n^2 = -P_n^2$ ($m_1 \ldots$ ground state)
- Properties of hadron ($P^2 = -m^2$) from eigenvector Γ with suitable base $\tau^{(i)}$

$$\Gamma_{\alpha\beta}(p,P) = \sum_i f_i(p^2, p\cdot P; -m^2)\, \tau^{(i)}_{\alpha\beta}(p,P)$$

- BS equation becomes a system of coupled integral equations for form factors f_i

Hadron properties from bound-state amplitude

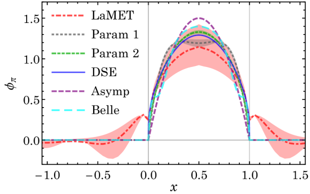
Bound-state amplitude gives access to . . .

- form factors: electromagnetic, transition
- PDAs, PDFs, GPDs, . . .

Ex 1: pion DA [e.g. Chang et al., PRL110(2013)092001]
(projection onto light front)

$$\phi_\pi(x) = \frac{1}{F_\pi} \mathrm{Tr}\, Z_2 \int_q \delta^x_\zeta(q_+)\, \gamma\cdot\zeta\, \gamma_5\, \Gamma_\pi(k, P)$$

LaMET: Zhang et al. (2017)



- DSE/BSE calculation via ∼ 50 Mellin moments
- Lattice calculation, either via moments (∼ 2) or directly (e.g., LaMET, RQCD)

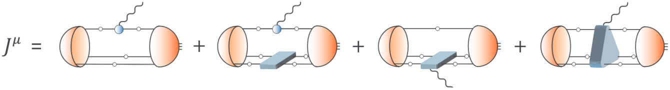
Ex 2: pion form factor $F_\pi(Q^2)$ [Maris/Tandy (2000)]
(impulse approximation) $(q_\pm = q \pm Q/2,\ k_\pm = q \pm Q/4)$

$$P_\mu F_\pi(Q^2) = -\int_q \mathrm{Tr}\left[\Gamma_\pi(k_+, -P_+)\, S(q)\, i \underbrace{\Gamma_\mu(q_+; Q)}_{\text{quark-photon vertex}}\, S(q+Q)\, \Gamma_\pi(k_-, -P_-) \right]$$

Hadron properties from bound-state amplitude

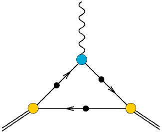
Nucleon electromagnetic current

(G. Eichmann, PRD84 (2011) 014014)



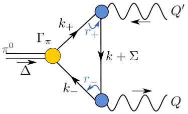
Pion form factor

(Maris, Tandy (2000), impulse approximation)



Meson transition form factor $\pi^0 \to \gamma^*\gamma^*$

(E. Weil et al. (2017), impulse approximation)



Require

- nonperturbative quark propagators (quarks DSE | lattice data)
- BS amplitudes Γ (truncated BSE | lattice: work in progress)
- Quark-photon vertex (quark-photon BSE | first lattice data → this talk)

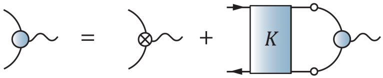
Tensor structure of quark bilinears

Quark-Photon (γ_μ) vertex

- full tensor structure required for electromag. elastic and transition form factors

$$\Gamma_\mu(k,Q) = \underbrace{i\gamma_\mu\lambda_1 + 2k^\mu[i\not{k}\,\lambda_2 + \lambda_3]}_{\Gamma_\mu^{BC}} + \sum_{j=1}^{8} i\tau_j T_\mu^{(j)}(k,Q) \quad = S^{-1} G_\mu(k,Q)\, S^{-1}$$

- Γ_μ satisfies inhomogeneous BSE, and Γ_μ^{BC} Vector-Ward identity



$$Q_\mu\Gamma_\mu = S^{-1}(k_-) - S^{-1}(k_+)$$

- requires truncation of system ...systematic error (?)

Lattice QCD can provide full tensor structure

$$G(x,y,z) = \left\langle D_U^{-1}(x,z)\,\Lambda\, D_U^{-1}(z,y)\right\rangle_U \qquad \text{where} \quad \Lambda = \gamma_\mu,\ \sigma_{\mu\nu},\ \gamma_5\gamma_\mu, \ldots$$

- lattice data for RI'(S)MOM renormalization program (Landau gauge)
- Map out full tensor structure (typically not considered)

Tensor structure of quark bilinears and the RI'SMOM scheme

Nonperturbative renormalization constants for hadronic operator

$$\left.\frac{\mathrm{Tr}\left[\Gamma_\Lambda \Gamma_0^{-1}\right]}{12 Z_2}\right|_{p^2=\mu^2} \stackrel{!}{=} \frac{Z_2^{RI}(\mu^2,a)}{Z_\Lambda^{RI}(\mu^2,a)} \overset{\mu^2 \gg 0}{\sim} \frac{C_{\Lambda,MS}^{RI}(\mu^2,a)\, Z_2^{MS}(\mu^2)}{C_{2,MS}^{RI}(\mu^2,a)\, Z_\Lambda^{MS}(\mu^2)}$$

- projection onto tree-level vertex, popular and straightforward
- lattice: **Monte Carlo averages** for quark propagator $S^{ab}(k_\pm)$ in Landau gauge and

$$G_\Lambda^{\alpha\beta}(k,Q) = \sum_{x,y,z} e^{ik_+(x-z)} e^{ik_-(z-y)} \left\langle [D_U^{-1}]_{xz}^{\alpha\gamma} \Lambda^{\gamma\delta} [D_U^{-1}]_{zx}^{\delta\beta} \right\rangle_U$$

Vertex from amputated 3-point function

$$\Gamma_\Lambda(k,Q) = S^{-1}(k_-) G_\Lambda(k,Q)\, S^{-1}(k_+) \qquad (k_\pm = k \pm Q/2)$$

New / beyond RI'SMOM: project onto full tensor structure, e.g., $\Lambda = \gamma_\mu$

$$\Gamma_\mu(k,Q) = i\gamma_\mu \lambda_1 + 2k^\mu [i\not{k}\, \lambda_2 + \lambda_3] + \sum_{j=1}^{8} i\tau_j\, T_\mu^{(j)}(k,Q)$$

Tensor structure of quark bilinears

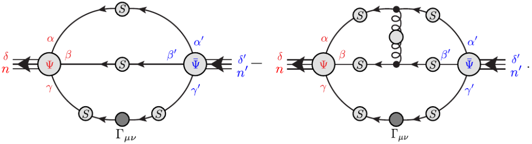
Other vertices are also needed, e.g.,

- $\boldsymbol{\sigma_{\mu\nu}}$ **vertex** for proton tensor charges from current $J_{\mu\nu}(Q^2 = 0)$
  [see recent paper Wang et al. (2018)]

$$J_{\mu\nu}(Q) = \sum_{k=1}^{3} \sum_{nn'} \left[J^{(k)}_{nn'}(Q) \right]_{\mu\nu} T^{(k)}_{nn'} \qquad T^{(k)}_{nn'} \ldots \text{isospin traces}, T^{(3)}_{nn'} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{3} \end{bmatrix}_{nn'}$$

and

$$[J^{(3)}_{nn'}(Q)]^{\delta\delta'}_{\mu\nu} =$$



- **Axialvector vertex** $(\gamma_5\gamma_\mu)$ required for axial form factors
  see, e.g., [Eichmann & Fischer, EPJA48 (2012) 9]

- **Pseudoscalar vertex**, ... **all the usual lattice hadron physics quantities**.

Parameters of our gauge field ensembles ($N_f = 0, 2$)

Lattice action

- Wilson gauge action
- Wilson clover fermions
- Landau gauge
  (after thermalization)

Can study:

- quark mass dependence
- discret. + volume effects

β	κ	$L_s^3 \times L_t$	a [fm]	m_π [MeV]
5.20	0.13584	$32^3 \times 64$	0.08	411
5.20	0.13596	$32^3 \times 64$	0.08	280
5.29	0.13620	$32^3 \times 64$	0.07	422
5.29	0.13632	$32^3 \times 64$	0.07	295
5.29	0.13632	$64^3 \times 64$	0.07	290
5.29	0.13640	$64^3 \times 64$	0.07	150
5.40	0.13647	$32^3 \times 64$	0.06	426
5.40	0.13660	$48^3 \times 64$	0.06	260

Consider:

- $G_\Lambda(k, Q)$ where $\Lambda = \gamma_\mu,\ \gamma_5\gamma_\mu,\ \sigma_{\mu\nu},\ \mathbf{1}$ (presently connected diagrams only)
- external quark momenta: $k_\pm = k \pm Q/2$
- twisted boundary condition: (a) $k \cdot Q = 0$ (b) $\frac{(k\cdot Q)^2}{|k||Q|} = const.$

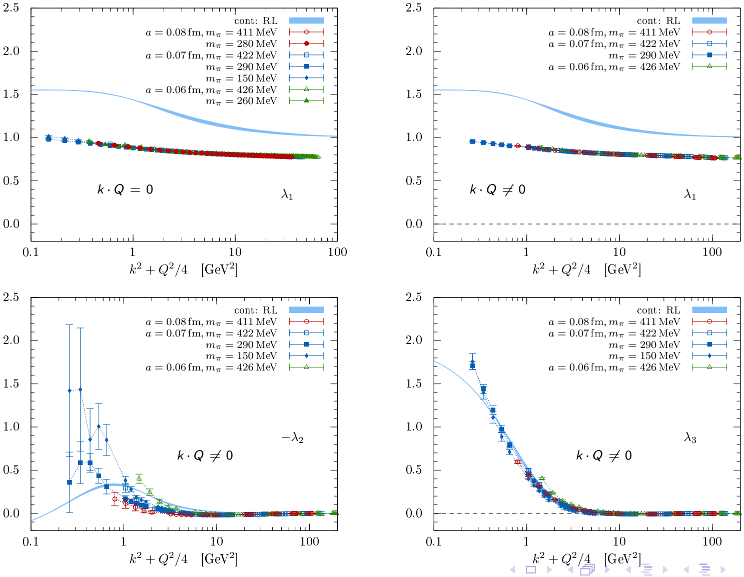
Acknowledgements

- $N_f = 2$ configurations provided by RQCD collaboration (Regensburg)
- Gauge-fixing and calculation of propagators at the HLRN, LRZ and FSU Jena

Ex: Quark-Photon Vertex $\Gamma_\mu = i\gamma_\mu\lambda_1 + 2k^\mu[i\not{k}\,\lambda_2 + \lambda_3] + \sum_{j=1}^{8} i\tau_j T_\mu^{(j)}(k,Q)$

lattice (preliminary) vs. continuum (rainbow-ladder)

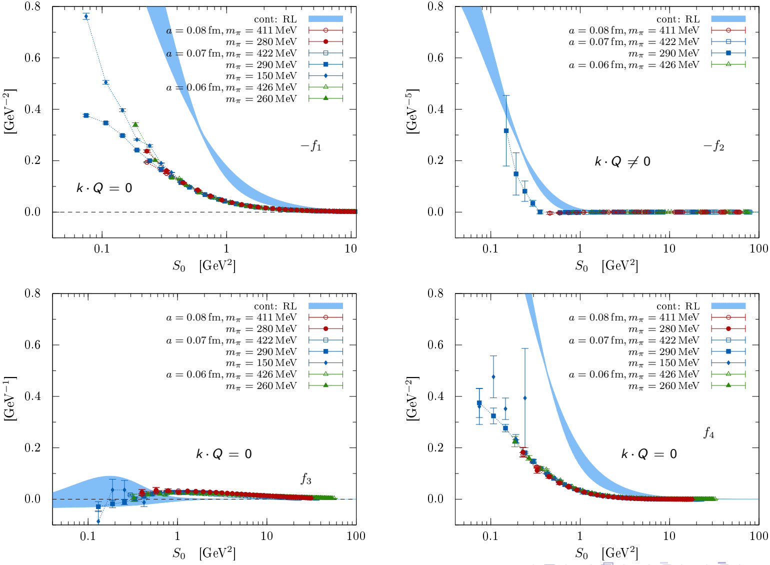
(not renormalized)



Ex: Quark-Photon Vertex $\Gamma_\mu = i\gamma_\mu\lambda_1 + 2k^\mu[i\not{k}\,\lambda_2 + \lambda_3] \; + \; \sum_{j=1}^{8} i\tau_j T_\mu^{(j)}(k,Q)$

lattice (preliminary) vs. continuum (rainbow-ladder)

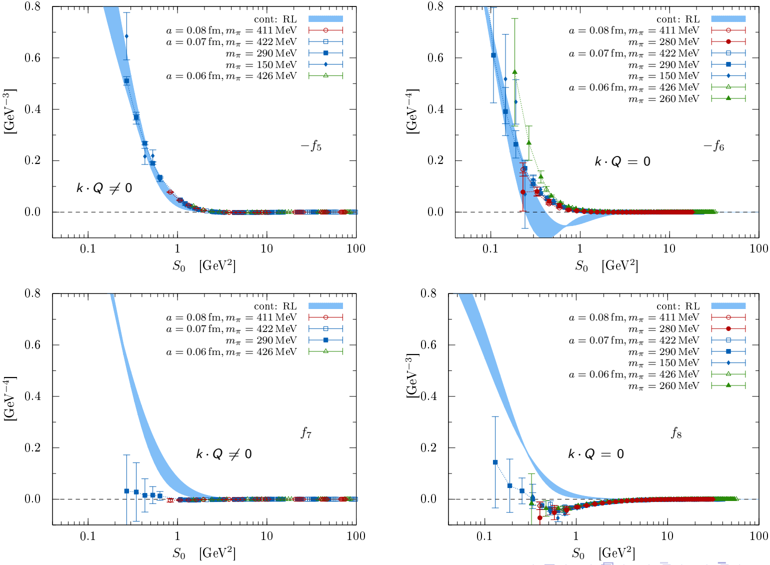
(not renormalized)



Ex: Quark-Photon Vertex $\Gamma_\mu = i\gamma_\mu\lambda_1 + 2k^\mu[i\not{k}\,\lambda_2 + \lambda_3] \; + \; \sum_{j=1}^{8} i\tau_j T_\mu^{(j)}(k,Q)$

lattice (preliminary) vs. continuum (rainbow-ladder)

(not renormalized)



Theoretical input

- needed for different quantities measured in upcoming HE experiments
- Also, BSM physics needs precise knowledge of the hadronic background

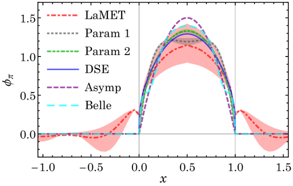
Lattice QCD

- Provides numerical access to many quantities
- Systematically improvable, manifestly gauge-invariant

Adding a gauge

- Access to QCD's *n*-point functions
- Continuum + lattice methods
  (→ synergy effects, complementary approach)
- address hadron physics in a different way
  (target: mechanism of underlying physical phenomena)

[Zhang et al. (2017)]



Vertex structure of some quark bilinears

- $\Lambda = \gamma_\mu, \gamma_\mu\gamma_5, \sigma_{\mu\nu}$
- First lattice data ever, well received

Next

- hadronic wave function

Thank you for your attention!