Finite-size scaling of Polyakov's loop in the 2D Abelian Higgs model

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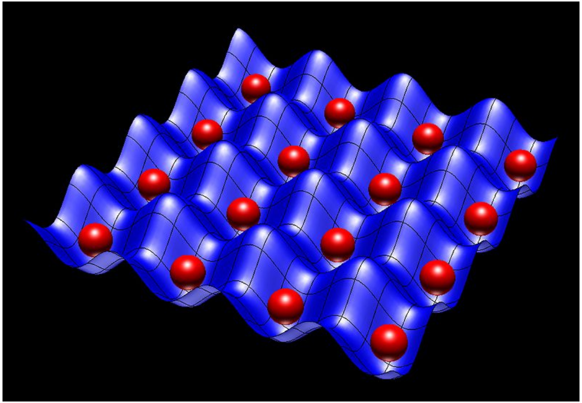
The proposal

We propose...

- a method to quantum simulate the lattice Abelian Higgs model in 2D.
- to use discrete integer-valued fields and maintain gauge invariance exactly (tensor formulation).
- to measure **universal features of the Polyakov loop**.
- to use a physical ladder built as an optical lattice.
- an even simpler model can be tested first using the same set-up (Ising).

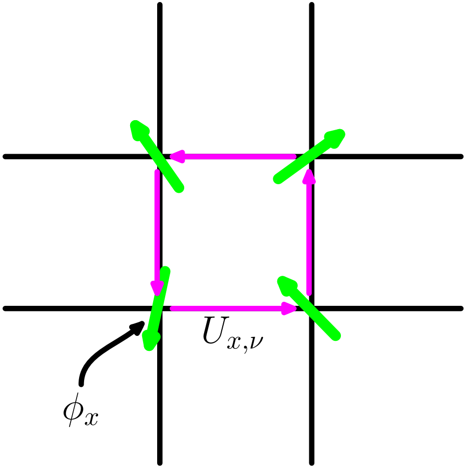


The model

The lattice gauge theory we focus on is the Abelian Higgs model in two Euclidean dimensions. This model

- is the Schwinger model with the fermion replaced by a complex scalar field.
- is believed to be confining, in the sense that there is a linear potential.
- has topological solutions.
- Here the Higgs mode is taken infinitely massive.

$$S = -\beta_{pl}\sum_{x}\sum_{\nu<\mu}\cos(A_{x,\mu}+A_{x+\mu,\nu}-A_{x+\nu,\mu}-A_{x,\nu})$$
$$-2\kappa\sum_{x}\sum_{\nu=1}^{2}\cos(\theta_{x+\nu}-\theta_{x}+A_{x,\nu})$$



The model

- The original partition function is a sum over the compact fields

$$Z = \int \mathcal{D}[A_{x,\mu}]\mathcal{D}[\theta_x]e^{-S}$$

- The Boltzmann weights can be Fourier expanded

$$e^{\beta_{pl}\cos(F_{x,\mu\nu})} = \sum_{m=-\infty}^{\infty} I_m(\beta_{pl})e^{imF_{x,\mu\nu}}$$

$$e^{2\kappa\cos(\theta_{x+\nu}-\theta_x+A_{x,\nu})} = \sum_{n=-\infty}^{\infty} I_n(2\kappa)e^{in(\theta_{x+\nu}-\theta_x+A_{x,\nu})}$$

giving

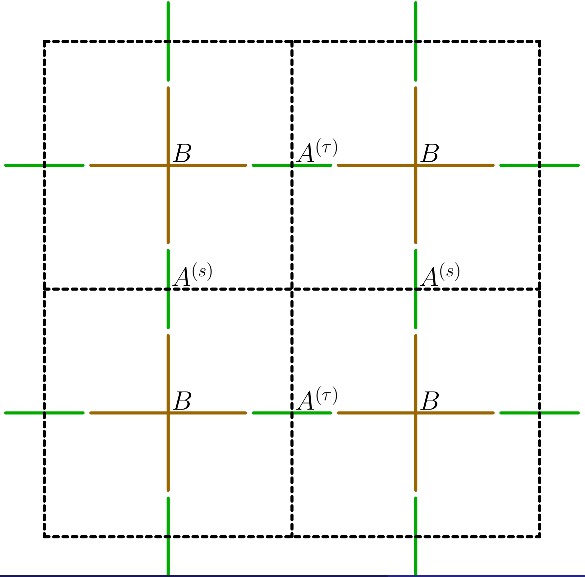
$$Z = \sum_{\{m\}} \left(\prod_{x,\mu\nu} I_m(\beta_{pl})\right)\left(\prod_{x,\mu} I_{m-m'}(2\kappa)\right)$$

For tensor people:

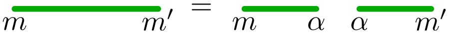
$$I_{m-m'}(2\kappa) = \sum_{\alpha} L_{m\alpha}L^T_{\alpha m'}(2\kappa)$$

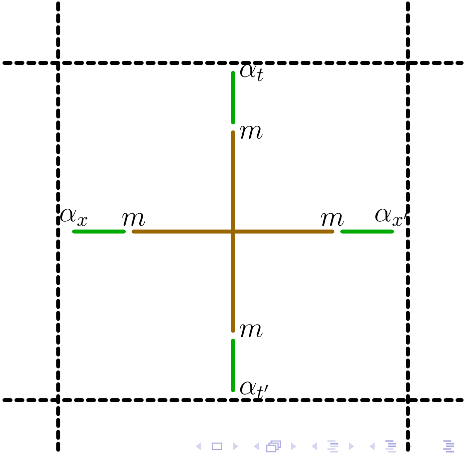
$$T_{ijkl}(\beta_{pl},\kappa) = \sum_m I_m(\beta_{pl})L_{mi}L_{mj}L_{mk}L_{ml}(2\kappa)$$

The model



$$I_{m-m'}(2\kappa_t) = U_{m\alpha}\sqrt{\lambda_\alpha}\sqrt{\lambda_\alpha}U^{\dagger}_{\alpha m'}$$





The observable

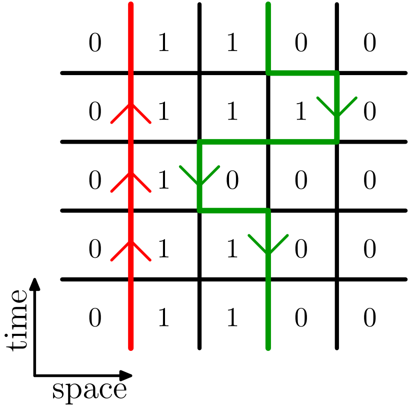
We looked at the Polyakov loop: A Wilson loop wrapped around the temporal direction of the lattice. This operator

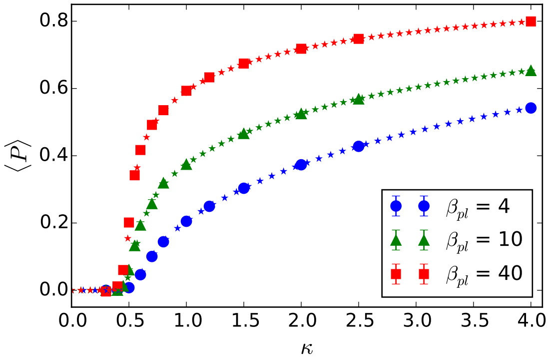
- is a product of gauge fields in the time direction.
- is an order parameter for confinement in gauge theories.

$$P = \prod_{n=1}^{N_\tau} U_{x^*+n\hat{\tau},\tau}.$$

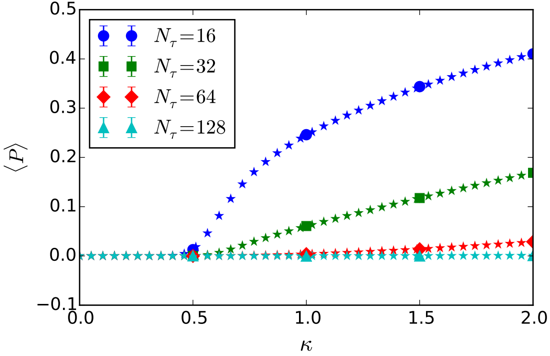
$$\langle P \rangle = \frac{1}{Z}\int \mathcal{D}[A_{x,\mu}]\mathcal{D}[\theta_x]e^{-S}P$$

$$= \sum_{\{m\}} \left(\prod_{x,\mu\nu} I_m(\beta_{pl})\right)\left(\prod_{x,\mu} I_{m-m'}(2\kappa)\right)\left(\prod_{n=1}^{N_\tau}\frac{I_{m-m'-1}(2\kappa)}{I_{m-m'}(2\kappa)}\right)$$





Varying β_{pl} keeping volume fixed.



Varying the temporal length kepping all else fixed.

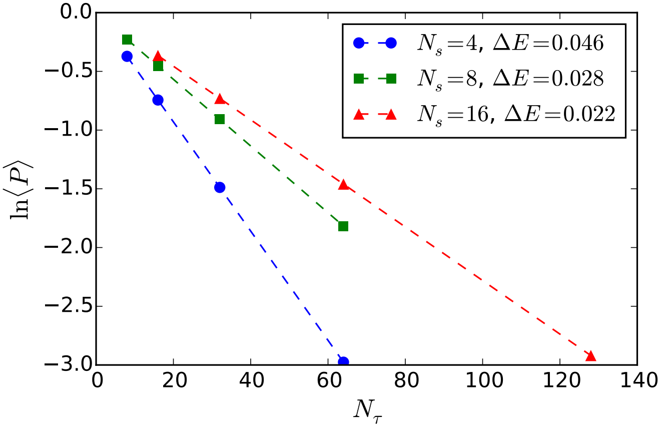
The energy gap

- Initial work led us to believe that

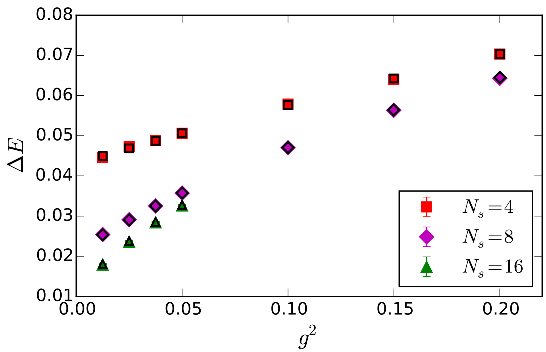
$$\langle P \rangle \simeq e^{-(\Delta E) N_\tau}$$

for large N_τ. $aN_\tau = \frac{1}{T}$.

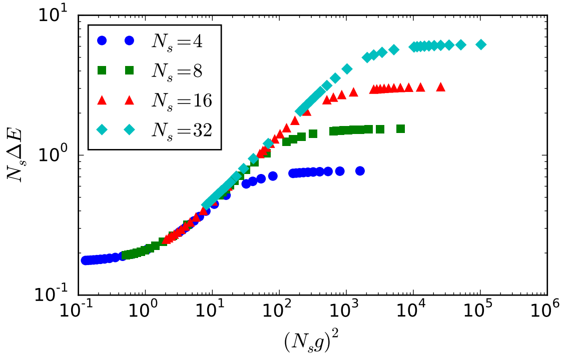
- ΔE is the energy gap between a system with a Polyakov loop, and one without.
- We further investigated the finite-size scaling of the gap and the dependence on β_{pl} and κ.



The energy gap & collapse

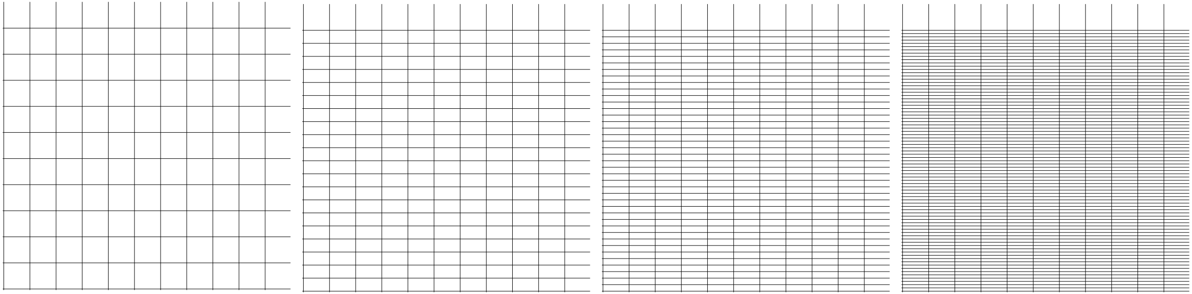


Comparison between MC and TRG.



Finite-size scaling collapse of ΔE

Continuous time limit



original lattice $\rightarrow$

a, κ_s smaller & β_{pl}, κ_T larger

a, κ_s smaller & β_{pl}, κ_T larger

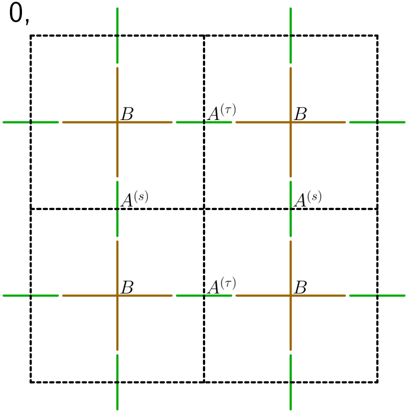
a, κ_s smaller & β_{pl}, κ_T larger

The quantum Hamiltonian

- This model has a continuous-time limit which is *gauge invariant*.
- The continuous-time limit: taking $\beta_{pl}, \kappa_\tau \to \infty$, and $\kappa_s, a \to 0$, such that

$$U \equiv \frac{1}{\beta_{pl} a} = \frac{g^2}{a}, \quad Y \equiv \frac{1}{2\kappa_\tau a}, \quad X \equiv \frac{2\kappa_s}{a}$$

are held constant.



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$$H = \frac{U}{2}\sum_{i=1}^{N_s}(L_i^z)^2 + \frac{Y}{2}\sum_i^{\prime}(L_{i+1}^z - L_i^z)^2 - X\sum_{i=1}^{N_s} U_i^x$$

with

$$L^z|m\rangle = m|m\rangle, \quad U^x = \frac{1}{2}(U^+ + U^-), \quad U^{\pm}|m\rangle = |m \pm 1\rangle.$$

The Polaykov loop

- The Polyakov loop has a continuous-time limit:

$$P = \prod_{n=1}^{N_\tau} \frac{I_{m-m'-1}(2\kappa)}{I_{m-m'}(2\kappa)} \mapsto -\frac{Y}{2}(2(L^z_{i^*+1} - L^z_{i^*}) - 1)$$

  This gets put into the quantum Hamiltonian.

- The Hamiltonian with the Polyakov loop inserted:

$$\tilde{H} = H - \frac{Y}{2}(2(L^z_{i^*+1} - L^z_{i^*}) - 1)$$

- In this form ΔE comes from the difference in the ground states of the two Hamiltonians.

Collapse across limits

- The energy gap between a system with a Polyakov loop, and one without:

$$\Delta E = E_{\text{PL}}^{(0)} - E^{(0)},$$

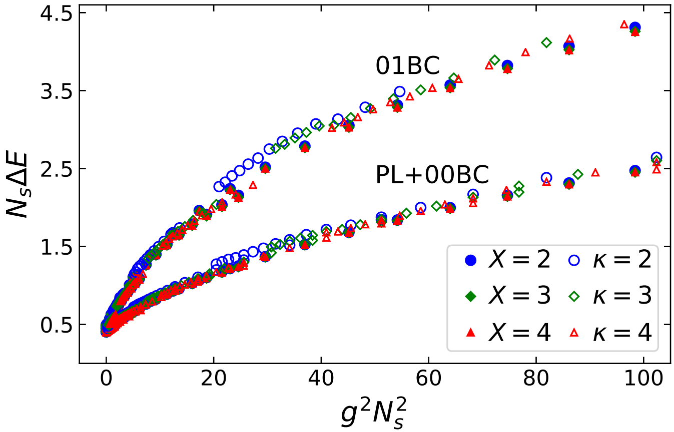
and a system with an external field, and one without:

$$\Delta E = E_{\text{01BC}}^{(0)} - E^{(0)}.$$

- We found for sufficiently small $(gN_s)^2$

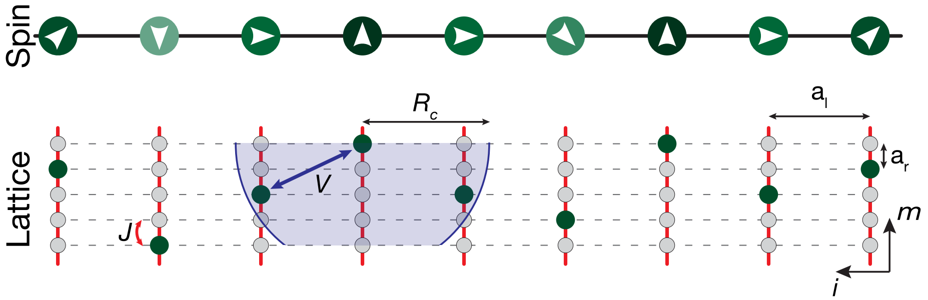
$$N_s \, \Delta E = f(g^2 N_s^2)$$

Furthermore, this collapse survives the continuous time limit!



Ladder system Hamiltonian

A 5-state truncation



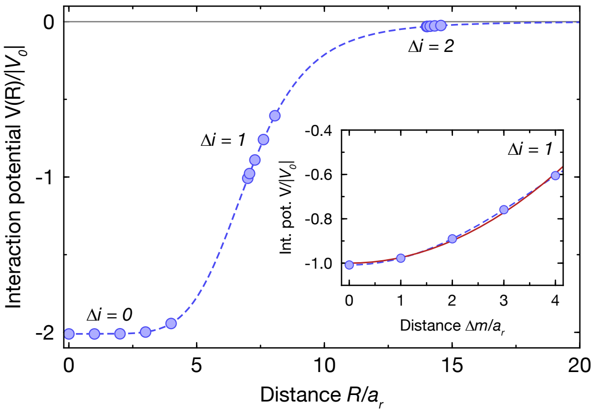
$$\hat{H} = -\frac{J}{2}\sum_{i=1}^{N_s}\sum_{m=-s}^{s-1}(\hat{a}^{\dagger}_{m,i}\hat{a}_{m+1,i} + \text{h.c.}) - \sum_{i=1}^{N_s}\sum_{m=-s}^{s}\epsilon_{m,i}\hat{n}_{m,i} + \sum_{i,i'=1}^{N_s}\sum_{m,m'=-s}^{s}V_{m,m',i,i'}\hat{n}_{m,i},\hat{n}_{m',i'}$$

The quadratic potential

- The local potentials and hopping map straightforwardly.
- The nearest-neighbor rung interactions:

$$V_{m,m',i,i'} = V_{m,m'}\delta_{i',i+1}$$
$$= (-|V_0| + \frac{Y}{2}(m-m')^2)\delta_{i',i+1}$$

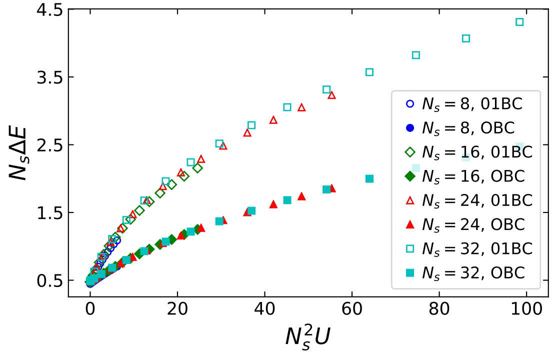
can be accomplished using an asymmetric ladder and a Rydberg-dressed potential.

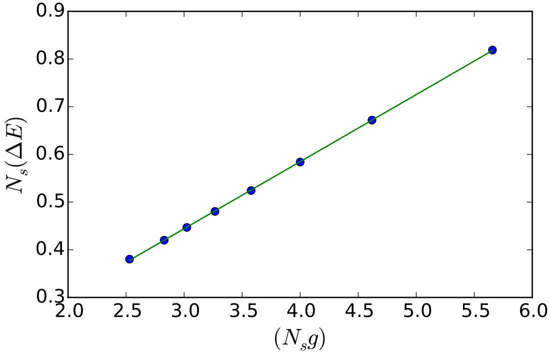


In conclusion

- The Abelian Higgs model, and the Polaykov loop, have a well-defined continuous-time limit which is gauge invariant.
- The Polyakov loop exhibits remarkable, universal finite-size scaling in both the direcrete *and* continuous-time limit.
- We propose a physical, multi-leg, optical-lattice ladder to quantum simulate the Abelian Higgs model in 2D.
- We can achieve the desired interactions for the lattice model using an asymmetric lattice and a Rydberg-dressed potential.
- The proposal could be tested with the simpler Ising model, where results are known exactly.
- **arXiv:1803.11166**, **1807.09186**

Thank you!





$$\Delta E \simeq \frac{a}{N_s} + b\, g^2 N_s$$

$$\Delta E \propto g$$