Contribution to the anomalous magnetic moment of the muon from the disconnected hadronic vacuum polarization with four-flavors of highly-improved staggered quarks

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Introduction

We describe the first steps in a computation of the contribution to the anomalous magnetic moment of the muon from the disconnected part of the hadronic vacuum polarization (HVP). We use the highly-improved staggered-quark (HISQ) formulation for the current density with gauge configurations generated with four flavors of HISQ sea quarks. Here we present our methodology and preliminary result from one lattice spacing $a \sim 0.15$fm.

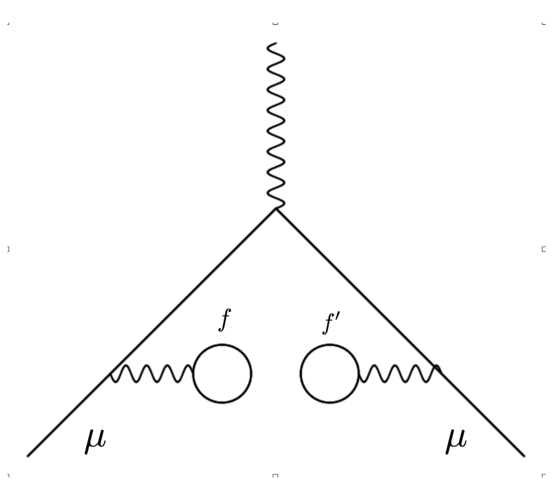

Figure 1: The Feynman diagram for the contribution to a_μ from the disconnected part of HVP. The fermion loops of different flavor f and f' are connected by virtual gluons and sea quarks (not shown).

The hadronic vacuum polarization at zero spatial momentum is given by

$$\begin{aligned}\Pi^{\mu\nu}(q^2) &= q^2\Pi(q^2)\\ &= \int d^4x e^{iqt}\,\langle J^\mu(t,\vec{x})J^\nu(0)\rangle\end{aligned}$$

The contribution to the anomalous magnetic moment is

$$a_\mu^{\rm HVP} = 4\alpha^2\int_0^\infty dq^2 f(q^2)\hat{\Pi}(q^2)$$

where

$$\begin{aligned}\hat{\Pi}(q^2) &= \Pi(q^2) - \Pi(0)\\ \Pi^{ii}(q^2) &= q^2\Pi(q^2)\\ f(q^2) &= \frac{m_\mu^2 q^2 A^3(1-q^2A)}{1+m_\mu^2 q^2 A^2}\\ A &= \frac{\sqrt{q^4+4m_\mu^2 q^2}-q^2}{2m_\mu^2 q^2}\end{aligned}$$

Motivation

- The goal is to compute the HVP with the precision needed to match the expected experimental precision.
- We aim here to improve on the precision of the disconnected HVP.
- We employ HISQ with $2+1+1$ flavors for both sea and valence quarks to help achieve this goal.

Methodology

We define the disconnected correlator

$$C_{\rm disc}(t) = \frac{1}{48}\sum_{i=1}^{3}\sum_{\vec{x}/a} Z_V^2\langle\langle J_i(t,\vec{x})\rangle_F\langle J_i(0)\rangle_F\rangle_G \qquad (1)$$

Then,

$$\Pi^{\rm disc\ HVP}(q^2) = a^4\sum_t e^{iqt}C_{\rm disc}(t)$$

In Eq. (1), V is the spatial lattice volume, and F indicates the integration over the fermionic degrees of freedom, which is performed explicitly while averaging over gauge configurations is indicated by G. Z_V is the vector-current renormalization factor. $J_i(r)$ is the one-link current with $\Gamma_\mu \otimes \Gamma_t = \gamma_\mu \otimes \mathbf{1}$, which in one-component basis,

$$J_\mu(r) = \frac{i}{2}\sum_f Q_f\bar{\chi}_f(r)\alpha_\mu(r)U_\mu(r)\chi_f(r+\hat{\mu}) + h.c.$$

Here, Q_f is the charge of the quark of flavor f in units of the electron charge e. We use stochastic estimation of the current density, the truncated solver method combined with low-mode deflation, and dilution with stride 2 to reduce the variance. The low-mode part is computed exactly by constructing it from the eigenvectors of $\not{D}$.

Parameter Optimization

The calculation presented here is carried out on a single gauge-field ensemble of size $32^3 \times 48$ with an approximate lattice spacing of 0.15 fm. There are several parameters in this simulation that need to be tuned to achieve the target uncertainty in the current-current correlation function at minimum computational cost. After tuning, the optimum parameter values are found to be

- The number of eigenpairs for deflation: 350
- The precision of the eigensolution: $\left|D_{eo}D_{eo}\tilde{v}_n^{(e)} - \lambda_n^2\tilde{v}_n^{(e)}\right| < 1\times 10^{-9}$ where $\tilde{v}_n^{(e)}$ and λ_n are the estimated $n^{\rm th}$ eigenpair
- The residual of fine and sloppy solve: 2.70×10^{-2} and 1×10^{-5}, respectively
- The number of fine and sloppy solves per configuration: 72 and 1408, respectively.

Result

$C_{\rm disc}(t)$ is computed using 758 gauge configurations with physical sea-quark masses. Fig. 2 shows statistically significant signal for $C_{\rm disc}(t)$ taking $Z_V = 1$.

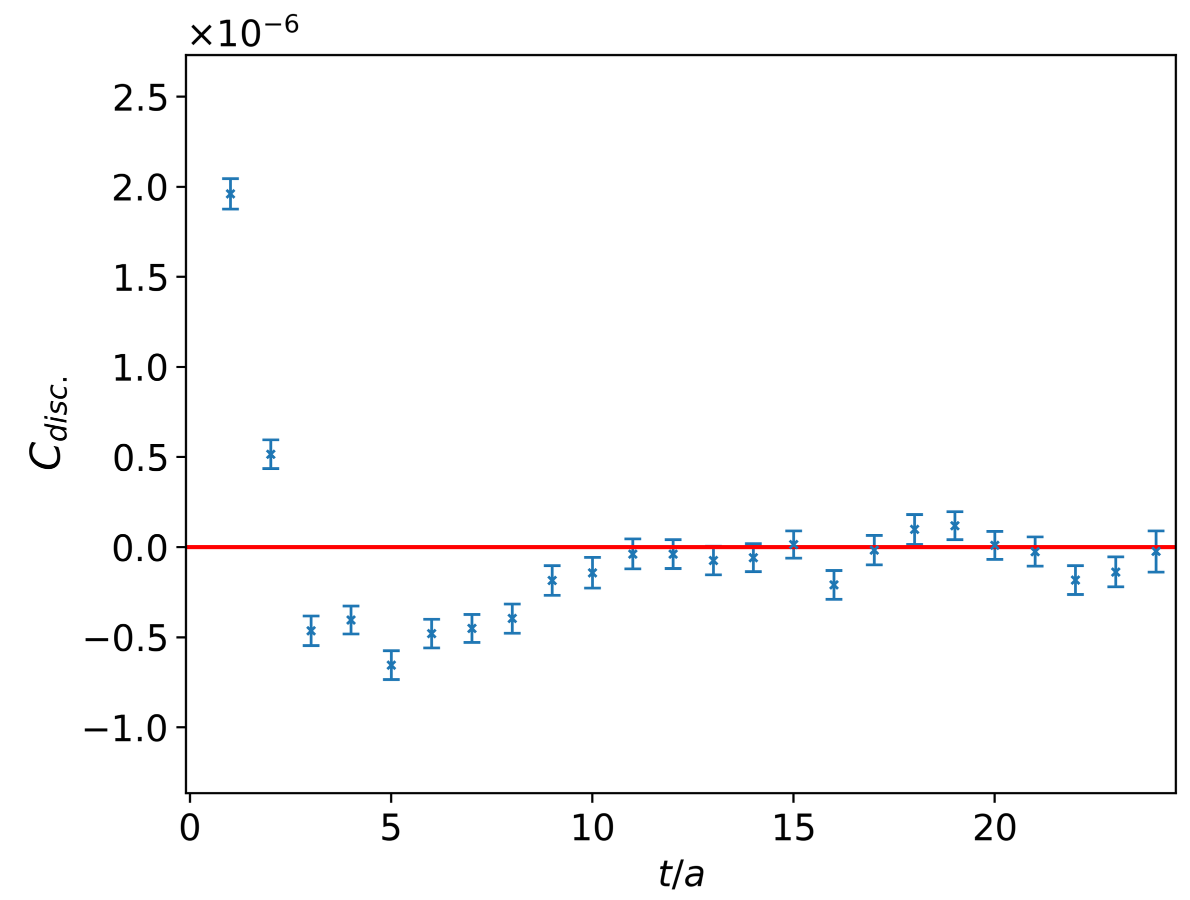

Figure 2: Time-slice disconnected current density correlator vs. the temporal separation in lattice units.

Outlook

The analysis of correlator and determination of $a_\mu^{\rm HVP}$ is in progress. For the next steps, we will increase the statistics, incorporate the renormalization factor Z_V, and continue to finer lattices.

References

[1] T. Blum et al. "Calculation of the Hadronic Vacuum Polarization Disconnected Contribution to the Muon Anomalous Magnetic Moment". In: *Phys. Rev. Lett.* 116 (23 June 2016), p. 232002. DOI: 10.1103/PhysRevLett.116.232002. URL: https://link.aps.org/doi/10.1103/PhysRevLett.116.232002.