Nucleon EDMs on a Lattice at the Physical Point

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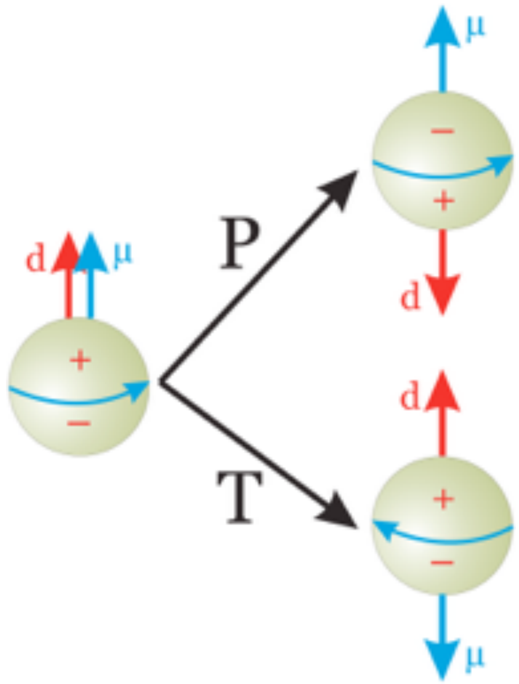
Outline

- Nucleon Electric Dipole Moments: Introduction
  - *Motivation*
  - *Experimental status & outlook*
  - *Lattice methodology*
- Physical point calculations with chiral quarks
  - *~~Form Factors~~* → *[T.Izubuchi's talk, July 27 5:30pm @106 (Hadron Structure)]*
  - *Electric dipole moments induced by quark chromo-EDM*
- Studies of θ_{QCD} -induced nucleon EDM
  - *Noise reduction with subvolume top.charge sampling*
  - *Results from $m_\pi \gtrsim 330$ MeV lattices*
  - *Outlook for physical point calculations*

Nucleon Electric Dipole Moments



$$\vec{d}_N = d_N \frac{\vec{S}}{S} \qquad \mathcal{H} = -\vec{d}_N \cdot \vec{E}$$

OR

$$\mathcal{L}_{int} = eA_\mu^{\rm em}\mathcal{V}^\mu \qquad \text{(P,T-even)}$$
$$+ eA_\mu^{\rm em}\mathcal{A}^\mu \qquad \text{(P,T-odd)}$$

EDMs are the most sensitive probes of CPv:

- Signals for beyond SM physics (SM = 10^{-5} of the current exp.bound)
- Prerequisite for Baryogenesis
- $\theta_{\rm QCD}$-induced EDM : Strong ~~CP~~ problem

$$\langle N_{p'}|J^\mu|\bar{N}_p\rangle_{\not{C}\not{P}} = \bar{u}_{p'}\Big[F_1\gamma^\mu + (F_2 + iF_3\gamma_5)\frac{\sigma^{\mu\nu}(p'-p)_\nu}{2m_N}\Big]u_p$$

Dirac (F_1)

Pauli (anom.magnetic) (F_2)

Electric dipole (F_3)

Experimental Outlook

Current nEDM limits:

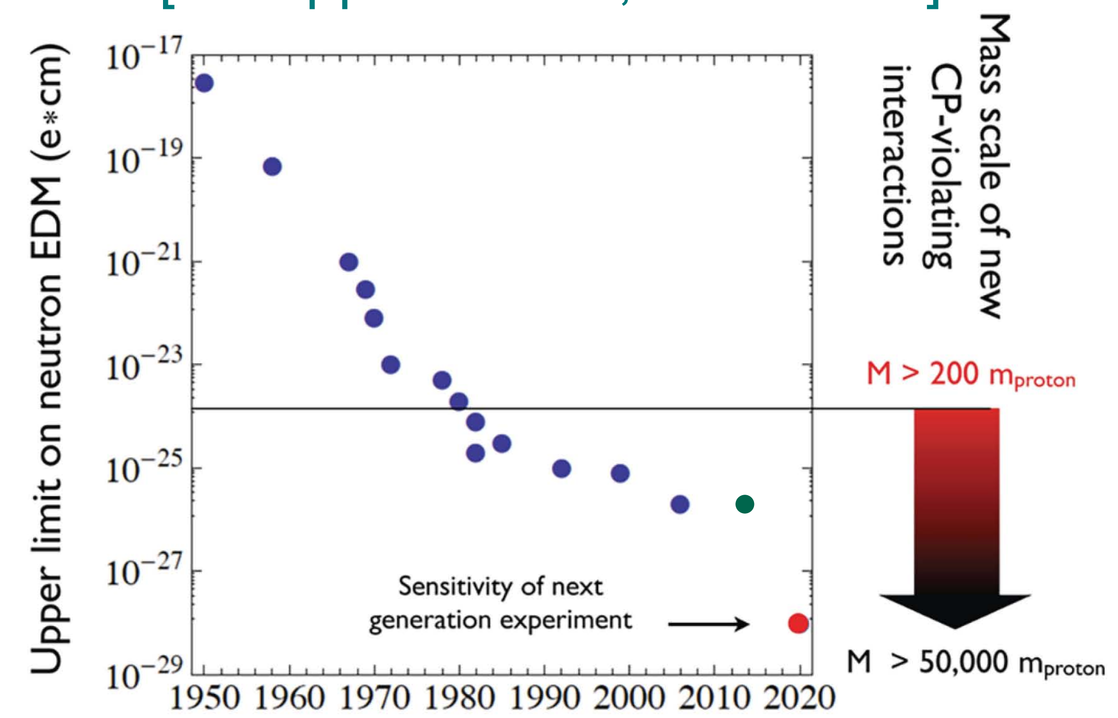
- $|d_n| < 2.9 \times 10^{-26}\, e \cdot \mathrm{cm}$
  [Baker et al, PRL97: 131801(2006)]
- $|d_n| < 1.6 \times 10^{-26}\, e \cdot \mathrm{cm}$
  [Graner et al, PRL116:161601(2016)]

Future nEDM sensitivity :

- 1–2 years : next best limit?
- 3–4 years : x10 improvement
- 7-10 years : x100 improvement

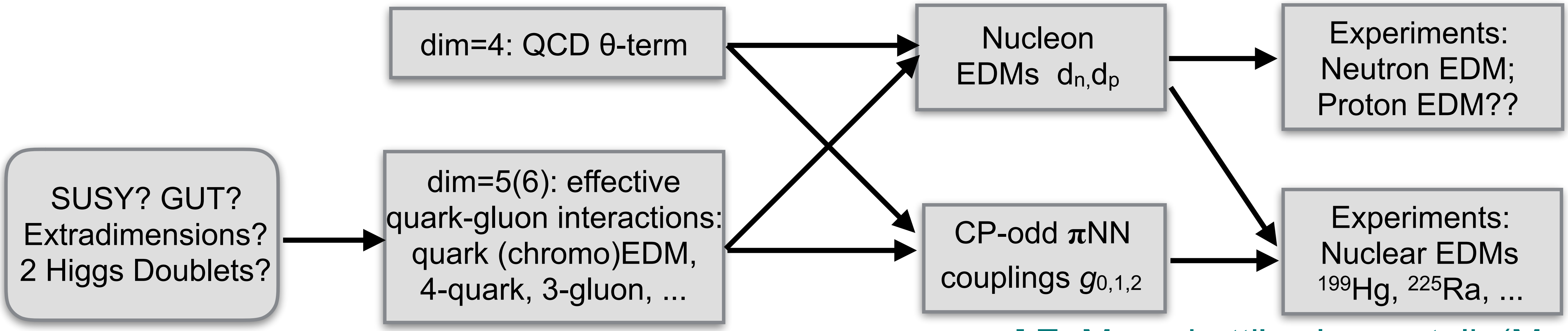
	10^{-28} *e* cm
CURRENT LIMIT	<300
Spallation Source @ORNL	**< 5**
Ultracold Neutrons @LANL	~30
PSI EDM	<50 (I), **<5 (II)**
ILL PNPI	<10
Munich FRMII	**< 5**
RCMP TRIUMF	<50 (I), **<5 (II)**
JPARC	**< 5**
Standard Model (CKM)	< *0.001*

[B.Filippone's talk, KITP 2016]



- Other experiments: light nuclei in storage rings, octupole-deformed ^{225}Ra, etc

Nucleon EDMs: a Window into New Physics



[E. Mereghetti's plenary talk (Mon)]

- Effective quark-gluon CPv interactions organized by dimension

$$\mathcal{L}_{eff} = \sum_i \frac{c_i}{[\Lambda_{(i)}]^{d_i-4}} \mathcal{O}_i^{[d_i]}$$

[J.Engel, M. Ramsey-Musolf, U. van Kolck, Prog.Part.Nucl.Phys. 71 (2013), pp. 21-74]

d=4 : θ_{QCD}

d=5(6) : quark EDM, quark-gluon chromo EDM

d=6 : 4-fermion CPv, 3-gluon (Weinberg)

$$d_{n,p} \qquad F_3^{n,p}(Q^2)$$

- $$d_{n,p} = d^\theta_{n,p}\theta_{\text{QCD}} + d^{cEDM}_{n,p} c_{cEDM} + \dots$$

$$c_i \iff d_{n,p} \quad ?$$

lattice QCD calculations are needed
to constrain θ_{QCD}, c_{cEDM}, ...

CP-odd Nucleon Structure on a Lattice

$$\langle \mathcal{O}\ldots\rangle_{\not{CP}} = \langle \mathcal{O}\ldots\rangle_{CP-even} - i\theta\langle Q\cdot\mathcal{O}\ldots\rangle_{CP-even} + O(\theta^2)$$

$\not{CP}$ coupling

$\not{CP}$ operator: $G\tilde{G}$, cEDM, $GG\tilde{G}$(Weinberg), etc

CPv interaction induces a chiral phase in fermion fields:

$$\langle \text{vac}|N|p,\sigma\rangle_{\not{CP}} = e^{i\alpha\gamma_5}u_{p,\sigma} = \tilde{u}_{p,\sigma}$$

$$u\,[u^T C\gamma_5 d]$$

$$(\not{\partial} + m_N e^{-2i\alpha\gamma_5})\tilde{u}_p = 0$$

$$\sum_\sigma \tilde{u}_{p,\sigma}\bar{\tilde{u}}_{p,\sigma} \sim \left(-i\not{p}_{\mathcal{E}} + m_N e^{2i\alpha\gamma_5}\right)$$

To determine $F_{2,3}$ correctly, one has to use positive-parity spinors
[M.Abramczyk, S.Aoki, S.N.S, *et al* (2017) arXiv:1701.07792]

$$\langle N_{p'}|\bar{q}\gamma^\mu q|N_p\rangle_{\not{CP}} = \bar{u}_{p'}\left[F_1\gamma^\mu + (F_2 + iF_3\gamma_5)\frac{i\sigma^{\mu\nu}(p'-p)_\nu}{2m_N}\right]u_p \,,\quad \text{with}\quad \begin{aligned}\gamma_4 u &= +u\\ \bar{u}\gamma_4 &= +\bar{u}\end{aligned}$$

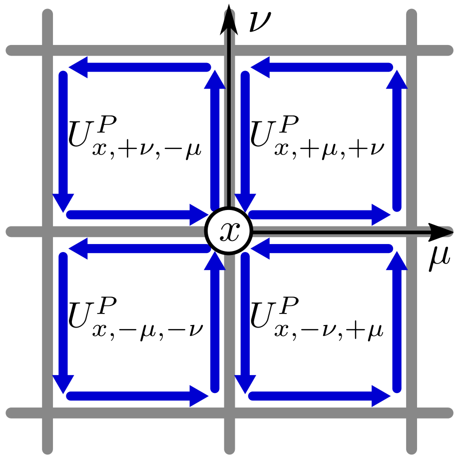
Prior to 2017, lattice determinations of EDM were subject to large bias from $F_{2,3}$ mixing

$$\text{“}F_3\text{”} \approx [F_3]_{\text{true}} - 2\alpha[F_2]_{\text{true}}$$

$$\text{“}d_{n,p}\text{”} \approx [d_{n,p}]_{\text{true}} - 2\alpha\frac{\kappa_{n,p}}{2m_N}$$

Quark Chromo-EDM on a Lattice

$$\mathcal{L}_{\rm cEDM} = \sum_{q=u,d} \frac{\tilde\delta_q}{2}\,\bar q\,[G_{\mu\nu}\sigma^{\mu\nu}\gamma_5]\,q$$



- dim-5 operator : O(a^{-2}) mixing with dim-3 pseudoscalar density
  ⇒ evaluate&subtract p,nEDM induced by PS density $P = \bar q\gamma_5 q$
  [T.Bhattacharya et al, 1502.07325]

- *Chiral symmetry is important:*
  O(a) clover term in, e.g., Wilson fermion action ≡ chromo-magnetic DM

$$\mathcal{L}^{\rm clover} = a\frac{c}{4}\bar q\,[G_{\mu\nu}\sigma^{\mu\nu}]\,q$$

In presense of CPv, condensate is realigned $q \to e^{i\gamma_5\Omega}q$
so that $\langle {\rm vac}|\mathcal{L}_m + \mathcal{L}_{\not{CP}}|\pi^a\rangle = 0$

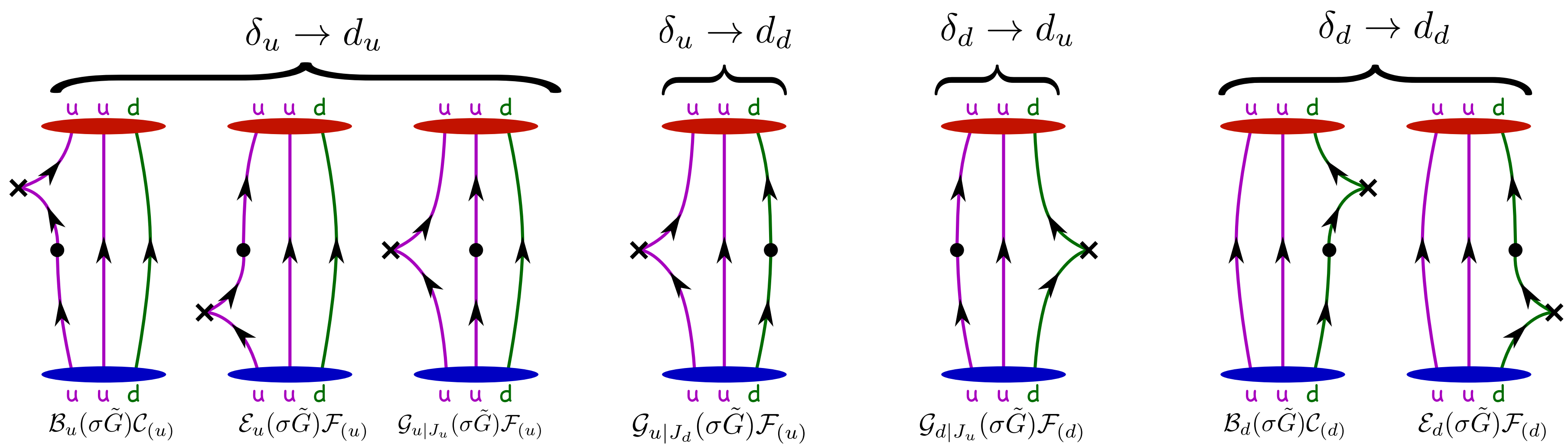
leading to mixing (chromo)EDM ⟺(chromo)MDM:

$$\delta\mathcal{L}_{\rm cEDM} = \delta(\bar q\,[\tilde D_q G_{\mu\nu}\sigma^{\mu\nu}\gamma_5]\,q) = \bar q\,[\{\Omega,\tilde D_q\}G_{\mu\nu}\sigma^{\mu\nu}]\,q) \sim \delta\mathcal{L}_{\rm cMDM}$$

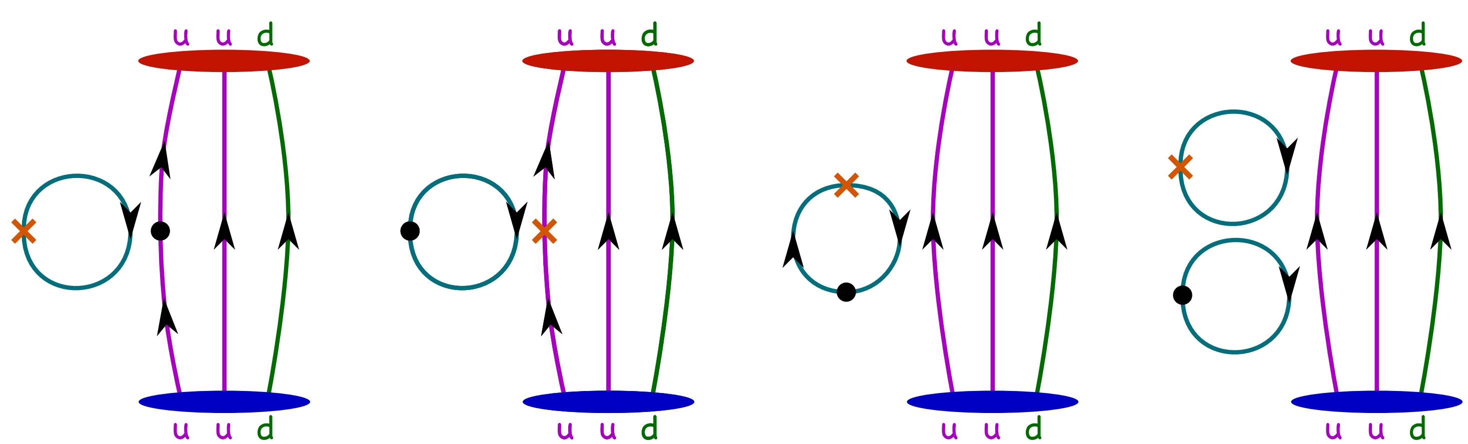
Quark-Gluon EDM: Insertions of dim-5 Operators

$$\mathcal{L}^{(5)} = \sum_q \tilde{d}_q \, \bar{q}(G\cdot\sigma)\gamma_5 q \quad \longrightarrow \quad \begin{cases} \langle N(y)\,\bar{N}(0) \int d^4x\, \bar{q}(G\cdot\sigma)\gamma_5 q\rangle \\ \langle N(y)\,[\bar{\psi}\gamma^\mu\psi]_z\,\bar{N}(0) \int d^4x\, \bar{q}(G\cdot\sigma)\gamma_5 q\rangle \end{cases}$$

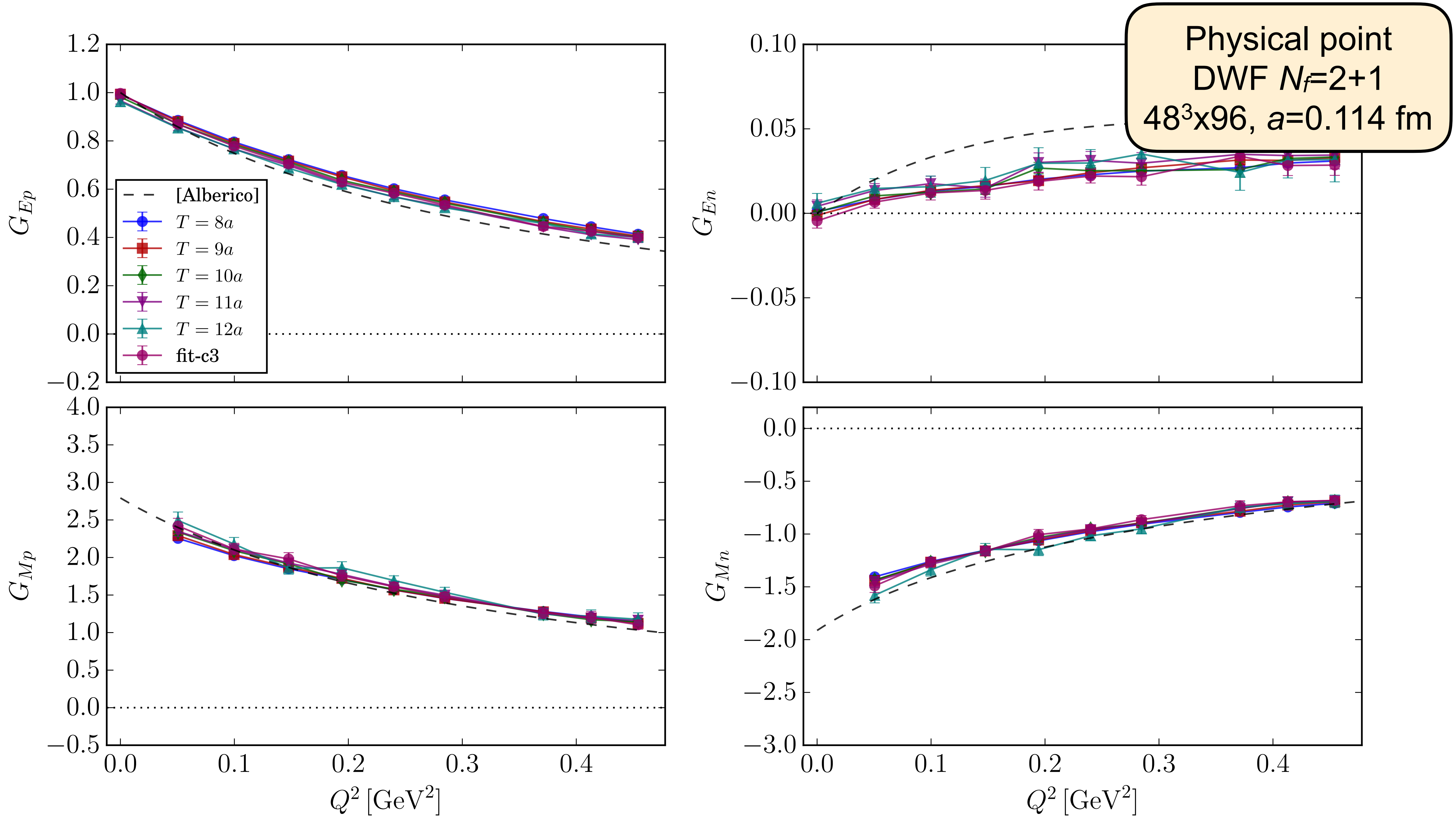
- This work: Only quark-connected insertions



- In future: Single- and double-disconnected diagrams (contribute to isosinglet cEDM, mix with θ-term)



Nucleon Vector (Sachs) Form Factors



$$G_E = F_1 - \frac{Q^2}{4m_N^2} F_2$$

$$G_M = F_1 + F_2$$

See [T.Izubuchi's talk, Fri 5:30pm @106 (Hadron Structure)]

Parity Mixing : cEDM and pseudoscalar(*)

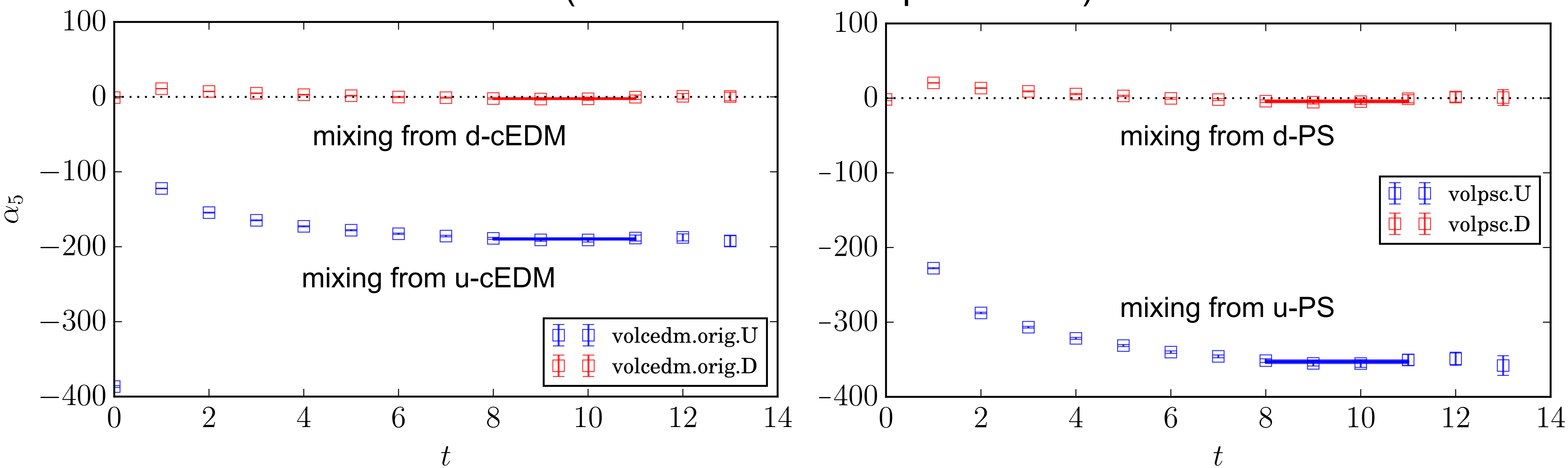
Physical point
DWF N_f=2+1
48^3x96, a=0.114 fm

$$N_\delta = \epsilon^{abc}\, u^a_\delta \left(u^{aT} \mathcal{C}\gamma_5 d^c\right)$$

$$\langle N(t)\bar{N}(0)\rangle_{\not{CP}} = \frac{-i\not{p} + m_N e^{2i\alpha_5\gamma_5}}{2m_N} e^{-E_N t}$$

$$\hat{\alpha}_5 = \frac{\alpha_5}{\tilde{d}} = -\frac{\mathrm{ReTr}\left[T^+\gamma_5\cdot C_{2pt}^{\overline{CP}}(t)\right]}{\mathrm{ReTr}\left[T^+\cdot C_{2pt}^{CP}(t)\right]}\,,\qquad t\to\infty$$

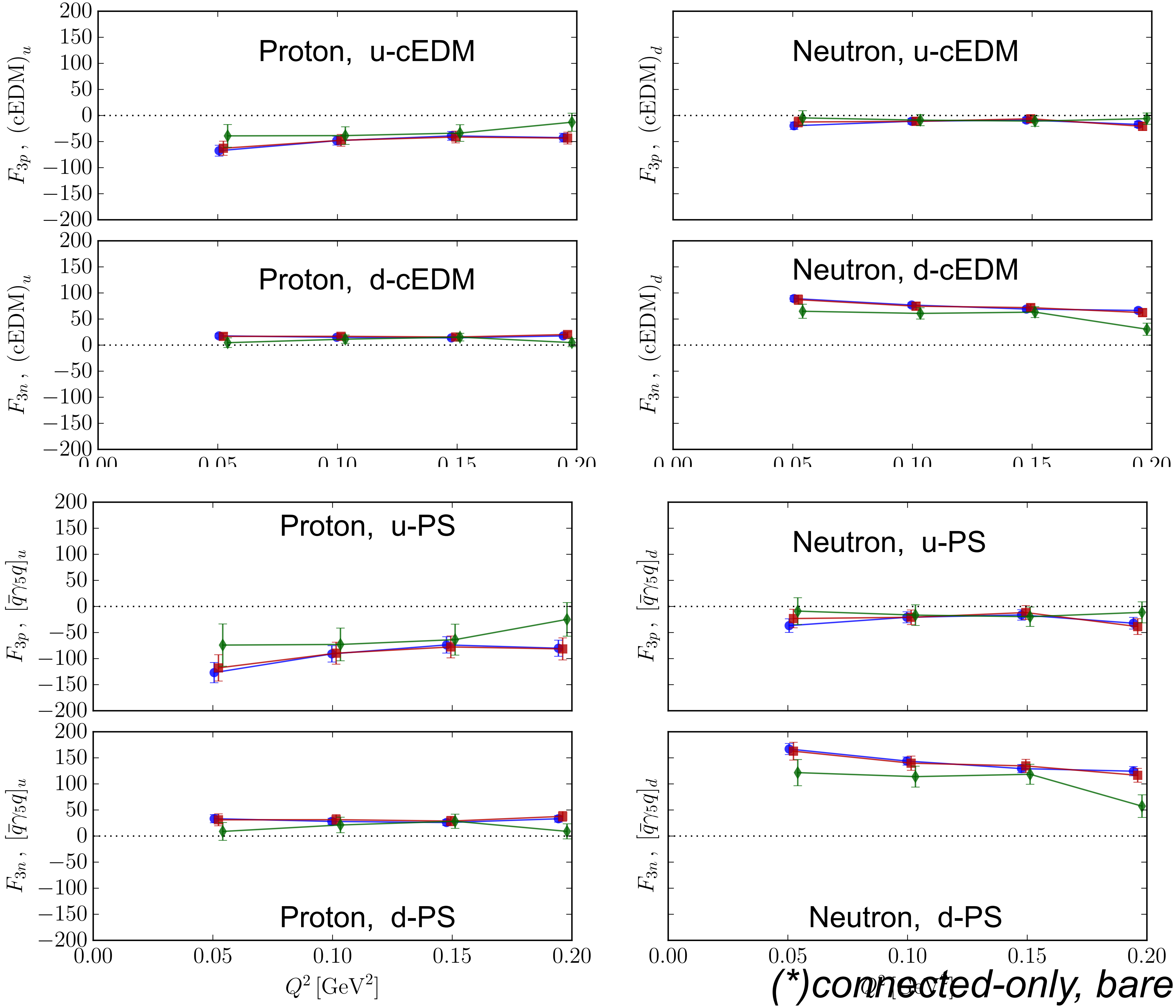
(flavor labels for the proton *uud*)



(*)connected-only, bare cEDM and PS operators

Proton & Neutron EDM Form Factors (*)

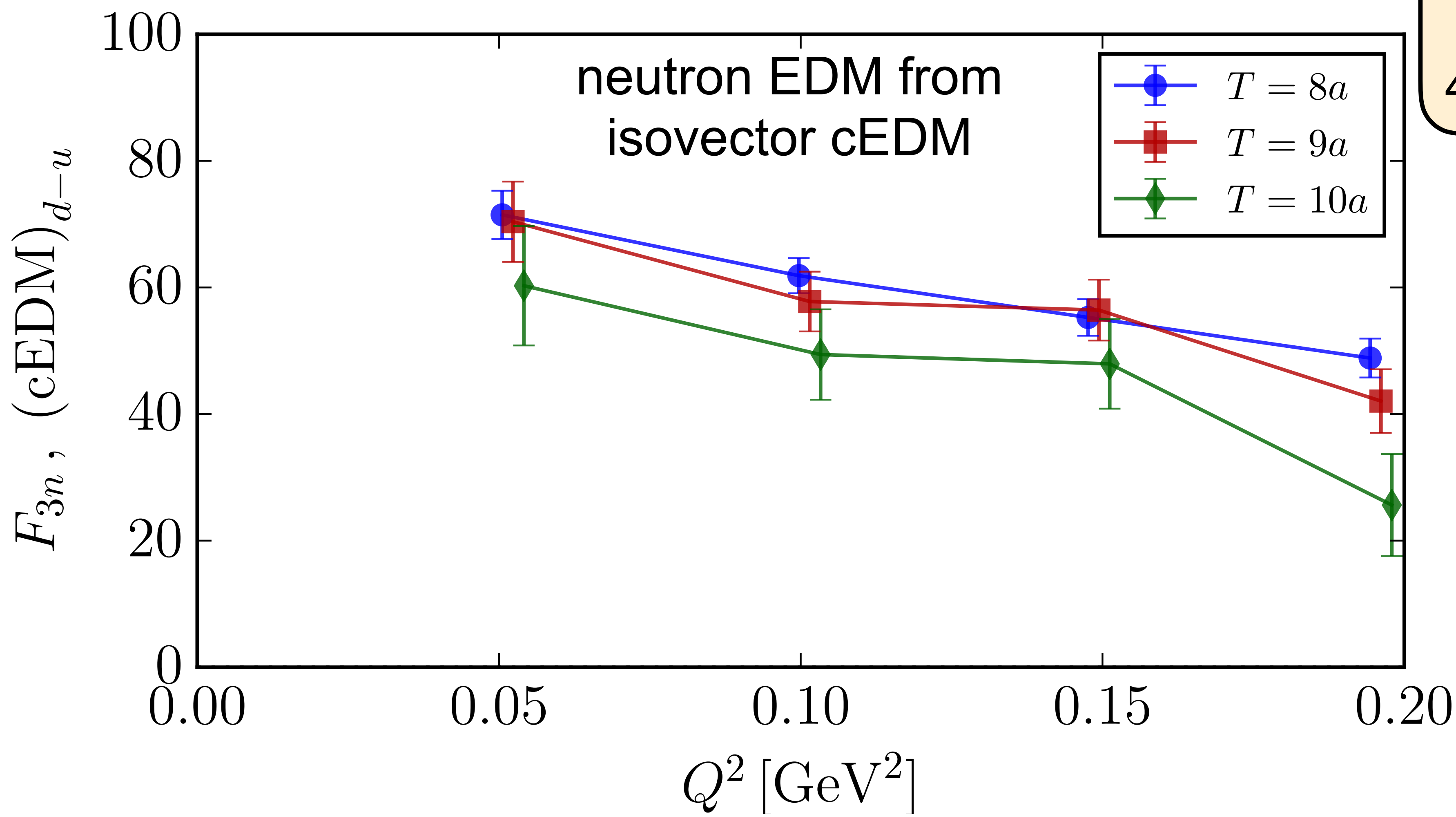
Physical point
DWF N_f=2+1
48^3x96, a=0.114 fm



(*)connected-only, bare cEDM and PS operators

Neutron EDM from Isovector Quark cEDM

Physical point
DWF N_f=2+1
48^3x96, a=0.114 fm



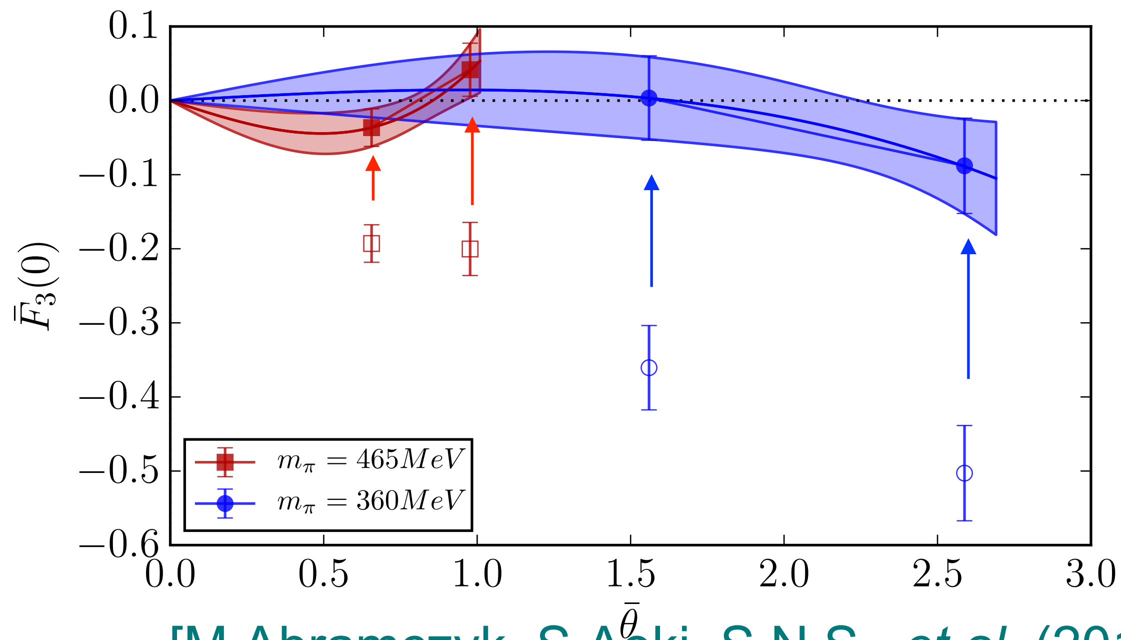
Outlook for cEDM-induced p,nEDM

- Renormalization & mixing subtractions : work underway using *position-space scheme*
- Flavor-dependent CPv from cEDM : disconnected diagrams are required, will be challenging due to noise and mixing with θ_{QCD} *term*

θ_{QCD}-induced nEDM : Status

Correction to previous results:

$$[F_3]_{\text{true}} = \text{``}F_3\text{''} + 2\alpha F_2$$



[M.Abramczyk, S.Aoki, S.N.S., *et al*, (2017)]

		m_π [MeV]	m_N [GeV]	F_2	α	$\tilde{F}_3$	F_3
[ETMC 2016]	n	373	1.216(4)	−1.50(16)[a]	−0.217(18)	−0.555(74)	0.094(74)
[Shintani et al 2005]	n	530	1.334(8)	−0.560(40)	−0.247(17)[b]	−0.325(68)	−0.048(68)
	p	530	1.334(8)	0.399(37)	−0.247(17)[b]	0.284(81)	0.087(81)
[Berruto et al 2006]	n	690	1.575(9)	−1.715(46)	−0.070(20)	−1.39(1.52)	−1.15(1.52)
	n	605	1.470(9)	−1.698(68)	−0.160(20)	0.60(2.98)	1.14(2.98)
[Guo et al 2015]	n	465	1.246(7)	−1.491(22)[c]	−0.079(27)[d]	−0.375(48)	−0.130(76)[d]
	n	360	1.138(13)	−1.473(37)[c]	−0.092(14)[d]	−0.248(29)	0.020(58)[d]

After removing the spurious contribution,

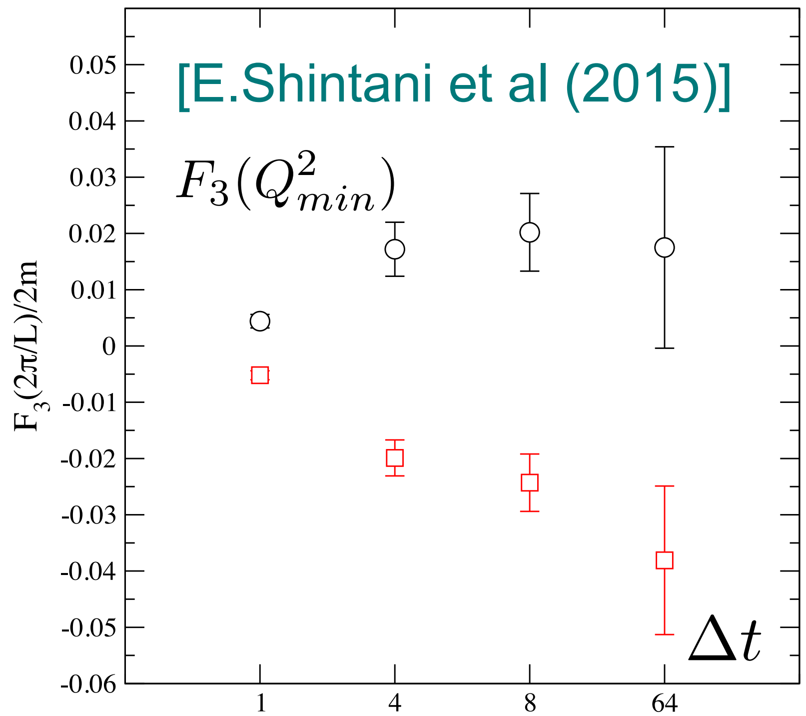
- *no lattice signal for θ_{QCD}-induced nEDM ⇨ d_N is very small*
- *no more conflict with phenomenology values or m_q scaling*

θ-Term Noise Reduction for EDM

Variance of lattice θ-induced nEDM signal ~ $(Volume)_{4d}$:

$$d_N \sim \langle Q \cdot (N J_\mu \bar{N}) \rangle$$

Top. charge $Q \sim \int_{V_4} (G\tilde{G})$, with $\langle |Q|^2 \rangle \sim V_4$



Constrain Q sum to the fiducial volume

- in time around current, $|t_Q - t_J| < \Delta t$ [E.Shintani et al (2015)]
- in time around source, $|t_Q - t_{source}| < \Delta t$ [J. Dragos, talk on Tue]
- 4-d sphere around sink, $|x_Q - x_{sink}| < R$ [K.-F.Liu et al, (2017)]:

$$Q \approx \int_{V_Q} d^4 z\, q(z)$$

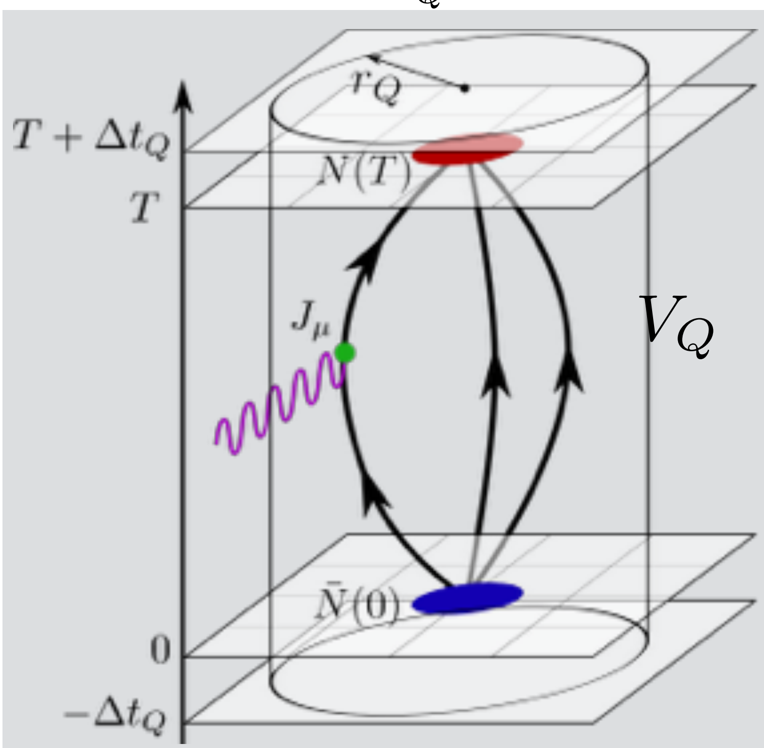
Proper treatment of nucleon parity mixing is critical for correct determination of F_3
$\Longrightarrow$ nucleon must "settle" in the new θ≠0 vacuum

$$N^{(+)} \to \tilde{N}^{(+)} \approx N^{(+)} + i\alpha N^{(-)}$$

$$N^{(-)} \to \tilde{N}^{(-)} \approx N^{(-)} - i\alpha N^{(+)}$$

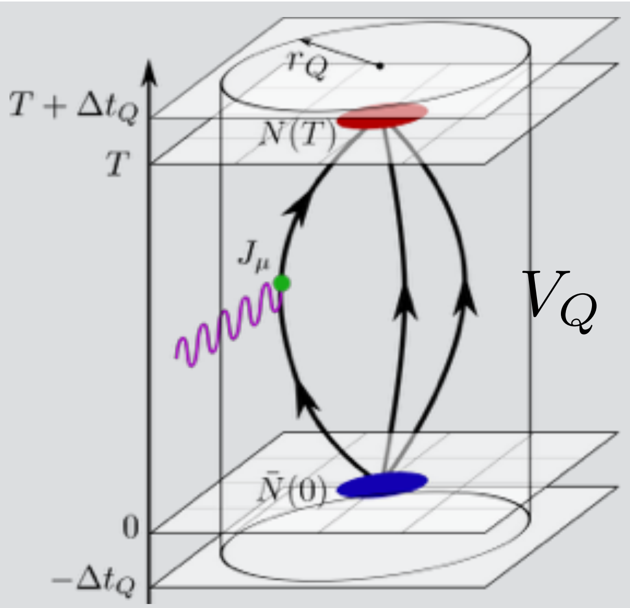
$\Longrightarrow$ *constrain time and space differently :*

4d "cylinder" $V_Q \; : \; |\vec{z}| < r_Q , \quad -\Delta t_Q < z_0 < T + \Delta t_Q$

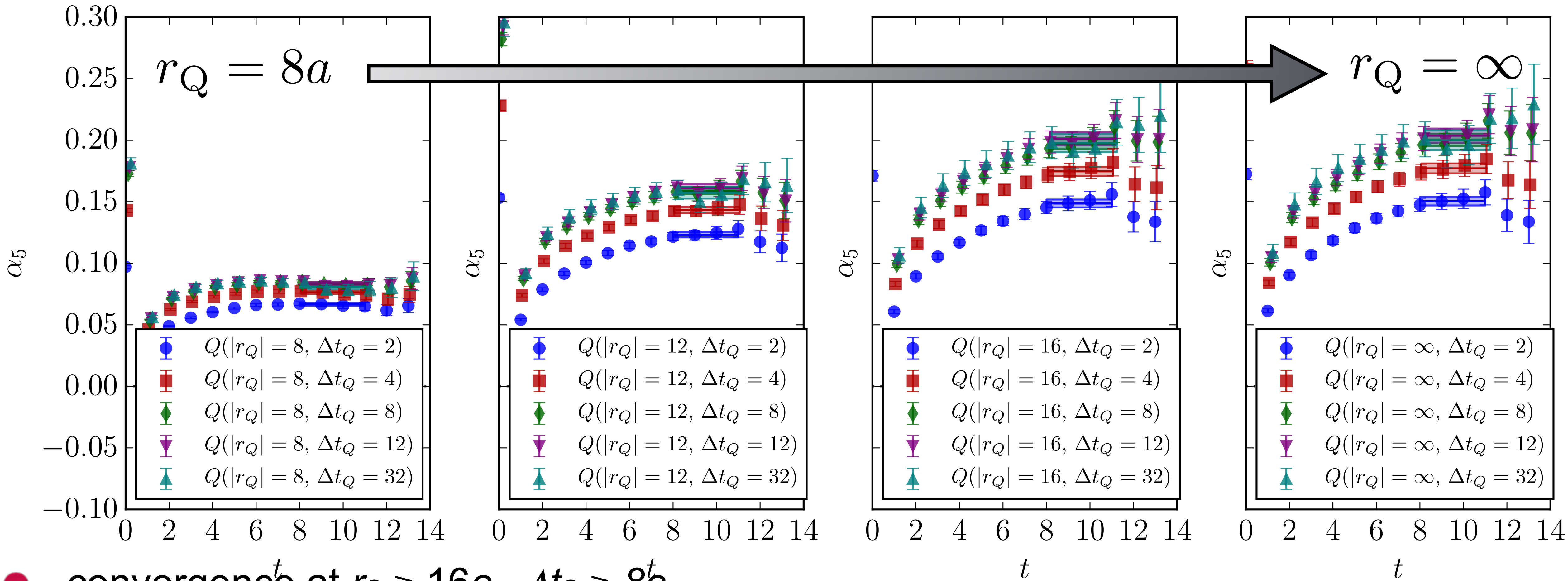


Tests on m_π=330 MeV Lattices: Parity Mixing

- N_f=2+1 Domain Wall (RBC/UKQCD) 24^3x64 a = 0.114 fm
- 1400 configs * (64sloppy+1exact) samples $\Longrightarrow$ 89.6k stat.
- Top.charge with 5-loop improved $G\tilde{G}$ [P. de Forcrand et al '97] on Wilson-flowed (t=8a^2) gauge links [M.Luscher, 1006.4518]

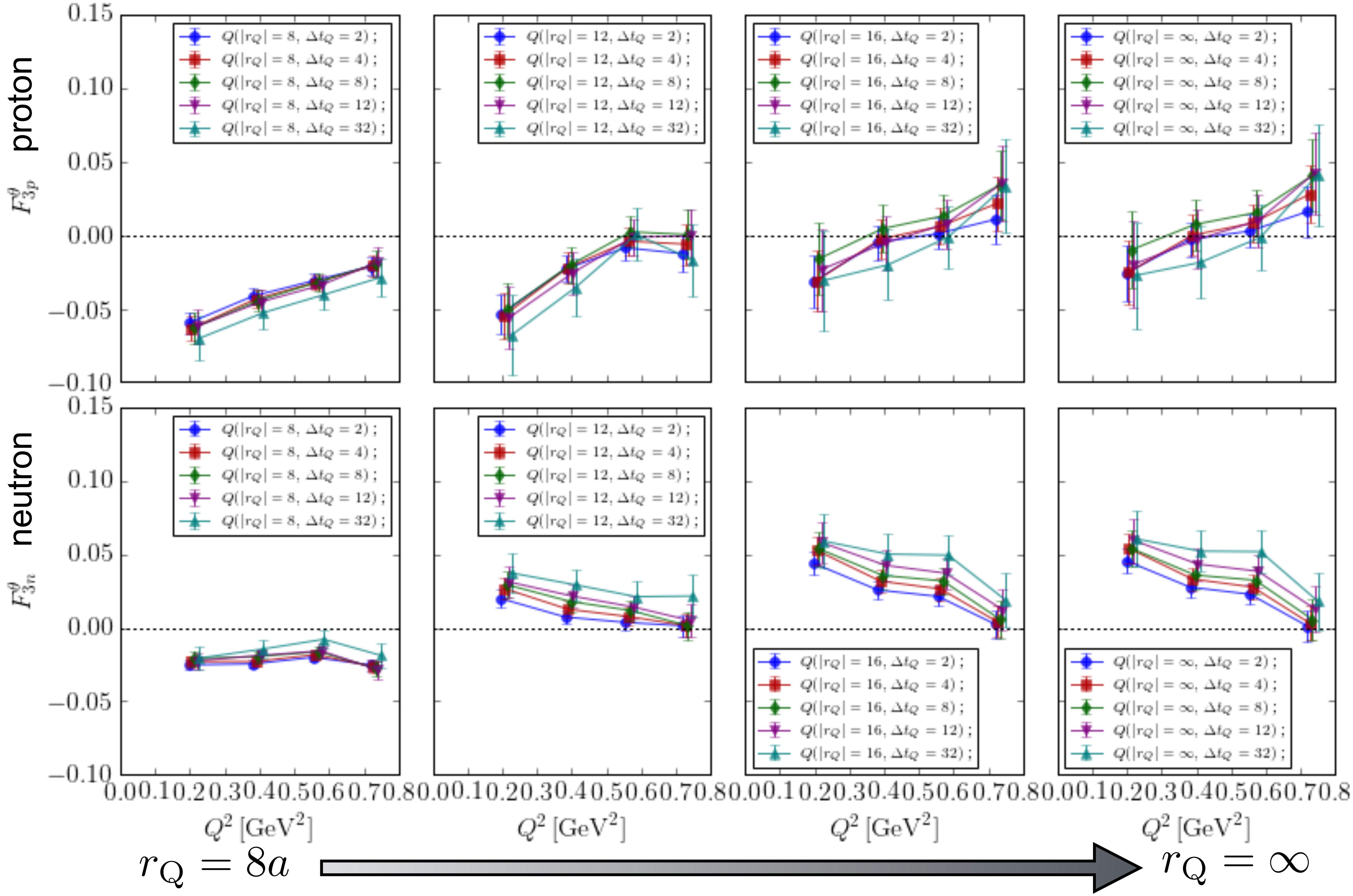


parity mixing angle α_N as a function of r_Q, Δt_Q



- convergence at $r_Q \gtrsim 16a$, $\Delta t_Q \gtrsim 8a$

Tests on m_π=330 MeV Lattices: EDM(Form Factor)



How Hard is θ-nEDM at the Physical Point?

- chiral fermions, $m_\pi = 330$ MeV [this work]

$$|2m_n d_n| = |F_{3n}(0)| \approx 0.05\cdot\theta$$

- Wilson fermions, m_π=360 MeV [Guo et al 2015] *after correction*

$$|2m_n d_n| = |F_{3n}(0)| \lessapprox 0.06\cdot\theta$$

- best guess for the physical point with $|d_n| \sim m_q \sim (m_\pi)^2$

$\Longrightarrow$ phys.point

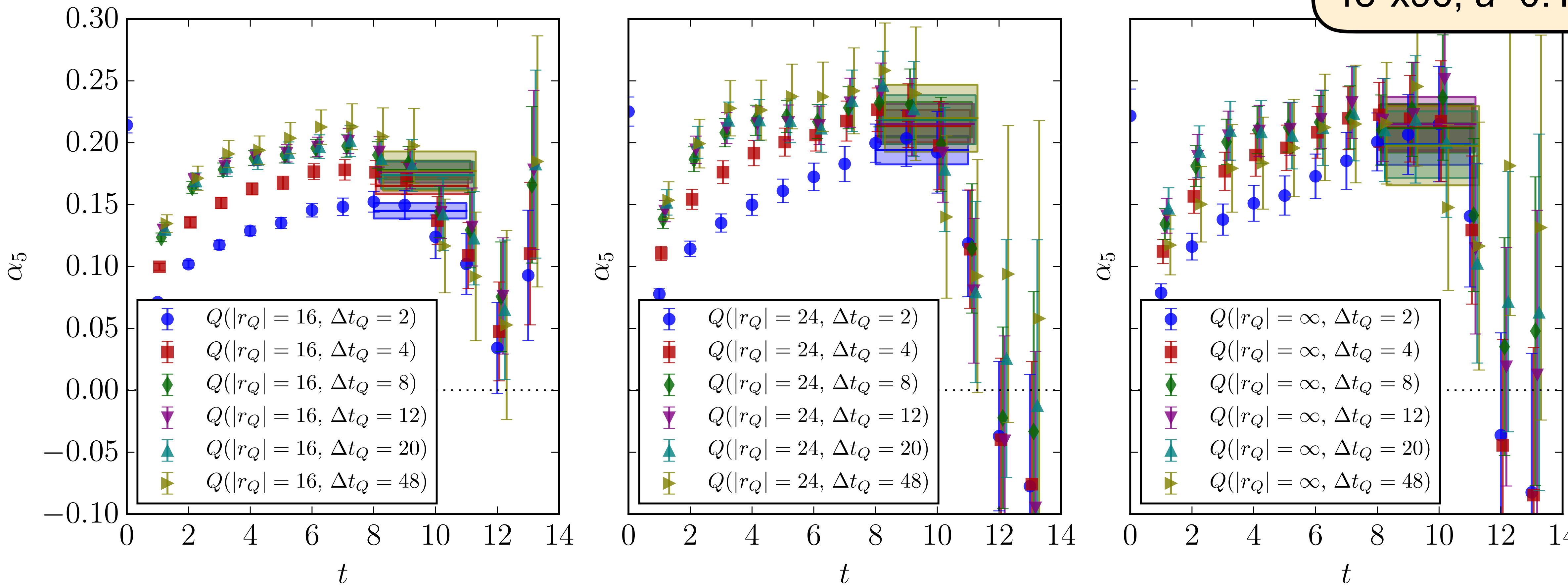
$$|F_{3n}(0)| \approx 0.01\cdot\theta,\quad |d_n| \approx 0.001\cdot\theta\; e\,\text{fm}$$

$$|F_{3n}^{\text{phys}}(0)| \sim O(10^{-2})\,\theta, \quad |d_n| \sim O(10^{-3})\,e\,\text{fm}\,\theta$$

Physical point : θ_{QCD}-induced Parity Mixing α_N

Physical point
DWF N_f=2+1
48^3x96, a=0.114 fm

Parity-mixing angle from constrained Q sum



48^3x96 m_π=139 MeV (PRELIMINARY)

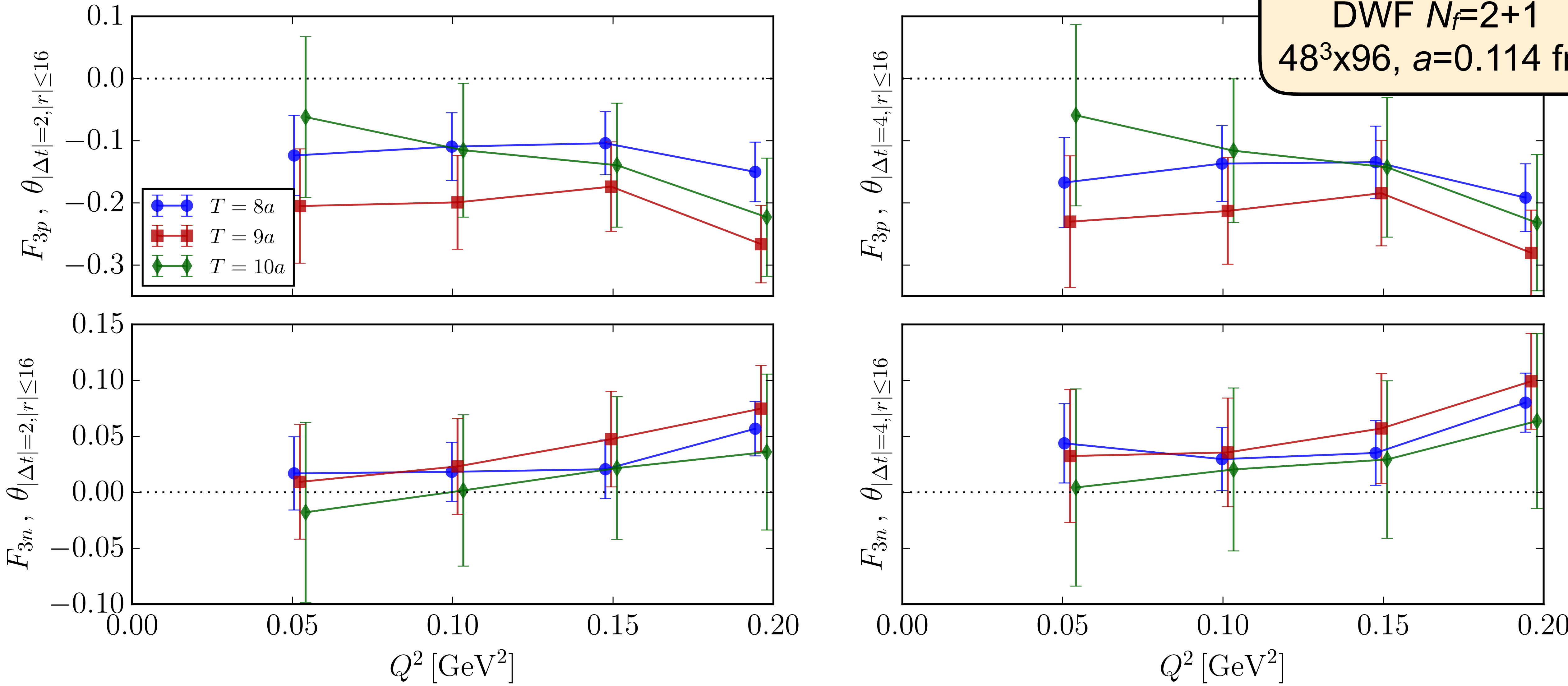
Reassuring results for noise reduction at the physical point

- time region $\Delta t_Q \gtrsim 8a \approx 1.2\,\mathrm{fm}$
- spatial region $r_Q \gtrsim 20a \approx 2.3\,\mathrm{fm}$

Physical point : θ_{QCD}-induced EDFF F_3

EDFF F_3 from constrained Q sum (*the most aggressive* Q cuts)

Physical point
DWF N_f=2+1
48^3x96, a=0.114 fm



48^3x96 m_π=139 MeV (PRELIMINARY)

- 33k lattice samples, ~ 30 M core-hours on Argonne BlueGene/Q
- connected diagrams only
- result compatible with zero, $|F_{3n}| \le 0.05$ constraint

Need x30..100 more statistics to constrain $|F_{3n}| \approx 0.01$:
θ-nEDM remains difficult at the physical point

Nucleon EDM : Summary

- Encouraging physical-point results for nucleon EDM induced by quark chromo-EDM
  - *~20% stochastic uncertainty for quark cEDM-induced EDM*
  - *Renormalization & mixing subtractions are underway*
  - *Full flavor dependence will require disconnected diagrams & θ_{QCD} -term*
- Clear signal for θ_{QCD}-induced nEDM at m_π = 330 MeV
  - *Variance-reduction for Q sampling is essential*
  - *Physical $|d_{n,p}| \approx 10^{-3}$ e fm values are in agreement with phenomenology*
- Constraining θ_{QCD}-induced nEDM at the physical point will be challenging
  - *O(300-1000) M core*hours may be required even with variance reduction*
  - *Shall look for alternative methods: dynamical θ-therm?*