

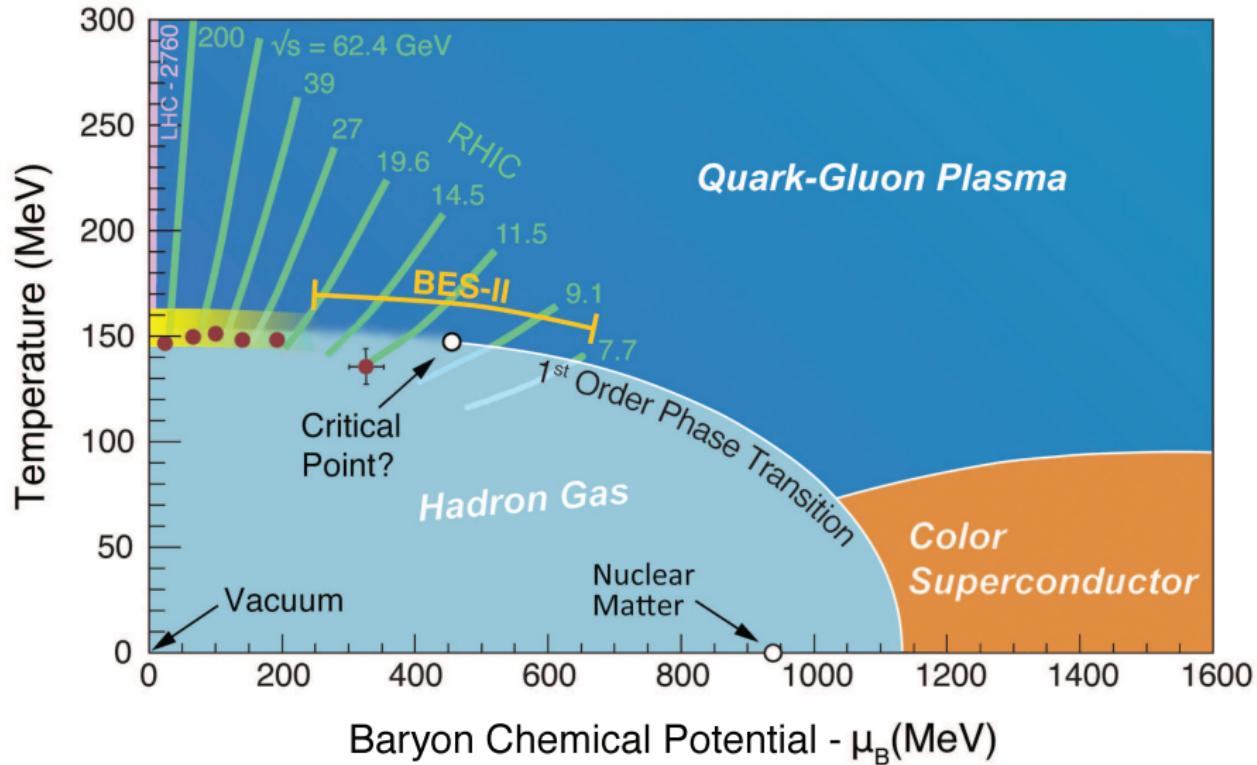


The QCD crossover from Lattice QCD

July 25, 2018 | Patrick Steinbrecher

HotQCD collaboration

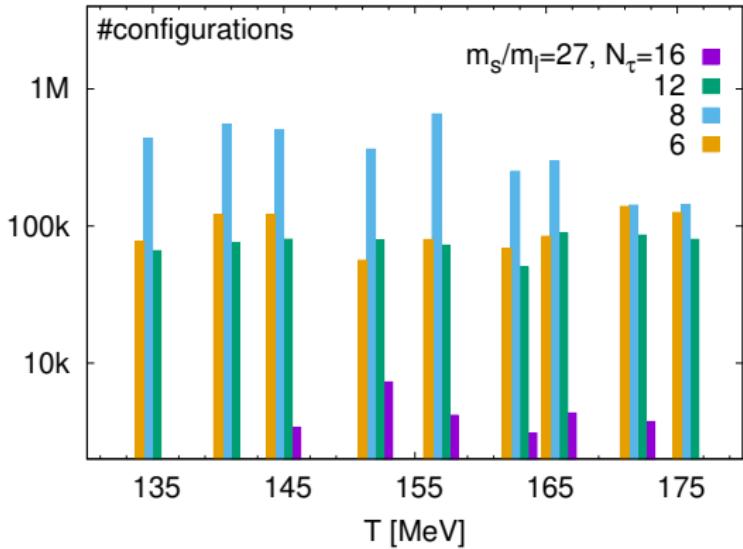
The QCD phase diagram



Quantum Chromodynamics

from first principles

- Lattice QCD
 - HISQ action
 - $N_\sigma = 4N_\tau$
 - sim. at $\mu = 0$
- physical quarks
 - 2 light quarks
 - 1 strange quark
 - $m_s/m_l = 27$
- $m_\pi \simeq 138$ MeV



everything continuum
extrapolated

Chiral observables in two-flavor formulation

- subtracted condensate

$$\Sigma_{\text{sub}} \equiv m_s(\Sigma_u + \Sigma_d) - (m_u + m_d)\Sigma_s$$

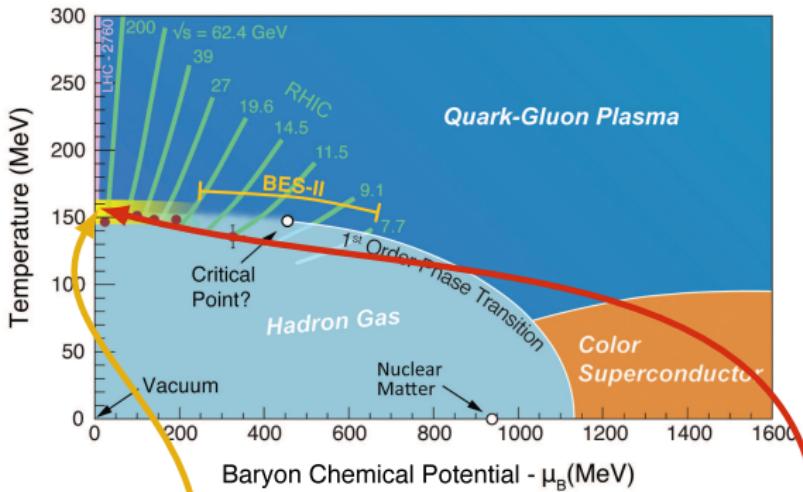
with $\Sigma_f = \frac{T}{V} \frac{\partial}{\partial m_f} \ln Z$

- subtracted susceptibility

$$\chi_{\text{sub}} \equiv \frac{T}{V} m_s \left(\frac{\partial}{\partial m_u} + \frac{\partial}{\partial m_d} \right) \Sigma_{\text{sub}}$$

- χ_{disc} is defined as χ_{sub} without connected part

Start of the QCD crossover line: T_0



$$\frac{d^2}{dT^2} \frac{\Sigma_{\text{sub}}}{f_K^4} \equiv 0 \quad \text{and} \quad \frac{d}{dT} \frac{\chi_{\text{sub}}}{f_K^4} \equiv 0$$



two crossover temperatures: $T_0(\Sigma_{\text{sub}})$ and $T_0(\chi_{\text{sub}})$

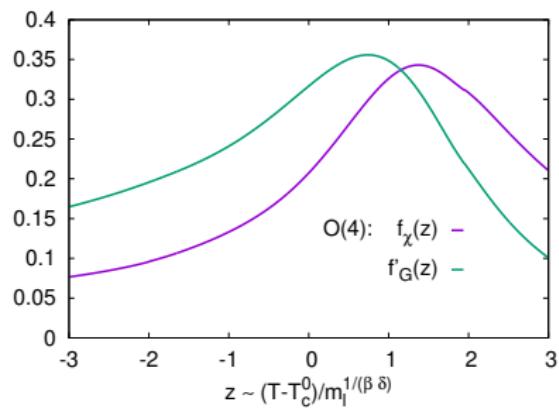
Pseudo-critical temperatures

- for $m_l \rightarrow 0$: pseudo-critical temperatures converge to the chiral transition temperature T_c^0
- at finite quark mass it is given by maximum of $O(4)$ universal scaling functions (Thursday talk, Sheng-Tai Li, Chiral phase transition)

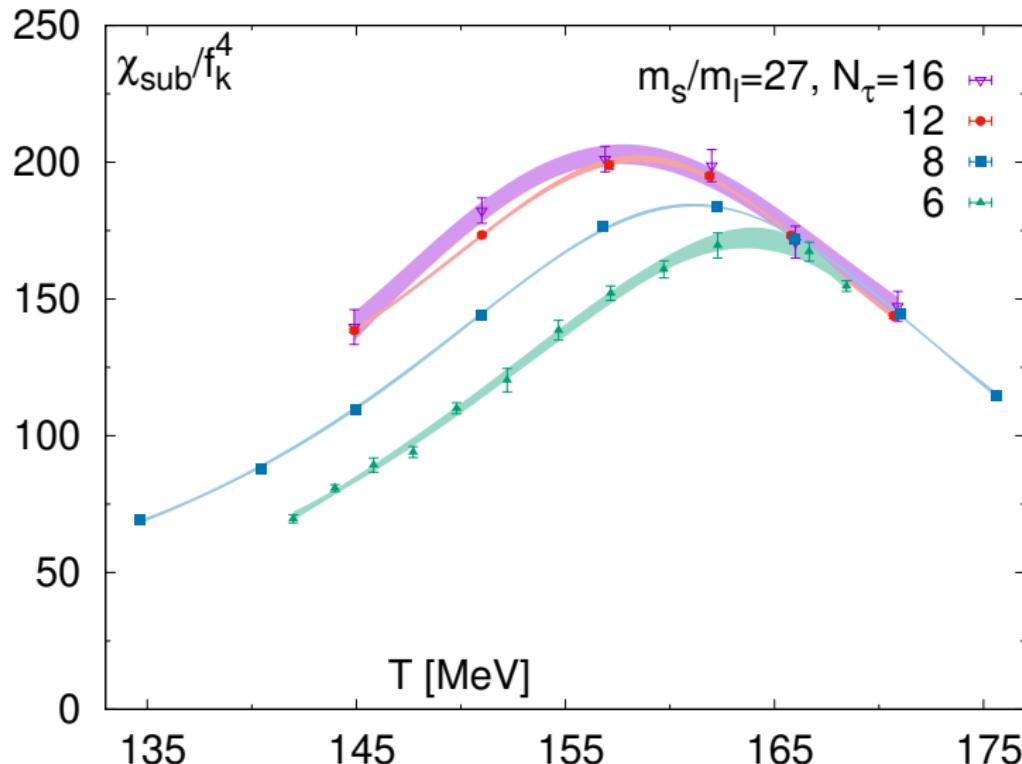
$$\chi_m = m_l^{1/\delta-1} f_\chi(z) + \text{reg.}$$

$$\chi_t = m_l^{(\beta-1)/\beta\delta} f'_G(z) + \text{reg.}$$

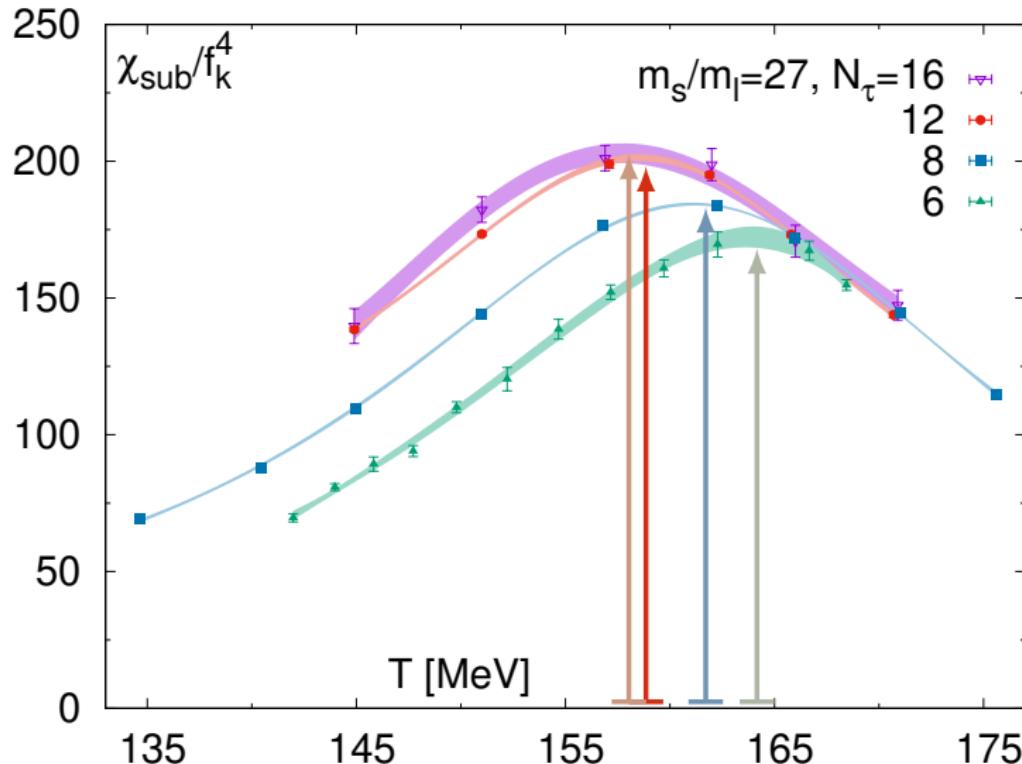
- for $m_l \rightarrow 0$
 - $\chi_t \sim \partial_T \Sigma_{\text{sub}}$ and $\chi_t \sim \partial_{\mu_B}^2 \Sigma_{\text{sub}}$
 - $\chi_m \sim \chi_{\text{sub}}$ and $\chi_m \sim \chi_{\text{disc}}$



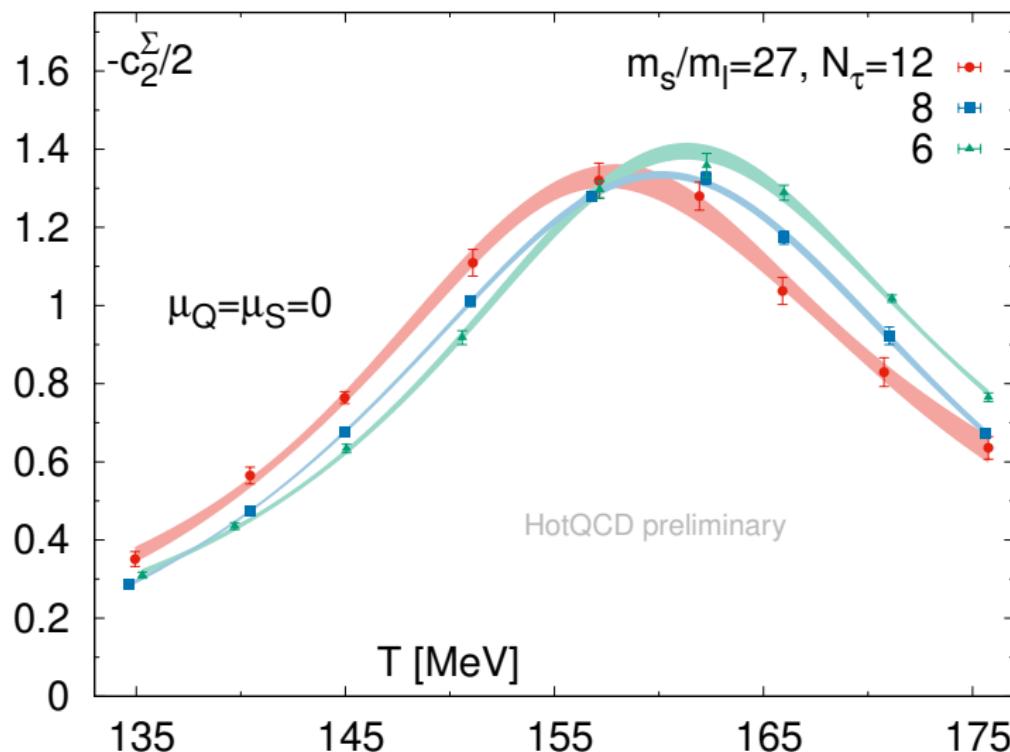
The subtracted chiral susceptibility



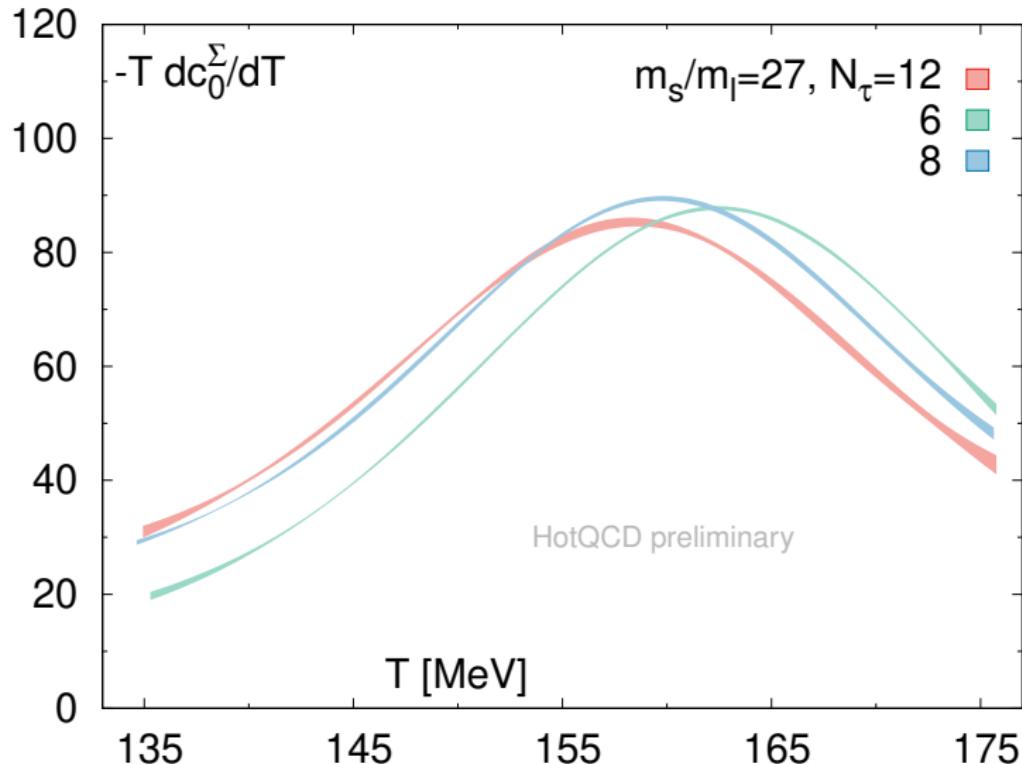
The subtracted chiral susceptibility



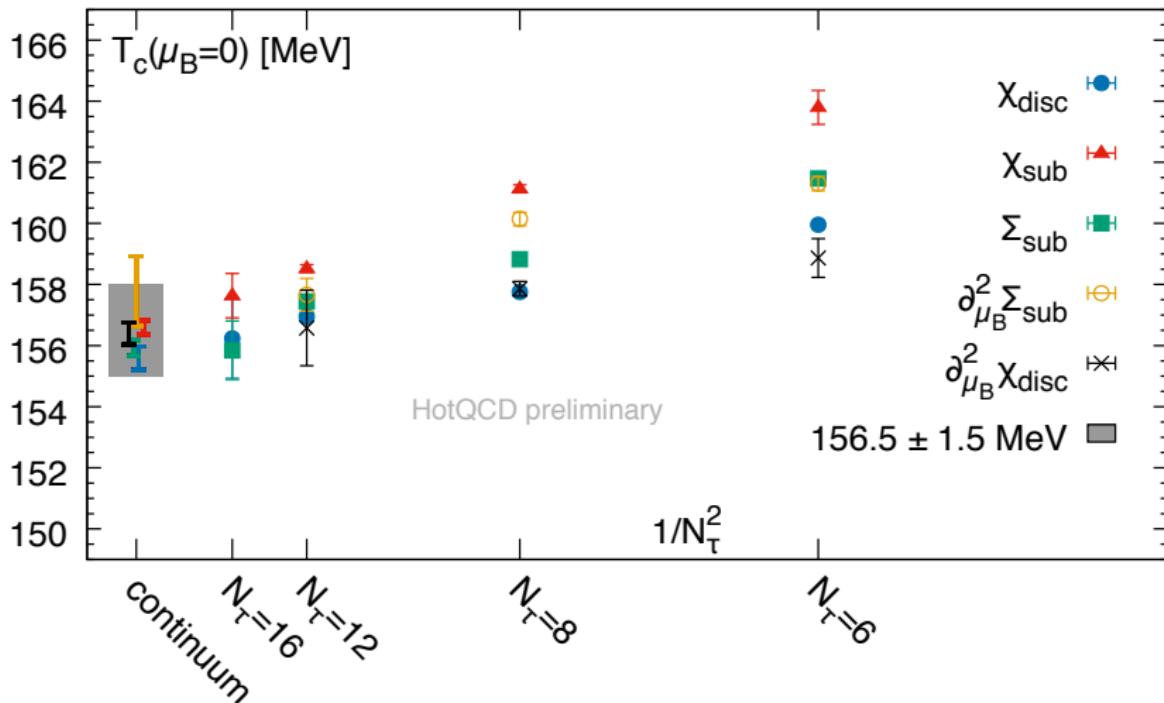
The 2nd μ_B derivative of chiral condensate Σ_{sub}



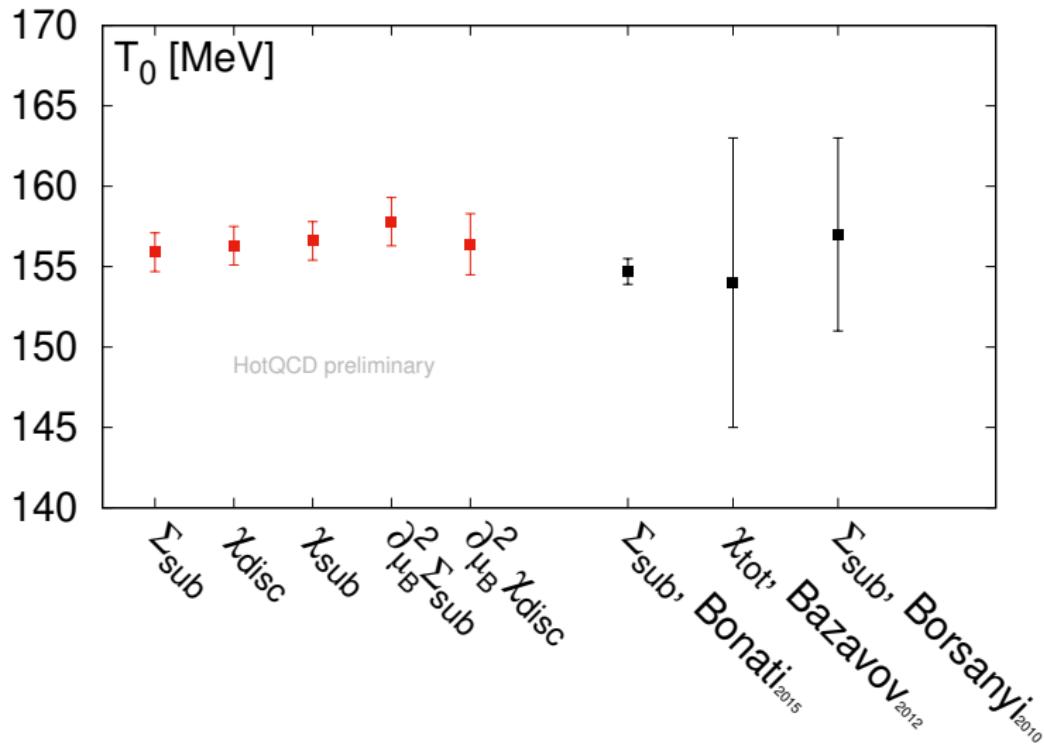
The 1st T derivative of chiral condensate Σ_{sub}



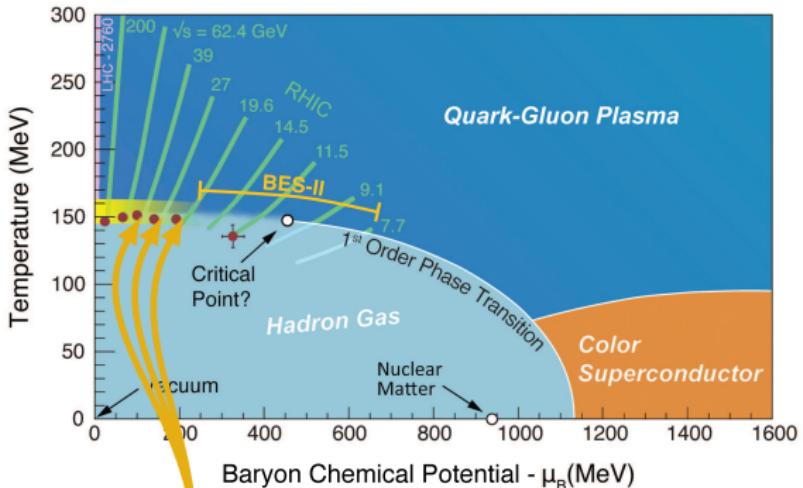
The T_0 continuum extrapolation



Crossover temperature T_0



The QCD crossover at $\mu \neq 0$



$$\frac{d^2}{dT^2} \frac{\Sigma_{\text{sub}}(T, \mu_B)}{f_K^4} \equiv 0$$

and

$$\frac{d}{dT} \frac{\chi_{\text{disc}}(T, \mu_B)}{f_K^4} \equiv 0$$



need Taylor expansion in T and μ_B around $(T_0, 0)$

Taylor expansion in chemical potentials

(just notation)

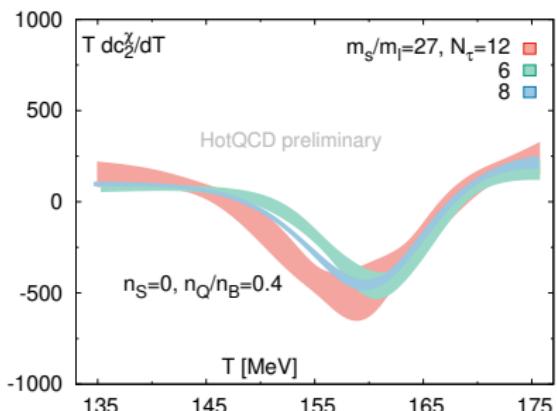
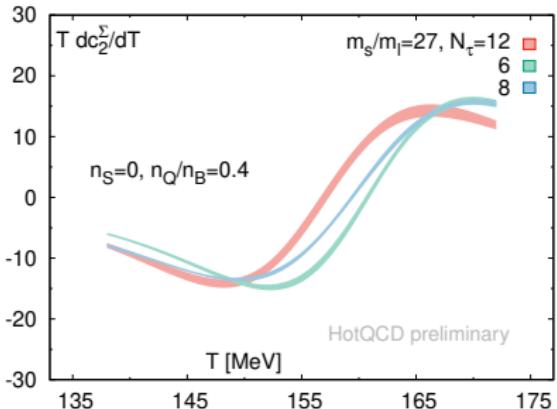
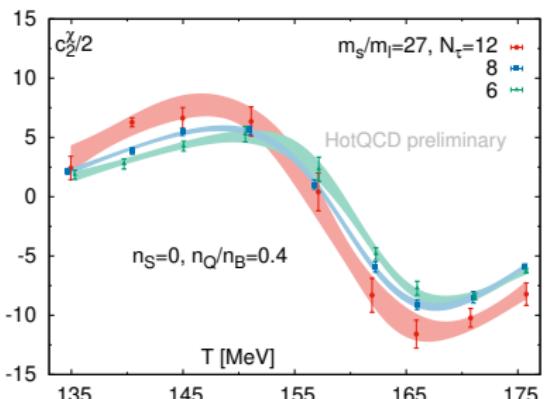
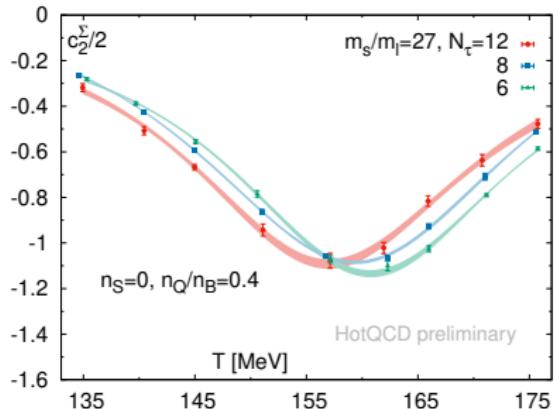
- simplest case $\mu_Q = \mu_S = 0$
- subtracted condensate

$$\frac{\Sigma_{\text{sub}}}{f_K^4} = \sum_{n=0}^{\infty} \frac{c_n^\Sigma}{n!} \hat{\mu}_B^n \quad \text{with} \quad c_n^\Sigma = \left. \frac{\partial \Sigma_{\text{sub}} / f_K^4}{\partial \hat{\mu}_B^n} \right|_{\mu=0}$$

- disconnected susceptibility

$$\frac{\chi_{\text{disc}}}{f_K^4} = \sum_{n=0}^{\infty} \frac{c_n^\chi}{n!} \hat{\mu}_B^n \quad \text{with} \quad c_n^\chi = \left. \frac{\partial \chi_{\text{disc}} / f_K^4}{\partial \hat{\mu}_B^n} \right|_{\mu=0}$$

Coefficients for a strangeness neutral system



The curvature of the crossover line

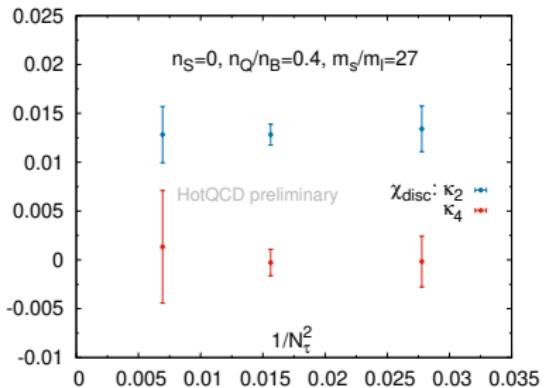
$$\frac{T_c(\mu_B)}{T_0} = 1 - \kappa_2 \left(\frac{\mu_B}{T_0} \right)^2 - \kappa_4 \left(\frac{\mu_B}{T_0} \right)^4 + \mathcal{O}(\mu_B^6)$$

- Taylor expansion in μ and T of:

$$\frac{d}{dT} \frac{\chi_{\text{disc}}(T, \mu_B)}{f_K^4} = (\dots) \mu_B^2 + (\dots) \mu_B^4 + \dots = \mathbf{0}$$

- has to be zero order by order

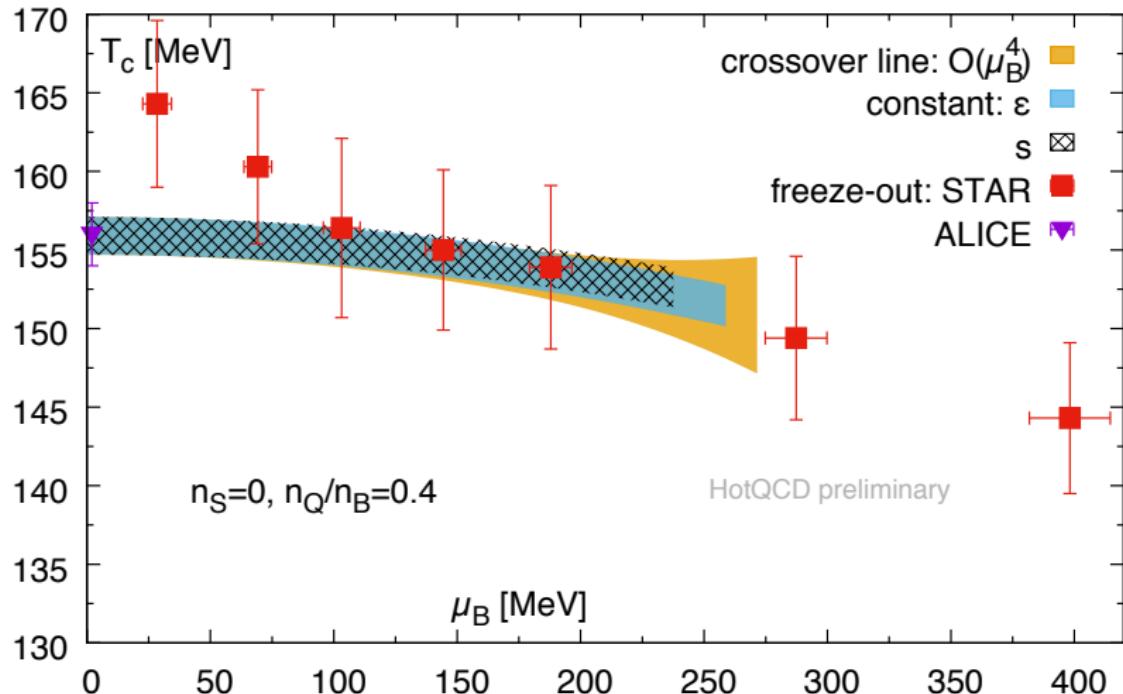
$$\kappa_2 = \frac{1}{2T_0^2} \frac{T_0 \frac{\partial c_2^\chi}{\partial T} \Big|_{(T_0,0)} - 2 c_2^\chi \Big|_{(T_0,0)}}{\frac{\partial^2 c_0^\chi}{\partial T^2} \Big|_{(T_0,0)}}$$



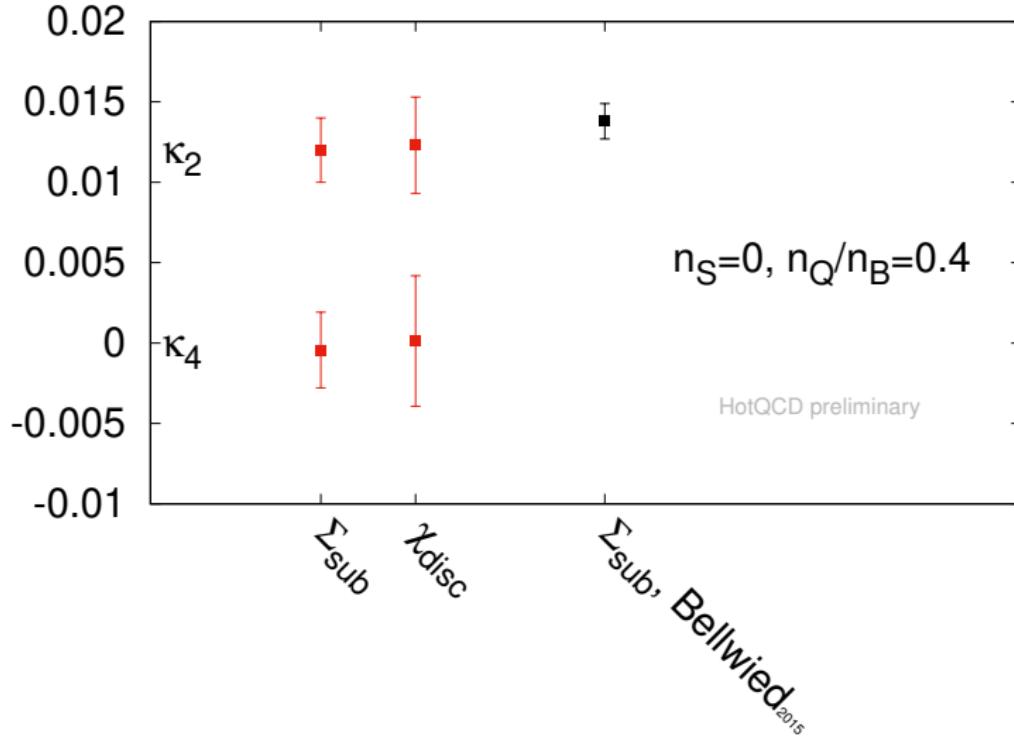
The QCD crossover line

STAR: arxiv:1701.07065

ALICE: arxiv:1408.6403

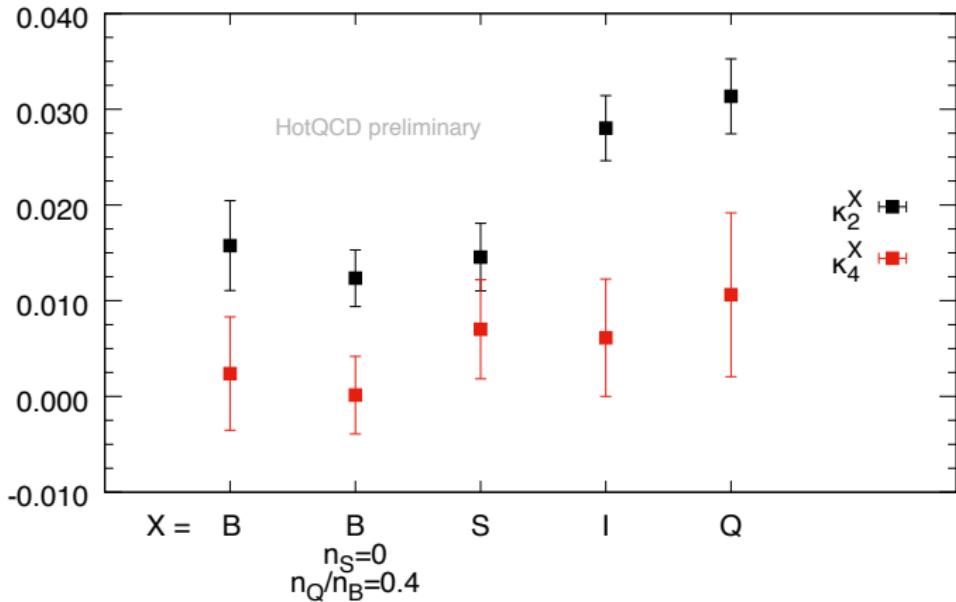


The curvature κ_n for strangeness neutral system



The crossover line

$$\frac{T_c(\mu_X)}{T_0} = 1 - \kappa_2^X \left(\frac{\mu_X}{T_0} \right)^2 - \kappa_4^X \left(\frac{\mu_X}{T_0} \right)^4 + \mathcal{O}(\mu_X^6)$$



Fluctuations along the QCD crossover $T_c(\mu_B)$

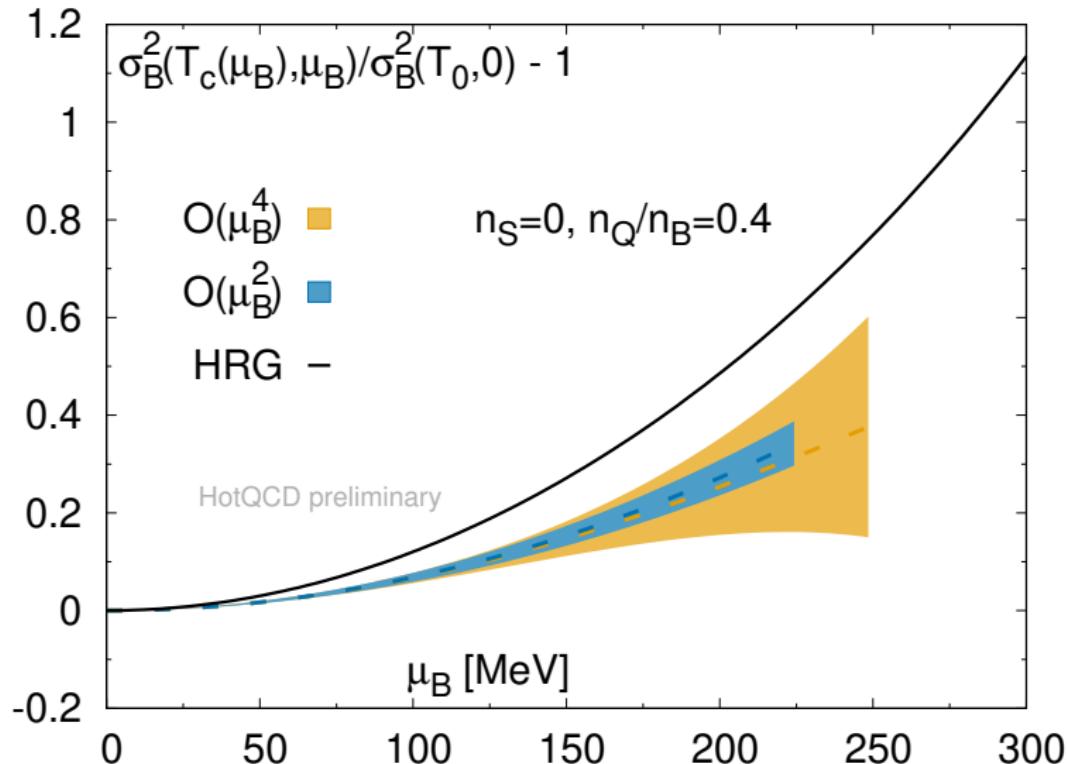
- Baryon-number fluctuations

$$\frac{\sigma_B^2}{Vf_K^3} = \frac{1}{Vf_K^3} \frac{\partial \ln Z}{\partial \hat{\mu}_B^2} = \sum_{n=0}^{\infty} \frac{c_n^B}{n!} \hat{\mu}_B^n \quad \text{with} \quad c_n^B = \left. \frac{1}{Vf_K^3} \frac{\partial \ln Z}{\partial \hat{\mu}_B^{n+2}} \right|_{\mu=0}$$

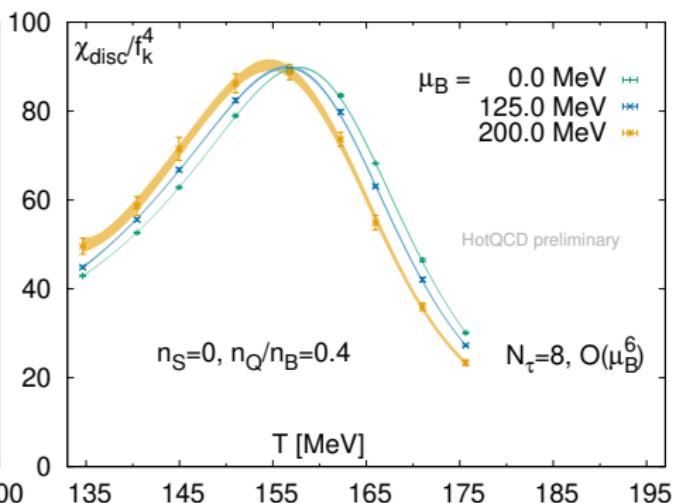
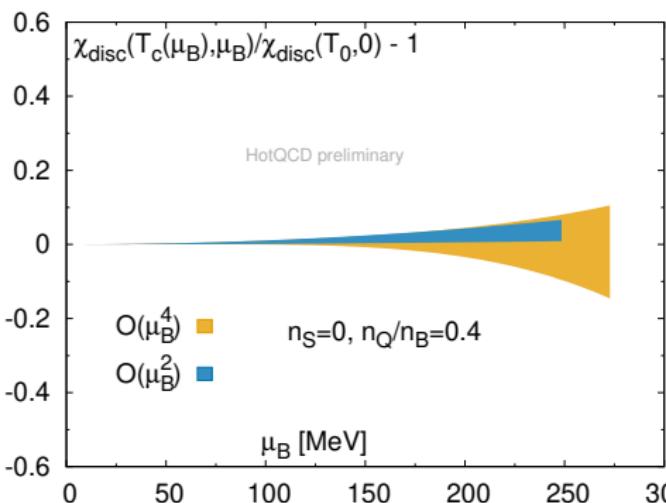
- σ_B^2 couples to condensate \rightarrow diverges at a critical point
- study increase along the crossover line

$$\frac{\sigma_B^2(T_c(\mu_B), \mu_B) - \sigma_B^2(T_0, 0)}{\sigma_B^2(T_0, 0)} = \lambda_2 \left(\frac{\mu_B}{T_0} \right)^2 + \lambda_4 \left(\frac{\mu_B}{T_0} \right)^4 + \dots$$

Baryon-number fluctuations \leadsto along $T_c(\mu_B)$



Susceptibility fluctuations ↵ along $T_c(\mu_B)$



σ_B^2 and χ_{disc} show no indication for a narrowing crossover

Critical point from Taylor expansions

- e.g. expansion of the pressure around $\mu_B = 0$ (for $\mu_Q \equiv \mu_S \equiv 0$)

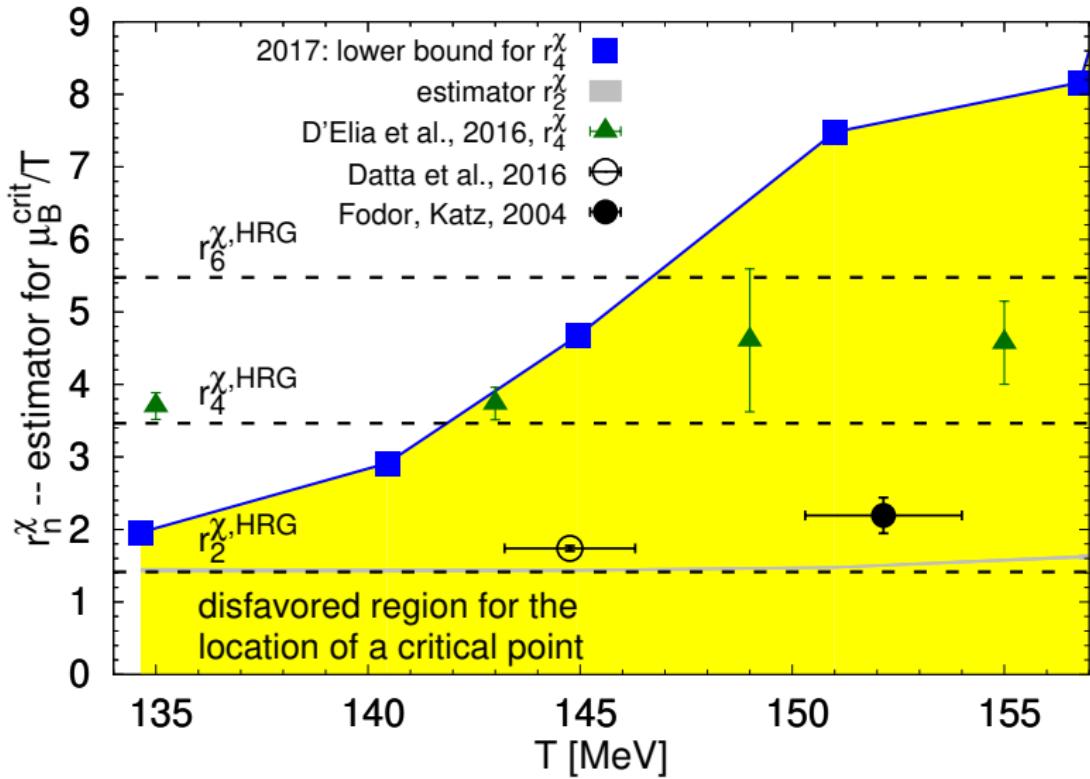
$$\frac{P}{T^4} = \sum_n \frac{1}{n!} \chi_n^B \hat{\mu}_B^n, \quad \chi_n^B = \frac{1}{VT^3} \left. \frac{\partial^n \ln Z}{\partial \hat{\mu}_B^n} \right|_{\mu_B=0}$$

- analysis of convergence radius can determine bound on the location of a critical point:

$$r_{2n}^P = \left| \frac{(2n+2)(2n+1)\chi_{2n}^B}{\chi_{2n+2}^B} \right|^{1/2}, \quad r_{2n}^\chi = \left| \frac{2n(2n-1)\chi_{2n}^B}{\chi_{2n+2}^B} \right|^{1/2}$$

- only if coefficients are **positive** for all $n \geq n_0$
 - if not \rightarrow no critical point on real axis

Critical point from Taylor expansions



Summary

- crossover starts at $T_0 = 156.5 \pm 1.5$ MeV
- crossover curvature for strangeness neutral system
 - $\frac{T_c(\mu_B)}{T_0} = 1 - \kappa_2 \left(\frac{\mu_B}{T_0} \right)^2 - \kappa_4 \left(\frac{\mu_B}{T_0} \right)^4 + \mathcal{O}(\mu_B^6)$
 - $\kappa_2 = 0.0123 \pm 0.003$
 - $\kappa_4 = 0.000131 \pm 0.0041$
- for $\mu_B < 250$ MeV and $n_s = 0$, $n_Q/n_B = 0.4$
 - crossover along const. entropy density and energy density
 - chemical freeze-out might be close to crossover
 - no indication for critical point

Thank you for your attention!