HVP contribution of the light quarks to (g_μ-2) including QED corrections with twisted-mass fermions

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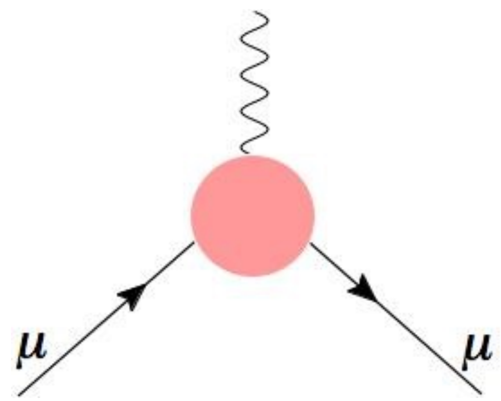
OUTLINE

- Isospin breaking effects on the lattice (RM123 method)
- Results for the light quark contribution to a_μ^{HVP}

In collaboration with:

V. Lubicz, G. Martinelli, S. Romiti, F. Sanfilippo, S. Simula, C. Tarantino

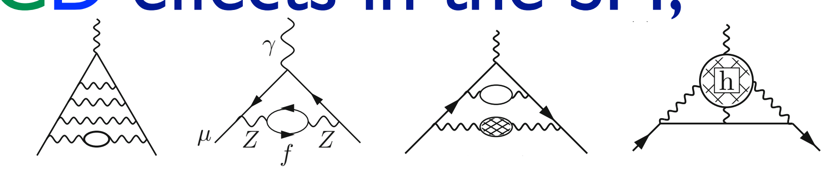
Muon magnetic anomaly



$$= (-i\,e)\,\bar{u}(p')\left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(q^2)\right]u(p)$$

<u>muon anomalous magnetic moment:</u> $a_\mu \equiv \frac{g_\mu - 2}{2} = F_2(0)$

- is generated by quantum loops;
- receives contribution from QED, EW and QCD effects in the SM;
- is a sensitive probe of new physics



PDG 2018

$$a_\mu^{\mathrm{SM}} = 116\ 591\ 823(1)(34)(26)\cdot 10^{-11}$$

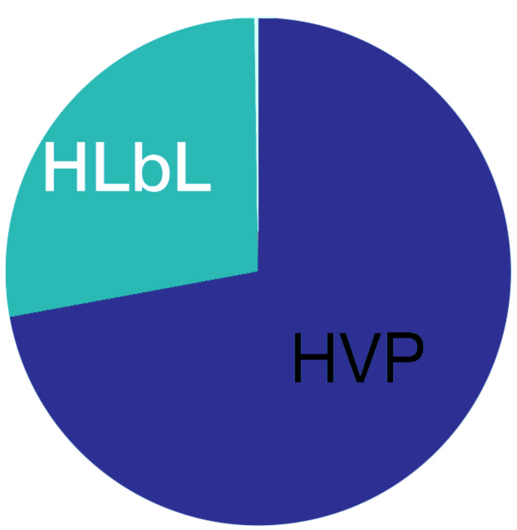
0.4ppm

QED+EW

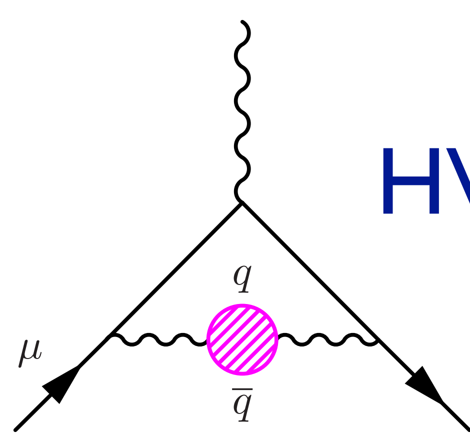
LO Had.

NLO/NNLO Had.

dispersion relations $e^+e^- \rightarrow$ *hadrons*



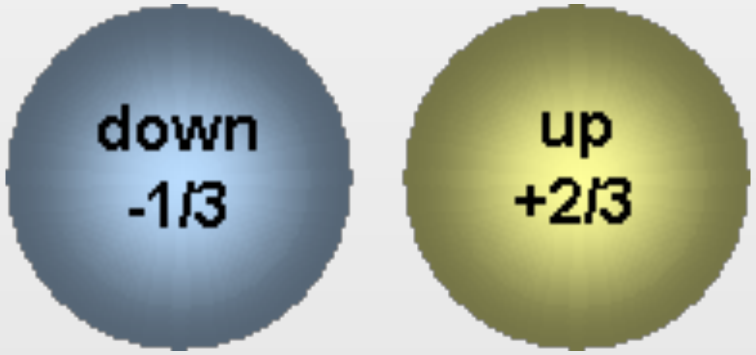
error budget



HVP → *ab-initio* **LQCD**

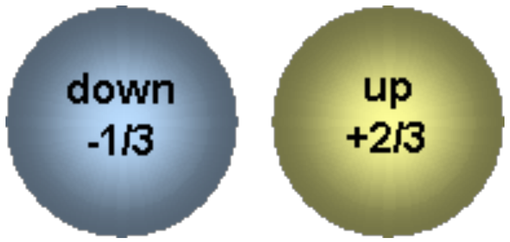
Phenomenological motivations

The determination of some hadronic observables in flavor physics has reached such an accurate degree of experimental and theoretical precision that electromagnetic and strong isospin breaking effects cannot be neglected anymore



ISOSPIN BREAKING EFFECTS

Isospin symmetry is an almost exact property of the strong interactions



Isospin breaking effects are induced by:

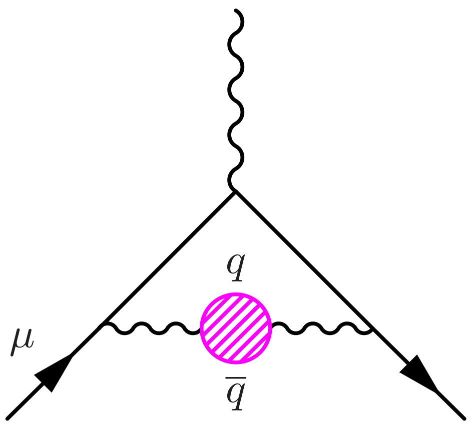
$m_u \neq m_d$: $O[(m_d - m_u)/\Lambda_{QCD}] \approx 1/100$ "Strong"

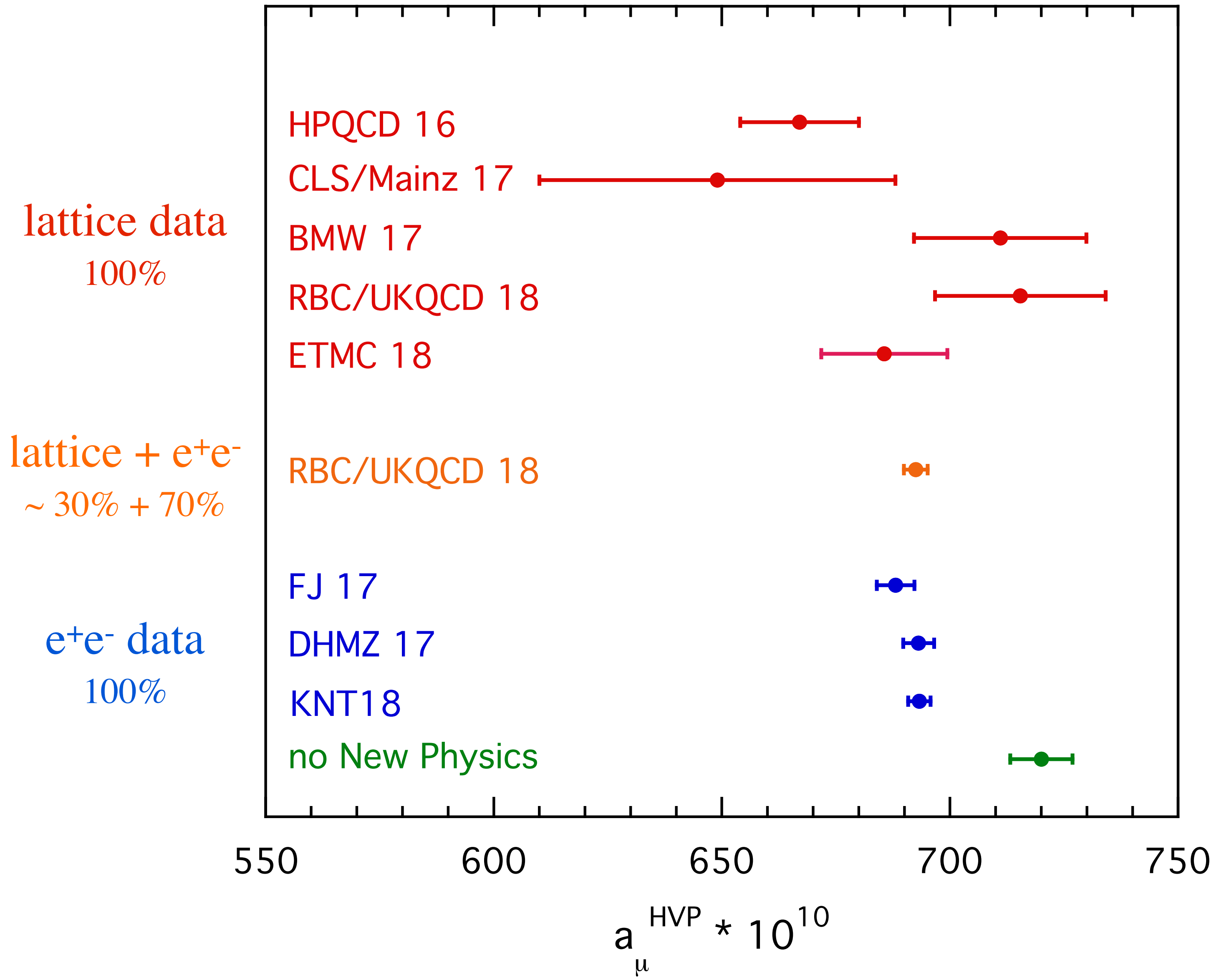
$Q_u \neq Q_d$: $O(\alpha_{e.m.}) \approx 1/100$ "Electromagnetic"

Since electromagnetic interactions renormalise quark masses the two corrections are intrinsically related

Though small, IB effects can play a very important role (quark masses, $M_n - M_p$, leptonic decay constants, vector form factor)

Hadronic Vacuum Polarisation

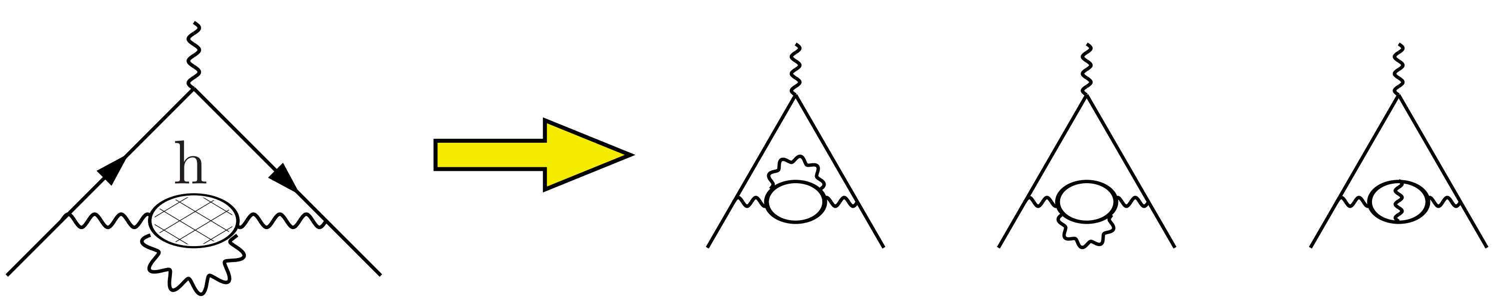




$\gtrsim 2\%$

$\gtrsim 0.4\%$

Given the present exper. and theor. (LQCD) accuracy, an important source of uncertainty are long distance electromagnetic and SU(2) breaking corrections



estimate in sQED

$$\delta a_\mu^{HVP} \sim 39(1)\cdot 10^{-11}$$

K. Melnikov, 2001

Isospin breaking effects on the lattice

RM123 method

A strategy for Lattice QCD:

The isospin breaking part of the Lagrangian is treated as a perturbation

Expand in: $m_d - m_u$ + α_{em}

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Isospin breaking effects due to the up-down mass difference in lattice QCD

RM123 collaboration

G.M. de Divitiis,[a,b] P. Dimopoulos,[c,d] R. Frezzotti,[a,b] V. Lubicz,[e,f] G. Martinelli,[g,d] R. Petronzio,[a,b] G.C. Rossi,[a,b] F. Sanfilippo,[c,d] S. Simula,[f] N. Tantalo[a,b] and C. Tarantino[e,f]

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Leading isospin breaking effects on the lattice

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(RM123 Collaboratio

arXiv:1303.4896

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RM123 Collaboration

The $(m_d - m_u)$ expansion

- Identify the isospin breaking term in the QCD action

$$S_m = \sum_x \left[m_u \bar{u}u + m_d \bar{d}d \right] = \sum_x \left[\frac{1}{2}(m_u + m_d)(\bar{u}u + \bar{d}d) - \frac{1}{2}(m_d - m_u)(\bar{u}u - \bar{d}d) \right] =$$

$$= \sum_x \left[m_{ud} (\bar{u}u + \bar{d}d) - \Delta m (\bar{u}u - \bar{d}d) \right] = S_0 - \Delta m \hat{S} \longleftarrow \boxed{\hat{S} = \Sigma_x(\bar{u}u - \bar{d}d)}$$

- Expand the functional integral in powers of Δm

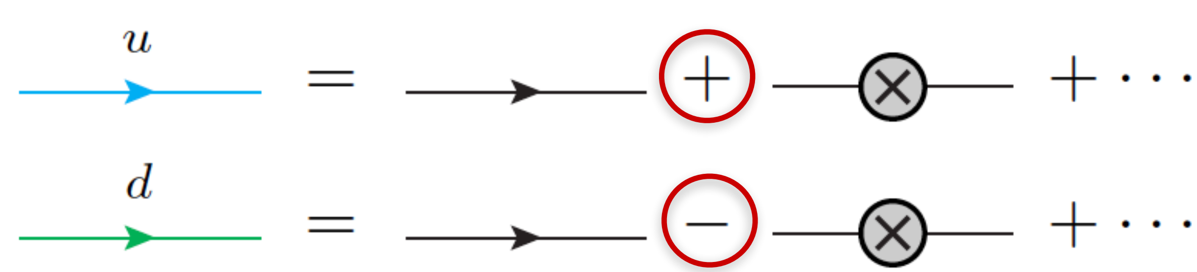
Advantage: factorised out

$$\langle O \rangle = \frac{\int D\phi\, O e^{-S_0 + \Delta m \hat{S}}}{\int D\phi\, e^{-S_0 + \Delta m \hat{S}}} \overset{1st}{\simeq} \frac{\int D\phi\, O e^{-S_0} (1 + \Delta m \hat{S})}{\int D\phi\, e^{-S_0} (1 + \Delta m \hat{S})} \simeq \frac{\langle O \rangle_0 + \Delta m \langle O \hat{S} \rangle_0}{1 + \cancel{\Delta m \langle \hat{S} \rangle_0}} = \langle O \rangle_0 + \Delta m \langle O \hat{S} \rangle_0$$

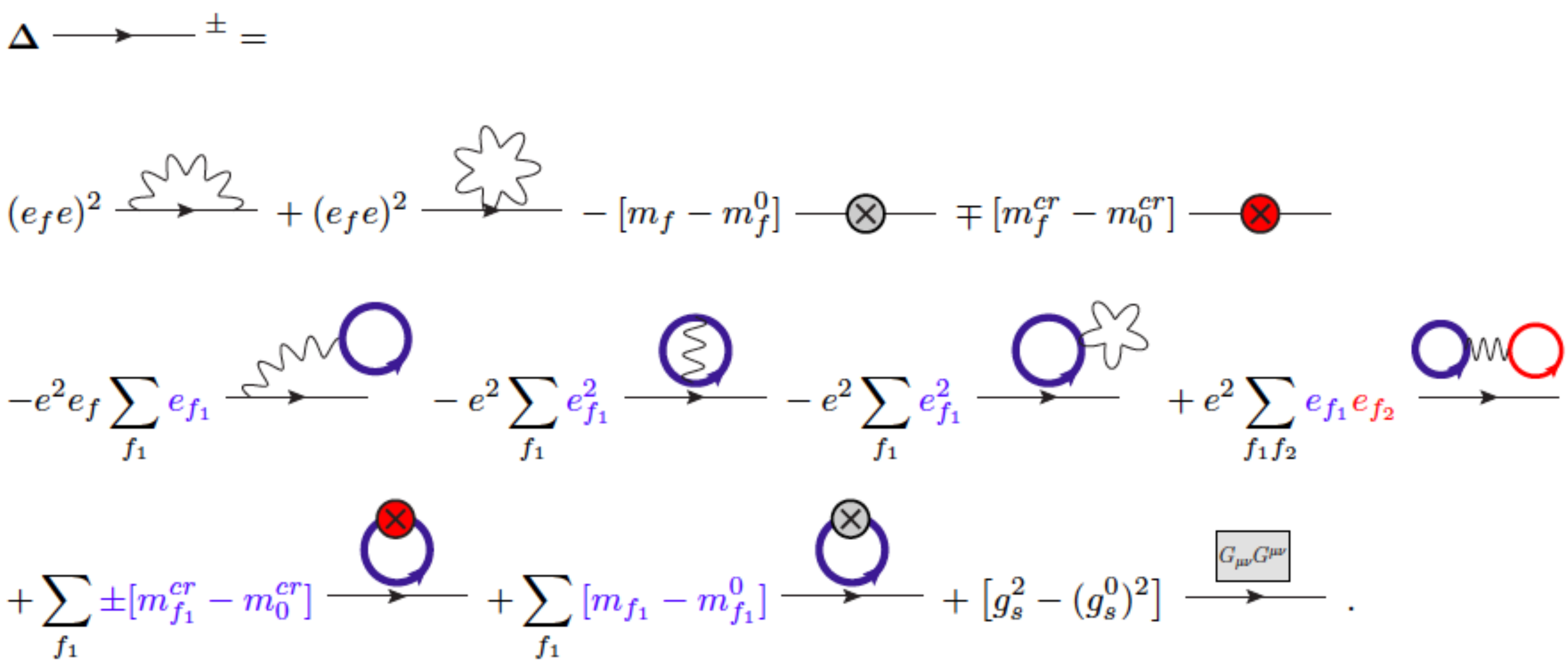
for isospin symmetry

- At leading order in Δm the corrections only appear in the valence quark propagators:

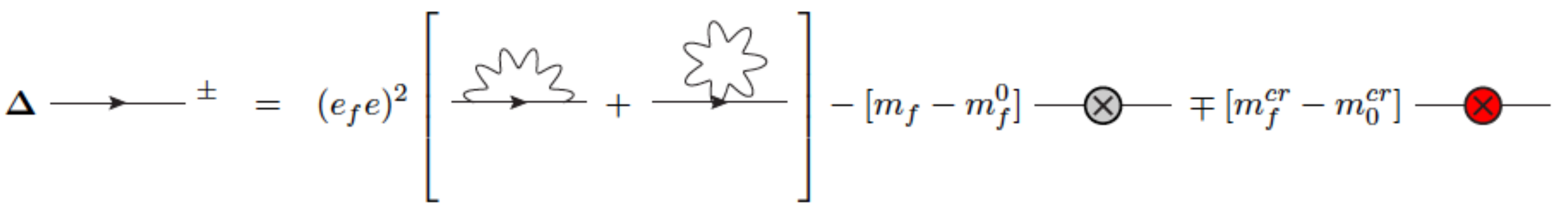
(disconnected contractions of $\bar{u}u$ and $\bar{d}d$ vanish due to isospin symmetry)



The QED expansion for the quark propagator

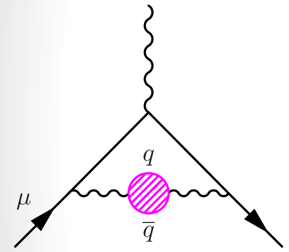


In the electro-quenched (*qQED*) approximation:



Results for the light quark contribution to a_μ^{HVP}

European Twisted Mass Collaboration

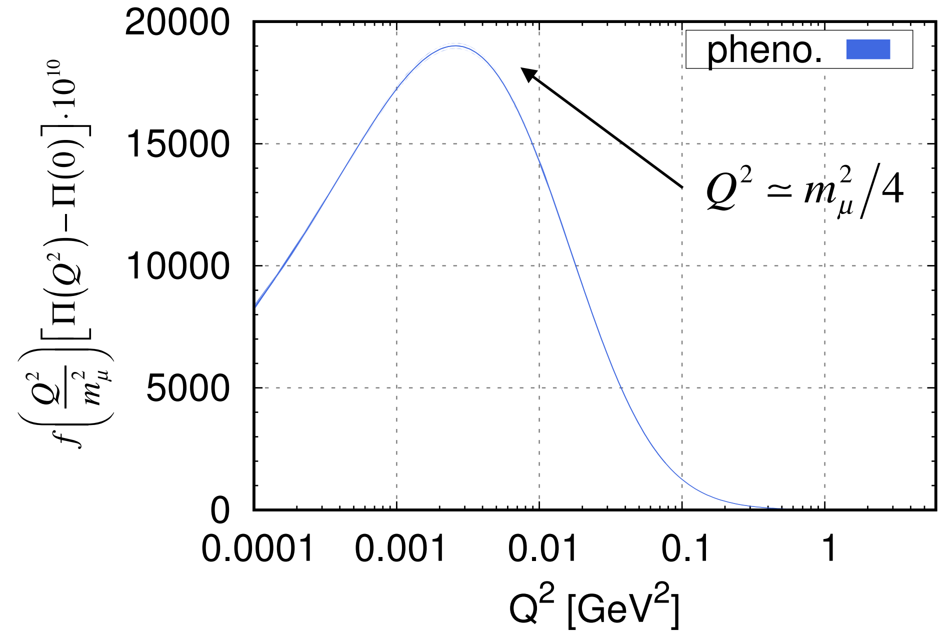


HVP from LQCD

$$\Pi_{\mu\nu}(Q) = \int d^4x\, e^{iQ\cdot x} \left\langle J_\mu(x) J_\nu(0) \right\rangle = \left[\delta_{\mu\nu} Q^2 - Q_\mu Q_\nu\right] \Pi\left(Q^2\right)$$

$$a_\mu^{HVP} = 4\alpha_{em}^2 \int_0^\infty dQ^2 \, \frac{1}{m_\mu^2} f\left(\frac{Q^2}{m_\mu^2}\right) \left[\Pi\left(Q^2\right) - \Pi(0)\right]$$

B. E. Lautrup and E. de Rafael, 1969; T. Blum, 2002



F. Jegerlehner, "alphaQEDc17"

Time-Momentum Representation

$$a_\mu^{HVP} = 4\alpha_{em}^2 \int_0^\infty dt \; \tilde{f}(t)\, V(t)$$

D. Bernecker and H. B. Meyer, 2011

$$V(t) \equiv \frac{1}{3} \sum_{i=1,2,3} \int d\vec{x} \left\langle J_i(\vec{x},t) J_i(0) \right\rangle$$

$$\boxed{a_\mu^{HVP} = \sum_{f=u,d,s,c} 4\alpha_{em}^2 \left\{ \sum_{t=0}^{T_{data}} \tilde{f}(t) V^f(t) + \sum_{t=T_{data}+a}^{\infty} \tilde{f}(t) \frac{G_V^f}{2M_V^f} e^{-M_V^f t} \right\}}$$

$t \leq T_{data} < T/2$ (avoid bw signals)

$t > T_{data} > t_{min}$ (ground-state dom.)

quark-connected terms only

lattice data
local vector currents

analytic representation
up to 10% for light quarks

Details of the lattice simulation

We have used the gauge field configurations generated by ETMC, European Twisted Mass Collaboration, in the pure isosymmetric QCD theory with Nf=2+1+1 dynamical quarks

ensemble	β	V/a^4	$a\mu_{ud}$	$a\mu_\sigma$	$a\mu_\delta$	N_{cf}	$a\mu_s$	M_π (MeV)	M_K (MeV)
$A40.40$	1.90	$40^3 \cdot 80$	0.0040	0.15	0.19	100	0.02363	317(12)	576(22)
$A30.32$		$32^3 \cdot 64$	0.0030			150		275(10)	568(22)
$A40.32$			0.0040			100		316(12)	578(22)
$A50.32$			0.0050			150		350(13)	586(22)
$A40.24$		$24^3 \cdot 48$	0.0040			150		322(13)	582(23)
$A60.24$			0.0060			150		386(15)	599(23)
$A80.24$			0.0080			150		442(17)	618(14)
$A100.24$			0.0100			150		495(19)	639(24)
$A40.20$		$20^3 \cdot 48$	0.0040			150		330(13)	586(23)
$B25.32$	1.95	$32^3 \cdot 64$	0.0025	0.135	0.170	150	0.02094	259 (9)	546(19)
$B35.32$			0.0035			150		302(10)	555(19)
$B55.32$			0.0055			150		375(13)	578(20)
$B75.32$			0.0075			80		436(15)	599(21)
$B85.24$		$24^3 \cdot 48$	0.0085			150		468(16)	613(21)
$D15.48$	2.10	$48^3 \cdot 96$	0.0015	0.1200	0.1385	100	0.01612	223 (6)	529(14)
$D20.48$			0.0020			100		256 (7)	535(14)
$D30.48$			0.0030			100		312 (8)	550(14)

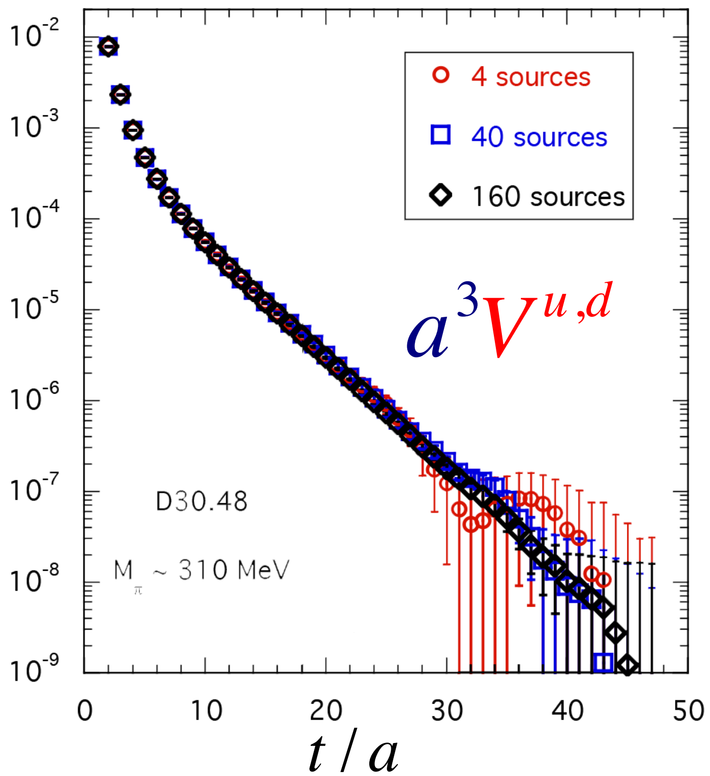
- Gluon action: Iwasaki
- Quark action: twisted mass at maximal twist (automatically O(a) improved)
  OS for *s* and *c* valence quarks

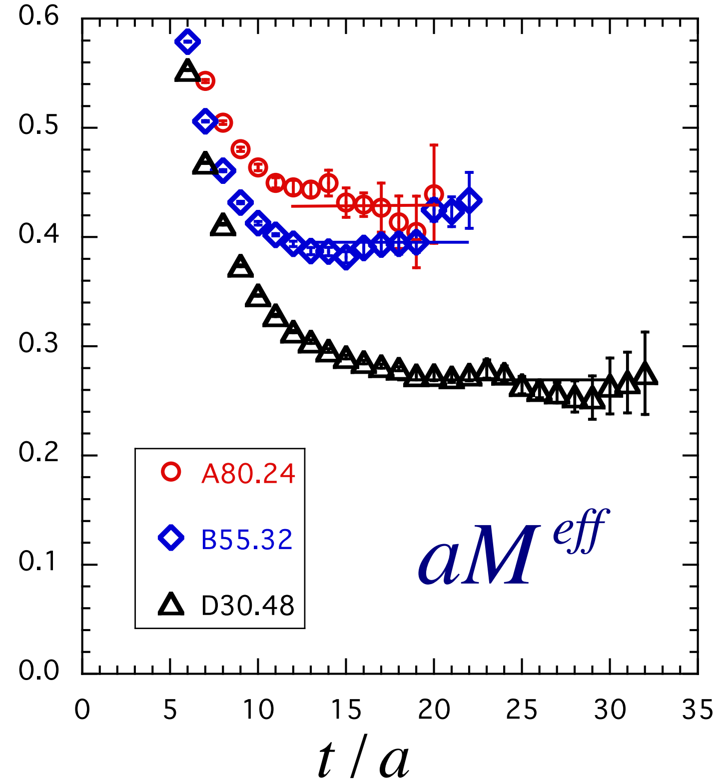
Pion masses in the range 220 - 490 MeV

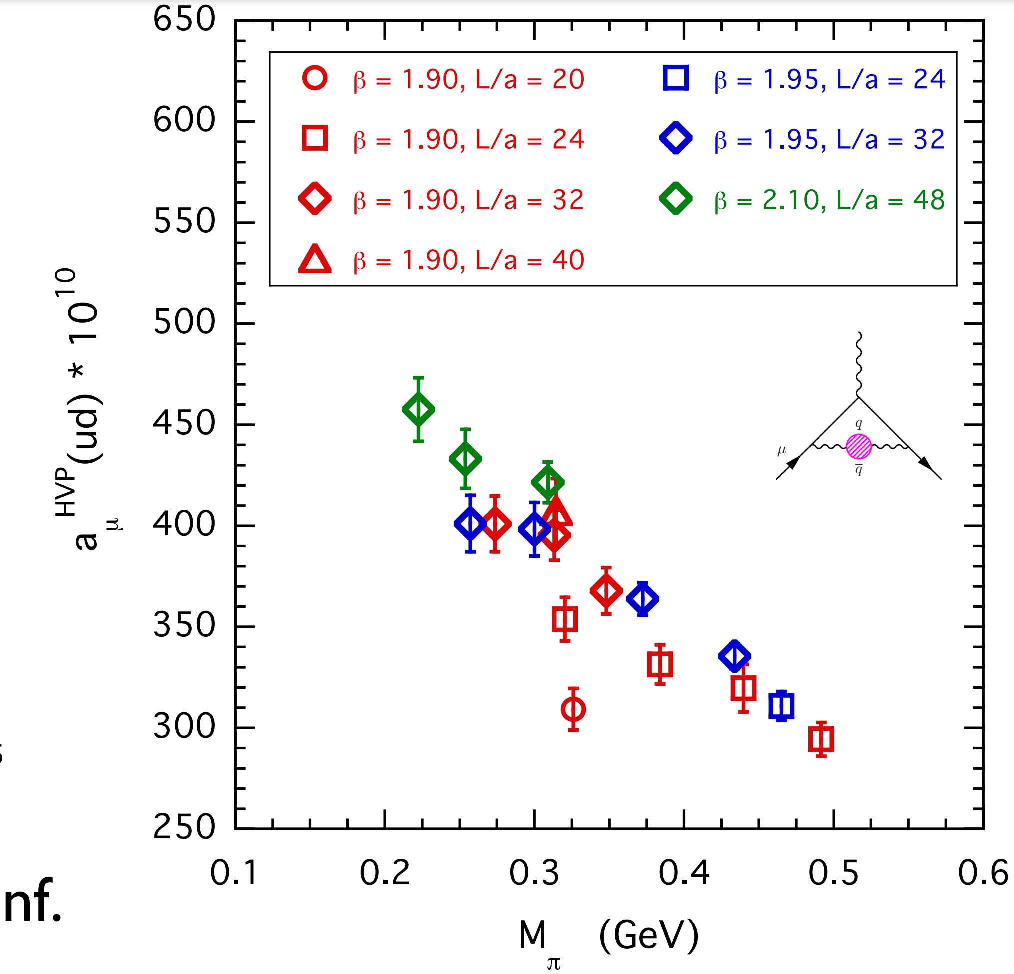
4 volumes @ $M_\pi \simeq 320$ MeV and $a \simeq 0.09$ fm

$M_\pi L \simeq 3.0 \div 5.8$

Light quark contribution







StN: $\propto e^{-(M_\rho - M_\pi)t}$

G. Parisi, 1984;
G. P. Lepage, 1989

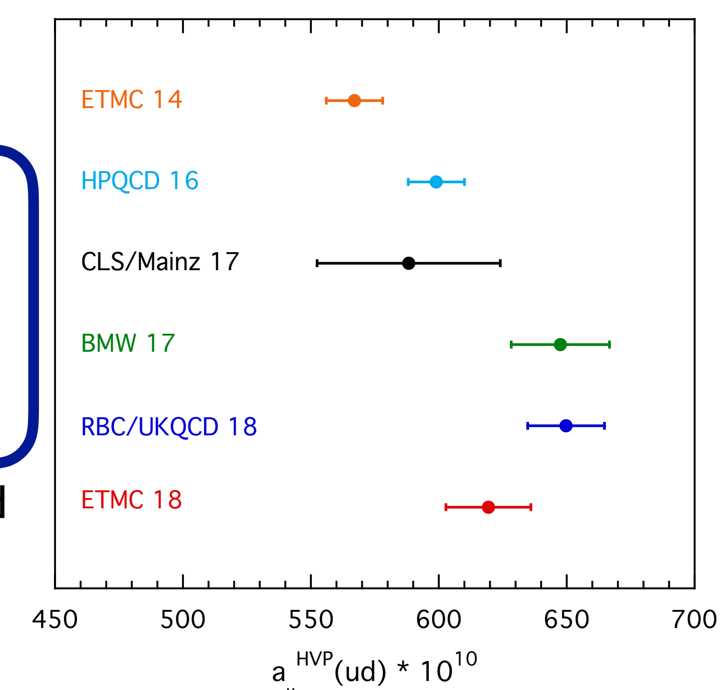
160 stoch. sources / gauge conf.

preliminary

$$a_\mu^{HVP}(ud) = 619.4(12.7)_{stat+fit}(6.8)_{chir}(6.2)_{FVE}(5.4)_{disc} \cdot 10^{-10}$$

$$= 619.4(16.6) \cdot 10^{-10}$$

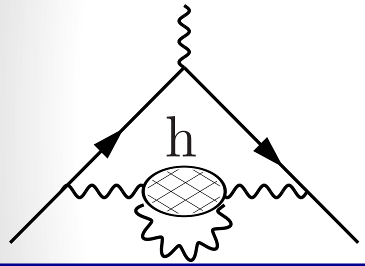
talk by S. Simula
@ (gμ-2) plenary workshop, Mainz 2018



quark-connected terms only

$a_\mu^{HVP}(s) = 53.1(2.5) \cdot 10^{-10}$ $\quad a_\mu^{HVP}(c) = 14.75(56) \cdot 10^{-10}$

DG *et al.*, 2017



LIB corrections

quark-connected terms only

$$\boxed{\delta a_\mu^{HVP} = \delta a_\mu^{HVP}(QCD) + \delta a_\mu^{HVP}(QED)}$$

photon zero-mode: QED_L

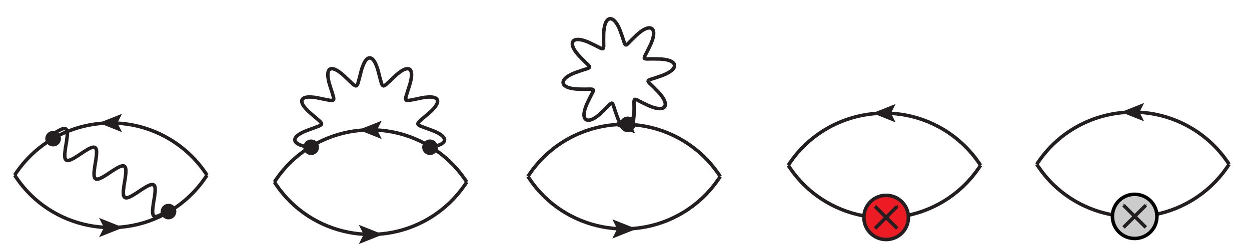
M. Hayakawa and S. Uno, 2008

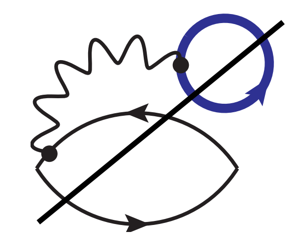
$$\delta a_\mu^{HVP}(QCD) = 4\alpha_{em}^2 \sum_{t=0}^{\infty} \tilde{f}(t)\,\delta V^{QCD}(t)$$

$$\delta a_\mu^{HVP}(QED) = 4\alpha_{em}^2 \sum_{t=0}^{\infty} \tilde{f}(t) \sum_{f=u,d,s,c} \delta V_f^{QED}(t)$$

<u>RM123 method</u>

G. M. de Divitiis *et al.*, 2012; 2013

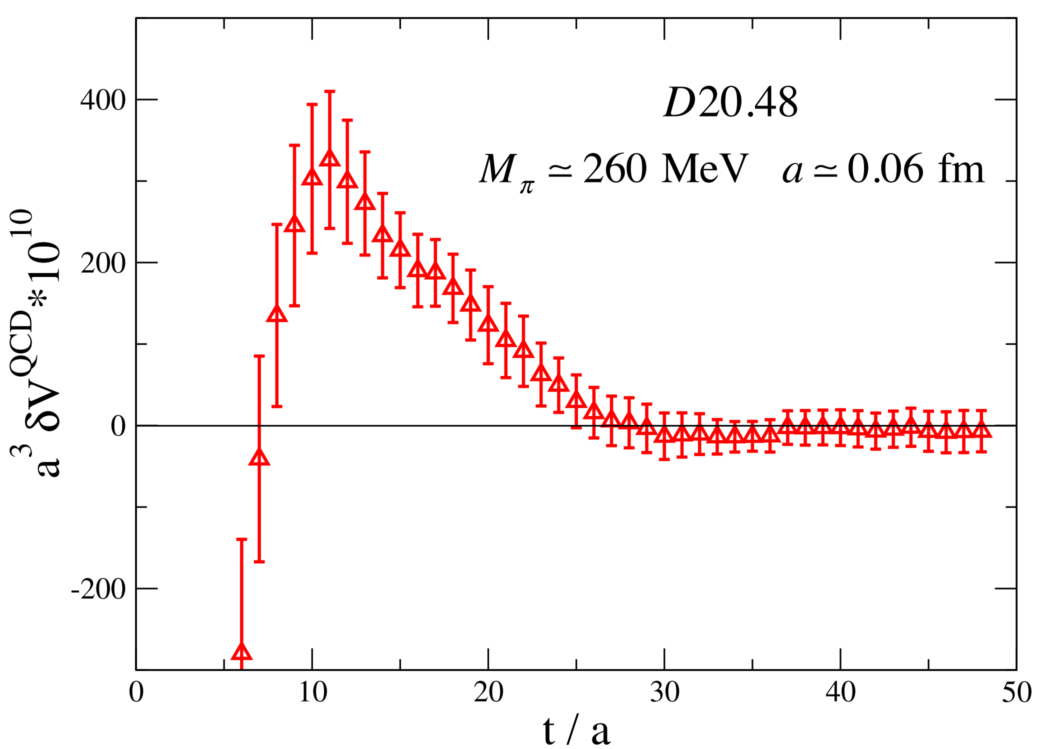




qQED approximation

QCD/QED separation is scheme and scale dependent $\longrightarrow$ $m_f(\overline{MS}, 2\ \text{GeV}) = m_f^0(\overline{MS}, 2\ \text{GeV})$

J. Gasser *et al.*, 2003

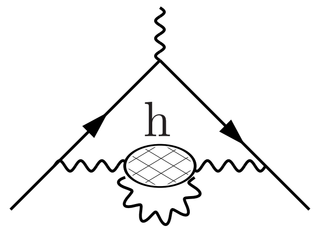


$$\delta a_\mu^{HVP}(QCD) = 4\alpha_{em}^2 \sum_{t=0}^{\infty} \tilde{f}(t)\,\delta V^{QCD}(t)$$

$$(m_d - m_u)\frac{Z_P}{3} \sum_{i=1,2,3} \sum_{\vec{x},y} \langle 0 | T\left\{ J_i^\dagger(\vec{x},t)\left[q_d^2 \bar{\psi}_d \psi_d - q_u^2 \bar{\psi}_u \psi_u \right] J_i(0) \right\} | 0 \rangle$$

$$[m_d - m_u](\overline{MS}, 2\ \text{GeV}) = 2.38(18)\ \text{MeV}$$

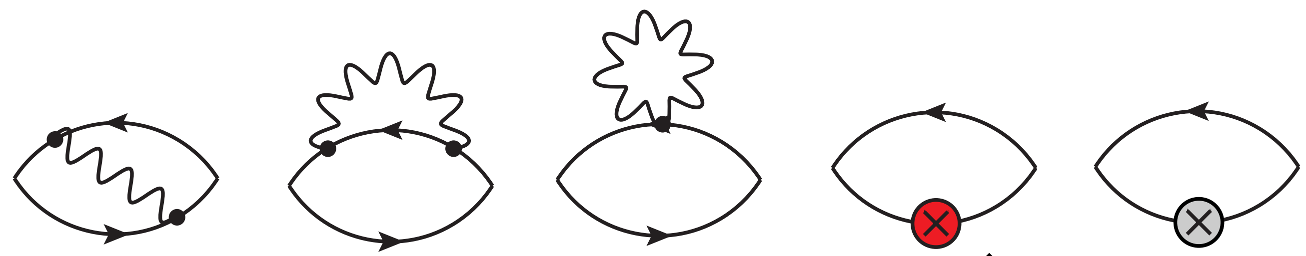
DG *et al.*, 2017



LIB corrections

$$\delta a_\mu^{HVP}(QED) = 4\alpha_{em}^2 \sum_{t=0}^{\infty} \tilde{f}(t) \sum_{f=u,d,s,c} \delta V_f^{QED}(t)$$

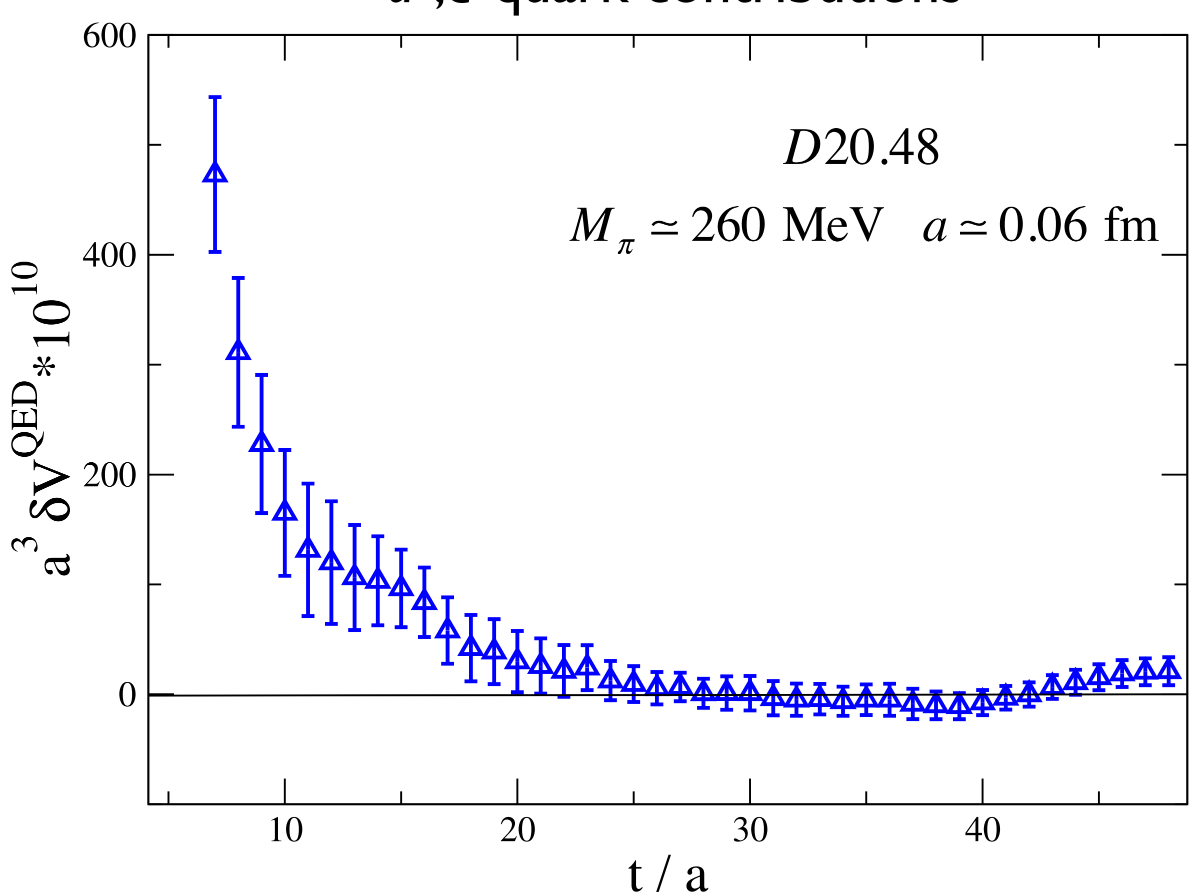
$$\delta V^{exch}(t) + \delta V^{self}(t) + \delta V^{tad}(t) + \delta V^{PS}(t) + \delta V^{S}(t) + \delta V^{Z_A}(t)$$



UV and IR finite

e.m. shift of the critical mass
G. M. de Divitiis *et al.*, 2013

u-,*d*-quark contributions



Mtm-LQCD setup + local $J_i(x)$ with opposite Wilson *r*-parameters

$$Z_A = Z_A^{(0)}\left(1 - 2.51406\ \alpha_{em} q_f^2\ Z_A^{fact}\right) + O\left(\alpha_{em}^2\right)$$

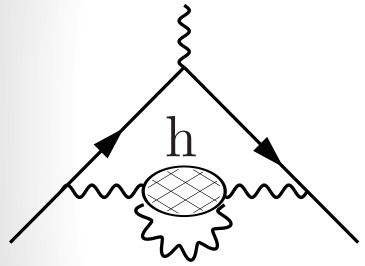
perturbative estimate at LO
G. Martinelli and Y.-C. Zhang, 1982

$$\delta V^{Z_A}(t) = -2.51406\ \alpha_{em} q_f^2\ Z_A^{fact}\ V(t)$$

$$Z_A^{fact} = 0.95(5)$$

preliminary

RI-MOM $O\left(\alpha_{em}\alpha_s^n\right)$



LIB corrections

$$\delta a_\mu^{HVP} = 4\alpha_{em}^2 \left\{ \sum_{t=0}^{T_{data}} \tilde{f}(t)\,\delta V(t) + \sum_{t=T_{data}+a}^{\infty} \tilde{f}(t)\,\delta\left[\frac{G_V^f}{2M_V^f} e^{-M_V^f t}\right] \right\}$$

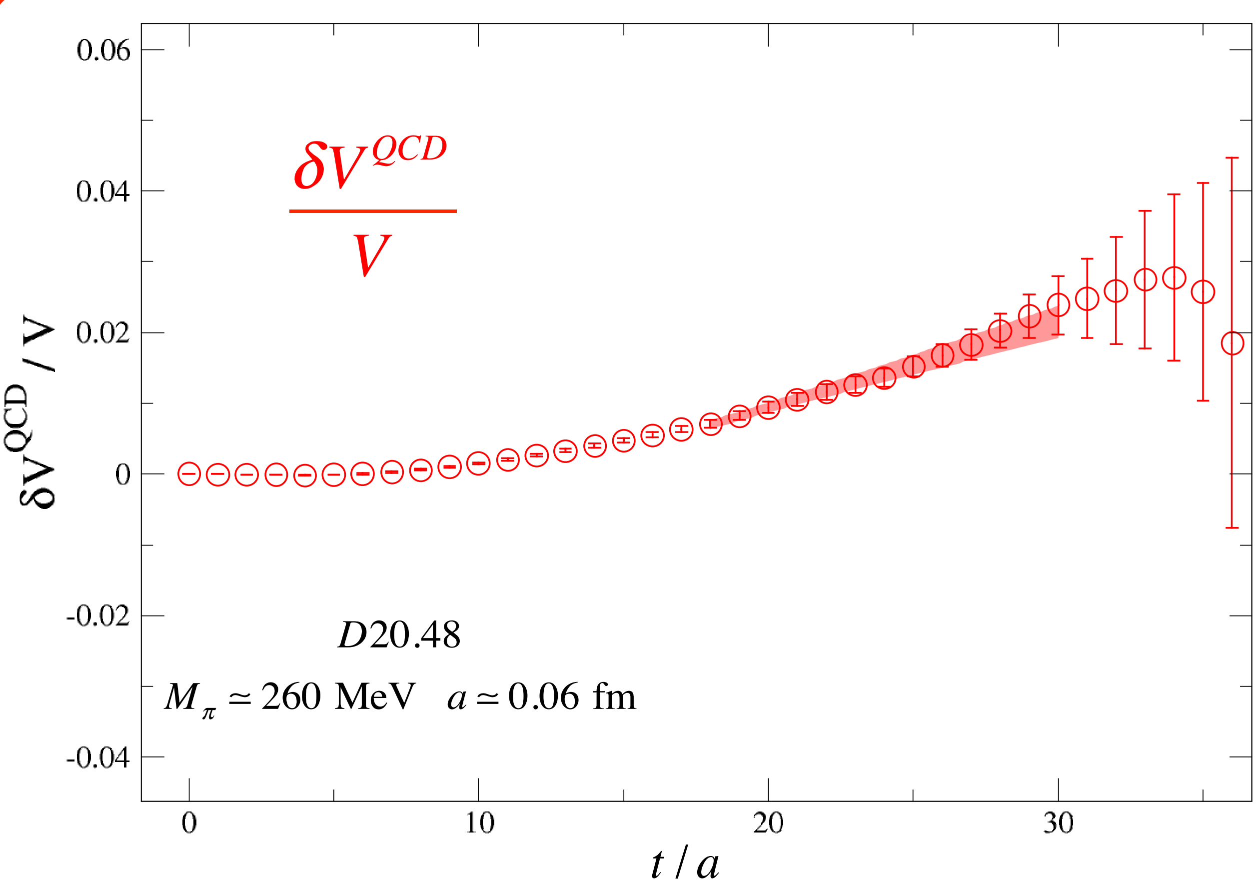
lattice data

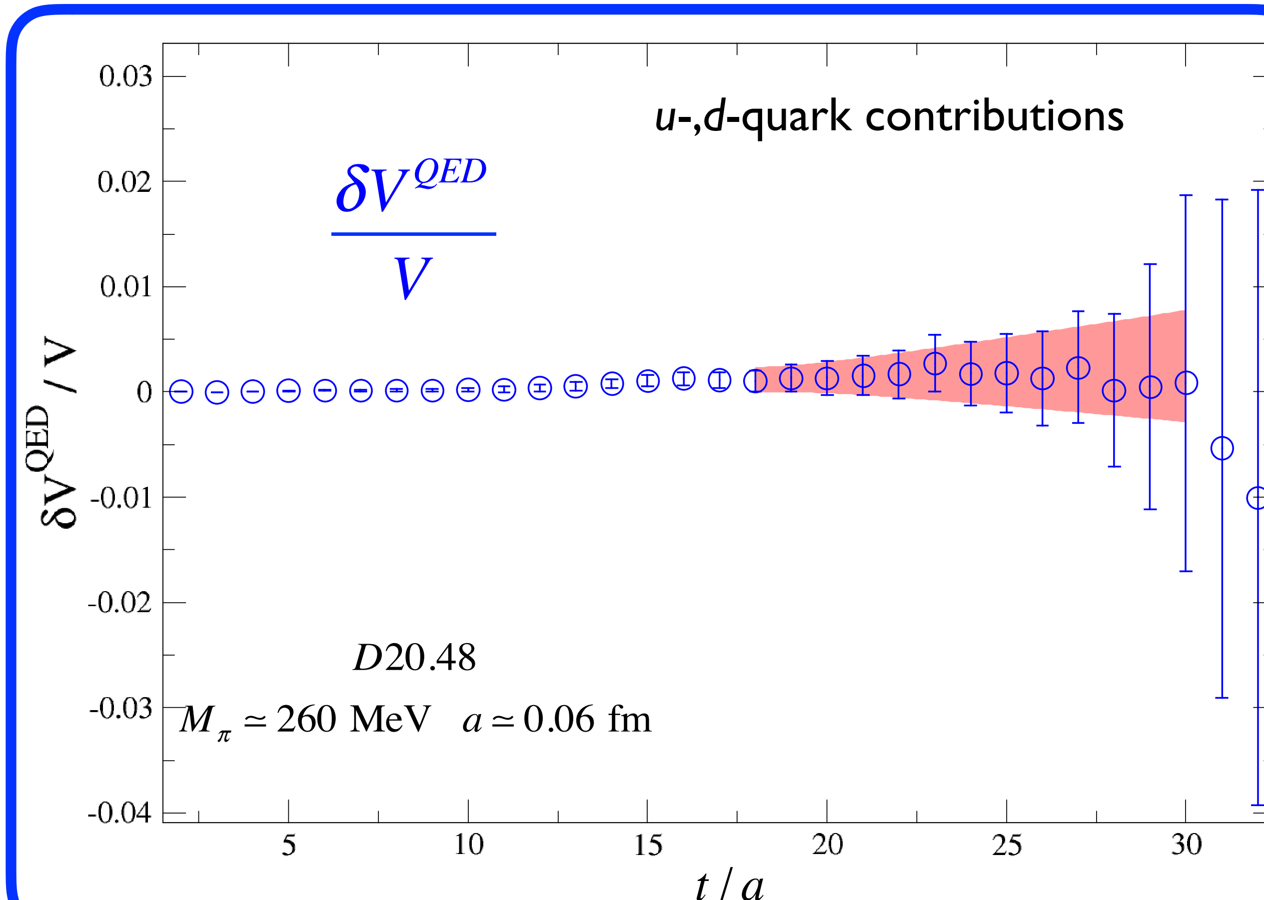
analytic repr.

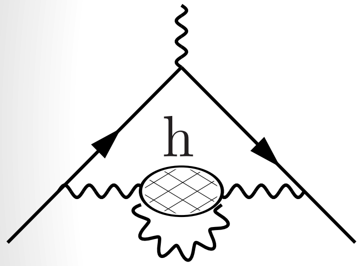
<u>RM123 method</u>

G. M. de Divitiis *et al.*, 2012; 2013

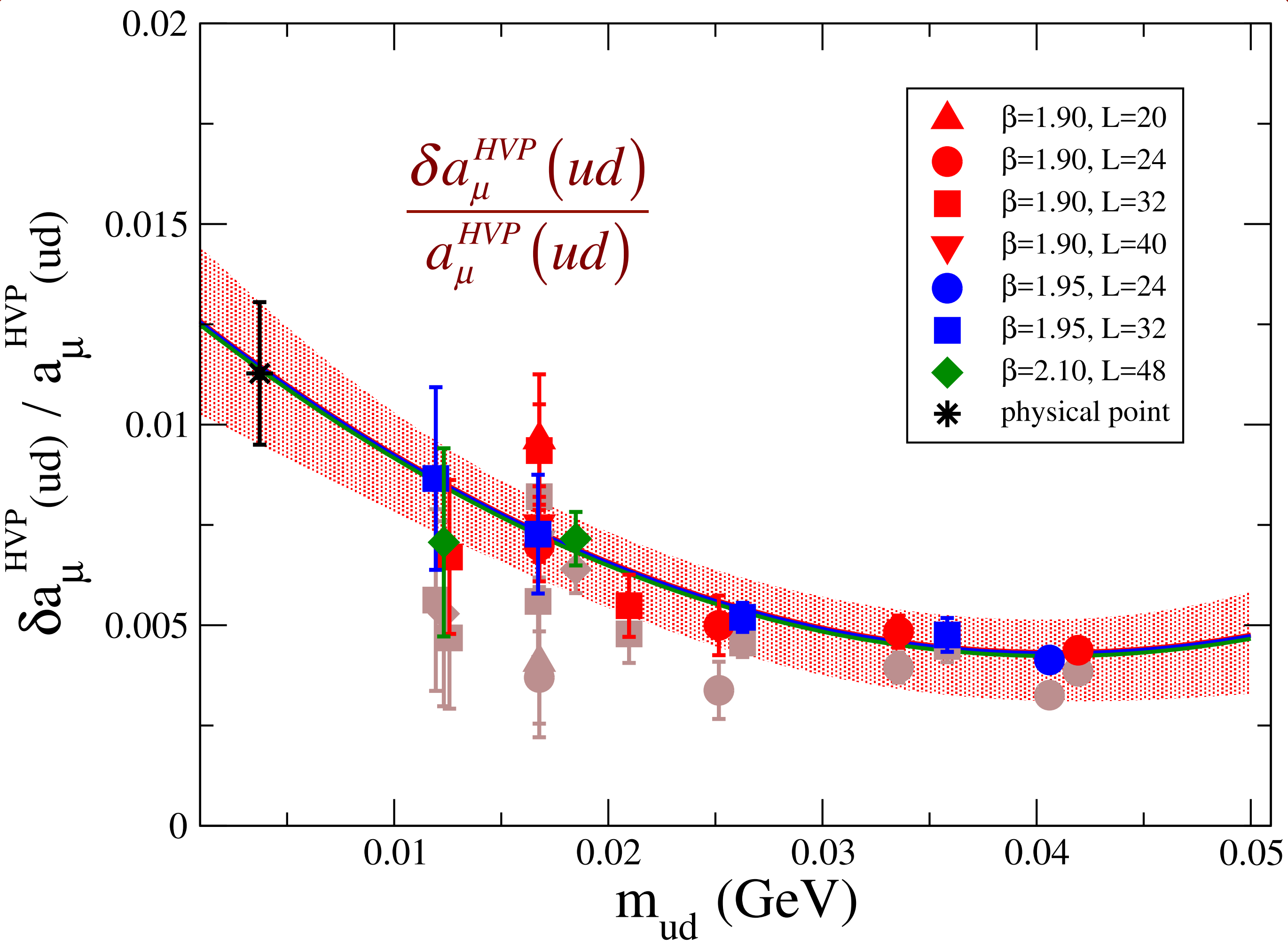
$$\frac{\delta V(t)}{V(t)} \xrightarrow[t \gg a]{} \frac{\delta G_V}{G_V} - \frac{\delta M_V}{M_V}\left(1 + M_V t\right)$$







LIB corr.: results



phenomenological fitting function

$$\frac{\delta a_\mu^{HVP}(ud)}{a_\mu^{HVP}(ud)} = \delta_0\left[1+\delta_1 m_{ud} + \delta_2\chi + Da^2 + FVE\right]$$

systematics

$$\chi = m_{ud}^2$$

$$\chi = m_{ud}\log(m_{ud})$$

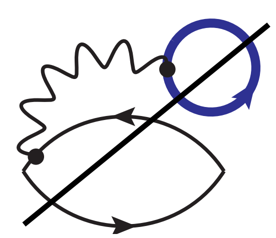
$$FVE = Fe^{-M_\pi L}$$

$$FVE = \hat{F}\,\frac{M_\pi^2}{16\pi^2 f_\pi^2}\,\frac{e^{-M_\pi L}}{(M_\pi L)^n}$$

$$FVE = \frac{\tilde{F}}{L^3}$$

FVEs expected to start at $O(1/L^3)$
(neutral mesons with vanishing charge radius)
DG *et al.*, 2017

in the ratio various systematics cancel out



quark-connected terms only

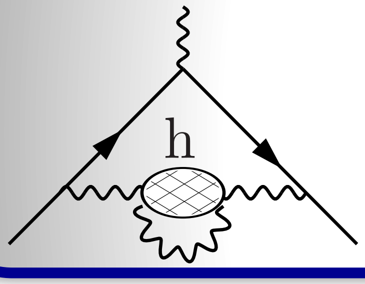
$$\frac{\delta a_\mu^{HVP}(ud)}{a_\mu^{HVP}(ud)} = 0.011(3)_{stat+fit}(2)_{chir}(2)_{FVE}(1)_{disc}(1)_{Z_A}(\ldots)_{qQED}$$

$$= 0.011(4)$$

preliminary

$$\boxed{\delta a_\mu^{HVP}(ud) = 7(2)\cdot 10^{-10}}$$

$$a_\mu^{HVP}(ud) = 619.4(16.6)\cdot 10^{-10}$$

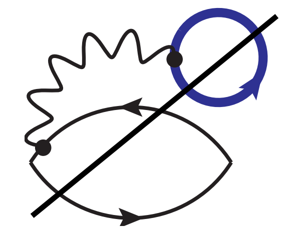


IB corr.: comparison

preliminary

quark-connected terms only

$$\delta a_\mu^{HVP}(ud) = 7(2)\cdot 10^{-10}$$



$$\delta a_\mu^{QCD}\left(\overline{MS}, 2\ \mathrm{GeV}\right) = 5.6(2.0)\cdot 10^{-10}$$

u-,*d*-quark contributions

$$\delta a_\mu^{QED}\left(\overline{MS}, 2\ \mathrm{GeV}\right) = 1.3(0.9)\cdot 10^{-10}$$

~80% due to strong IB

$$\delta a_\mu^{HVP}(s) = -0.018(11)\cdot 10^{-10}$$

$$\delta a_\mu^{HVP}(c) = -0.030(13)\cdot 10^{-10}$$

DG *et al.*, 2017

negligible *s*-,*c*-quark contributions

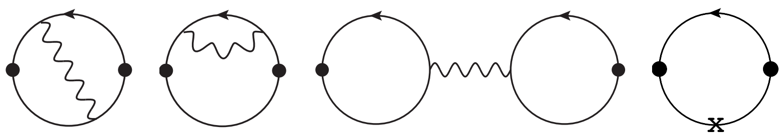
$$\delta a_\mu^{HVP}(ud) = 7.8(5.1)\cdot 10^{-10}$$

Sz. Borsanyi *et al.*, 2018

estimate from $\pi^0\gamma, \eta\gamma, \rho-\omega$ mixing, $M_{\pi^\pm}$

$$\delta a_\mu^{HVP}(ud) = 9.5(10.2)\cdot 10^{-10}$$

T. Blum *et al.*, 2018



$$\delta a_\mu^{HVP}(ud; QCD) = 9.0(4.5)\cdot 10^{-10}$$

B. Chakraborty *et al.*, 2018

strong IB only

Conclusions

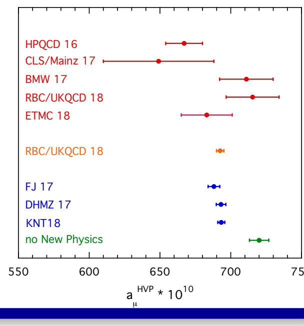
- The HVP contribution is currently one of the most important sources of the theoretical uncertainty to the muon (g-2) ➡ LQCD

RM123 method

- We have completed our lattice calculation of a_μ^{HVP}, by determining the light quark contribution at $O(\alpha_{em}^2)$ and $O(\alpha_{em}^3)$ (*s*- and *c*-quark contributions already published on JHEP). The results will appear soon on arXiv

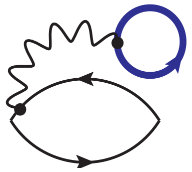
$$a_\mu^{HVP} = 683(18)\cdot 10^{-10} \qquad \delta a_\mu^{HVP} = 7(2)\cdot 10^{-10}$$

preliminary



In progress...

- evaluation of the quark-disconnected terms and relaxation of the *qQED* approximation



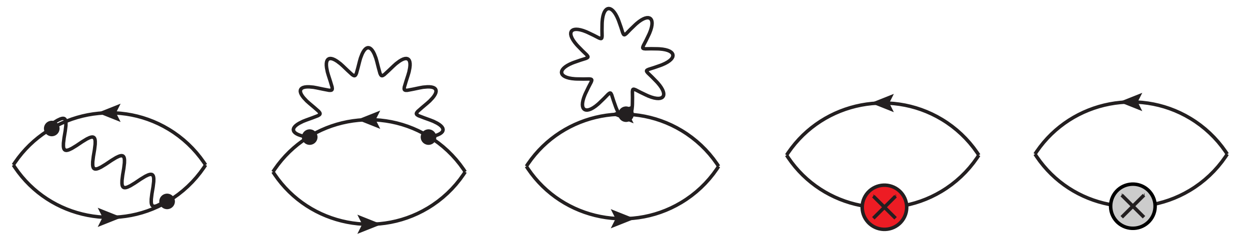
- non-perturbative determinations of the e.m. corrections to the RCs of bilinear operators
- systematic study of FVEs in the strong and QED IB corrections

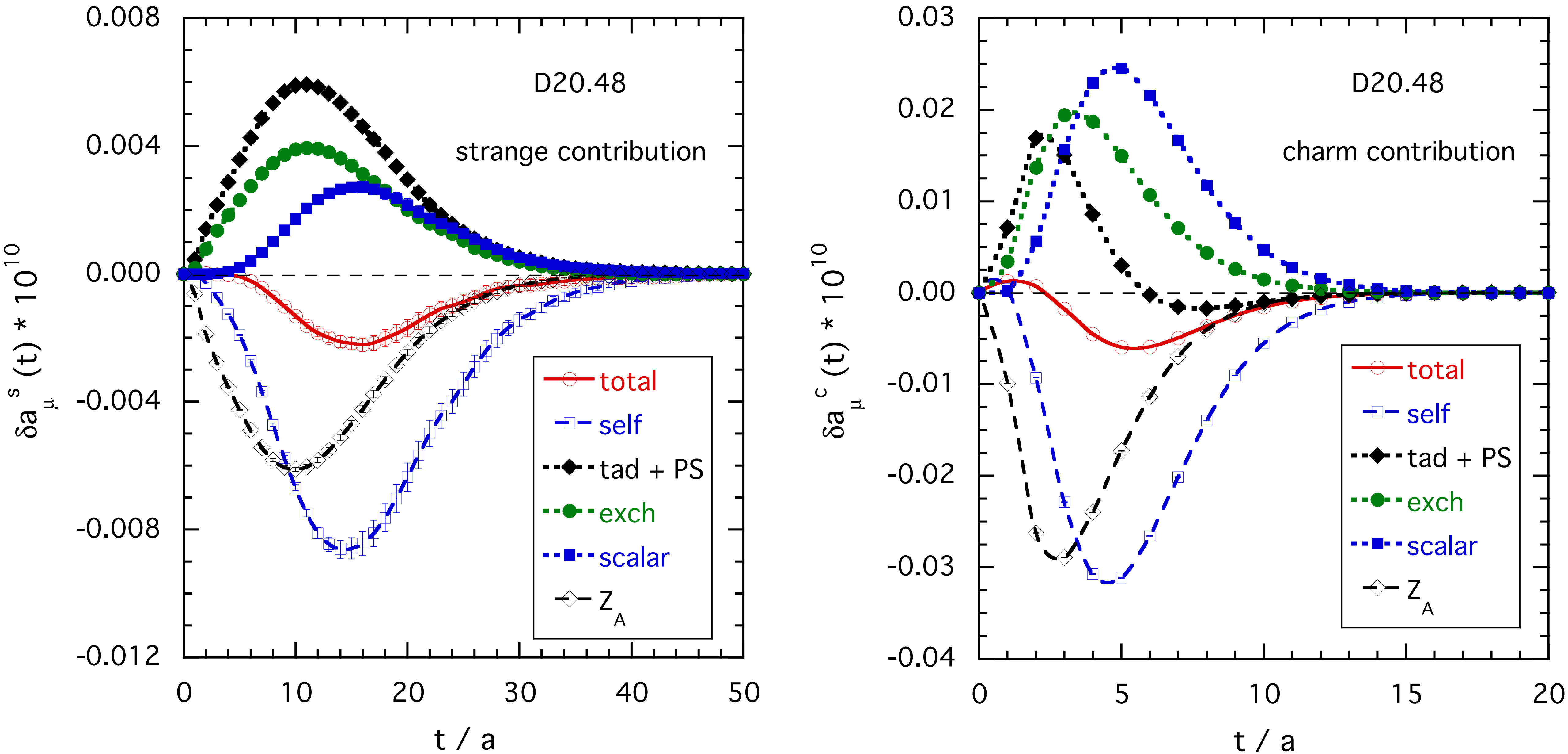
Future perspectives

- use of the new ETMC lattice setup @ the physical pion point

Backup slides

IB corr.: *s*-,*c*-quark contributions

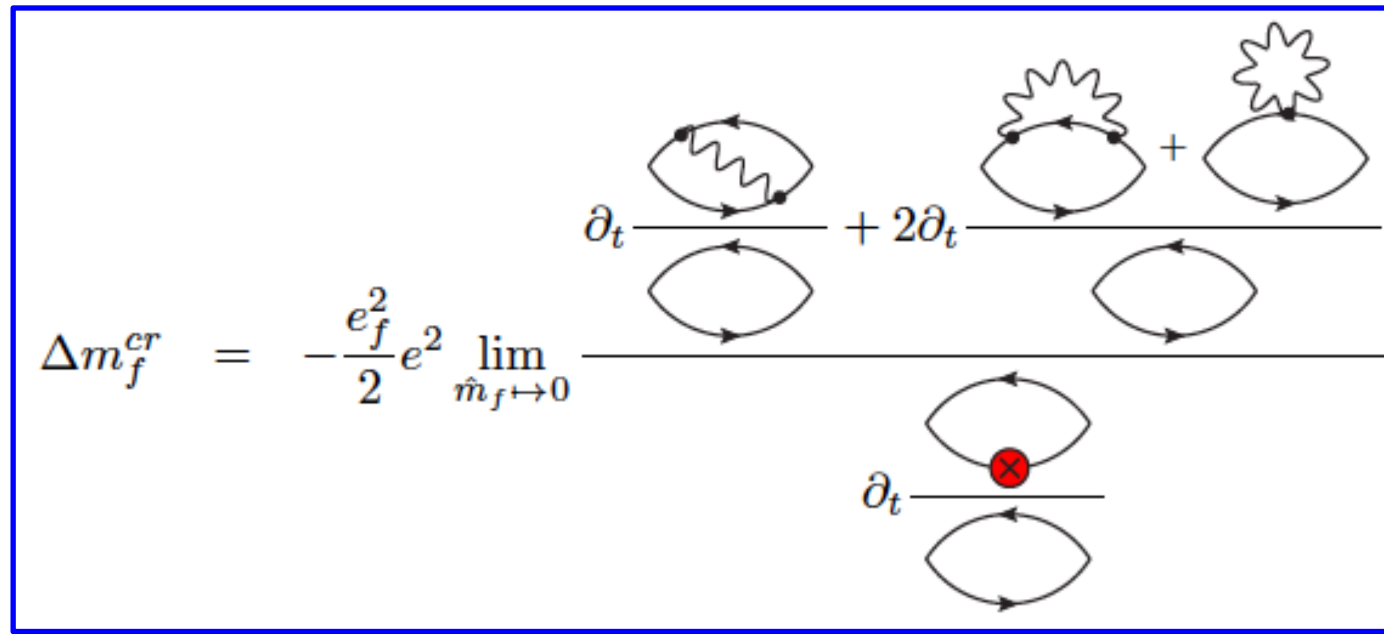




Tuning the critical mass

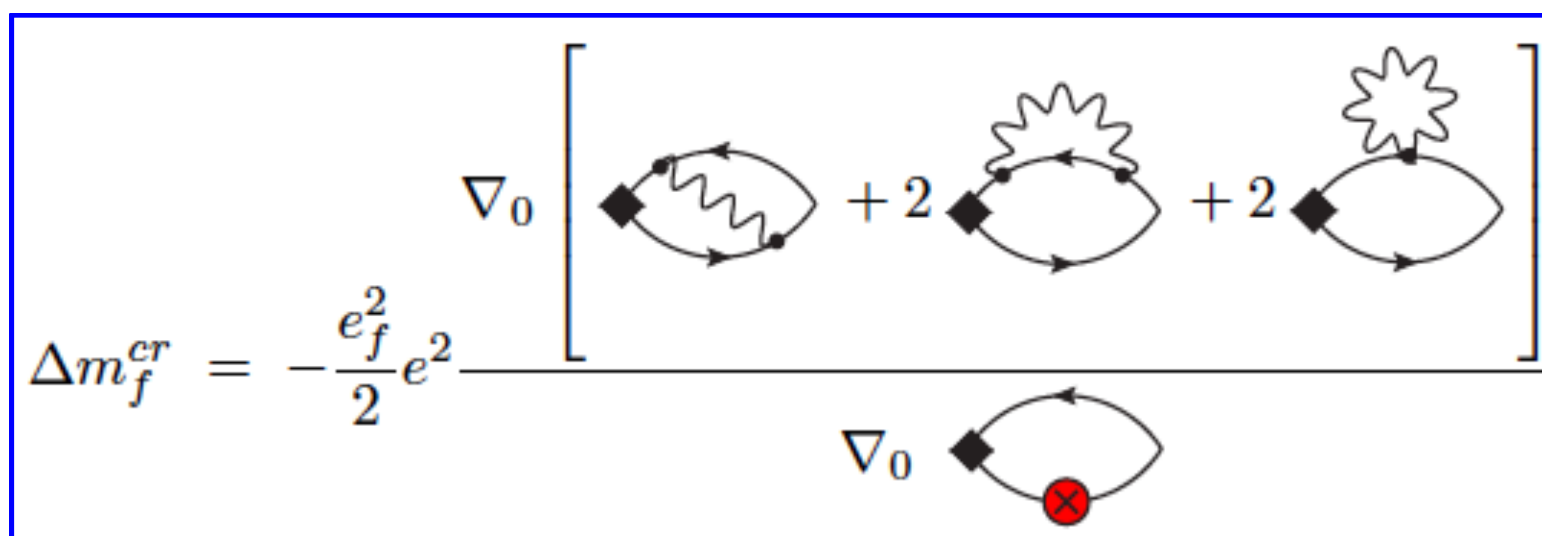
1 The Dashen theorem: in the massless theory, the neutral pion and kaon are Goldstone bosons even in the presence of electromagnetic interactions:

$$\boxed{\lim_{m_f \to 0} M_{\pi^0} = \lim_{m_f \to 0} M_{K^0} = 0}$$



2 With twisted mass fermions, one can extend the method used also in the isosymmetric QCD case, based on a specific Ward-Takahashi identity:

$$\boxed{\nabla_\mu \left\langle V_\mu^1(x) P_5^2(0) \right\rangle = 0}$$



More precise: it does not require a chiral extrapolation