Real-time Evolution of $U(1)$ Chiral Charge

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Lattice 2018, 23th of July 2018

Overview

Lattice Model

Results/Outlooks

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Anomalous Processes

Gauge bosons

↕

Anomaly

Axion Physics

Chiral Magnetic Effect

Baryogenesis

Fermionic charge

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Model

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\Psi}\not{D}\Psi$$
$$+(D_\mu\phi)^* D_\mu\phi - V(\phi)$$

with: $j_5^\mu = \bar{\Psi}\gamma^\mu\gamma^5\Psi$

$$V(\phi) = m^2|\phi|^2 + \lambda|\phi|^4$$

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Integrate out $\Psi \longrightarrow F_{CS} = \mu N_{CS}$

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Continuum of vac. $N_{CS} \propto \vec{A} \cdot \vec{B}$

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Sum-Up

- $B = 0$: Instability
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$$-\frac{1}{2c_s^2}(\partial_0 a)^2 + \frac{1}{2}(\partial_i a)^2$$

with a a scalar field

U(1)+Axion !

Reproduce EoM

Real-Time Simulations

MC generated thermal ensemble

← **Measure**

t_f

Solve **classical** EoM

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Technical Comments

- Lattice topological $F\tilde{F}$
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Refs.: JHEP04(2018)026
j.nuclphysb.2017.12.001

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- $\mu = 0, B = 0$

$\vec{A} \cdot \vec{B}$ costs $E \implies \langle Q^2(t) \rangle \to cst$

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RD walk

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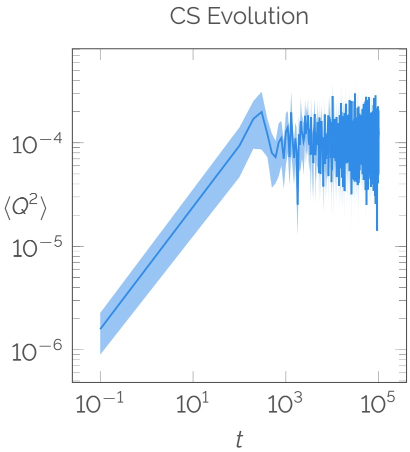

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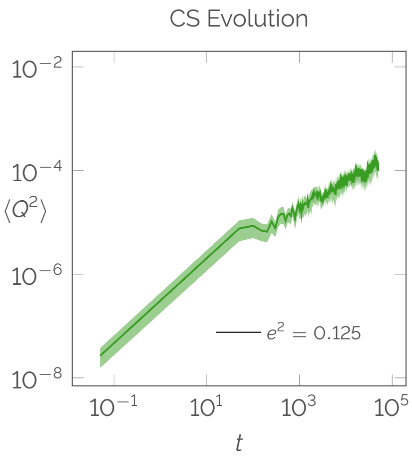

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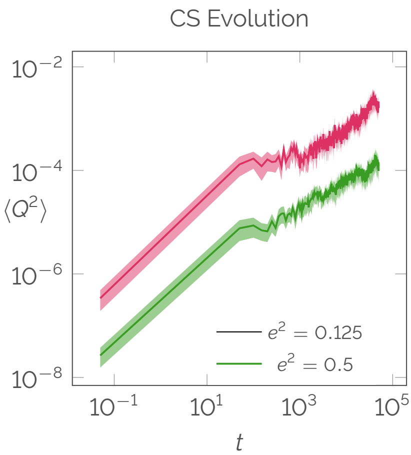

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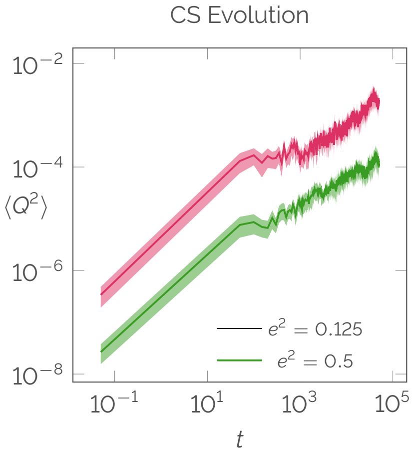

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Agree on parametric

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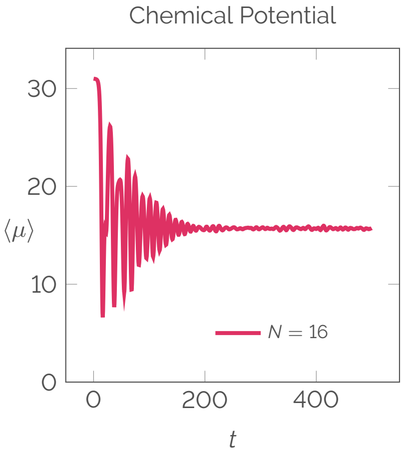

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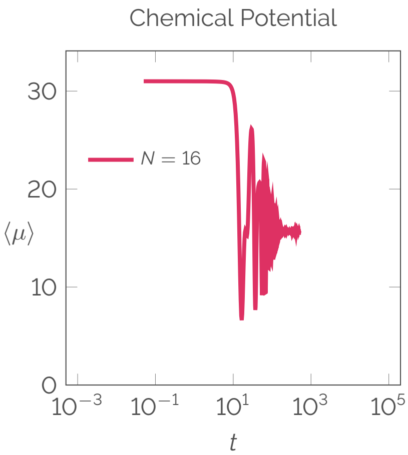

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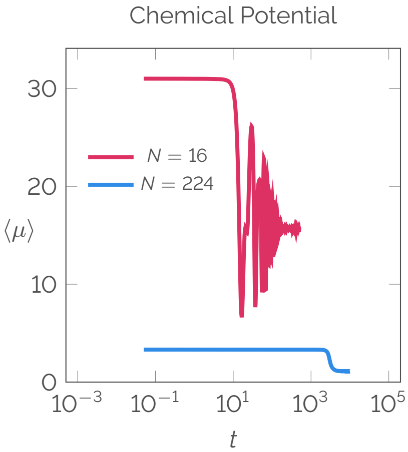

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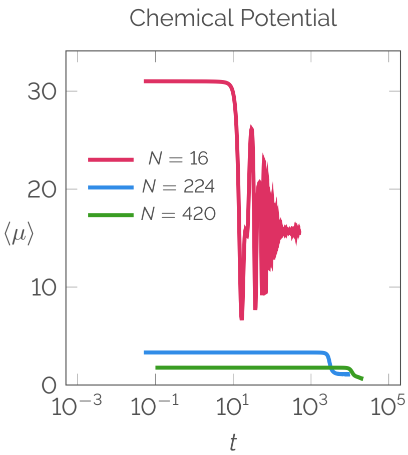

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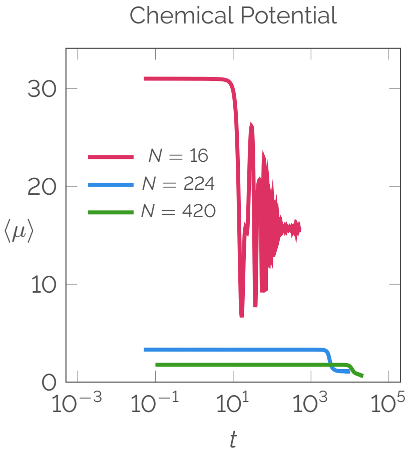
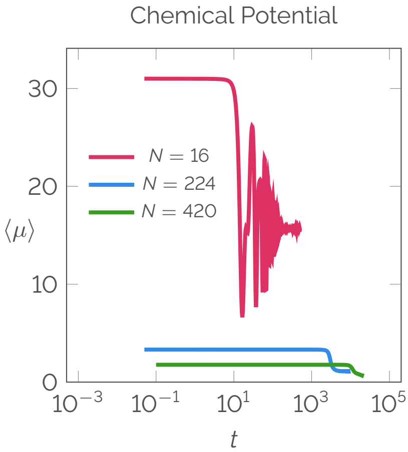
Question:

μ independent rate at small μ ?



Further Outlooks



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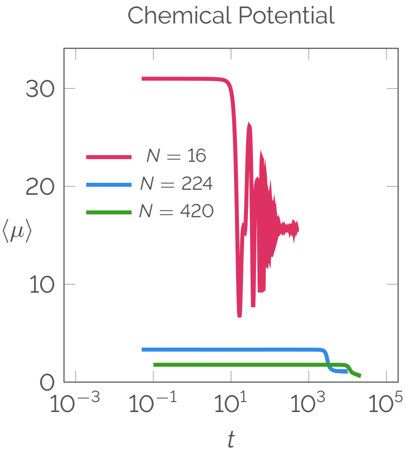
- Non-homogeneous axion



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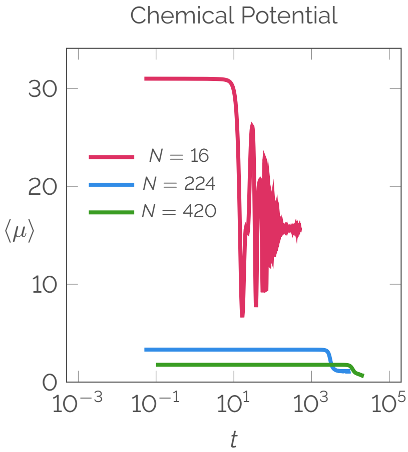

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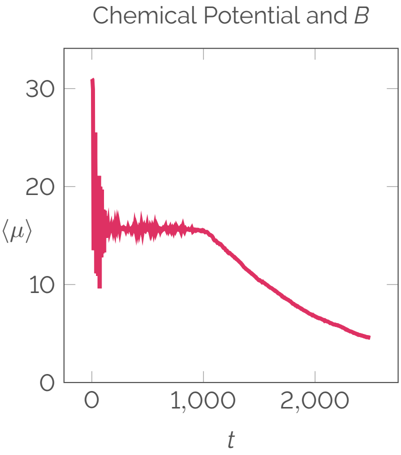
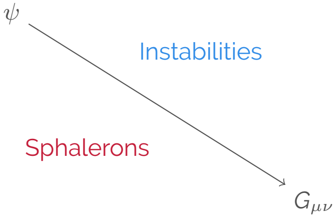
Thank you!

Non-Abelian





[Rubakov,1986]

[Rummukainen,2014]