Corrections to Dashen's theorem from lattice QCD and quenched QED

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Outline

1. Introduction
2. QCD+QED
3. Definitions
4. Computational details
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Dashen's Theorem

- According to Dashen's theorem, in the $SU(3)$ limit [Dashen, 1969]

$$\Delta_{\mathrm{QED}} M_K^2 = \Delta_{\mathrm{QED}} M_\pi^2$$

where $\quad \Delta_{\mathrm{QED}} M_K^2 = \left(M_{K^-}^2 - M_{K^0}^2\right)_{\mathrm{QED}} \quad \Delta_{\mathrm{QED}} M_\pi^2 = \left(M_{\pi^-}^2 - M_{\pi^0}^2\right)_{\mathrm{QED}}$

- Corrections are large away from $SU(3)$ limit

- Correcton can be characterized by

$$\epsilon = \frac{\Delta_{\mathrm{QED}} M_K^2 - \Delta_{\mathrm{QED}} M_\pi^2}{\Delta M_\pi^2}$$

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QCD+QED

- Noncompact photon action

$$S_\gamma[A] = \frac{1}{4} \sum_{\mu,\nu,x} (\partial_\mu A_{x,\nu} - \partial_\nu A_{x,\mu})^2$$

- Gauge fixing: Coulomb gauge $\underline{\boldsymbol{\partial}}^\dagger \cdot \underline{\mathbf{A}}_x = 0$
- Zero mode subtraction: QED_L [Hayakawa & Uno, 2008]

$$\sum_{\underline{\mathbf{x}}} A_{\underline{\mathbf{x}},t,\mu} = 0 \qquad \text{for all } t \text{ and } \mu$$

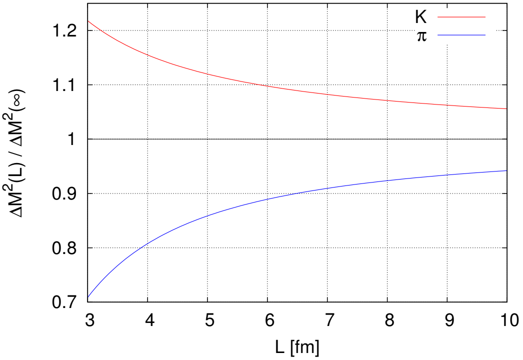
- Coupling to quarks

$$U_{x,\mu} \quad \longrightarrow \quad U_{x,\mu} \cdot e^{i\, e\, q\, A_{x,\mu}}$$

FV correction in QED$_L$

$$m(T,L) \sim m \cdot \left(1 - \frac{q^2\,\alpha\,\kappa}{2\,m\,L}\left[1 + \frac{2}{m\,L}\right] + \mathcal{O}\left(\frac{\alpha}{L^3}\right)\right)$$

with $\kappa \approx 2.837297$ [Borsanyi *et.al.*, 2015]



Quenched QED

- QCD + QED

$$\left\langle \mathcal{O} \right\rangle_{U,A} = \frac{1}{Z} \int \mathrm{d}U \int \mathrm{d}A \;\; \mathcal{O}(U,A) \;\; e^{-S_{\mathrm{g}}[U]} \, e^{-S_{\gamma}[A]} \; \det M(U,A)$$

- Quenched QED

$$\begin{aligned} \left\langle \mathcal{O} \right\rangle_{\substack{U,\mathrm{dynamical} \\ A,\mathrm{quenched}}} &= \frac{1}{Z} \int \mathrm{d}U \underbrace{\int \mathrm{d}A \;\; \mathcal{O}(U,A) \;\; e^{-S_{\gamma}[A]}}_{\left\langle \mathcal{O}(U,A) \right\rangle_{A,\mathrm{q.}}} \, e^{-S_{\mathrm{g}}[U]} \; \det M(U) \\ &= \left\langle \left\langle \mathcal{O}(U,A) \right\rangle_{A,\mathrm{q.}} \right\rangle_{U} \end{aligned}$$

- In practice
  - Generate $SU(3)$ configurations
  - For each $SU(3)$ configuration, generate $U(1)$ configurations with S_{γ}
  - Measure $\mathcal{O}(U,A)$

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Physical point

bare parameters $\longrightarrow$	measurable observables
$m_l = \frac{m_d+m_u}{2}$	$M^2_{\pi^+}$
$\delta m = m_d - m_u$	$\Delta M^2_K = M^2_{K^0} - M^2_{K^+}$
m_s	$M^2_{K\chi} = M^2_{K^0} + M^2_{K^+} - M^2_{\pi^+}$
α	α
a	M_Ω

- We are at the physical point, if

$$\frac{M^2_{\pi^+}}{M^2_\Omega} = \left(\frac{M^2_{\pi^+}}{M^2_\Omega}\right)_{\text{phys}}, \quad \frac{\Delta M^2_K}{M^2_\Omega} = \left(\frac{\Delta M^2_K}{M^2_\Omega}\right)_{\text{phys}}, \quad \frac{M^2_{K\chi}}{M^2_\Omega} = \left(\frac{M^2_{K\chi}}{M^2_\Omega}\right)_{\text{phys}}, \quad \alpha = \alpha_{\text{phys}}$$

- In vicinity of physical point

$$\mathcal{O}(\{m_l, \delta_m, m_s, \alpha\}) = \mathcal{O}_{\text{phys}} + B\left(\frac{M^2_{\pi^+}}{M^2_\Omega} - \left(\frac{M^2_{\pi^+}}{M^2_\Omega}\right)_{\text{phys}}\right) + C\left(\frac{\Delta M^2_K}{M^2_\Omega} - \left(\frac{\Delta M^2_K}{M^2_\Omega}\right)_{\text{phys}}\right) + \\ + D\left(\frac{M^2_{K\chi}}{M^2_\Omega} - \left(\frac{M^2_{K\chi}}{M^2_\Omega}\right)_{\text{phys}}\right) + E\left(\alpha - \alpha_{\text{phys}}\right)$$

m_u/m_d

- Introduce

  M_{uu}: meson with quark content u, $\bar{u}$, w/o disconnected
  M_{dd}: meson with quark content d, $\bar{d}$, w/o disconnected

- Connection to quark masses

$$\begin{aligned} M_{uu}^2 &\propto m_u \\ M_{dd}^2 &\propto m_d \\ \Delta M^2 = M_{dd}^2 - M_{uu}^2 &\propto m_d - m_u \\ M_{\pi^0}^2 &\propto \frac{m_d + m_u}{2} \end{aligned}$$

- Define

$$\frac{m_u}{m_d} = \frac{2M_{\pi^0}^2 - M_{dd}^2 + M_{uu}^2}{2M_{\pi^0}^2 + M_{dd}^2 - M_{uu}^2} = \frac{2M_{\pi^0}^2 - \Delta M^2}{2M_{\pi^0}^2 + \Delta M^2}$$

Strategy to obtain m_u/m_d

- Measure $\Delta M^2 = M^2_{dd} - M^2_{uu}$ in continuum, at physical point

$$\left(\frac{\Delta M^2}{M^2_\Omega}\right)(\{m_l, \delta m, m_s, \alpha\}) = \left(\frac{\Delta M^2}{M^2_\Omega}\right)_{\text{phys}} + B\left(\frac{M^2_{\pi^+}}{M^2_\Omega} - \left(\frac{M^2_{\pi^+}}{M^2_\Omega}\right)_{\text{phys}}\right) +$$
$$+C\left(\frac{\Delta M^2_K}{M^2_\Omega} - \left(\frac{\Delta M^2_K}{M^2_\Omega}\right)_{\text{phys}}\right) + D\left(\frac{M^2_{K\chi}}{M^2_\Omega} - \left(\frac{M^2_{K\chi}}{M^2_\Omega}\right)_{\text{phys}}\right) + E\left(\alpha - \alpha_{\text{phys}}\right)$$

- Then

$$\frac{m_u}{m_d} = \frac{2\left(\frac{M^2_{\pi^0}}{M^2_\Omega}\right)_{\text{phys}} - \left(\frac{\Delta M^2}{M^2_\Omega}\right)_{\text{phys}}}{2\left(\frac{M^2_{\pi^0}}{M^2_\Omega}\right)_{\text{phys}} + \left(\frac{\Delta M^2}{M^2_\Omega}\right)_{\text{phys}}}$$

Correction to Dashen's Theorem

- Correction is characterised by

$$\epsilon = \frac{\Delta_{\text{QED}} M_K^2 - \Delta_{\text{QED}} M_\pi^2}{\Delta M_\pi^2}$$

where $\quad \Delta_{\text{QED}} M_K^2 = \left(M_{K^-}^2 - M_{K^0}^2\right)_{\text{QED}} \quad \Delta_{\text{QED}} M_\pi^2 = \left(M_{\pi^-}^2 - M_{\pi^0}^2\right)_{\text{QED}}$

- $\Delta_{\text{QED}} M_\pi^2$ is difficult to measure
  ΔM_π^2 is dominated by $\Delta_{\text{QED}} M_\pi^2$ [FLAG, 2017]

- Measure instead

$$\epsilon' = \frac{\Delta_{\text{QED}} M_K^2 - \Delta M_\pi^2}{\Delta M_\pi^2}$$

- We need to obtain $\Delta_{\text{QED}} M_K^2$ in continuum, at physical point

Strategy to obtain $\Delta_{\mathrm{QED}} M_K^2$

- $\frac{\Delta M_K^2}{M_\Omega^2} = \left(\frac{\Delta_{\mathrm{QED}} M_K^2}{M_\Omega^2}\right)$ if

$$\left(\frac{\Delta M^2}{M_\Omega^2}\right) = 0, \quad \frac{M_{\pi^+}^2}{M_\Omega^2} = \left(\frac{M_{\pi^+}^2}{M_\Omega^2}\right)_{\mathrm{phys}}, \quad \frac{M_{K\chi}^2}{M_\Omega^2} = \left(\frac{M_{K\chi}^2}{M_\Omega^2}\right)_{\mathrm{phys}}, \quad \alpha = \alpha_{\mathrm{phys}}$$

- Measure $\left(\frac{\Delta M^2}{M_\Omega^2}\right)_{\mathrm{phys}}$ and C

$$\left(\frac{\Delta M^2}{M_\Omega^2}\right) = \left(\frac{\Delta M^2}{M_\Omega^2}\right)_{\mathrm{phys}} + B\left(\frac{M_{\pi^+}^2}{M_\Omega^2} - \left(\frac{M_{\pi^+}^2}{M_\Omega^2}\right)_{\mathrm{phys}}\right) + \\ + C\left(\frac{\Delta M_K^2}{M_\Omega^2} - \left(\frac{\Delta M_K^2}{M_\Omega^2}\right)_{\mathrm{phys}}\right) + D\left(\frac{M_{K\chi}^2}{M_\Omega^2} - \left(\frac{M_{K\chi}^2}{M_\Omega^2}\right)_{\mathrm{phys}}\right) + E\left(\alpha - \alpha_{\mathrm{phys}}\right)$$

- Obtain $\frac{\Delta_{\mathrm{QED}} M_K^2}{M_\Omega^2}$ from

$$0 = \left(\frac{\Delta M^2}{M_\Omega^2}\right)_{\mathrm{phys}} + C\left(\frac{\Delta_{\mathrm{QED}} M_K^2}{M_\Omega^2} - \left(\frac{\Delta M_K^2}{M_\Omega^2}\right)_{\mathrm{phys}}\right)$$

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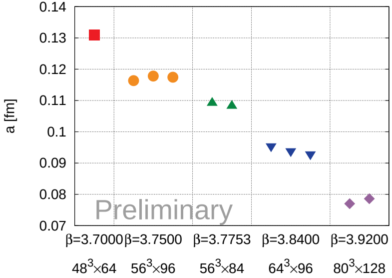
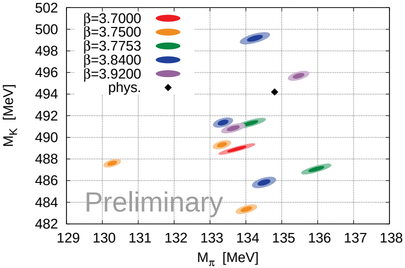
Ensembles

- Subset of [1612.02364,1711.04980]
- Tree-level Symanzyk gauge action
- $N_f = 2+1+1$ staggered fermions
- stout smearing 4 steps, $\varrho = 0.125$
- m_l and m_s are chosen to give approx.
  $\overline{M}_\pi = 134.8(3)\,\text{MeV}$
  $\overline{M}_K = 494.2(3)\,\text{MeV}$
  [FLAG, 2017]
- m_c is fixed via $\frac{m_c}{m_s} = 11.85$
- α is set to Thomson limit
- a is fixed via $M_\Omega = 1672.45\,\text{MeV}$





Fit strategy

- Measurements at 3 parameter sets:

$$\begin{aligned} \text{ISO:}\quad & \delta m = 0, \quad \alpha = 0 \quad \longrightarrow \quad \Delta M^2 = 0, \quad \Delta M_K^2 = 0 \\ \text{QED:}\quad & \delta m = 0, \quad \alpha = \alpha_{\text{phys}} \\ \text{SIB:}\quad & \delta m \neq 0, \quad \alpha = 0 \end{aligned}$$

- Fully correlated fit for $\left(\frac{\Delta M^2}{M_\Omega^2}\right)_{\text{phys}}$, A_γ, A_s, B, C, D

$$\left(\frac{\Delta M^2}{M_\Omega^2}\right)(QED) = \left(\frac{\Delta M^2}{M_\Omega^2}\right)_{\text{phys}} + A_\gamma \cdot (aM_\Omega)^2 + B\left(\frac{M_{\pi^+}^2}{M_\Omega^2}(QED) - \left(\frac{M_{\pi^+}^2}{M_\Omega^2}\right)_{\text{phys}}\right) + \\ + C\left(\frac{\Delta M_K^2}{M_\Omega^2}(QED) - \left(\frac{\Delta M_K^2}{M_\Omega^2}\right)_{\text{phys}}\right) + D\left(\frac{M_{K\chi}^2}{M_\Omega^2}(QED) - \left(\frac{M_{K\chi}^2}{M_\Omega^2}\right)_{\text{phys}}\right)$$

$$\left(\frac{\Delta M^2}{M_\Omega^2}\right)(SIB) - \left(\frac{\Delta M^2}{M_\Omega^2}\right)(ISO) = A_s \cdot (aM_\Omega)^2 + \\ + B\left(\frac{M_{\pi^+}^2}{M_\Omega^2}(SIB) - \frac{M_{\pi^+}^2}{M_\Omega^2}(ISO)\right) + C\left(\frac{\Delta M_K^2}{M_\Omega^2}(SIB) - \frac{\Delta M_K^2}{M_\Omega^2}(ISO)\right)$$

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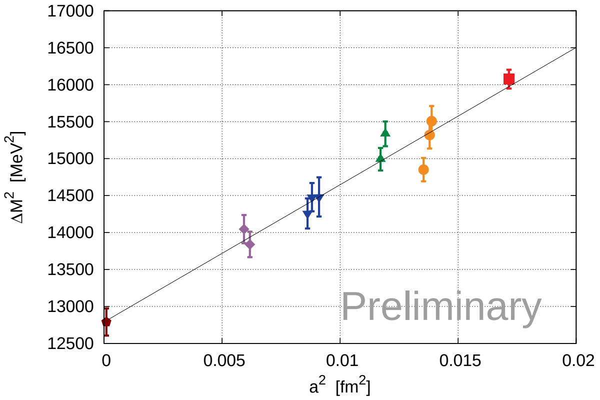
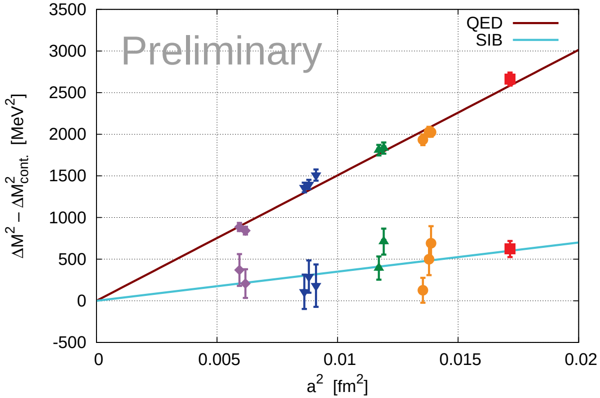
Continuum limit



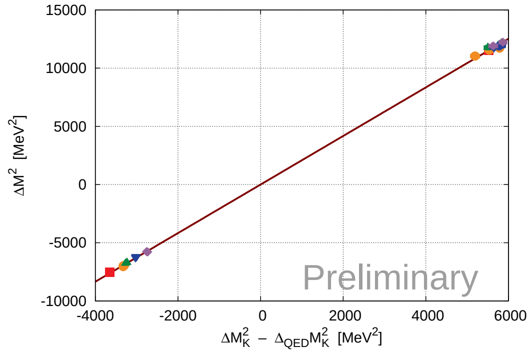
$$\Delta M^2 = 1.279(18)(17) \times 10^4 \text{ MeV}^2$$

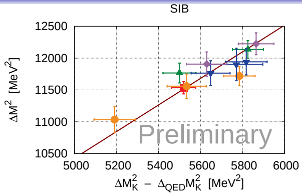
$$\frac{m_u}{m_d} = \frac{2M_{\pi^0}^2 - \Delta M^2}{2M_{\pi^0}^2 + \Delta M^2} = 0.4804(55)(52)$$

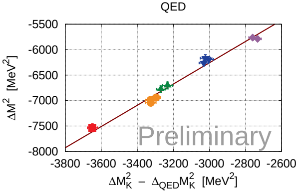
A_γ vs. A_s


QED
SIB
Preliminary
3500
3000
2500
2000
1500
1000
500
0
-500
0
0.005
0.01
0.015
0.02
$\Delta M^2 - \Delta M^2_{cont.}$ [MeV^2]
a^2 [fm^2]

$\Delta_{\mathrm{QED}} M_K^2$







$$C = 2.085(29)(12)$$

$$\Delta M^2 = \Delta M^2_{\mathrm{phys}} + C\left(\Delta M_K^2 - \Delta M^2_{K,\mathrm{phys}}\right) + \dots$$

$$\Delta_{\mathrm{QED}} M_K^2 = \Delta M_K^2 - \frac{1}{C} \cdot \Delta M^2 = 2228(35)(47)\,\mathrm{MeV}^2$$

$$\epsilon' = \frac{\Delta_{\mathrm{QED}} M_K^2 - \Delta M_\pi^2}{\Delta M_\pi^2} = 0.767(28)(38)$$

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Summary

- Computation of m_u/m_d and $\Delta_{\mathrm{QED}} M_K^2$ using $N_f = 2+1+1$ staggered fermions

- QED is still quenched

- Preliminary results, compatible with previous calculations