Update on $\bar{B}_{(s)} \to D^*_{(s)} \ell \bar{\nu}$ form factor at zero-recoil using the Oktay-Kronfeld action

Sungwoo Park
(on behalf of the LANL/SWME Collaboration)

T-2, Los Alamos National Laboratory

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Collaborators

Los Alamos National Laboratory

- Sungwoo Park (speaker)
- Rajan Gupta
- Tanmoy Bhattacharya

Brookhaven National Laboratory

- Yong-Chull Jang

Seoul National University

- Weonjong Lee
- Jon A. Bailey
- Jaehoon Leem
- Jaedon Choi
- Sunkyu Lee
- Seung-Yeob Jwa

Motivation for V_{cb}

- Standard model evaluation of $|\varepsilon_K^{\text{SM}}|$ with lattice QCD inputs : $\hat{B}_K$, $|V_{cb}|$ (exclusive, HFLAV averaged), and etc. has a strong tension from the experiment [Weonjong Lee (next talk)]

$$|\varepsilon_K^{\text{Exp}}| = 2.228 \pm 0.011$$

$$\Delta\varepsilon_K = \varepsilon_K^{\text{Exp}} - \varepsilon_K^{\text{SM}} \approx 4.2\sigma, \qquad |\varepsilon_K^{\text{SM}}| = 1.570 \pm 0.156$$

(In units of 1.0×10^{-3})

- The error budget for $\varepsilon_K^{\text{SM}}$ from various inputs

source	error (%)	memo
V_{cb}	31.3	Exclusive channel, Lattice
$\bar{\eta}$	26.7	apex of UT, AOF
η_{ct}	21.4	$c - t$ box diagram
η_{cc}	9.0	$c - c$ box diagram
$\bar{\rho}$	4.0	apex of UT, AOF
⋮	⋮	⋮

- Belle II, the new B-factory starts running fully on Dec. 2018 and the target statistics is 50 times larger than the previous Belle experiment.

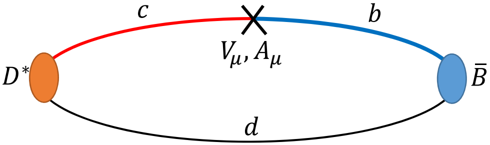
$|V_{cb}|$ from the exclusive $\bar{B} \to D^* \ell \bar{\nu}$ at zero recoil

1. **Experiment**: $\frac{d\Gamma}{dw}(\bar{B} \to D^* \ell \bar{\nu}) \propto (w^2 - 1)^{1/2} |V_{cb}|^2 \cdot |\mathcal{F}(w)|^2$,
   where the recoil parameter $1 \leq (w \equiv v_{\bar{B}} \cdot v_{D^*}) \leq \frac{M_{\bar{B}}^2 + M_{D^*}^2}{2 M_{\bar{B}} M_{D^*}} \approx 1.5$.

2. **Lattice QCD**: Calculate the zero recoil form factor $\mathcal{F}(w = 1) = h_{A_1}(1)$

$$\frac{1}{\sqrt{M_{\bar{B}} M_{D^*}}} \langle D^*(p_{D^*}, \epsilon) | A^\mu | \bar{B}(p_{\bar{B}}) \rangle = -i h_{A_1}(w)(w+1)\epsilon^{*\mu} + i h_{A_2}(w)(\epsilon^* \cdot v_{\bar{B}}) v_{\bar{B}}^\mu + i h_{A_3}(w)(\epsilon^* \cdot v_{\bar{B}}) v_{D^*}^\mu$$

$$\frac{1}{\sqrt{M_{\bar{B}} M_{D^*}}} \langle D^*(p_{D^*}, \epsilon) | V_\mu | \bar{B}(p_{\bar{B}}) \rangle = h_V(w) \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu} v_{D^*}^\alpha v_{\bar{B}}^\beta$$



3. **Determine $|\mathbf{V_{cb}}|$** by combining experiment with lattice QCD results

Current status of $h_{A_1}(w = 1)$

Collaboration	Fermilab-MILC	HPQCD	LANL-SWME
Action for b	Fermilab	NRQCD	Oktay-Kronfeld
Action for c	Fermilab	HISQ	Oktay-Kronfeld
Action for ℓ	AsqTad	HISQ	HISQ
$h_{A_1}(w = 1)$	0.906(4)(12)	0.895(10)(24)	-
error (%)	1.4	2.9	-
Year	2014	2018	-

[Fermilab-MILC Collab., PRD 89, 114504 (2014)]
[HPQCD Collab., PRD 97, 054502 (2018)]

Improved Fermilab action: OK action

- For the $\bar{B} \to D^* \ell \bar{\nu}$ study, the heavy quark discretization error, especially for charm is dominant ($\lambda_c \sim \frac{\Lambda_{QCD}}{2m_c} \sim \frac{500\text{ MeV}}{2 \times 1.3\text{ GeV}} \sim \frac{1}{5}$)

[Fermilab-MILC (2014)] $h_{A_1}(w=1)$	
source	error (%)
statistics	0.4
matching	0.4
χPT	0.5
$g_{D^* D_\pi}$	0.3
c discretization	$1.0 \to (0.2)_{\rm OK}$
etc.	0.1
total	$1.4 \to (0.8)_{\rm OK}$

- Fermilab action calculation of $h_{A_1}(w=1)$ has $\mathcal{O}(\lambda_c^3) \sim 1\%$ discretization error.

- To achieve the precision below 1%, one solution we use is the Oktay-Kronfeld (OK) action, $\mathcal{O}(\lambda^3)$ improved action where its discretization error appears at $\mathcal{O}(\lambda^4)$. [Oktay and Kronfeld, PRD78, 014504 (2008)]

OK-HISQ Action

- **Sea quarks**: HISQ action with $N_f = 2+1+1$ dynamical flavors. The following ensembles are generated by the [MILC collaboration].

ID	a (fm)	Volume	$M_\pi L$	M_π (MeV)	$N_{\text{conf}} \times N_{\text{src}}$
a12m310	0.12	$24^3 \times 64$	4.54	305	1053×3
$a12m220$	0.12	$32^3 \times 64$	4.29	217	
$a12m130$	0.12	$48^3 \times 64$	3.88	132	
a09m310	0.09	$32^3 \times 96$	4.50	313	1001×3
$a09m220$	0.09	$48^3 \times 96$	4.71	220	
$a09m130$	0.09	$64^3 \times 96$	3.66	128	
⋮			⋮		

- **Valence light quarks** (u, d, s): HISQ action
- **Valence heavy quarks** (c, b): Oktay-Kronfeld action
  - Nonperturbative Tuning of κ_{crit}, κ_c, κ_b.

$\bar{B} \to D^* \ell \bar{\nu}$ at zero recoil: $h_{A_1}(1)$ and R

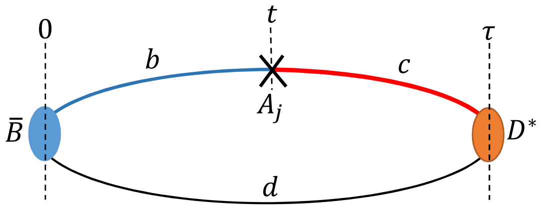
- $h_{A_1}(1)$: semileptonic form factor for $\bar{B} \to D^* \ell \bar{\nu}$ at zero recoil, [Fermilab-MILC, PRD79, 014506 (2009)]

$$|h_{A_1}(1)|^2 = \frac{\langle D^* | A^j_{cb} | \bar{B} \rangle \langle \bar{B} | A^j_{bc} | D^* \rangle}{\langle D^* | V^4_{cc} | D^* \rangle \langle \bar{B} | V^4_{bb} | \bar{B} \rangle} \times \rho^2_{A_j}$$

- $\rho^2_{A_j} = \frac{Z^{cb}_{A_j} Z^{bc}_{A_j}}{Z^{cc}_{V_4} Z^{bb}_{V_4}}$: Matching factor, expected to be very close to 1 [Fermilab-MILC, PRD89, 114504 (2014)]

- We extract the ground state matrix elements from multi-state fits of 3pt correlation functions using various source-sink separation time τ
  - $\langle D^* | A^j_{cb} | B \rangle$: $C^{B \to D^*}_{A_1}(t, \tau)$ and $C^{D^* \to B}_{A_1}(t, \tau)$ simultaneous fit
  - $\langle B | V^4_{bb} | B \rangle$: $C^{B \to B}_{V_4}(t, \tau)$ simultaneous fit
  - $\langle D^* | V^4_{cc} | D^* \rangle$: $C^{D^* \to D^*}_{V_4}(t, \tau)$ simultaneous fit

3-point correlation function: current improvement



$$O_B(0) = \bar{\psi}_b(0)\gamma_5\Omega(0)\chi_d(0)$$
$$O_{D^*}(x) = \bar{\psi}_c(x)\gamma_j\Omega(x)\chi_d(x)$$

$$A_j^{cb}(y) = \bar{\Psi}_c(y)\gamma_j\gamma_5\Psi_b(y),$$

Current operator using the improved field $\Psi(x)$: [Jaehoon Leem, Lattice 2017]

$$\begin{aligned}
\Psi(x) = e^{M_1/2}\Big[1 &+ d_1\gamma\cdot\boldsymbol{D} && \rightarrow \mathcal{O}(\lambda^1)\\
&+ d_2\Delta^{(3)} + d_B i\boldsymbol{\Sigma}\cdot\boldsymbol{B} + d_E\boldsymbol{\alpha}\cdot\boldsymbol{E} && \rightarrow \mathcal{O}(\lambda^2)\\
&+ d_{rE}\{\gamma\cdot\boldsymbol{D},\boldsymbol{\alpha}\cdot\boldsymbol{E}\} + d_3\sum_i\gamma_i D_i\Delta_i + d_4\{\gamma\cdot\boldsymbol{D},\Delta^{(3)}\} &&\\
&+ d_5\{\gamma\cdot\boldsymbol{D}, i\boldsymbol{\Sigma}\cdot\boldsymbol{B}\} + d_{EE}\{\gamma_4 D_4,\boldsymbol{\alpha}\cdot\boldsymbol{E}\} && \rightarrow \mathcal{O}(\lambda^3)\\
&+ d_6[\gamma_4 D_4,\Delta^{(3)}] + d_7[\gamma_4 D_4, i\boldsymbol{\Sigma}\cdot\boldsymbol{B}]\Big]\psi(x).
\end{aligned}$$

Excited state analysis on $C_J^{X\to Y}(t,\tau)$

- We include $2+1$ states for $|B_m\rangle$ and $|D_n^*\rangle$ where $n, m = 0(\text{ground}), 1, 2$

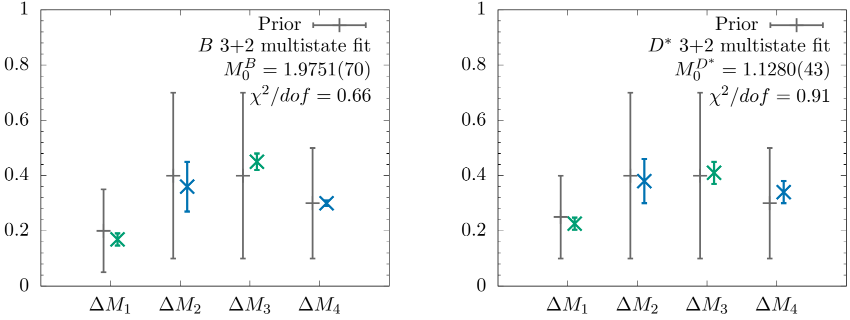
$$
\begin{aligned}
C_{A_j}^{B\to D^*}(t,\tau) &= \langle O_{D^*}^\dagger(0) A_j^{cb}(t) O_B(\tau)\rangle \qquad (0 < t < \tau) \\
&= \mathcal{A}_0^{D^*}\mathcal{A}_0^{B}\langle D_0^*|A_j^{cb}|B_0\rangle e^{-M_{B_0}(\tau-t)} e^{-M_{D_0^*}t} \\
&\quad + \mathcal{A}_0^{D^*}\mathcal{A}_1^{B}\langle D_0^*|A_j^{cb}|B_1\rangle (-1)^{\tau-t} e^{-M_{B_1}(\tau-t)} e^{-M_{D_0^*}t} \\
&\quad + \mathcal{A}_1^{D^*}\mathcal{A}_0^{B}\langle D_1^*|A_j^{cb}|B_0\rangle (-1)^{t} e^{-M_{B_0}(\tau-t)} e^{-M_{D_1^*}t} \\
&\quad + \mathcal{A}_1^{D^*}\mathcal{A}_1^{B}\langle D_1^*|A_j^{cb}|B_1\rangle (-1)^{\tau} e^{-M_{B_1}(\tau-t)} e^{-M_{D_1^*}t} \\
&\quad + \mathcal{A}_2^{D^*}\mathcal{A}_0^{B}\langle D_2^*|A_j^{cb}|B_0\rangle e^{-M_{B_0}(\tau-t)} e^{-M_{D_2^*}t} \\
&\quad + \mathcal{A}_0^{D^*}\mathcal{A}_2^{B}\langle D_0^*|A_j^{cb}|B_2\rangle e^{-M_{B_2}(\tau-t)} e^{-M_{D_0^*}t} \\
&\quad + \mathcal{A}_2^{D^*}\mathcal{A}_1^{B}\langle D_2^*|A_j^{cb}|B_1\rangle (-1)^{\tau-t} e^{-M_{B_1}(\tau-t)} e^{-M_{D_2^*}t} \\
&\quad + \mathcal{A}_1^{D^*}\mathcal{A}_2^{B}\langle D_1^*|A_j^{cb}|B_2\rangle (-1)^{t} e^{-M_{B_2}(\tau-t)} e^{-M_{D_1^*}t} \\
&\quad + \mathcal{A}_2^{D^*}\mathcal{A}_2^{B}\langle D_2^*|A_j^{cb}|B_2\rangle e^{-M_{B_2}(\tau-t)} e^{-M_{D_2^*}t} + \cdots .
\end{aligned}
$$

- We fit the ground state matrix element and excited state contaminations. All the meson 2pt amplitudes and masses are determined from the separate 2-point function analysis.

B, D^*-meson $3+2$ excited states

$$C^{2\text{pt}}(t) = |\mathcal{A}_0|^2 e^{-M_0 t}\left(1 + \left|\frac{\mathcal{A}_2}{\mathcal{A}_0}\right|^2 e^{-\Delta M_2 t} + \left|\frac{\mathcal{A}_4}{\mathcal{A}_0}\right|^2 e^{-(\Delta M_2+\Delta M_4)t} - (-1)^t\left|\frac{\mathcal{A}_1}{\mathcal{A}_0}\right|^2 e^{-\Delta M_1 t} - (-1)^t\left|\frac{\mathcal{A}_3}{\mathcal{A}_0}\right|^2 e^{-(\Delta M_1+\Delta M_3)t}\right) + \cdots + (t \leftrightarrow T - t)$$

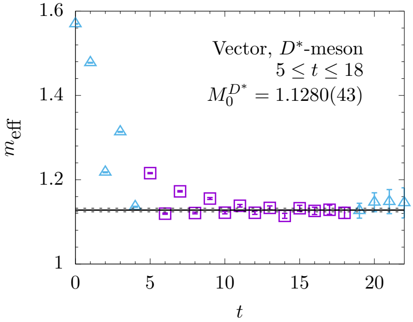
Empirical Bayesian method to stabilize the excited states [PNDME collab., PRD95, 074508 (2017)]

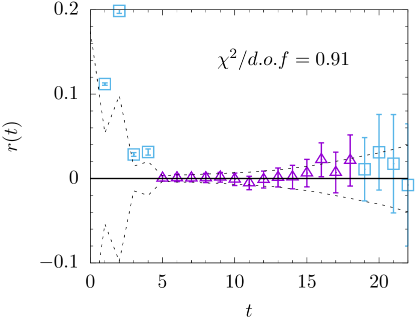


(a12m310 lattice)

D^*-meson 3 + 2 excited states

$$m_{\text{eff}}(t) \equiv \frac{1}{2} \ln \frac{C^{2pt}(t)}{C^{2pt}(t+2)}, \qquad r(t) \equiv \frac{C^{2pt}(t) - f(t)}{|C^{2pt}(t)|}$$





- (a12m310 lattice)
- Similar for B-meson.

Simulation details

ID	m_x/m_s	$\{\sigma, N\}$	$N_{\text{cfg}} \times N_{\text{src}}$	τ
$a12m310$	0.1, $0.2^{\dagger}$, 0.3, 0.4, 1.0	$\{1.5, 5\}$	1053×3	10, 11, 12, 13, (14, 15)
$a09m310$	$0.2^{\dagger}$, 1.0	$\{2, 10\}$	1001×3	15, 16, 17, 18

- m_x: spectator quark mass
- $\dagger$: degenerate sea quark mass point
- $\{\sigma, N\}$: covariant Gaussian smearing parameters
- τ: source-sink time separation.

- We use nonperturbatively tuned hopping parameters κ_{crit}, κ_b, κ_c.
- We do the simultaneous fits using 4 τ values.

Isolation of the matrix element

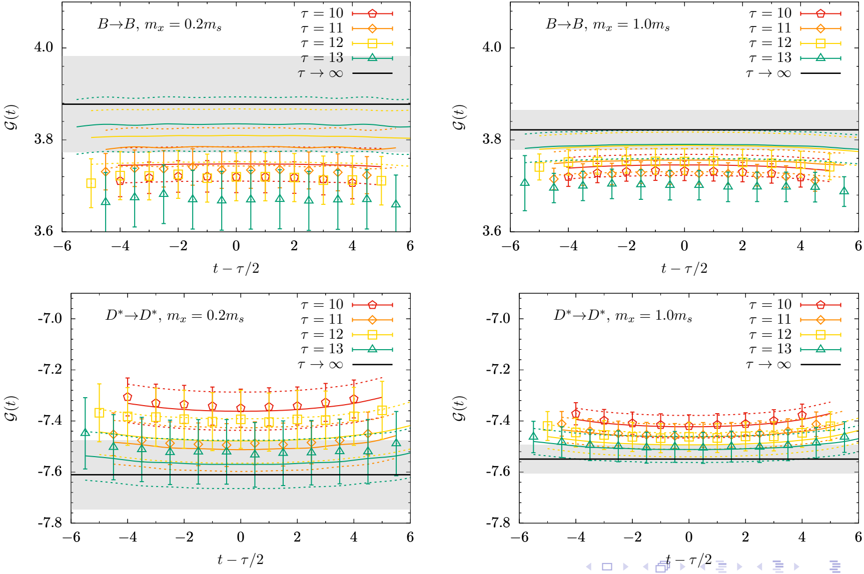
- We introduce the ratio $\mathcal{G}$ which gives the desired ground state matrix element in $\tau \to \infty, t \to \infty$ and $\tau - t \to \infty$ limit, [PNDME Collaboration, PRD96, 114503 (2017)]

$$\mathcal{G}(t,\tau) \equiv \frac{C_J^{X\to Y}(t,\tau)}{C^Y(\tau)} \times \left[\frac{C^Y(t)C^Y(\tau)C^X(\tau-t)}{C^X(t)C^X(\tau)C^Y(\tau-t)}\right]^{1/2}$$
$$= \langle Y|J|X\rangle + \cdots .$$

- **We do not fit** $\mathcal{G}(t,\tau)$. Only used to display the 3pt function plots
- $C_J^{X\to Y}(t,\tau)$: 3-point function, source at 0, current J at t, sink at τ
- $C^X(\tau)$: Meson 2-point function, source at 0, sink at τ

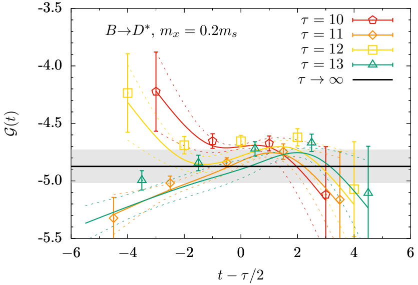
2+1-state fit results of 3pt functions

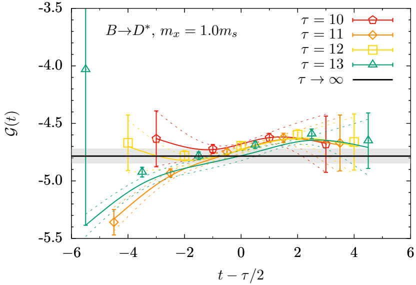
(a12m310 lattice, $\mathcal{O}(\lambda^2)$-improved currents)



2+1-state fit results of 3pt functions

(a12m310 lattice, $\mathcal{O}(\lambda^2)$-improved currents)





$\mathcal{O}(\lambda^\ell)$ improved matrix element from 2+1-state fit

- a12m310 lattice
- $\tau = 10, 11, 12, 13$ simultaneous fit. We do not include $\tau = 14, 15$ data which are noisier.
- No $\mathcal{O}(\lambda)$-improvement correction on $\langle B|V_4|B\rangle$ and $\langle D^*|V_4|D^*\rangle$

$(m_x = 0.2 m_s)$

Current	$\langle B\|V_4\|B\rangle$	cdf	p	$\langle D^*\|V_4\|D^*\rangle$	cdf	p	$\langle D^*\|A_j\|B\rangle$	cdf	p
Unimp.	4.035(114)	0.94	0.53	8.597(179)	0.67	0.87	4.974(155)	1.04	0.40
$\mathcal{O}(\lambda)$-imp.	4.035(114)	0.94	0.53	8.597(179)	0.67	0.87	5.088(154)	1.06	0.37
$\mathcal{O}(\lambda^2)$-imp.	3.878(105)	0.93	0.56	7.610(135)	0.89	0.60	4.874(144)	1.08	0.34

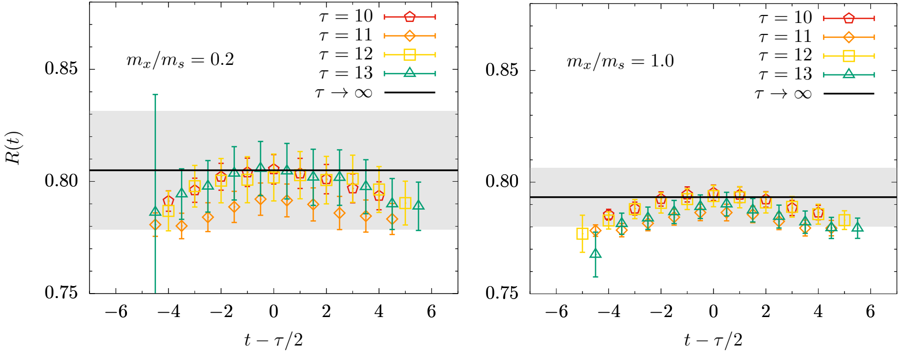
$(m_x = 1.0 m_s)$

Current	$\langle B\|V_4\|B\rangle$	cdf	p	$\langle D^*\|V_4\|D^*\rangle$	cdf	p	$\langle D^*\|A_j\|B\rangle$	cdf	p
Unimp.	3.972(48)	1.24	0.20	8.484(68)	0.55	0.95	4.863(66)	1.29	0.10
$\mathcal{O}(\lambda)$-imp.	3.972(48)	1.24	0.20	8.484(68)	0.55	0.95	4.981(67)	1.25	0.13
$\mathcal{O}(\lambda^2)$-imp.	3.822(43)	1.24	0.20	7.549(56)	0.70	0.84	4.784(64)	1.20	0.18

$|h_{A_1}(1)/\rho_{A_j}|^2$ result and double ratio data $R(t)$

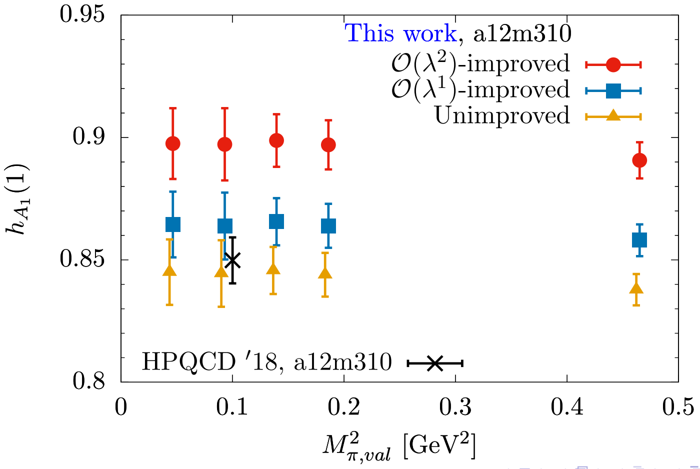
$$R(t,\tau) = \frac{C_{A_1}^{B\to D^*}(t,\tau)\, C_{A_1}^{D^*\to B}(t,\tau)}{C_{V_4}^{B\to B}(t,\tau)\, C_{V_4}^{D^*\to D^*}(t,\tau)}$$

- a12m310 lattice, $\mathcal{O}(\lambda^2)$-improved currents
- We do not fit $R(t,\tau)$,
- but horizontal line and band corresponds to $|h_{A_1}(1)/\rho_{A_j}|^2$ from the 2+1 state fit results of $C^{B\to D^*}$, $C^{B\to B}$ and $C^{D^*\to D^*}$.



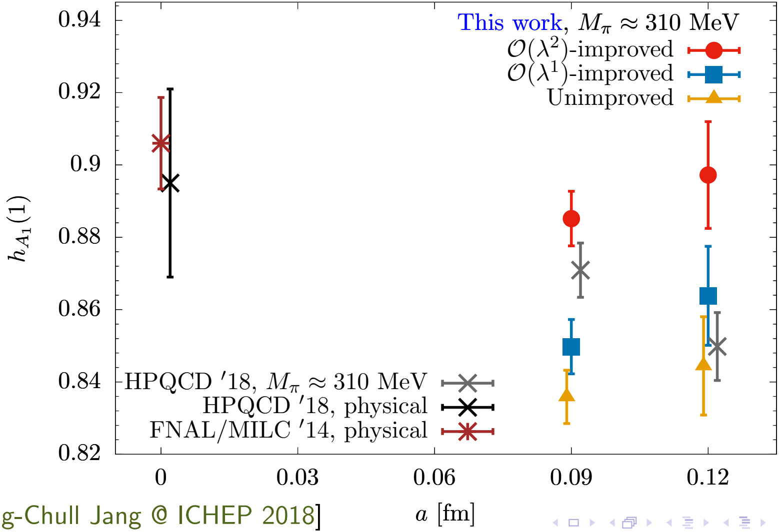
Preliminary results of $h_{A_1}(1)$

- a12m310 lattice ($M_{\pi,sea} \approx 310$ MeV)
- Light spectator mass dependence is appeared to be small.
- Disclaimer: $\rho_{A_j} = 1$ for this work. No chiral extrapolation yet.



Preliminary results of $h_{A_1}(1)$

- This work shows results on a12m310 and a09m310 HISQ lattices.
- Disclaimer: $\rho_{A_j} = 1$ for this work. No chiral-continuum extrapolation yet.



[Yong-Chull Jang @ ICHEP 2018]

Summary

- We obtained preliminary result of $B_{(s)} \to D^*_{(s)} \ell\nu$ semileptonic form factor at zero recoil.
- where the excited contamination is controlled by multistate fits.

Followings are underway:

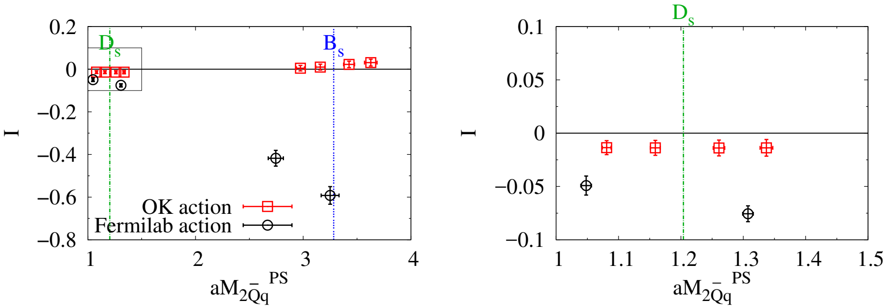
- Analysis for the nonzero recoil form factors of $B \to D^{(*)}\ell\nu$ decays
- Analysis for the decay constants f_D, f_{D_s}, f_B, f_{B_s} and f_{B_c}
- Calculation of the matching factor ρ_{A_μ}, ρ_{V_μ}, Z_A^{hl}, Z_V^{hl}, $\cdots$.
- Chiral-continuum extrapolation

Back-up

Improvements in OK action: Inconsistency

[Yong-Chull Jang et al., EPJC 77:768 (2017)]
Inconsistency parameter I should vanish in the continuum limit. The nonzero value of I shows the nontrivial discretization error, $(\delta M \equiv M_2 - M_1)$

$$I \equiv \frac{2\delta M_{\bar{Q}q} - (\delta M_{\bar{Q}Q} + \delta M_{\bar{q}q})}{2M_{2\bar{Q}q}} \sim \mathcal{O}(\lambda^4) \qquad \text{for OK action}$$



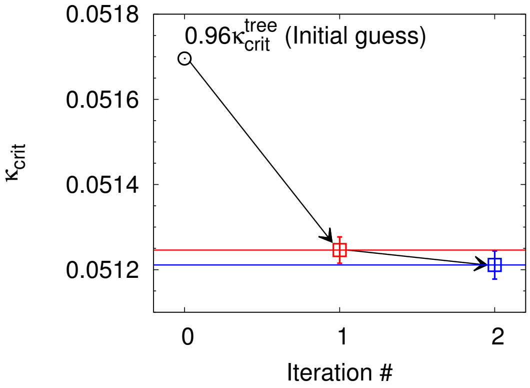
$(a \approx 0.12$ fm, $M_\pi \approx 310$ MeV$)$

Nonperturbative Tuning: κ_{crit}

- The OK action depends on the parameter κ_{crit} through the bare mass

$$am_0 = \frac{1}{2u_0}\left(\frac{1}{\kappa} - \frac{1}{\kappa_{\text{crit}}}\right).$$

κ_{crit}: The value of κ where the pion mass vanishes.



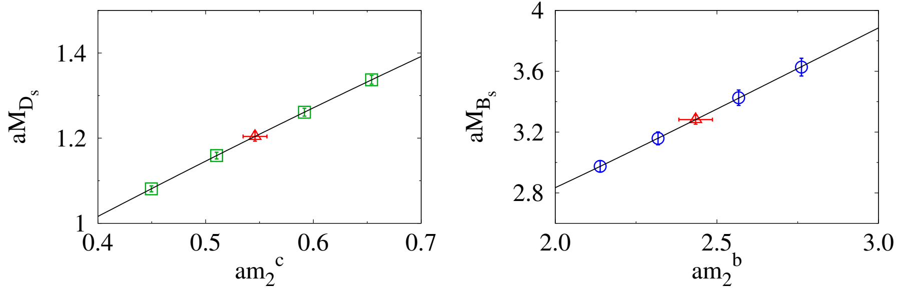
a12m310 lattice
($N_{\text{conf}} = 130$, $N_{\text{src}} = 1$)

Using the iteration, we determine $\kappa_{\text{crit}} = 0.051211(33)(4)$.

Nonperturbative Tuning: κ_c, κ_b

- We find κ_c, κ_b such that the simulated D_s, B_s meson masses reproduce the experimental values.

$$\kappa_c = 0.048524(33)(43), \qquad \kappa_b = 0.04102(14)(9)$$



- Here m_2 (kinetic quark mass) is related to the $m_0(\kappa, \kappa_{crit})$ by the tree-level formula.

$$\frac{1}{am_2} = \frac{2\zeta^2}{am_0(2+am_0)} + \frac{r_s\zeta}{1+am_0}$$