Proton Spin Decomposition

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LATTICE 18
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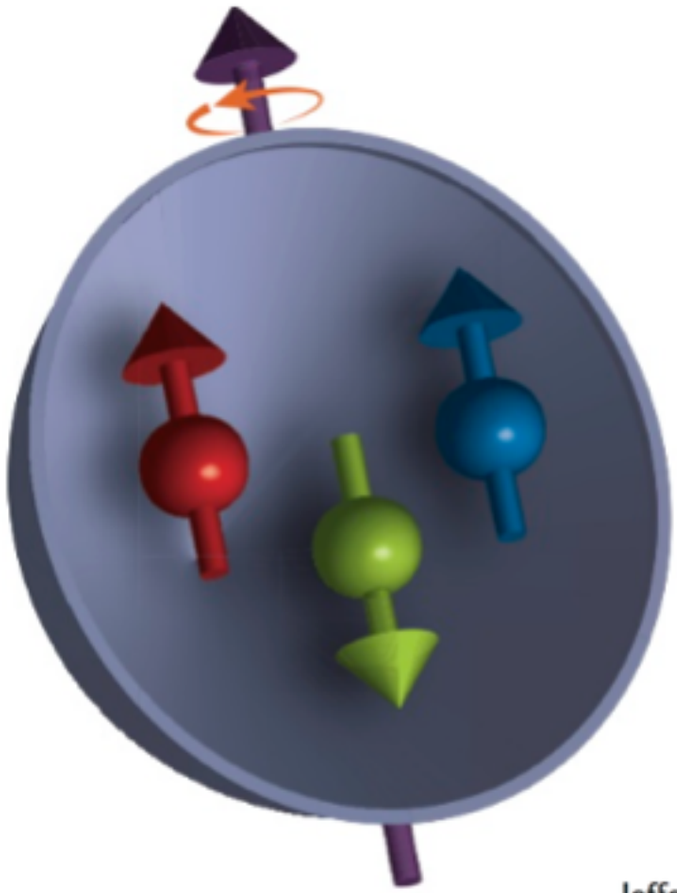
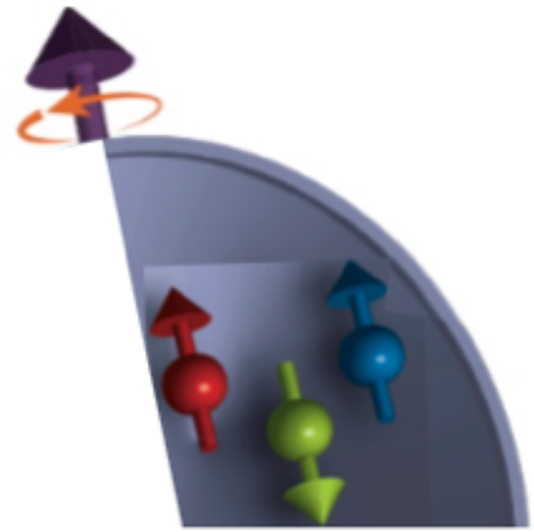
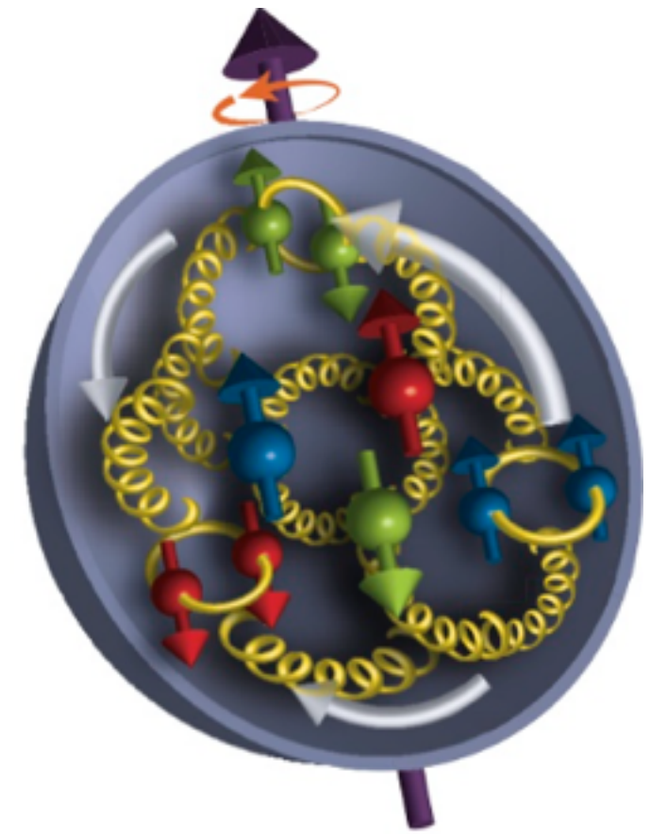
Where does the proton spin come from?



1980s





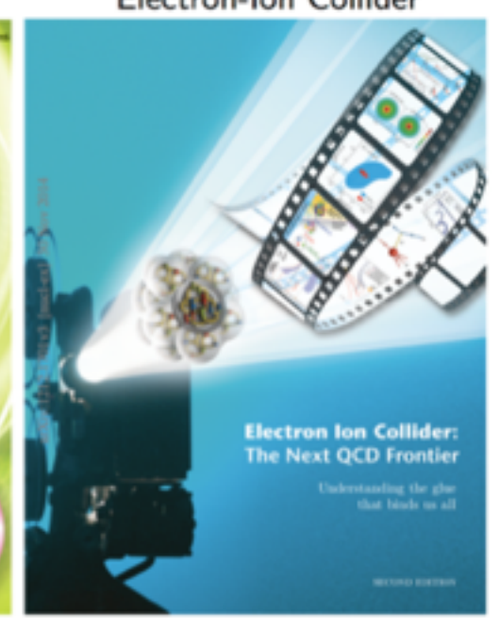
now

Only 30% of the proton spin comes from the quark spin, based on the experiments

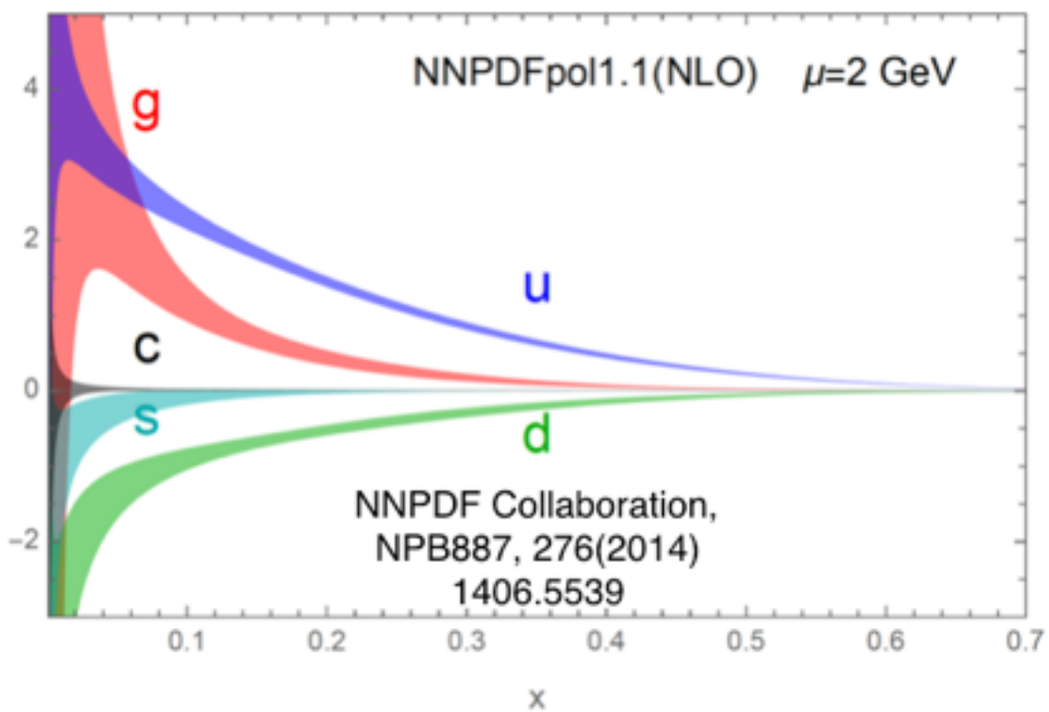
Jefferson Laboratory

Relativistic Heavy Ion Collider

A future Electron-Ion Collider



Quark and glue spin



*Quark spin/helicity (**u**,**d**,**s**…): the **integration** of the quark polarized parton distribution function (PDF)*

$$\Delta q = \int_0^1 dx \Delta q(x)$$

*Glue helicity (**g**): that of the gluon polarized PDF*

$$\Delta G = \int_0^1 dx \Delta g(x)$$

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Proton spin

Quark model prediction vs. experiment

- The quark model (agrees with the lattice simulation at heavy quark limit):
  $\Delta u \to 4/3$, $\Delta d \to -1/3$, $\Delta s \to 0$, $\Delta g \to 0$;

- **The polarized neutron decay:**
  $\Delta u - \Delta d \approx$ **1.2723(23);**

  PDG, CPC40, 100001 (2016)

- **The phenomenology fit of quark distribution based on Exp.:**
  $\Delta u \sim 0.8$, $\Delta d \sim -0.4$, $\Delta s \sim -0.1$, $\Delta g \sim 0.4(?)$;

  EPJA52, 268 (2016), 1212.1701

  D. Florian, PRL 113, 012001 (2014), 1404.4293

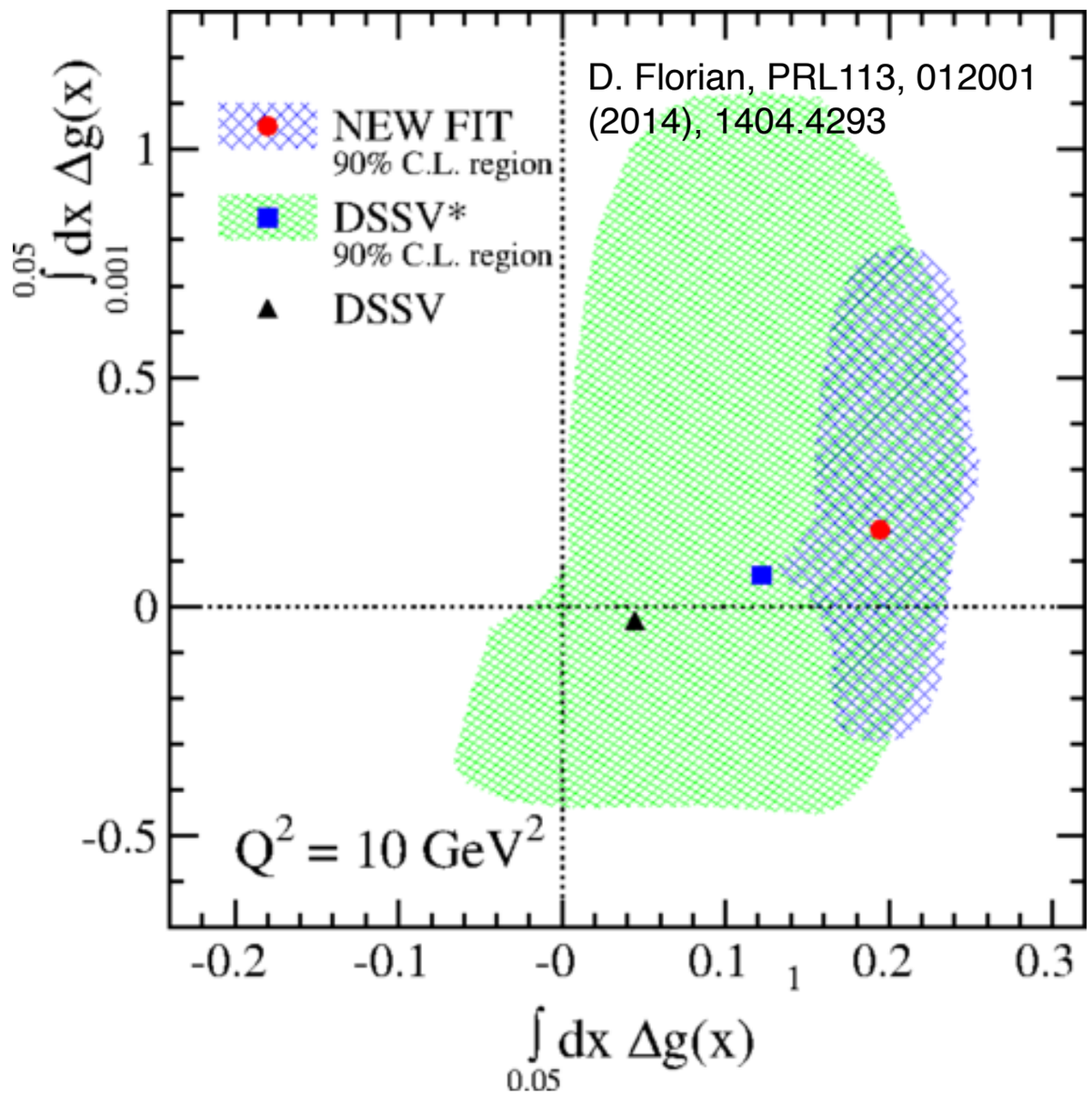
- **The experiments are quite different from the naive theoretical understanding!**

Proton spin

Glue helicity

Glue helicity $\Delta G = \int_0^1 dx \Delta g(x) = \int_0^1 dx \frac{i}{2xP^+} \int \frac{d\xi^-}{2\pi}$

$$e^{-ixP^+\xi^-} \langle PS|F_a^{+\alpha}(\xi^-)\mathcal{L}^{ab}(\xi^-,0)\tilde{F}^+_{\alpha,b}(0)|PS\rangle$$



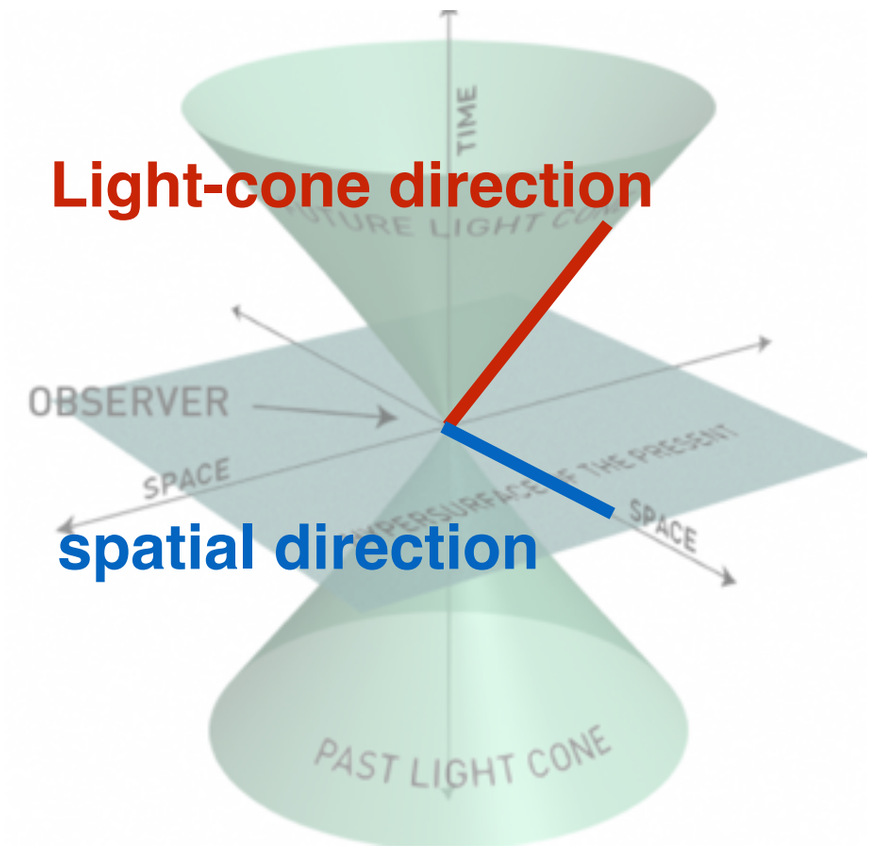
has been shown to be
~0.2 or 40%
of the proton spin
with large uncertainty,
based the global analysis
with experimental results
from:

STAR NPA932, 500(2014),1404.5134
PHENIX PRD90, 012007(2014), 1402.6296
COMPASS PLB690, 466(2010), 1001.4654

Glue spin

Large momentum effective theory (LaMET)

$$O_{\Delta_G} = \left[\vec{E}^a(0) \times (\vec{A}^a(0) - \frac{1}{\nabla^+}(\vec{\nabla} A^{+,b})\mathcal{L}^{ba}(\xi^-, 0))\right]^z = \vec{E}_{LC} \times \vec{A}_{LC},\ A^+_{LC} = 0$$



When nucleon is boosted:

- The Coulomb and Temporal gauge conditions become the light-cone one.
- **Glue spin** below becomes **glue helicity,** the integration of the glue polarized PDF, **at tree level**.

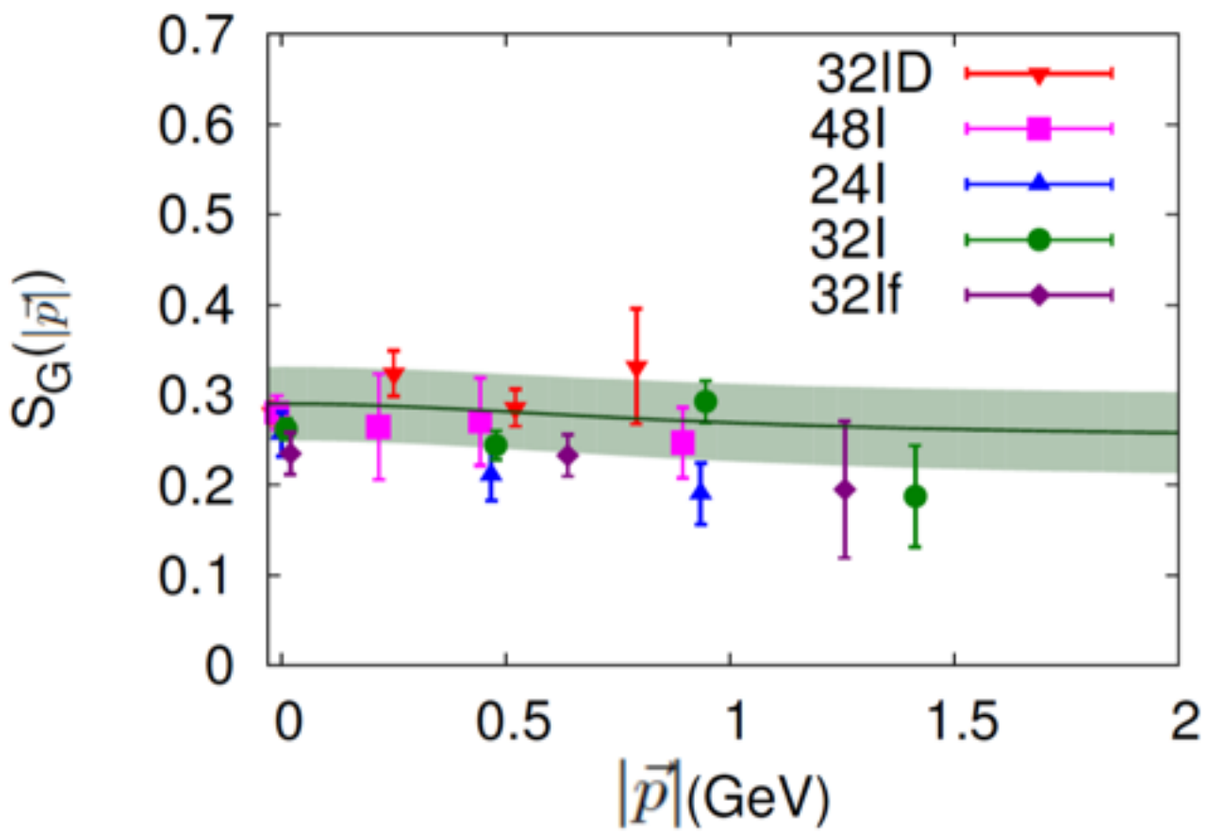
$$O_{S^c_G} = \vec{E}^c \times \vec{A}^c,\ \partial_i A^c_i = 0$$ ***Coulomb gauge***

or $$O_{S^t_G} = \vec{E}^t \times \vec{A}^t,\ A^t_0 = 0$$ ***Temporal gauge***

X. Ji, J. Zhang, and Y. Zhao, PRL111 112002 (2013), 1304.6708

Glue spin

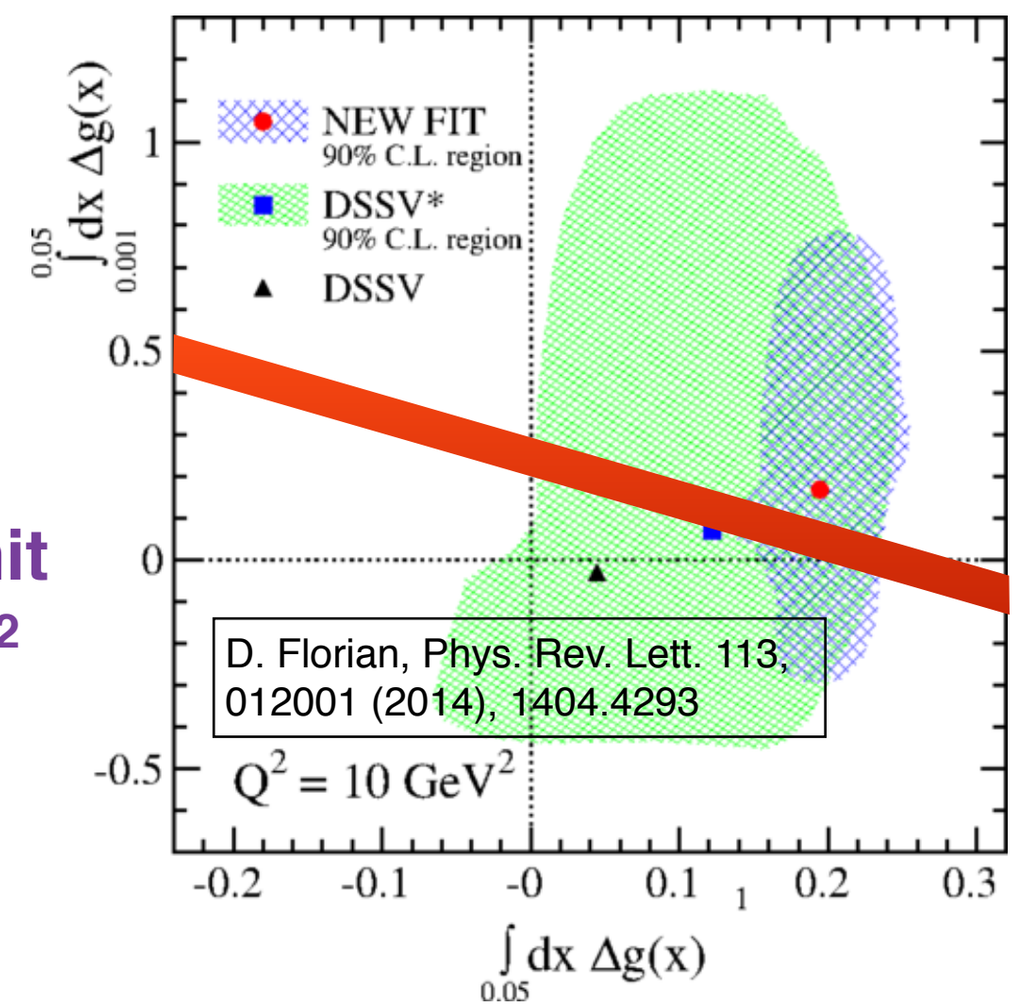
YBY, R. Sufian, et al., χQCD collaboration, PRL118, 042001(2017), 1609.05937
ViewPoint and Editor's suggestion



Results

Neglect the matching and apply an empirical form to fit the data,

$$\int_{0.001}^{0.05} \mathrm{d}x \Delta g(x) + \int_{0.05}^{1} \mathrm{d}x \Delta g(x) \simeq S_g$$



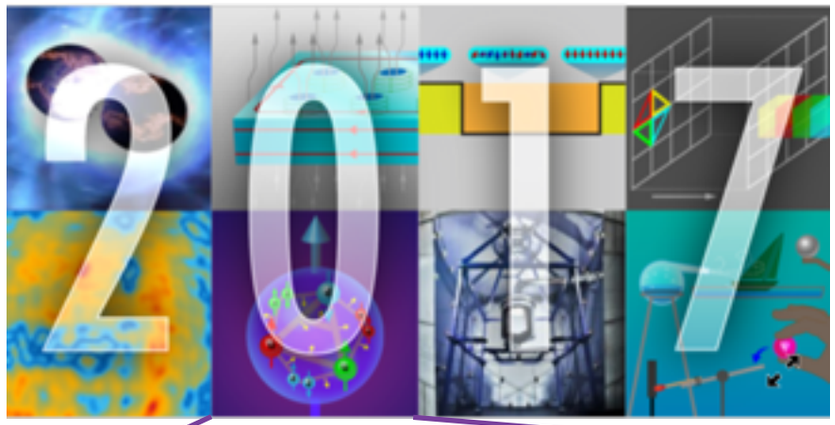
D. Florian, Phys. Rev. Lett. 113, 012001 (2014), 1404.4293

the glue spin at the large momentum limit for the renormalized value at μ^2=10GeV^2 will be

S_G=0.251(47)(16).

One of **eight**

YBY, R. Sufian, et. al., χQCD collaboration, PRL118, 042001(2017), 1609.05937
ViewPoint and Editor's suggestion



APS Highlights of 2017

https://physics.aps.org/articles/v10/137

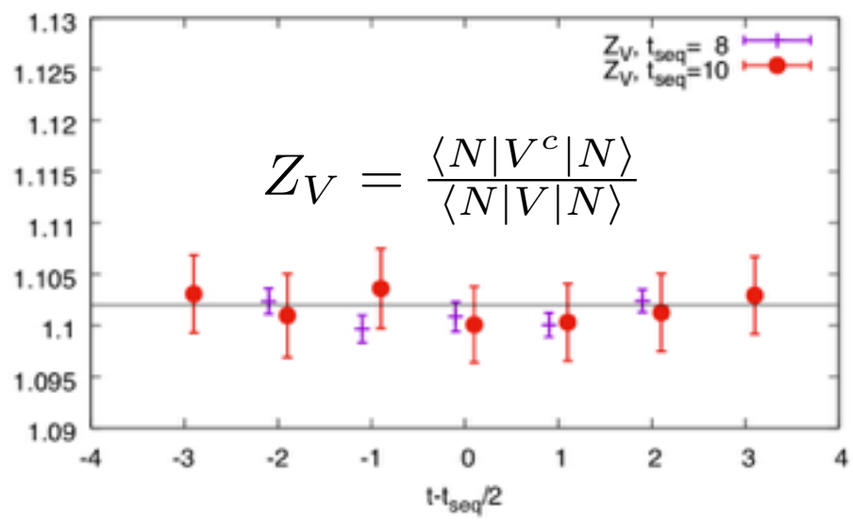
Gluons Provide Half of the Proton's Spin

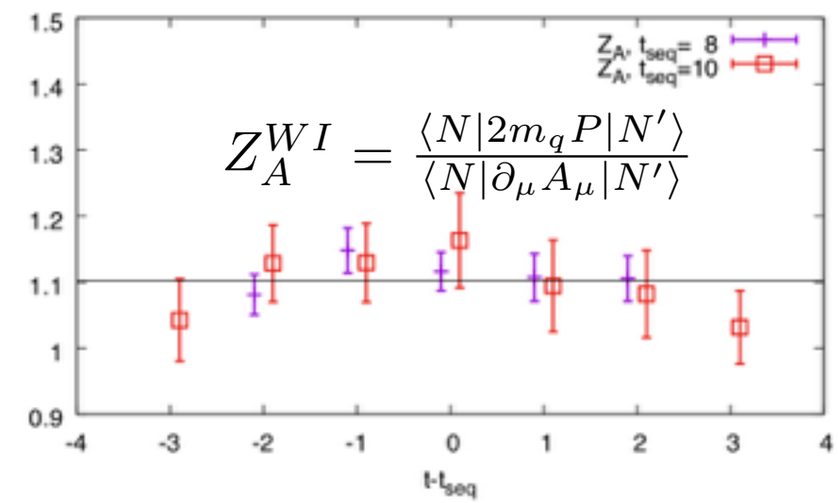
The gluons that bind quarks together in nucleons provide a considerable chunk of the proton's total spin. That was the conclusion reached by Yi-Bo Yang from the University of Kentucky, Lexington, and colleagues (see Viewpoint: **Spinning Gluons in the Proton**). By running state-of-the-art computer simulations of quark-gluon dynamics on a so-called spacetime lattice, the researchers found that 50% of the proton's spin comes from its gluons. The result is in agreement with recent experiments and shows how such lattice simulations can now accurately predict an increasing number of particle properties. The simulations also indicate that, despite being substantial, the gluon spin contribution is too small to play a major part in "screening" the quark spin contribution—which according to experiments is only 30%—through a quantum effect called the axial anomaly. The remaining 20% of the proton spin is thought to come from the orbital angular momentum of quarks and gluons.

Quark spin

Normalization

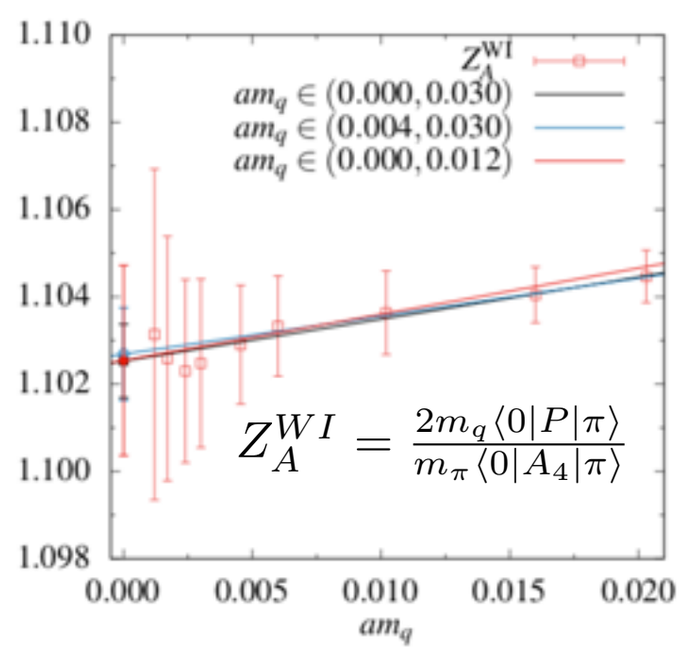
- *The normalization is the ratio between the **local** current and **conserved/ chiral** current;*

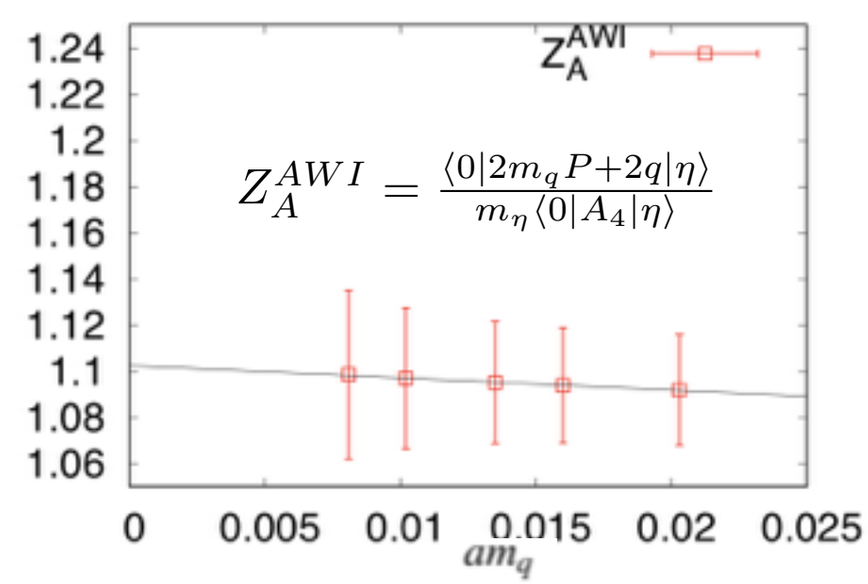




With chiral fermion, all the cases provide the same normalization Z_V=Z_A.

- *Can also be obtained with the (anomalous) Ward Identity.*

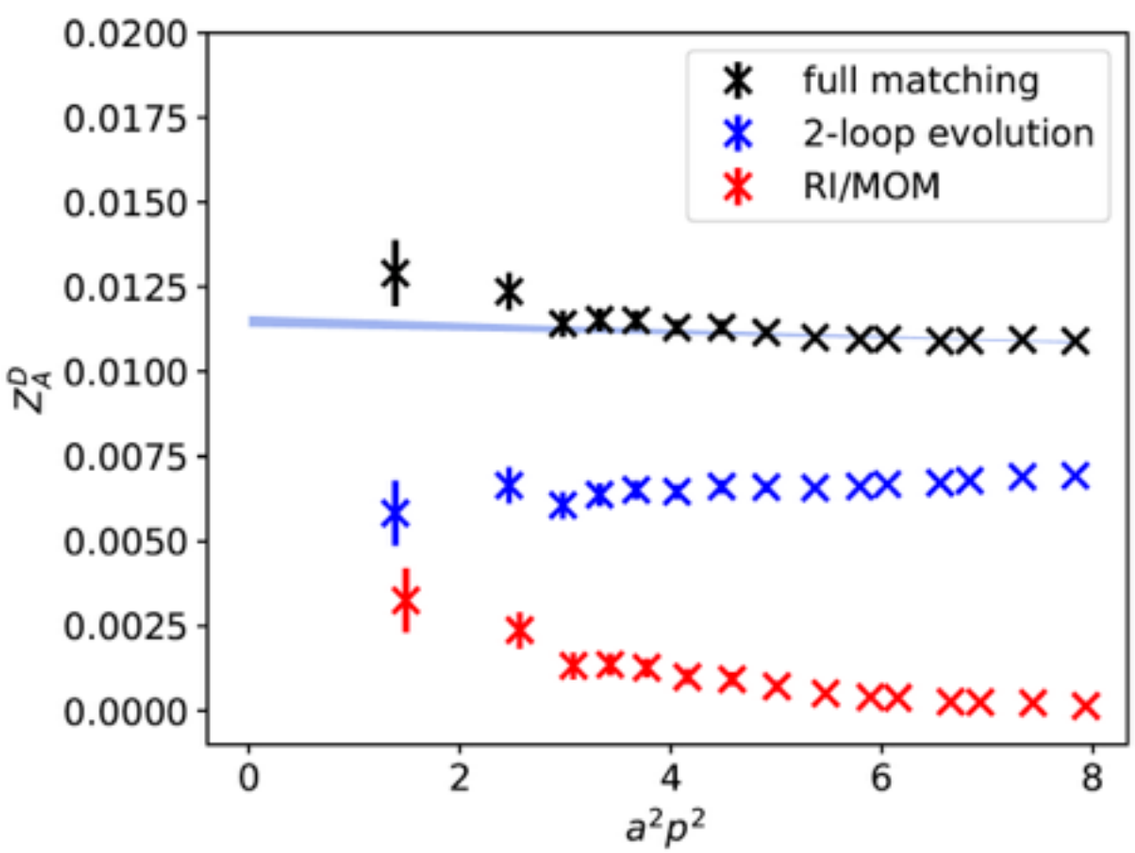




Quark spin

Renormalization

$$\begin{pmatrix} \Delta u^{\overline{\rm MS}}(\mu) \\ \Delta d^{\overline{\rm MS}}(\mu) \\ \Delta s^{\overline{\rm MS}}(\mu) \end{pmatrix} = \begin{pmatrix} Z_A + Z_A^{{\rm D},\overline{\rm MS}}(\mu) & Z_A^{{\rm D},\overline{\rm MS}}(\mu) & Z_A^{{\rm D},\overline{\rm MS}}(\mu) \\ Z_A^{{\rm D},\overline{\rm MS}}(\mu) & Z_A + Z_A^{{\rm D},\overline{\rm MS}}(\mu) & Z_A^{{\rm D},\overline{\rm MS}}(\mu) \\ Z_A^{{\rm D},\overline{\rm MS}}(\mu) & Z_A^{{\rm D},\overline{\rm MS}{\rm I}}(\mu) & Z_A + Z_A^{{\rm D},\overline{\rm MS}}(\mu) \end{pmatrix} \begin{pmatrix} \Delta u \\ \Delta d \\ \Delta s \end{pmatrix}$$



J. Liang, YBY, et al., χQCD collaboration, 1806.08366

- *An accurate renormalization of the singlet axial vector current requires the **RI/MOM** calculation of the **quark loop** and also **the 2-loop matching.***
- *The only complete renormalization calculation with 2-loop finite piece so far.*

Parton distributions and lattice QCD calculations: a community white paper
Huey-Wen Lin et.al, Prog.Part.Nucl.Phys. 100 (2018) 107-160, 1711.07916

Quark spin

Rating system

	★	○
Discretization	$\mathcal{O}(a^2)$ action with three ***a***, two ***a*** <0.1fm, $(a_{max}/a_{min})^2 \geq 2$	$\mathcal{O}(a^2)$ action with two ***a***, one ***a*** <0.1fm, $(a_{max}/a_{min})^2 \geq 1.4$, or $\mathcal{O}(a)$ action
pion mass	One physical $\mathbf{m_\pi}$ or one $\mathbf{m_\pi}$ <200 MeV and two $\mathbf{m_\pi}$ <250 MeV	Three $\mathbf{m_\pi}$ with two of them <300 MeV
Finite volume	One ensemble with $\mathbf{m_{\pi,min}}\,\mathbf{L} \geq 4$, or three ensembles with $\mathbf{L} \geq 2.5$ fm	One ensemble with $\mathbf{m_{\pi,min}}\,\mathbf{L} \geq 3.4$, or two ensembles with $\mathbf{L} \geq 2.5$ fm
Excited states	**Three** source-sink separations or variation method with **3x3** matrix	**Two** source-sink separations or variation method with **2x2** matrix
Renormalization	Non-perturbative renormalization	Perturbative renormalization

■ for uncertainties that have not met the criteria.

Quark spin

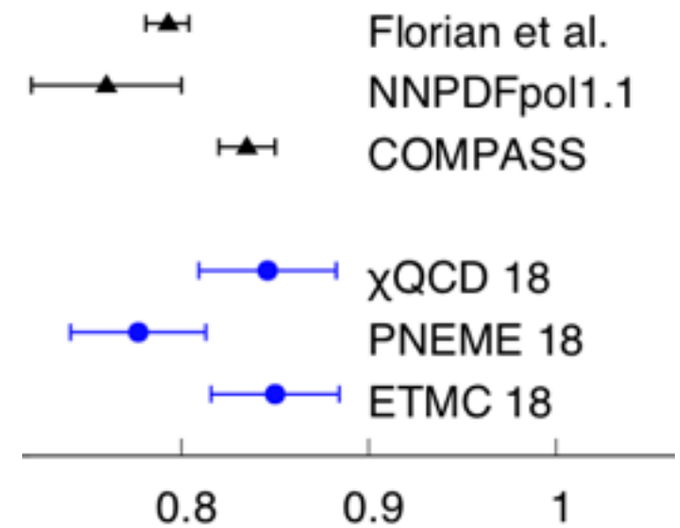
Summary of the present results

	N_f	Disc	m_π	FV	Ren	ESC
χQCD 18	2+1	o	o	★	★	★
PNDME 18	2+1+1	o	★	★	■	★
ETMC 18	2+1+1	■	★	★	★	★

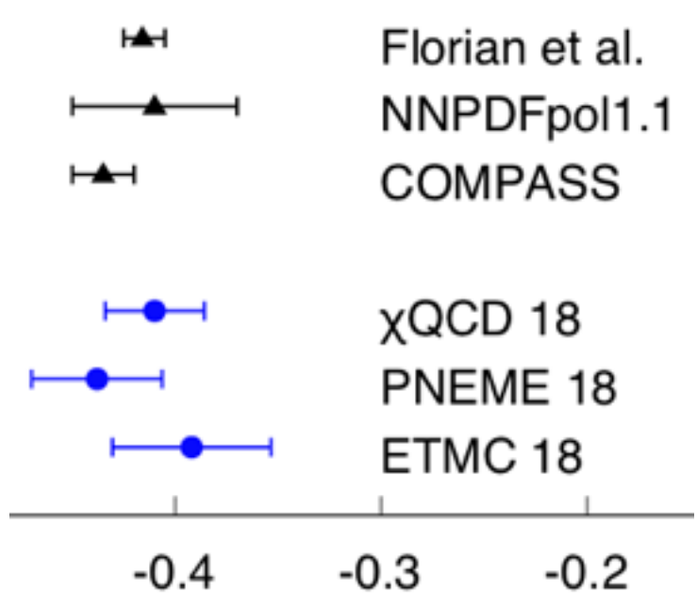
As the result at just one lattice spacing

As the non-perturbative normalization only

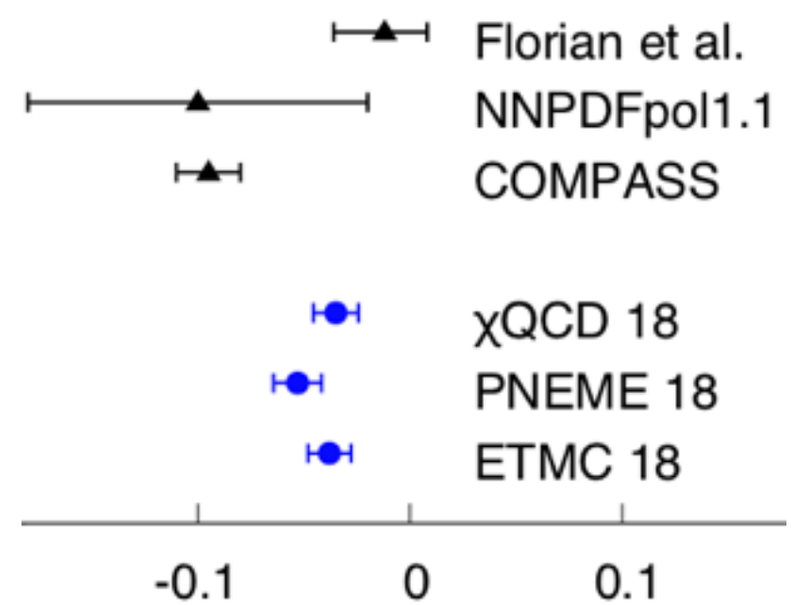
★ for NP renormalization + (A)WI normalization



Δu



Δd



Δs

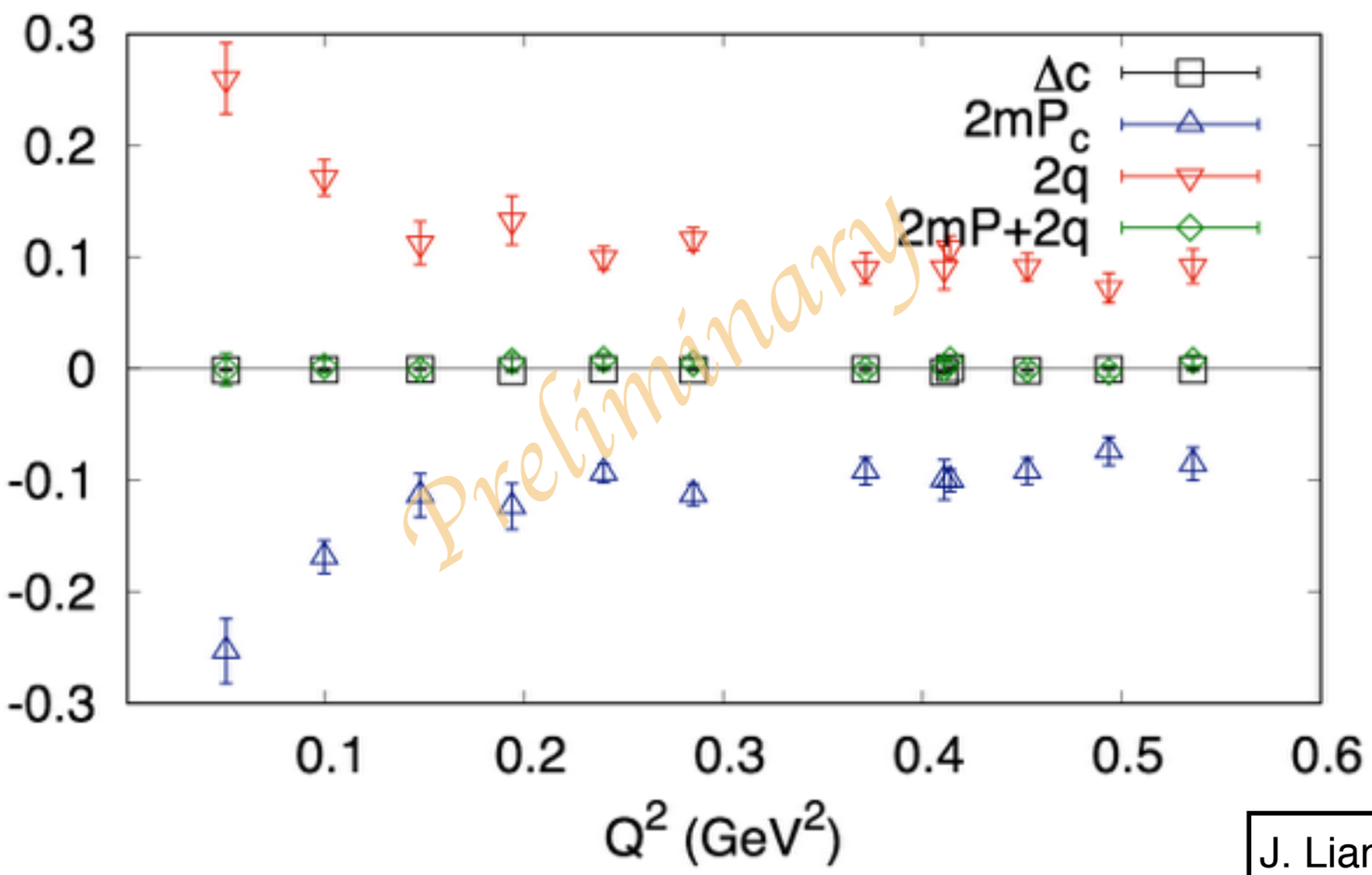
χQCD 18: 1806.08366
PNEME 18: 1806.10604
ETMC 18: in preparation

Quark spin

a story behind Δc

$$\langle ps \left| \vec{\mathcal{A}}_\mu \cdot \vec{s} \right| ps \rangle = \lim_{p'-p\to 0} \frac{i|\vec{s}|}{(\vec{p}' - \vec{p}) \cdot \vec{s}} \langle p', s | 2m_f \mathcal{P} - 2i \frac{\alpha_s}{4\pi} F\tilde{F} \, |p, s\rangle$$

$$= 2m_f \langle p, s | \int d^3x \; \vec{x} \cdot \vec{s} \; \mathcal{P}(x) | p, s \rangle - 2i \langle p, s | \int d^3x \; \vec{x} \cdot \vec{s} \; \frac{\alpha_s}{4\pi} F(x)\tilde{F}(x) \, |p, s\rangle$$

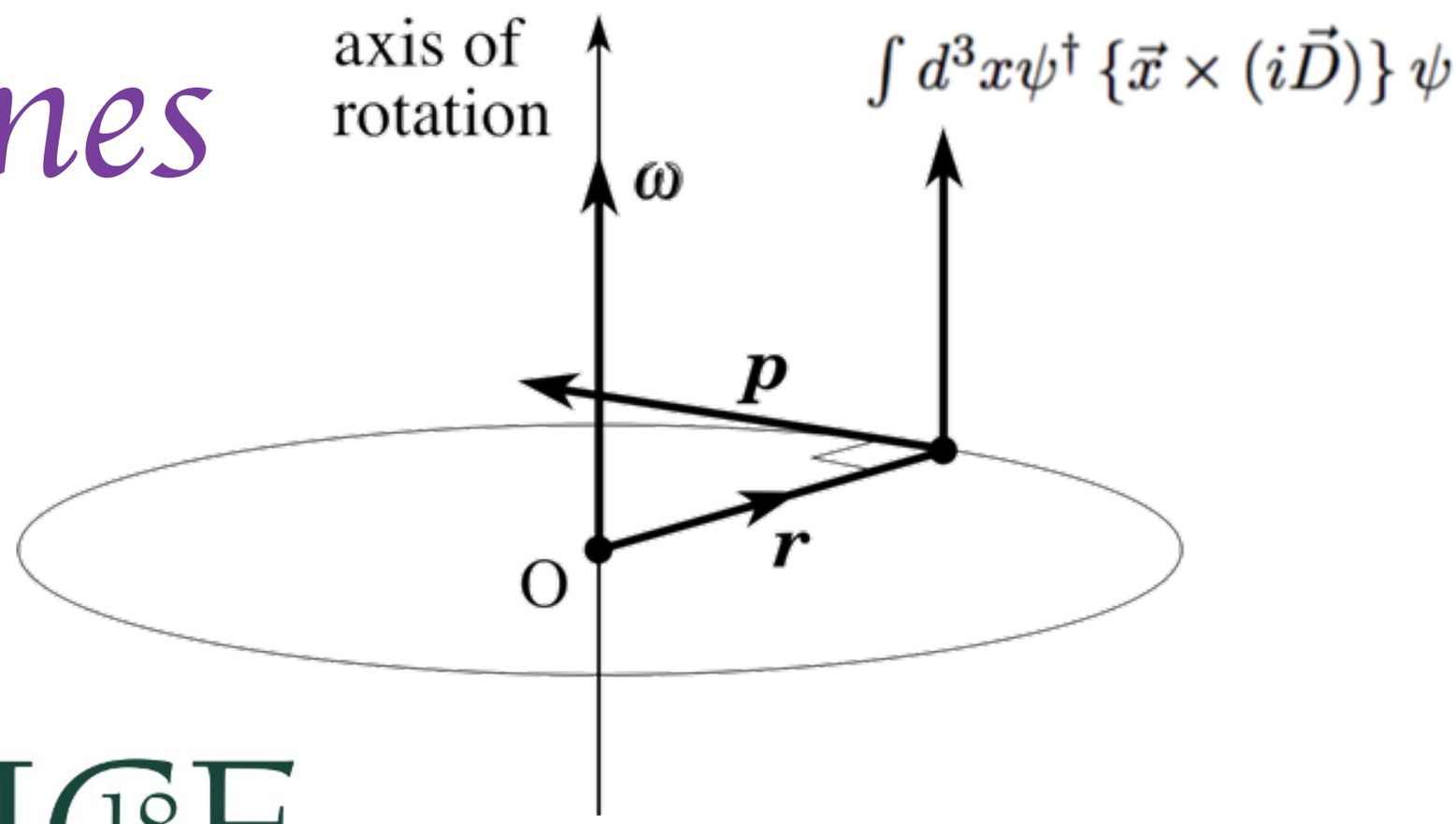


- The contribution from the glue anomaly turns out to be **very large** (~0.2-0.3 per flavor)!
- Accurate calculation of the light flavor cases is ongoing.

J. Liang, **YBY**, et. al., χQCD collaboration, in preparation

M. Gong, **YBY**, et. al., χQCD collaboration, PRD95, 114509 (2017), 1511.03671

Angular momenta and Orbital ones



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Proton spin

Connections between decompositions

X. Ji, PRL78, 610 (1997)

$$\vec{J} = \int d^3x \, \tfrac{1}{2} \, \overline{\psi} \, \vec{\gamma} \, \gamma^5 \, \psi + \int d^3x \psi^\dagger \, \{\vec{x} \times (i\vec{D})\} \, \psi + \int d^3x 2\{\vec{x} \times \mathrm{Tr}[\vec{E} \times \vec{B}]\}$$

$$\int d^3x \psi^\dagger \, \{\vec{x} \times \overrightarrow{A}\} \, \psi$$

quark spin

Different definitions of the quark OAM

Further decomposition of the glue AM

$$\vec{J} = \int d^3x \, \tfrac{1}{2} \, \overline{\psi} \, \vec{\gamma} \, \gamma^5 \, \psi + \int d^3x \psi^\dagger \, \{\vec{x} \times (i\vec{\nabla})\} \, \psi$$

$$\int d^3x 2\mathrm{Tr}[\vec{E} \times \vec{A}] + \int d^3x 2\mathrm{Tr}[E^i \vec{x} \times \vec{\nabla} A^i]$$

glue spin *glue OAM*

R. L. Jaffe and A. V. Manohar, NPB337, 509 (1990)

Proton spin

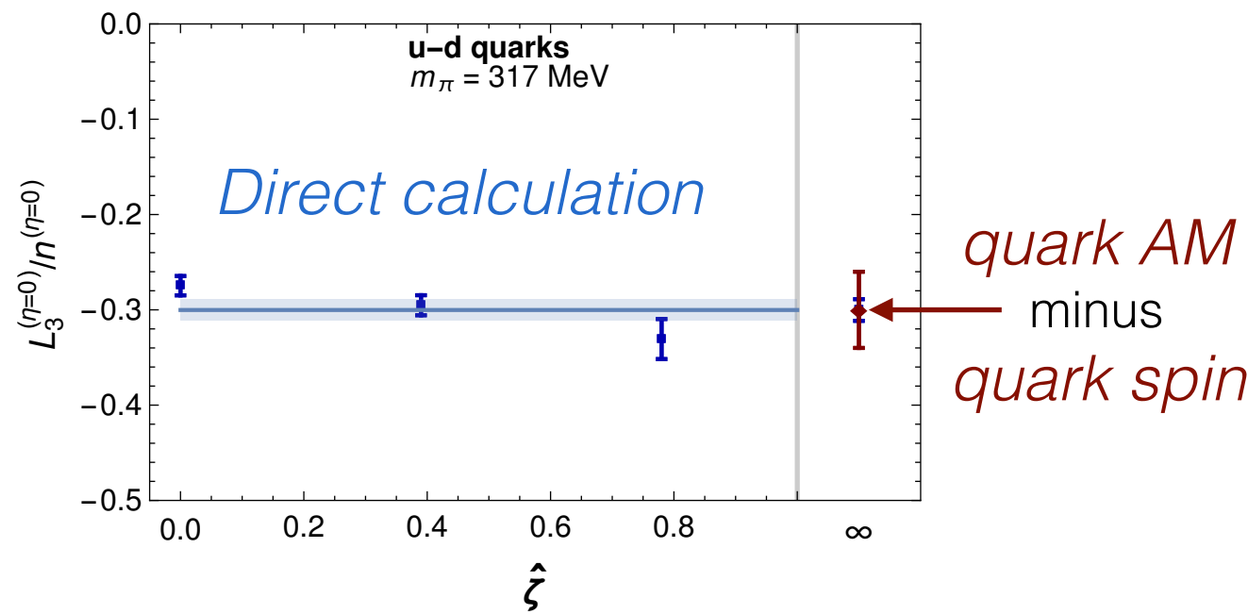
M. Engelhardt, et. al., in preparation
Jul. 25, 3:40PM, 106

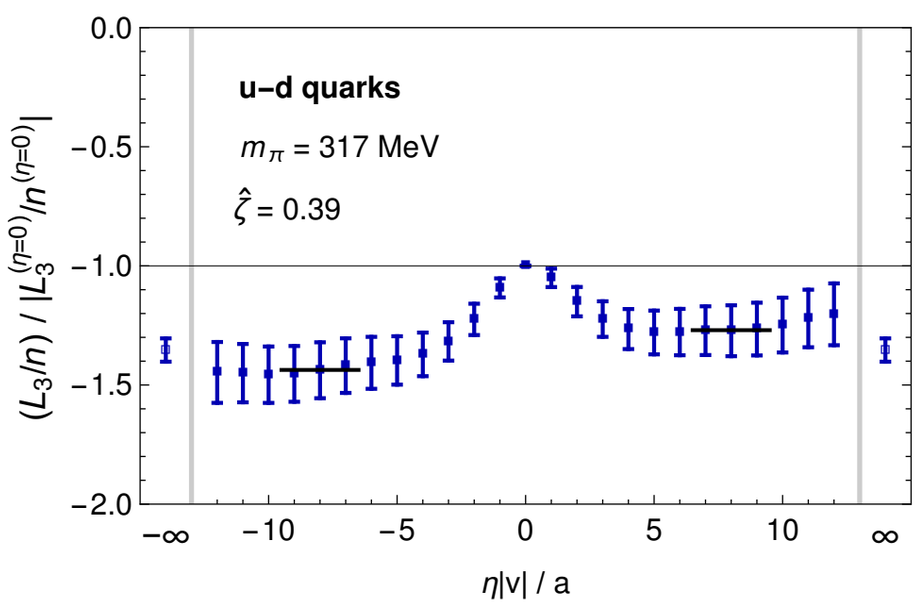
Direct calculation on OAM

The Ji quark orbital angular momentum,

$$\int d^3x \psi^\dagger \{\vec{x} \times (i\vec{D})\} \psi \ ,$$

can be calculated with the quark bilinear operator with wilson links:





$$\frac{\text{Jaffe-Manohar OAM}}{\text{Ji OAM}}$$

Similarly, Jaffe-Manohar quark OAM,

$$\int d^3x \psi^\dagger \{\vec{x} \times (i\vec{\nabla})\} \psi$$

can be calculated and ***the ratio over Ji OAM*** *seems to deviate from* ***one****.*

Proton spin

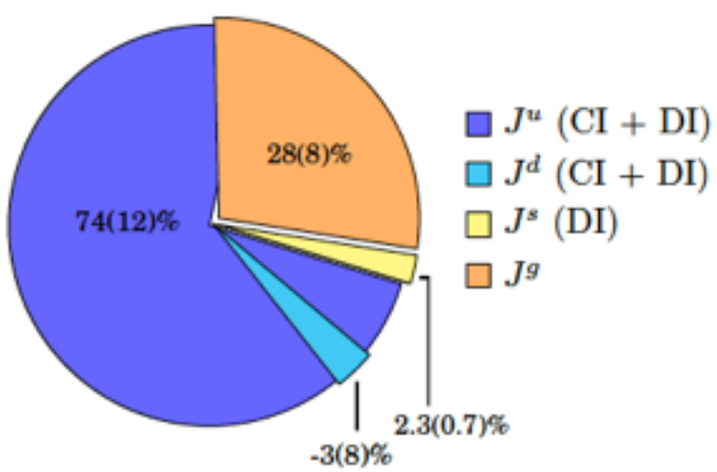
Lattice result of Ji AM

*Quark **AM***

*Glue **AM***

$$\vec{J} = \underbrace{\int d^3x \, \tfrac{1}{2} \overline{\psi} \vec{\gamma} \gamma^5 \psi + \int d^3x \psi^\dagger \{\vec{x} \times (i\vec{D})\} \psi}_{} + \underbrace{\int d^3x 2\{\vec{x} \times \mathrm{Tr}[\vec{E} \times \vec{B}]\}}_{}$$

Quenched result

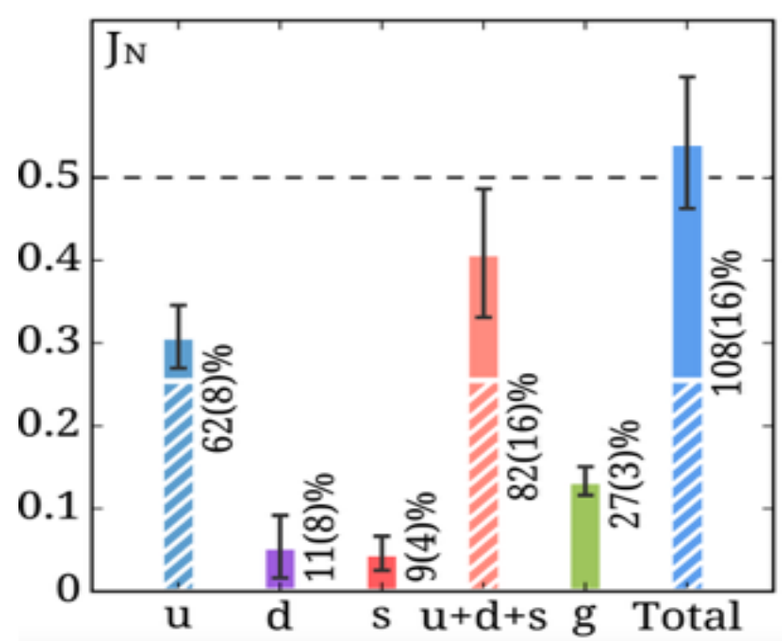


M. Deka, T. Doi, **YBY**, et. al., χQCD collaboration, PRD91, 014505 (2015), 1312.4816

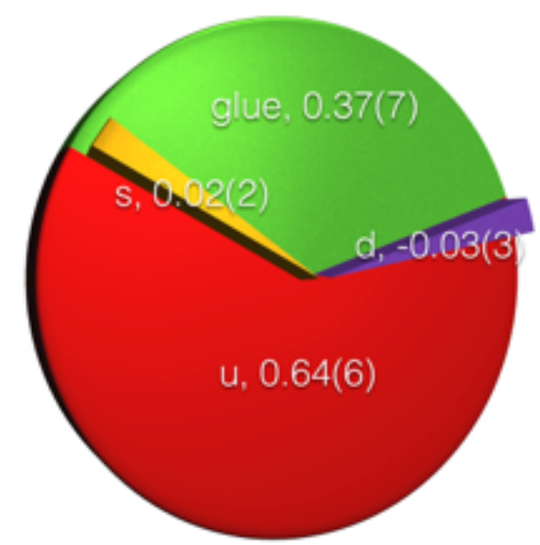
2-flavor result

(neglecting the difference between the glue momentum and AM fractions)

EMTC 17: PRL119(2017), 1706.02973



***Preliminary** 2+1 flavors result*



YBY, et. al., χQCD collaboration, in preparation

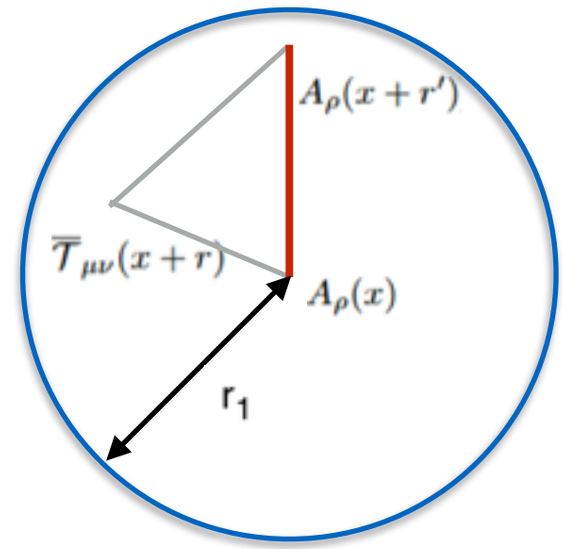
*All of them are based on the **perturbative** renormalization;*
***Non-perturbative** renormalization should be applied on them to get the state-of-the-art result.*

Gluon renormalization with CDER

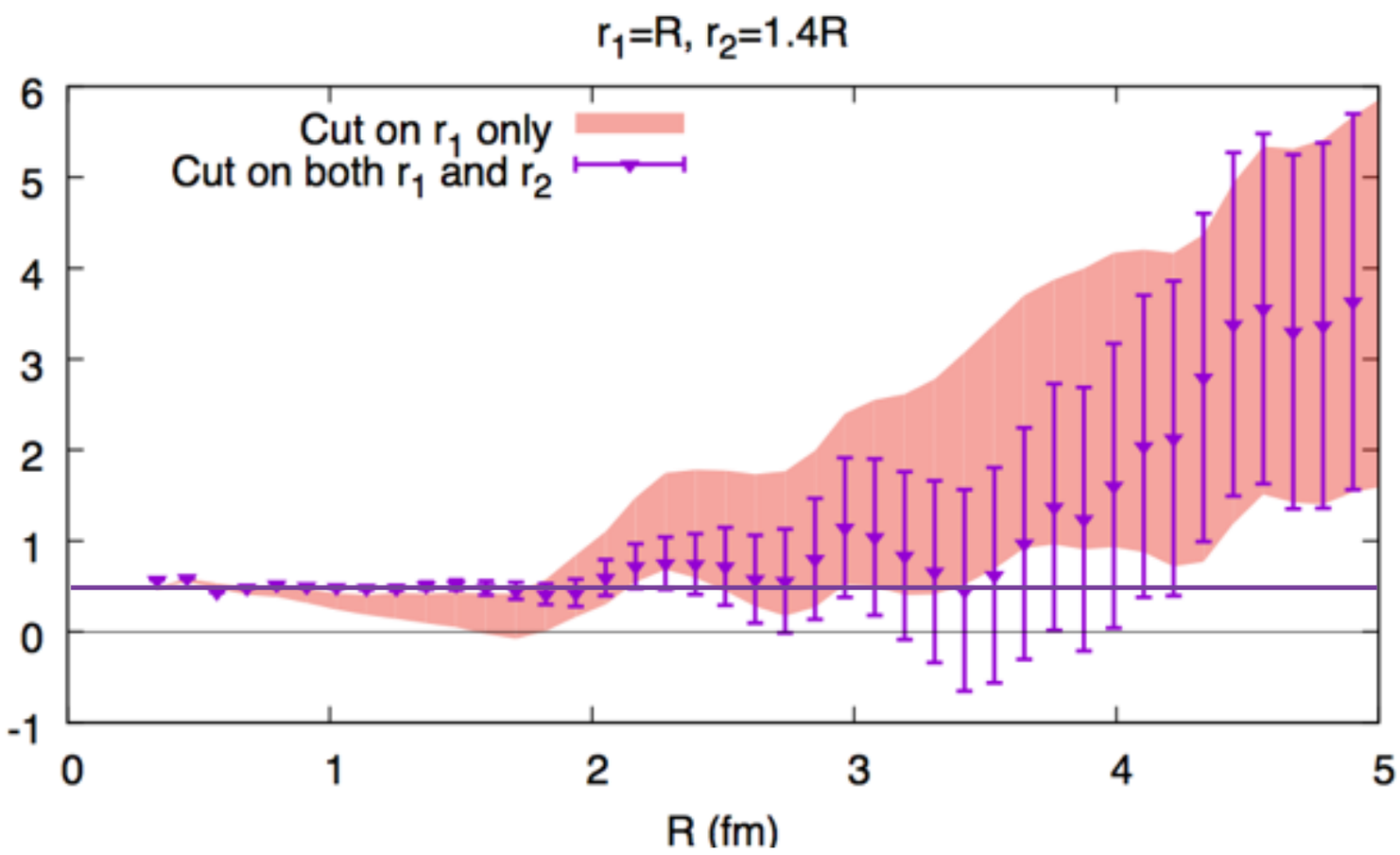
K. Liu, J. Liang, **YBY**, PRD96, 114504(2017), 1805.00531

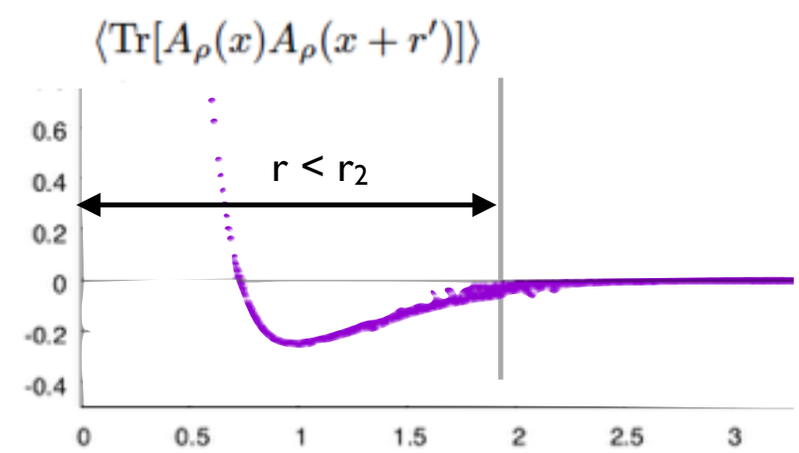
$$Z_T^{-1}(\mu_R^2) = \frac{p^2\langle(\overline{T}_{\mu\mu} - \overline{T}_{\nu\nu})\,\mathrm{Tr}[A_\rho(p)A_\rho(-p)]\rangle}{2p_\mu^2\langle\mathrm{Tr}[A_\rho(p)A_\rho(-p)]\rangle}\Bigg|_{\substack{p^2=\mu_R^2,\\ \rho\neq\mu\neq\nu,\\ p_\rho=0,\\ p_\nu=0}}$$

$$C_3(p) = \left\langle \int d^4x\, d^4y\, d^4z\, e^{ip\cdot(x-y)}\overline{T}_{\mu\nu}(z)\,\mathrm{Tr}[A_\rho(x)A_\rho(y)]\right\rangle$$
$$\simeq \left\langle \int_{|r|<r_1} d^4r \int_{|r'|<r_2} d^4r' \int d^4x\, e^{ip\cdot r'}\overline{T}_{\mu\nu}(x+r)\,\mathrm{Tr}[A_\rho(x)A_\rho(x+r')]\right\rangle$$



r_1: Cut on the $T_{\mu\nu}$ - A_ρ correlation

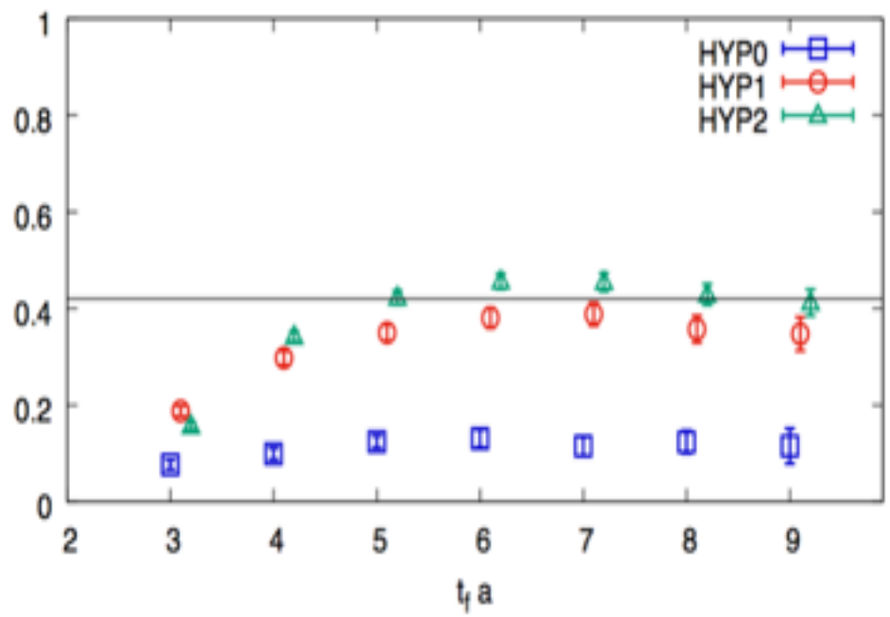




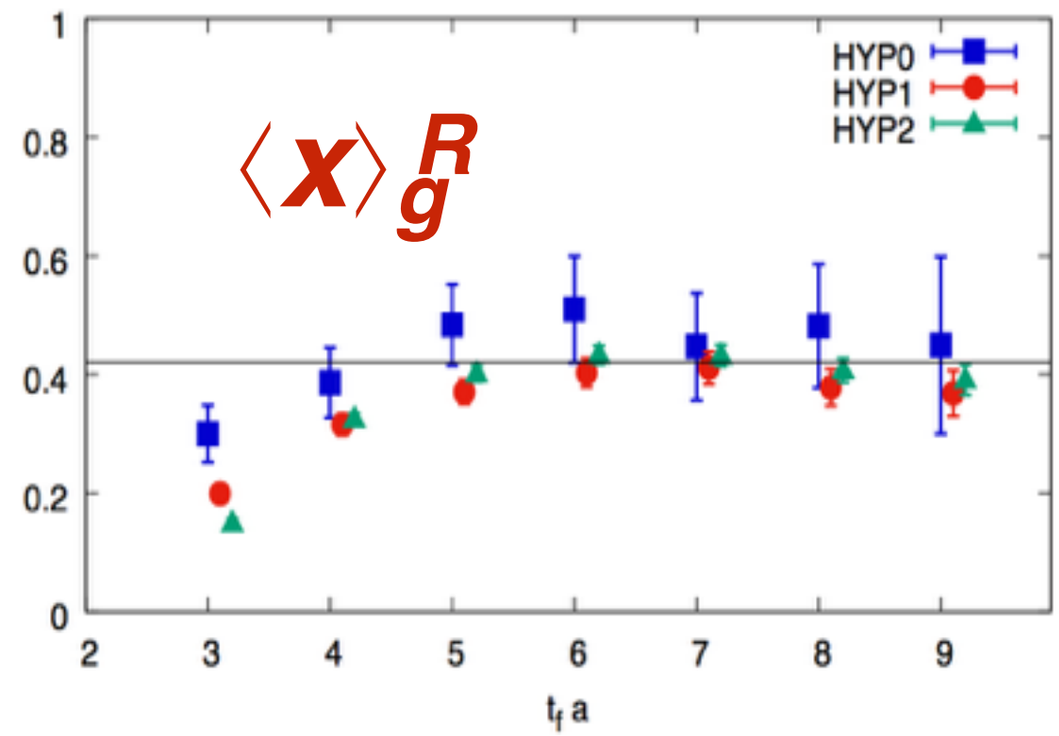
r_2: Cut on the A_ρ - A_ρ correlation

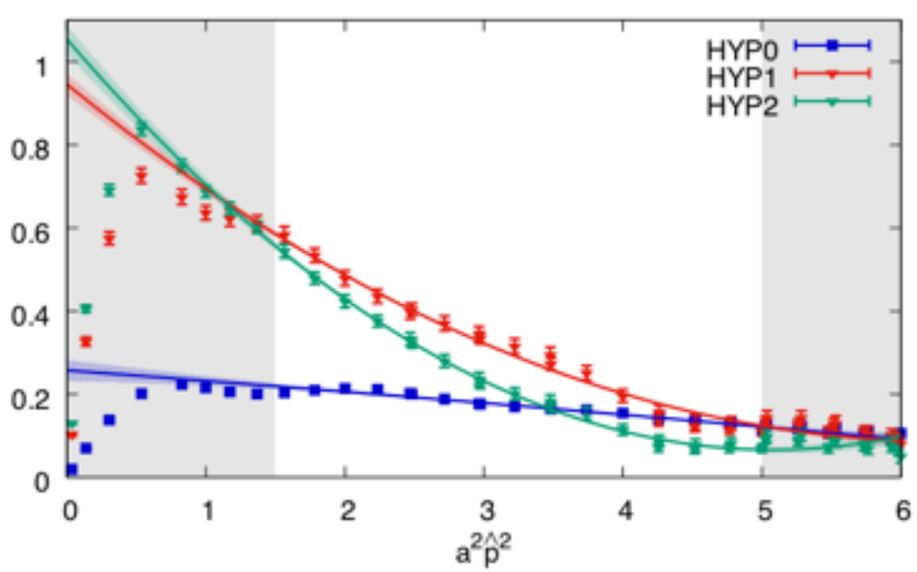
YBY, et. al., χQCD collaboration, 1805.00531

Non-perturbative renormalized glue momentum fraction



$$\frac{\langle x \rangle_g^{bare}}{Z^{-1}} = \langle x \rangle_g^{R}$$





- The lattice regularization effects are **fully cancelled** within the statistical uncertainties;
- The **non-perturbative renormalized quark/glue AM results** will come out soon.

YBY, et. al., χQCD collaboration, 1805.00531

Summary

- We are very close to a state-of-the-art theoretical picture of the proton spin.
- **Increasing computation resources** and **technique breakthroughs** open the gates on the direct calculation of the glue spin and OAM.
- Fully non-perturbative renormalization becomes possible now.

At the right time, which will be soon, I myself will explain—and it will be an explanation that you'll think is reasonable—about everything that's occurred. Until then, be cheerful and think of each thing well.

Shakespeare: The Tempest: Act 5, Scene 1, 299-303

Quark spin

a story behind Δc

$$\langle ps \left| \vec{\mathcal{A}}_\mu \cdot \vec{s} \right| ps \rangle = \lim_{p'-p\to 0} \frac{i|\vec{s}|}{(\vec{p}' - \vec{p}) \cdot \vec{s}} \langle p', s | 2m_f \mathcal{P} - 2i \frac{\alpha_s}{4\pi} F\tilde{F} \, |p, s\rangle$$

$$= 2m_f \langle p, s | \int d^3x \; \vec{x} \cdot \vec{s} \; \mathcal{P}(x) | p, s \rangle - 2i \langle p, s | \int d^3x \; \vec{x} \cdot \vec{s} \; \frac{\alpha_s}{4\pi} F(x) \tilde{F}(x) \, |p, s\rangle$$

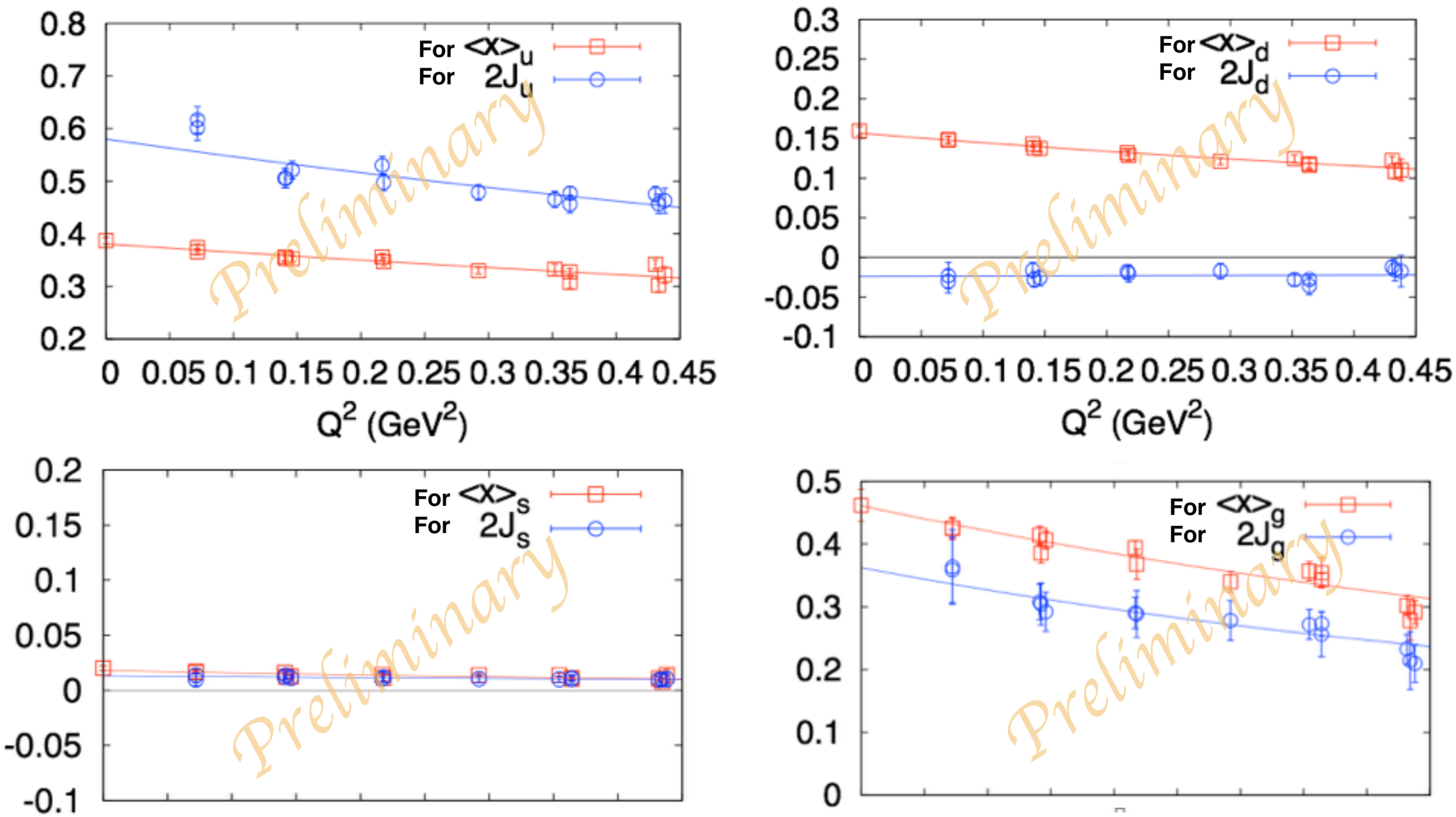
- The quark spin can be further decomposed into the contributions from 2mP and triangle anomaly;
- Require the calculation with p≠p', and then extrapolate the result to the forward limit — likes the other GPD moments calculation;
- If the triangle anomaly contribution is small, then such a decomposition is trivial.

M. Gong, **YBY**, et. al., χQCD collaboration, PRD95, 114509 (2017), 1511.03671

YBY, et. al., χQCD collaboration, in preparation

Angular momenta

as the second moment of GPDs

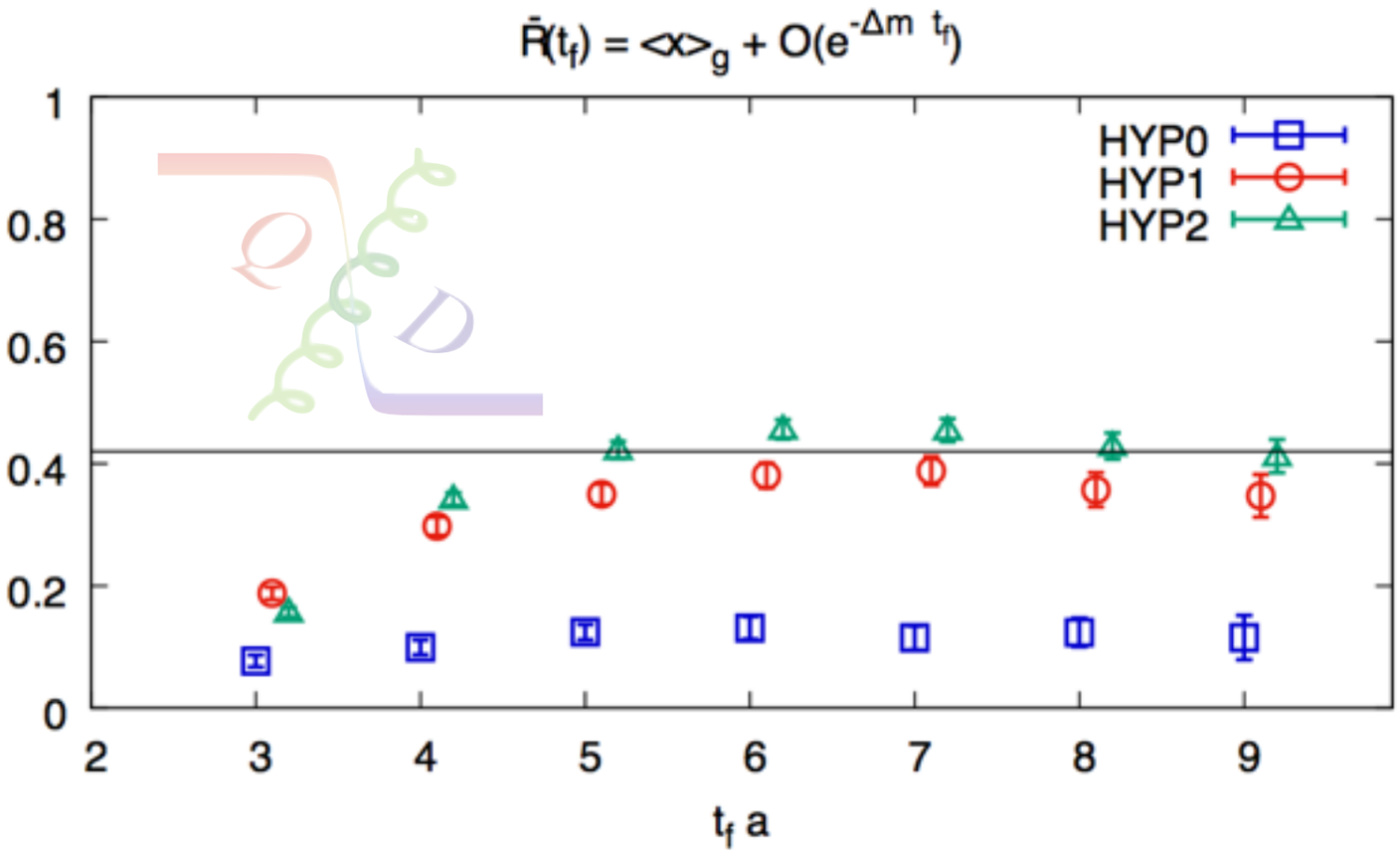


Lattice regularization dependence of $\langle x \rangle_g^{bare}$

The bare glue momentum fraction $\langle x \rangle_g^{bare}$ can be obtained by,

$$\bar{R}(t_f) \equiv \sum_{0<t<t_f} R(t_f,t) - \sum_{0<t<t_f-1} R(t_f-1,t) = \langle x \rangle_g^{\rm bare} + \mathcal{O}(e^{-\delta m\, t_f}),$$

$$R(t_f,t) = \frac{4\langle 0|\Gamma^e \int d^3y\, \chi(\vec{y},t_f)\overline{\mathcal{T}}_{g,44}(t)\bar{\chi}(\vec{0},0)|0\rangle}{3M_N\langle 0|\Gamma^e \int d^3y\, \chi(\vec{y},t_f)\bar{\chi}(\vec{0},0)|0\rangle},$$



- HYP smearing can be used to improve the signal to noise ratio of $\langle x \rangle_g^{bare}$, as in the quark case.
- But the central values will also be changed dramatically after the smearing is applied.
- The non-perturbative renormalization is imperative in the state-of-the-art glue matrix element calculation.

YBY, et. al., χQCD collaboration, 1805.00531