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ALPHA Collaboration

$\mathcal{O}(a)$ improved quark mass renormalization for a non-perturbative matching of HQET to three-flavor QCD

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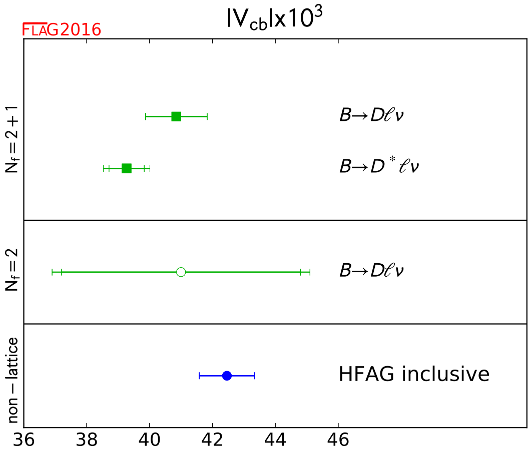
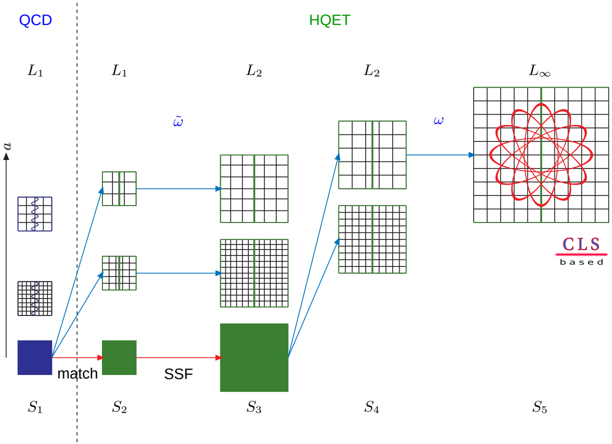
$N_f = 2 + 1$ form factors of semi-leptonic B-decays



- Few number of lattice determinations in particular for semi-leptonic $B \to D$
- $N_f = 2 + 1$ seems to be necessary
- **HQET**: clean method with controlled systematics
- HQET studies for semi-leptonic $B \to K$, $N_f = 2$:
  - *M. Della Morte, S. Dooling, J. Heitger, D. Hesse and H. Simma* [1312.1566] ALPHA Collaboration
  - *F. Bahr, D. Banerjee, F. Bernardoni, A. Joseph, M. Koren, H. Simma and R. Sommer* [1601.04277] ALPHA Collaboration

$N_f = 2+1$ form factors of semi-leptonic B-decays

ALPHA Collaboration's strategy for B–physics via non-perturbative HQET at $\mathcal{O}(1/m_h)$



- non-perturbative finite-volume matching of QCD and HQET in L_1:

$$\Phi_i^{HQET}(L_1, z, a) \stackrel{!}{=} \lim_{a\to 0} \Phi_i^{QCD}(L_1, z, a)$$

- Φ_i^{HQET} depend on HQET-parameters $\omega_i(z, a)$

- Ensembles on line of constant physics with fixed $L_1 \approx 0.5$ fm:

$$\bar{g}^2_{GF}(L_1/2) \equiv g^2_\star \approx 4.0$$
$$m_{sea}(L_1/2) \approx 0$$

HQET–QCD matching for $N_\mathrm{f} = 2 + 1$

Importance of quark mass renormalization in the $\mathcal{O}(a)$ improved theory for the non-perturbative finite-volume matching step

- Fix renormalized & improved heavy quark mass from the bare subtracted mass
- HQET parameters at RGI $z = z_\mathrm{b} = L_1 M_\mathrm{b}$ (and $z_\mathrm{c} = L_1 M_\mathrm{c}$ for $\mathrm{B} \to \mathrm{D}$ decays later)

HQET–QCD matching for $N_f = 2 + 1$

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$$M_h = h(L)\, Z_m(g_0, L/a)\, \left(1 + b_m(g_0)\, a m_{q,h}\right)\, m_{q,h} + \mathcal{O}(a^2)$$

$$Z_m(g_0, L/a) = \frac{Z(g_0)\, Z_A(g_0)}{Z_P(g_0, L/a)}\,, \quad a m_{q,h} = \frac{1}{2}\left(\frac{1}{\kappa_h} - \frac{1}{\kappa_{cr}}\right)\,, \quad L = L_0 = \frac{L_1}{2}$$

- Z_A: *J. Bulava, M. Della Morte, J. Heitger and C. Wittemeier* [1604.05827] ALPHA Collaboration
  or via χ-SF: *M. Dalla Brida, T. Korzec, S. Sint and P. Vilaseca*, in prep.
  Z_P and $h(L)$: *I. Campos, P. Fritzsch, C. Pena, D. Preti, A. Ramos and A. Vladikas* [1802.05243] ALPHA Collaboration

Determination of the improvement coefficients

- To impose fixed $z \equiv L_1 M_h$, we determine b_m and Z for

$$L_0^3 \times T \qquad L_0 = L_1/2 \qquad T = L_0 \qquad 0.008\,\text{fm} \lesssim a \lesssim 0.02\,\text{fm}$$

- Status: Test measurements on tuning ensembles with $L_0/a = 12, 16, 20, 24, 32$
- Plan: measure on tuned ensembles with $L_0 = \frac{L_1}{2}$ on a line of constant physics

Determination of the improvement coefficients

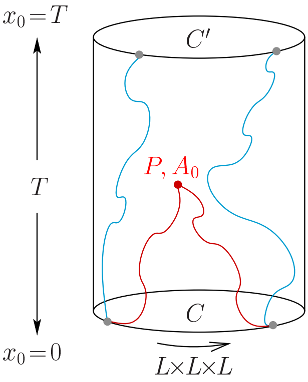
- Calculation based on non-degenerate heavy-light boundary-to-bulk Schrödinger functional correlation functions $f_A^{ij}(x_0)$ and $f_P^{ij}(x_0)$ and the PCAC-relation:

$$m_{ij}(x_0) \equiv m_{ij}(x_0; L/a, T/L, \theta) = \frac{\tilde{\partial}_0 f_A^{ij}(x_0) + a c_A \partial_0^* \partial_0 f_P^{ij}(x_0)}{2 f_P^{ij}(x_0)}$$

- Bare quark masses:

$$am_{q,1} \equiv am_{q,l} \qquad am_{q,2} \equiv am_{q,h} \qquad am_{q,3} \equiv \frac{am_{q,1}+am_{q,2}}{2}$$

- Impose different mass dependent improvement conditions: Measure a range of heavy valence quark masses



Improvement coefficients: quenched and $N_f = 2$

M. Guagnelli, R. Petronzio, J. Rolf, S. Sint, R. Sommer and U. Wolff [hep-lat/0009021] ALPHA Collaboration
P. Fritzsch, J. Heitger and N. Tantalo [1004.3978] ALPHA Collaboration

Definition of the estimators up to $\mathcal{O}(a^2)$ effects

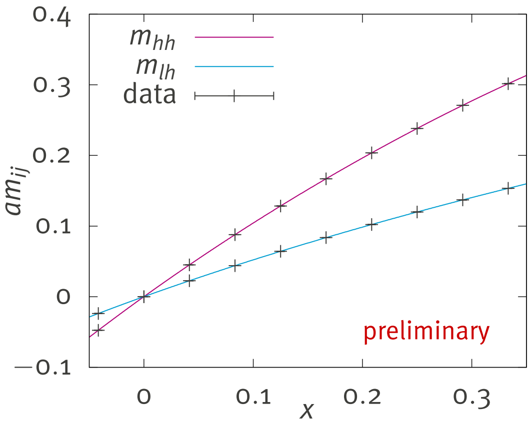
$$R_{\mathrm{AP}} \equiv \frac{2\,(2m_{12} - m_{11} - m_{22})}{(m_{11} - m_{22})\,(am_{\mathrm{q},1} - am_{\mathrm{q},2})} = (b_{\mathrm{A}} - b_{\mathrm{P}})\left[1 + \mathcal{O}(am_{\mathrm{q},1} + am_{\mathrm{q},2})\right]$$

$$R_{\mathrm{m}} \equiv \frac{4\,(m_{12} - m_{33})}{(m_{11} - m_{22})\,(am_{\mathrm{q},1} - am_{\mathrm{q},2})} = b_{\mathrm{m}}\left[1 + \mathcal{O}(am_{\mathrm{q},1} + am_{\mathrm{q},2})\right]$$

$$R_Z \equiv \frac{m_{11} - m_{22}}{m_{\mathrm{q},1} - m_{\mathrm{q},2}} + (R_{\mathrm{AP}} - R_{\mathrm{m}})\,(am_{11} + am_{22}) = Z\left[1 + \mathcal{O}(am_{sea})\right]$$

Improvement coefficients: $N_f = 2 + 1$

- New approach in collaboration with *G. de Divitiis, C. C. Köster and A. Vladikas*, Lat'17: [1710.07020] ALPHA Collaboration and in progress
- Simultaneously fit the bare current quark masses m_{ij} in
  $$x \equiv \Delta m_h = am_{q,h} - am_{q,l} = \frac{1}{2}\left(\frac{1}{\kappa_h} - \frac{1}{\kappa_l}\right)$$
  with $\kappa_l = \kappa_{sea}$
- heavy-heavy:
  $$m_{hh} = D_0 + D_1 x + D_2 x^2 + \cdots + D_k x^k$$
- light-heavy:
  $$m_{lh} = D_0 + \tfrac{1}{2} D_1 x + N_2 x^2 + \cdots + N_k x^k$$



Improvement coefficients: $N_f = 2 + 1$

$$m_{hh} = D_0 + D_1 x + D_2 x^2 + \cdots + D_k x^k \qquad m_{lh} = D_0 + \frac{1}{2} D_1 x + N_2 x^2 + \cdots + N_k x^k$$

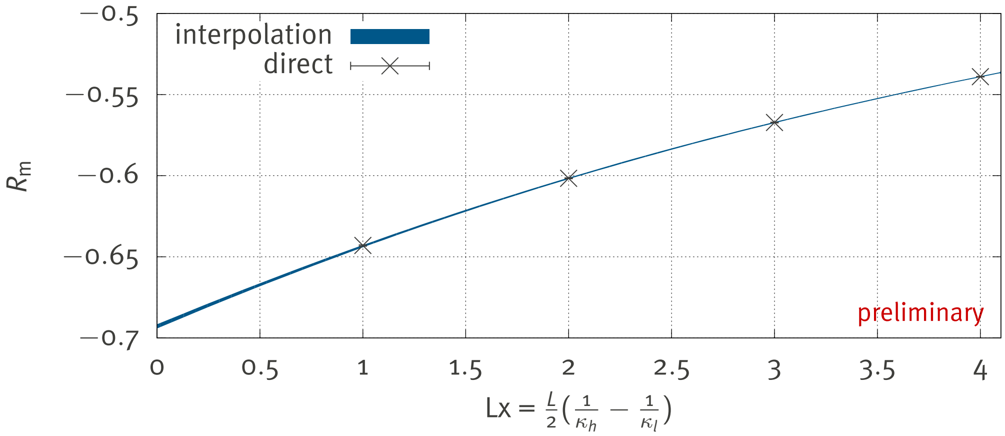
Express the current masses by the fit parameters

$$R_{AP} \equiv 2 \frac{(2N_2 - D_2) + (2N_3 - D_3)x + \cdots + (2N_k - D_k)x^{k-2}}{D_1 + D_2 x + \cdots + D_k x^{k-1}} = (b_A - b_P)\left[1 + \mathcal{O}(am_{q,1} + am_{q,2})\right]$$

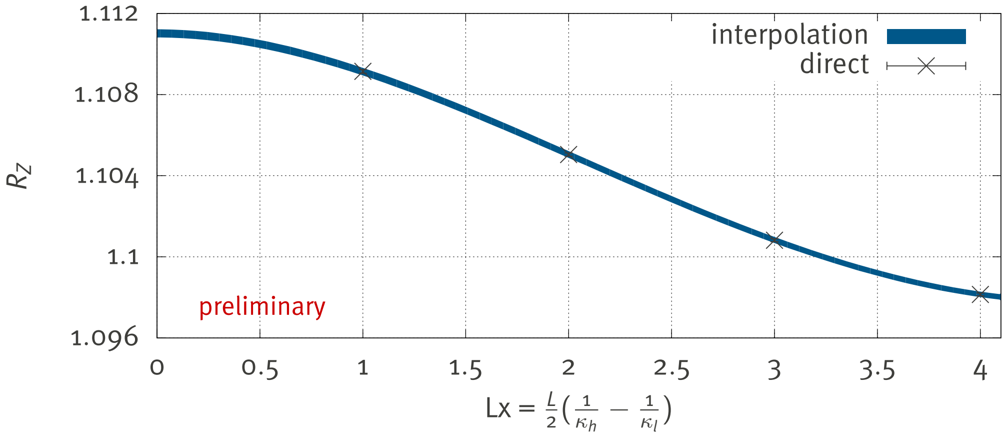
$$R_m \equiv \frac{(4N_2 - D_2) + (4N_3 - \frac{D_3}{2})x + \cdots + (4N_k - \frac{D_k}{2^{k-2}})x^{k-2}}{D_1 + D_2 x + \cdots + D_k x^{k-1}} = b_m \left[1 + \mathcal{O}(am_{q,1} + am_{q,2})\right]$$

$$R_Z \equiv \frac{D_1 + D_2 x + \cdots + D_k x^{k-1}}{1 + 2\frac{D_0 D_2}{D_1 D_1}} = Z\left[1 + \mathcal{O}(am_{sea})\right]$$

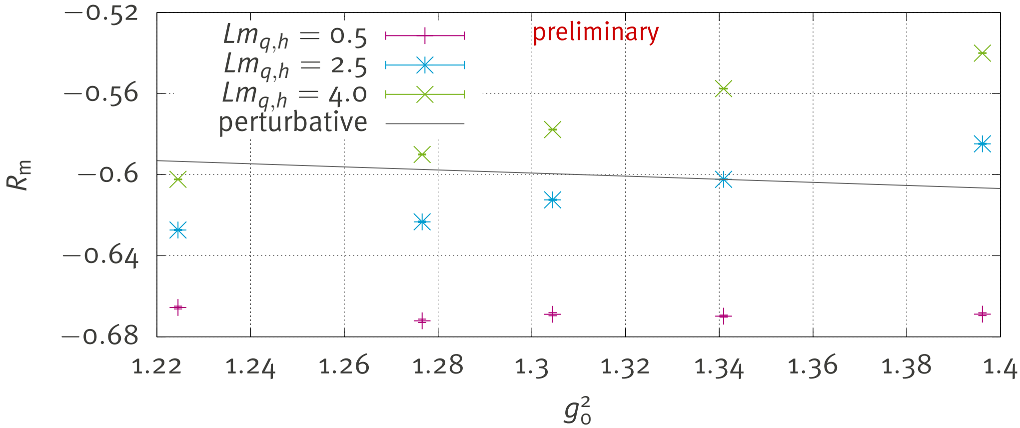
Improvement coefficients: representative example $L/a = 12$



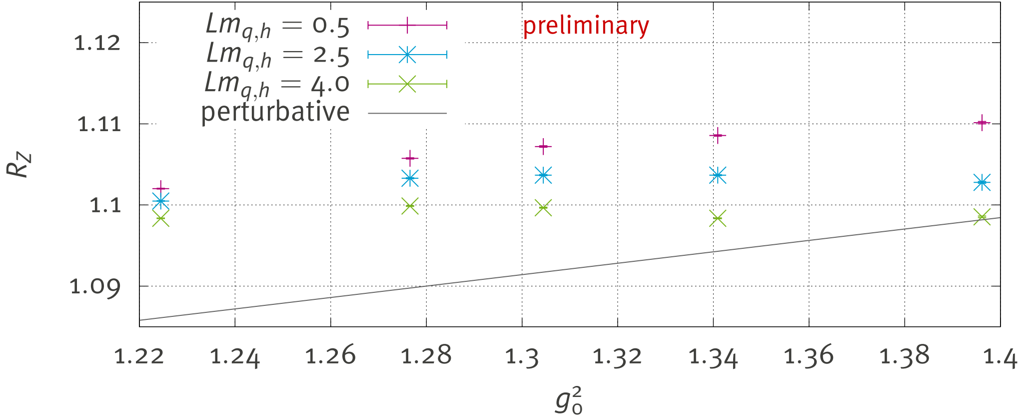
Renormalization constant: representative example $L/a = 12$



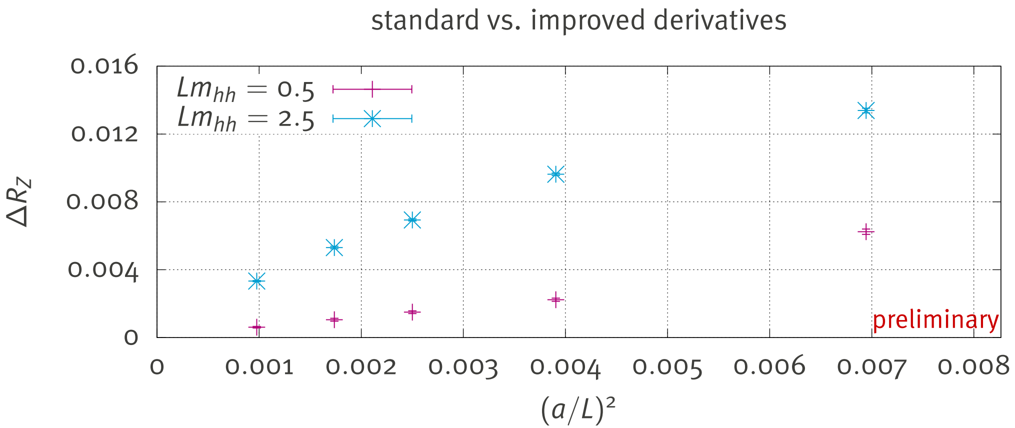
Improvement coefficients: dependence on the coupling



Renormalization constant: dependence on the coupling



Renormalization constant: $\mathcal{O}(a^2)$ ambiguities



Summary

- Improvement coefficients necessary for the finite-volume matching of HQET to QCD
- First test measurements done on tuning ensembles, new strategy applied
- Expected behavior of $\mathcal{O}(a)$ ambiguities

Summary

- Improvement coefficients necessary for the finite-volume matching of HQET to QCD
- First test measurements done on tuning ensembles, new strategy applied
- Expected behavior of $\mathcal{O}(a)$ ambiguities
- Plans for the future:
    - Measurements on all tuned ensembles with $L = L_0$
    - Improvement conditions for a large range of heavy valence quark masses
    - Determination of the z dependence of the matching parameters

Improvement coefficients: $N_f = 2+1$

$$m_{hh} = D_0 + D_1 x + D_2 x^2 + \cdots + D_k x^k \qquad m_{lh} = D_0 + \frac{1}{2} D_1 x + N_2 x^2 + \cdots + N_k x^k$$

Bare subtracted quark mass: $(\kappa_l = \kappa_{sea})$

$$m_{q,i} = m_{q,l} + \Delta m_i, \qquad \Delta m_i = \frac{1}{2}\left(\frac{1}{\kappa_i} - \frac{1}{\kappa_{sea}}\right)$$

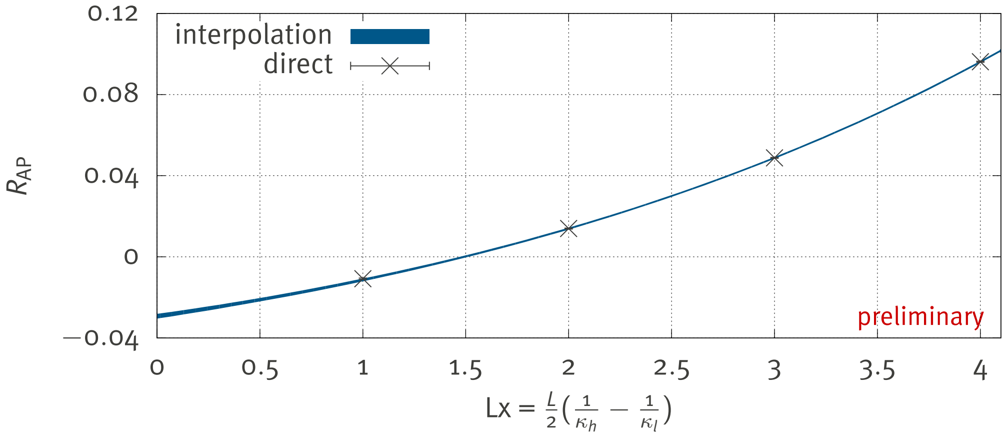
Definition of the current quark masses up to $\mathcal{O}([am]^2)$

$$m_{ii} = m_{ll} + Z \left[\left\{1 + B m_{q,l}\right\} \Delta m_i + \left\{b_m - [b_A - b_P]\right\} [\Delta m_i]^2\right]$$

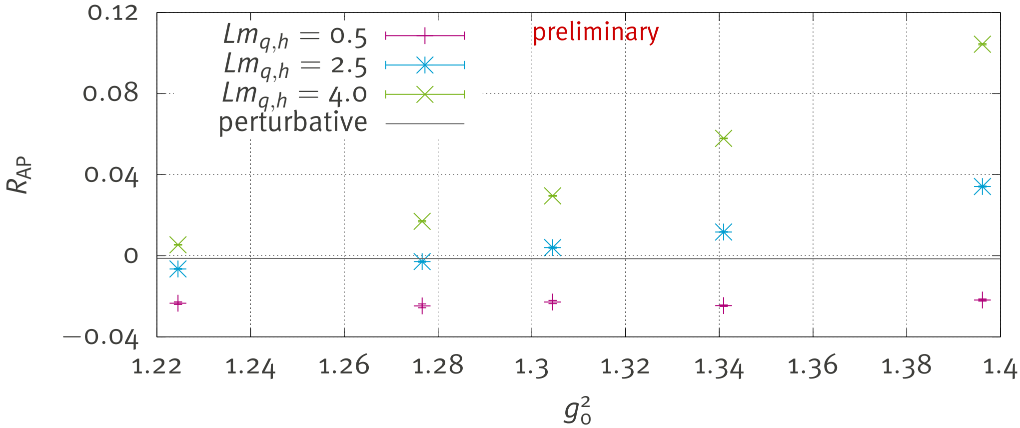
$$m_{li} = m_{ll} + Z \frac{1}{2} \left[\left\{1 + B m_{q,l}\right\} \Delta m_i + \left\{b_m - \frac{1}{2}[b_A - b_P]\right\} [\Delta m_i]^2\right]$$

with $B = 2(b_m - [b_A - b_P]) + N_f(\bar{b}_m - [\bar{b}_A - \bar{b}_P]) - (r_m - 1)[b_A - b_P]$

Improvement coefficients: representative example $L/a = 12$



Improvement coefficients: dependence on the coupling



Improvement coefficients: $\mathcal{O}(am_q)$ ambiguities

standard vs. improved derivatives